Status Externalities in Education and Low Birth Rates in Korea*

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Abstract

East Asians, especially South Koreans, appear to be preoccupied with their offspring's education—most children spend time in expensive private institutes and in cram schools in the evenings and on weekends. At the same time, South Korea currently has the lowest total fertility rate in the world. In this paper, we propose a theory with status externalities and endogenous fertility that connects these two facts. Using a quantitative heterogeneous-agent model calibrated to Korea, we find that fertility would be 16% higher in the absence of the status externality and that childlessness in the poorest quintile would fall from five to less than one percent. We further show that the externality amplifies the fertility decline over time. We then explore the effects of various government policies. A pro-natal transfer increases fertility and reduces education while an education tax reduces both education and fertility, with heterogeneous effects across the income distribution. The policy mix that maximizes the current generation's welfare consists of an education tax of 12% and moderate pro-natal transfers—a monthly child allowance of 3% of average income for 18 years. This would raise average fertility by about 5% and decrease education spending by 16%. Although this policy increases the welfare of the current generation, it may not do the same for future generations as it lowers their human capital.

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1 Introduction

Forced to decide between giving her daughter siblings or an expensive education, Hong Sung-ok saw little choice. "I can't afford not to send my child to private tuition, because everyone else does," says the 47-year-old insurance saleswoman. "I spend more than half my income on tutors and childcare expenses - it's really expensive... That's why I decided to have only one child." (Financial Times, January 2, 2013)

South Korea (henceforth Korea) has an extremely low total fertility rate that has hovered around 1.2 for the last two decades. In 2019, it reached a record low of 0.92, less than half the replacement rate. In fact, for almost two decades now, Korea's fertility rate has been among the lowest in the world, below that of other low-fertility countries such as Germany or Italy. The Korean government has expressed concerns over low birth rates, as the latter imply rapid population aging and pose a considerable challenge to the public pension system. Starting in 2006, the government launched a billion-dollar program to reverse the decline. In 2020, 37 billion USD (2.1% of annual GDP) was spent on policies aimed at boosting fertility.[1]

Another notable feature of Korean society is that children's education is very highly valued by parents. This preoccupation with education in Korea is sometimes called "education fever," echoing the title of a popular book by Seth (2002). Many teenagers attend math and English classes in private education institutes called *hagwons*, often as late as midnight. Others, meanwhile, spend numerous hours each week with a private tutor. Participation rates in after-school programs are around 75%. These private education investments are so expensive that an individual family with two children, both participating in private education, may spend as much as 26% of their disposable income and 36% of consumption expenditure on private education (even though most children attend public schools).[2]

[1] Source: "The Fourth Basic Plan on Low Fertility and Aging Society," published by the Korean Presidential Committee on Aging Society and Population Policy in 2020 (available only in Korean at `https://www.betterfuture.go.kr`).

[2] In 2019, the average monthly spending for private education was approximately 378 USD per child for families with two children participating in any after-school programs. Source: authors' calculation based on "Monthly Private Education Expenditures per Participation Student by School Level and Characteristics" and "Average Monthly Income and Expenditure (Whole Households)" from Statistics Korea (2020).

The strong emphasis on education seems related to a highly competitive university entrance exam, but is also rooted in Confucian values. de Silva (2018) provides empirical evidence for a rat race for human capital in Korea based a 10 p.m. curfew that was introduced in some but not all states. The curfew substantially lowered private tutoring expenses without affecting college entrance much.

In this paper, we propose a new mechanism that connects high education spending with low birth rates. The novel ingredient is a status externality in which parents value the education of their children *relative* to the education of other children. The concern for status seems particularly relevant in Korea, and East Asia more generally, as documented in several empirical papers.[3] A passion for education coupled with extremely low fertility rates appears to a certain extent in other East Asian countries as well, such as in China, Hong Kong, Singapore and Taiwan. While our paper uses data from Korea, we believe that the mechanism, qualitative findings, and policy implications may apply throughout East Asia. More specifically, we incorporate this concern for status in education into a quantitative general equilibrium model and calibrate it to data from Korea. We use the model to explore how the externality affects parents along the income distribution, as well as examine welfare consequences. Intuitively, one would expect the preoccupation with relative education to lead to an over investment in the latter, making children costly and thus causing inefficiently low birth rates. We accordingly also explore various government policies aimed at addressing this distortion and compute the policy that maximizes the welfare of the first generation. We find that addressing the distortion caused by such an externality is less straightforward than might be imagined.

To analyze the connection between education and fertility in Korea, we build an overlapping generations model with endogenous fertility and concern for the relative quality of children. The model features heterogeneous agents, where potential parents choose the number of children and how much to invest in the latter's edu-

[3] Jinkins (2016) and Podoshen, Li, and Zhang (2011) find that Chinese consumers care more about peer beliefs and conspicuous consumption than American consumers. These differences between East Asian and Western consumers are considered to be related to Confucianism and a culture of interdependence in East Asia (Wong and Ahuvia 1998). Although they do not discuss Koreans or East Asians, Charles, Hurst, and Roussanov (2009) document racial differences in status-seeking behavior and argue that these differences are economically large and relatively constant over time.

cation, incorporating a quantity-quality trade-off. The children's human capital is a function of parental spending on education, parental human capital, and luck, as children are born with different learning abilities. Agents are heterogeneous along several dimensions. First, parents differ in terms of their human capital, which endogenously arises due to their parental choices and random components. Agents also differ in the extent to which they care about their own consumption relative to their own leisure and utility from children. This latter preference is randomly determined. The novel feature of our model is a concern for status in the quality dimension. Specifically, parents derive utility from the human capital of their children relative to the human capital of other people's children. Given the status externality, the key object of the equilibrium both in the steady state and along the transition path is the distribution of human capital, which is endogenously determined as a fixed point. That is, agents take the future evolution of human capital distributions as given when making decisions such as the number of children and human capital investments. This expectation must be consistent with the actual evolution in equilibrium.

We calibrate our model to a recent cohort of women in the Korean Labor and Income Panel Study (KLIPS). We then use the calibrated model to understand fertility choices and education investments in Korea and to conduct policy experiments. The first finding is that the status externality plays an important role in fertility decisions: in its absence, fertility increases by 16.4%. The reason being that the status externality drives up education investments, which makes children costly and induces parents to have fewer offspring. This channel especially affects the poor. Indeed, childlessness falls from five percent to less than one percent in the poorest quintile when the status externality is eliminated. Removing the externality therefore changes the slope of the fertility-income relationship from positive to negative. This is an interesting result in that Korea displays a positive fertility-income relationship, whereas fertility and income are negatively related in most other countries.[4] We also use our model to explore the role of status concerns for the fertility decline over time. We find that the status externality amplified the

[4]See Jones and Tertilt (2008). Another rare exception to the generally negative fertility-income relationship is the United States in recent years. To this regard, Bar et al. (2018) document that US fertility is upward-sloping for very high incomes. They relate this phenomenon to the marketization of time, though concerns about relative education may also play a role.

fertility decline, in particular by increasing childlessness among the poor.

We use the quantitative model to study the effect of several government policies. Motivated by actual policies recently introduced in Korea, we consider pro-natal transfers and Pigouvian-style taxes on private investment in education.[5] We find that pro-natal transfers increase the fertility rate as intended, with a magnitude that is in line with recent empirical estimates of the fertility effects of the pro-natal cash bonus in Korea (Kim 2020). At the same time, they decrease education spending per child, and do so to a greater extent among low-income than high-income families. As a result, human capital, output and consumption are lower in the new steady state. We also investigate the effect of a tax on education spending, as an alternative method of addressing the education externality. Our results indicate that such a tax does indeed reduce education spending, but fertility falls as well. This is because the total cost of raising children increases due to the higher effective price of education. The effect is larger for high-income families, which demand more education, meaning that the income elasticity of fertility and education investment decreases as the tax rate rises. As parents choose to invest less per child, they experience an increase in consumption, while future generations experience lower human capital and consumption. These findings indicate that both policies would necessarily involve some intergenerational conflict by lowering future generations' human capital and output.

Finally, we explore the optimal policy mix that maximizes the welfare of the initial parent generation. We find that the optimal policy is an education investment tax rate of 12% and moderately large pro-natal transfers—a monthly child allowance of 71 USD (3% of average income) for 18 years. This optimal policy increases the fertility rate by 5.6%, lowers the childlessness rate by more than half a percentage point and decreases education spending by 16%. The effects on fertility and education are heterogeneous along the income distribution, with the largest effects

[5]The Korean government has introduced pro-natal transfers in various forms, such as monthly allowances or a one-time birth bonus, starting in 2006 (source: "The Fourth Basic Plan for Low Fertility and Aging Society" published by the Korean Presidential Committee on Aging Society and Population Policy in 2020). These programs have been growing in size, a trend expected to continue in the coming years: https://www.koreatimes.co.kr/www/opinion/2020/12/137_300988.html. As for education policy, the Korean government banned all private education in 1980. In 2000, however, the Constitutional Court ruled that the comprehensive prohibition of private education was unconstitutional, though there remain restrictions on the hours of operation of *hagwons* (Choi and Cho 2016).

felt among the poorest quintile of parents. While this policy addresses the distortion caused by the status externality and is optimal from the perspective of the first generation, we find that it does decrease human capital and output over time. Thus, although the welfare of the parents' generation measurably increases, future generations are worse off compared to a world without the policy, at least in terms of average utility. We discuss several modifications to the setup that would likely alter this result. In particular, if (part of) education spending serves purely signaling purposes without increasing human capital, then reducing overinvestment without hurting future generations would clearly be feasible.

Our research builds upon the economic analysis of fertility pioneered by Becker (1960) and, more specifically, the quantity-quality trade-off first modeled by Becker and Tomes (1976). Many subsequent analyses have used this framework in quantitative models to understand fertility differences over time and across countries (Greenwood and Seshadri 2002; Greenwood, Seshadri, and Vandenbroucke 2005; Manuelli and Seshadri 2009) and to study the aggregate and distributional implications of fertility in the presence of endogenous human capital investments and heterogeneous households (de la Croix and Doepke 2003; Cordoba, Liu, and Ripoll 2016; Cavalcanti, Kocharkov, and Santos 2020). Some recent work, including that of Sommer (2016), Guner, Kaya, and Sánchez-Marcos (2019) and Daruich and Kozlowski (2020), consider rich heterogeneous agent life-cycle models while abstracting from the quantity-quality trade-off. Recently, policies such as government-subsidized daycare and subsidies for having children have also been analyzed in quantitative models, though these typically use models with exogenous fertility (Domeij and Klein 2013; Hannusch 2019; Guner, Kaygusuz, and Ventura 2020).

The key novel ingredient in our fertility model is a status externality in education. While a sizable body of work on relative status concerns does exist, dating back to Veblen (1899), the importance of status has not been explored in the context of fertility choices.[6] Though much of the literature analyzes conspicuous consumption, the idea that people may overinvest in education because private returns are greater than social returns has been around for a long time (e.g. Akerlof (1976)). Ramey and Ramey (2010) argue that competition in college admissions leads to an education externality in the United States, but their analysis also leaves out fertil-

[6]See Clark, Frijters, and Shields (2008) for a review of the more recent literature.

ity implications. The macroeconomic implications of status concerns (e.g., catching or keeping up with the Joneses) have been analyzed by several authors—e.g. Abel (1990), Gali (1994) and Ljungqvist and Uhlig (2000)—while Genicot and Ray (2017) theoretically explore the effect of status externalities on inequality. None of these papers considers the implications for fertility. The only exception may be the verbal ideas of Easterlin (1966), who argues that aspirations formed during childhood affect fertility choices as adults.

In contrast to a large part of the fertility literature concerned with excessively high birth rates (e.g., Lee and Miller (1990)), our paper provides a framework in which birth rates can be too low in the presence of a status externality. A few other reasons for fertility being below socially optimal levels have been set forth. Schoonbroodt and Tertilt (2014) argue that the lack of property rights that parents have over their (future) children causes the social benefit from child-bearing to exceed the private benefit, leading parents to choose too few children. Doepke and Kindermann (2019) argue that fertility might be inefficiently low when men and women disagree about fertility and husbands and wives bargain about having children. A similar argument is made in Ashraf, Field, and Lee (2014). Jones (2020) points out that fertility may be too low in a growth model with idea spillovers. Note that these papers use various notions of optimality. Indeed, the definition of efficient levels of endogenous fertility is beset by conceptual and philosophical difficulties. Golosov, Jones, and Tertilt (2007) propose the alternative concept of $\mathcal{A}$-efficiency, which we apply in this paper.

Finally, there is a largely empirical literature that studies fertility in Korea specifically. For example, Lee (2009) provides a general survey of potential reasons for low fertility and policy responses in this country. The possible connection between the East Asian "Education Fever" and low fertility was perhaps first pointed out by Anderson and Kohler (2013) in a purely empirical study, also of Korea. Hong et al. (2016) and Kim (2020) both study the recent introduction of pro-natal transfers in Korea and find small but positive effects on fertility. Ma (2016) and Myong, Park, and Yi (2020) set forth a complementary explanation for low fertility in Korea by noting the recent increase in female labor force participation together with strong family values that expect mothers to stay home with their children.[7]

[7] A family values explanation has similarly been given for certain low-fertility European coun-

The remainder of the paper is organized as follows. In Section 2, we describe the data and present our empirical findings on the relationship between fertility and education in Korea. In Section 3, we set up the model. Section 4 explains how we calibrate the model. Section 5 quantifies the importance of the status externality. In Section 6, we study the effects of pronatal transfers and education taxes. In Section 7 we solve for the optimal policy mix that maximizes the utility of the first generation. Section 8 concludes.

2 Fertility and Private Education in Korea

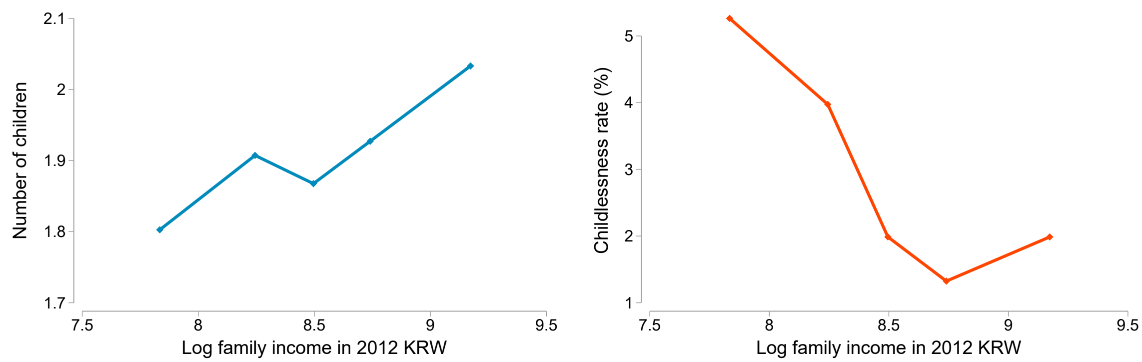
In this section, we provide stylized facts about fertility and private education expenditures in Korea. In particular, we focus on the heterogeneity in both fertility and education expenditures across the income distribution, which forms the basis for the quantitative analysis in the subsequent sections. First, we discuss how fertility differs across the income distribution. We then present some evidence on private education and document the cross-sectional relationship between private education expenditures and household income.

We use the longitudinal samples from the Korean Labor and Income Panel Study (KLIPS), which is similar to the Panel Study of Income Dynamics (PSID) in the United States. Our baseline results are based on cohorts of women born between 1970 and 1975. A robustness analysis in Appendix A.4 shows similar findings using women born earlier. Fertility is measured by completed fertility.[8] We will document income gradients in fertility and education investments. As we are interested in a permanent measure of income at the household level, we construct long-term household income as in Chetty et al. (2014) and focus on two-parent

tries (Billari and Kohler 2004). However, these countries have comparatively higher birth rates than Korea, and in some cases, fertility has recently been trending upward again. Thus, Anderson and Kohler (2013) argue that strong social norms alone cannot fully explain the Korean case.

[8] The completed fertility is the average number of children ever born to women belonging to the same cohort, which is around 1.9 in our data for the 1970-1975 cohort. This is higher than the total fertility rate (TFR) of Korea, which was around 0.9 in 2019. The total fertility rate is defined as the number of children who would be born per woman if they were to pass through the childbearing years delivering children according to the current schedule of age-specific fertility rates. Therefore, the total fertility rate is lower than the completed fertility rate, reflecting the declining fertility trend in Korea.

Figure 1: Fertility by Income Quintile



Panel A. Completed fertility

Panel B. Childlessness rate

Notes: We group households into quintiles based on their long-term income and calculate average completed fertility and the childlessness rate in each quintile for cohorts born between 1970 and 1975.

households for consistency with the model analyzed in the following sections.[9] Appendix A.1 provides further details on the data employed herein.

2.1 Fertility across the Income Distribution

Panel A of Figure 1 shows how fertility varies across the income distribution. More precisely, this figure depicts the relationship between the average number of children and average household income in each income quintile. We see that the relationship is generally positive: the number of children increases from 1.80 for the lowest-income quintile to 2.03 in the highest-income quintile. This finding suggests that low birth rates in Korea are related to factors affecting low-income households.

Another way to quantify the relationship between fertility and income is to es-

[9]Specifically, we use the average income of two-adult households in which the woman's age is between 40 and 43. Focusing on two-adult families enables us to abstract from the different marriage and fertility choices of varying types of households and to circumvent comparisons between the income of one- and two-adult households (Bar et al. 2018). Though, as the fraction of one-adult households is relatively small (8%) in our data, including single-parent families does not significantly change our results, as shown in Appendix A.5.

timate the income elasticity of demand for children following Jones and Tertilt (2008), through which we obtain an estimated elasticity of 0.082. This finding may appear puzzling given that estimates of elasticity from the United States are negative, as reported in Jones and Tertilt (2008). Elasticity estimates from the US cohorts that most closely correspond to our data are around -0.2.[10]

Given this positive relationship between family income and fertility in Korea, a natural question is the extent to which it is due to the intensive margin (number of children conditional on having at least one child) or the extensive margin (fraction of women without any children, as indicated by the childlessness rate). Panel B of Figure 1 shows that the childlessness rate has a negative relationship with family income, decreasing from 5.3% for the lowest-income quintile to 2.0% in the highest-income quintile. Poor families thus appear substantially more likely to have no children. Although the absolute level of childlessness is somewhat low in Korea, its clear income gradient confirms that the extensive margin is also important to understanding the positive relationship between fertility and family income, as shown in Panel A of Figure 1.

2.2 Education Fever and Private Education

The obsession with education among Korean parents, sometimes termed "education fever" (Seth 2002; Anderson and Kohler 2013), has been widely discussed. As a result of this focus on education, Korean children spend much time in after-school programs and with private tutors. While some of these activities are related to physical education, art and music (and thus may be considered leisure activities), a large portion of children also attend after-school programs to learn English and improve their math and computer skills.

To illustrate the widespread participation across age groups, Table 1 reports the percentage of children from each age group attending after-school programs, according to the Private Education Expenditures Survey (Statistics Korea 2020). We see that more children attend after-school programs focusing on "main subjects"

[10] Jones and Tertilt (2008)'s estimates are based on men's income. When male income is used instead of family income in our dataset, the relationship between fertility and income remains positive, with an estimated elasticity of 0.096.

Table 1: Private Education Participation Rate (2019, %)

	Average	Elementary	Middle	High
Any subject	74.8	83.5	71.4	67.9
A. Main subjects	56.7	57.9	61.8	57.8
a. Individual tutoring	8.9	6.4	10.4	14.3
b. Group tutoring	9.3	10.5	10.0	7.6
c. Hagwon	41.1	37.6	50.7	46.0
d. Others	20.4	32.1	11.3	6.9
B. Art, music, physical activities	44.0	67.4	26.2	15.3
a. Individual tutoring	5.6	7.3	4.6	3.6
b. Group tutoring	5.0	7.8	3.1	1.0
c. Hagwon	34.8	55.9	17.1	10.3
d. Other	5.8	9.2	3.4	1.3

Notes: The main subjects include Korean, English, math, science, second foreign language, writing and computer science. Other activities may include online education programs or home-based sessions with tutors from education companies. Note that the total is not the sum of the lower categories, since a single child may, for example, attend a hagwon and have individual tutoring lessons. Source: "Private Education Participation Rate by School Level" from Statistics Korea (2020).

than those related to arts, sports and music. These private education programs start as early as elementary school but continue all the way through high school. While a sizable fraction of these activities occur in so-called *Hagwons*, several other arrangements exist as well, such as private tutoring by college students. Overall, 74.8% of children engage in some kind of private education activity.

This high participation rate is even more striking given the fact that private education is expensive in Korea. In our KLIPS sample, we find that total education expenditures per child amount to 9.2% of family income.[11] A natural question is how this ratio varies across the income distribution. To assess this, we use detailed information on private education expenditures, disaggregated by child's age, from the individual-level survey data. This data, however, is missing some components—such as school tuition paid by households—that are included in our

[11] Specifically, we begin by calculating the education expenditures and incomes separately by child's age from birth to age 24, which corresponds to the age range we use in our model. We then divide the sum of expenditures by the sum of after-tax income. See Appendix A.2 for details.

Table 2: Percentage of Income Spent on Private Education

Income quintile	Pre-school	Elementary school	Middle school	High school	Weighted average
1st	8.9	9.0	8.4	5.7	8.4
2nd	6.8	8.0	8.5	6.1	7.4
3rd	6.1	7.7	7.6	6.6	7.0
4th	5.6	6.7	7.4	6.9	6.5
5th	4.6	5.0	5.8	5.8	5.1

Notes: This table shows the percentage of income spent on private education per child at each stage of education. The shares are calculated by dividing the average expenditures by the average income of the corresponding income quintile. The last column shows the average expenditures across different education stages weighted by the number of years spent in each education stage. Source: authors' calculation based on KLIPS data.

comprehensive measure of total expenditures at the household level.

Table 2 displays the percentage of family income spent on private education by quintile for children at different stages of their education. We find that poorer families tend to spend a higher percentage of their income on private education at most stages. The last column, which shows overall education spending over a child's entire education, reveals that parents in the bottom income quintile spend 3.3 percentage points more of their family income than those in the top income quintile. The income elasticity of demand for private education—analogous to the income elasticity of fertility estimated in Section 2.1—is equal to 0.698, which is substantially less than one.[12]

Koreans often associate the burden of these education expenditures with low fertility—as exemplified by the parent's remark quoted in the Financial Times and presented at the beginning of this paper. According to our empirical evidence, this channel appears particularly relevant for low-income families whose expenditures on education represent a sizeable fraction of their lifetime income, with respect to an additional child. This is consistent with the markedly lower fertility rates among such families, as documented in Section 2.1. In the following sections, we formally ex-

[12]We first calculate average spending on education for each income quintile and education stage, and then estimate an average for each income quintile weighted by the number of years spent in each education stage. Details are given in Appendix A.2.

plore the role of the status externality in a quantitative heterogeneous-agent model that replicates these empirical observations on fertility and education expenditures across the income distribution.

3 The Model

Our model builds on the quantity-quality literature where parents choose the number of children and how much to invest in each.[13] The model is an overlapping generations model, where each generation lives for two periods: as children and as adults. As in most of the fertility literature, we assume asexual reproduction; that is, we abstract from marriage and model only a generic parent. Similarly, we do not distinguish between boys and girls and thus abstract from gender differences in parental inputs.[14] In contrast to much of the literature, we assume that fertility, n, is a discrete choice. This is not only realistic, but also naturally leads to childlessness as an equilibrium outcome for some parents.[15] For some parents, the cost of having a child will be so high (relative to income) that they may prefer not to have any offspring.

As documented in Section 2, there are marked cross-sectional patterns in fertility and private education investment across the income distribution. Similar to other developed countries, in Korea there is a considerable degree of income inequality across families. Since we would like our model to reproduce an empirically reasonable degree of heterogeneity, we introduce some heterogeneity in order to generate a cross-sectional income distribution. Agents (or potential parents) differ in their own levels of human capital, and hence income, which leads them to make different fertility and education investment choices for their children. However, since human capital is endogenous in the model (each parent is a child of a previous parent), we introduce additional exogenous heterogeneity so that the

[13] See Becker (1960), Becker and Tomes (1976) and de la Croix and Doepke (2003) among others.

[14] In reality, parental inputs (including spending on private academic education) are higher for boys than girls. But the gaps have substantially declined in Korea over time, so that current gender gaps are small (Choi and Hwang 2020).

[15] The importance of distinguishing the extensive and intensive fertility choice margins was recently emphasized by Aaronson, Lange, and Mazumder (2014) and Baudin, de la Croix, and Gobbi (2015).

income distribution in equilibrium is non-degenerate. Specifically, we assume two exogenous sources of heterogeneity. First, parents differ in their relative taste for own consumption relative to leisure and utility from offspring. Second, parents draw their child's idiosyncratic ability that is correlated across generations. This idiosyncratic ability enters into the human capital production function and will be described in detail below.[16]

The novel feature of our model is a *status externality*. There are two standard ways to model such externalities in the literature. In Abel (1990)'s *catching* up with Joneses, utility depends on the *previous* period's consumption, whereas in Gali (1994)'s *keeping* up with the Joneses, utility depends on *current* aggregate consumption. In our model, parents care not only about the quality of their children (as in standard models), but also about their quality relative to the other children in the economy summarized by the *future* aggregate human capital. Specifically, we model this by assuming the following functional form in the utility parents derive from the number, n, and the human capital, h, of their children:

$$\phi(n)\log(h' - \chi \bar{h}') \tag{1}$$

where $\bar{h}'$ is the mean human capital of their children's generation—which is the endogenous benchmark quality to which parents compare their children—and $\chi \in [0, 1)$ is the strength of the externality.[17] The status externality may in fact originate from deeper sources such as intense aspirations (e.g., Genicot and Ray (2017)) or the distinct school grading system in Korea that heavily uses norm-referenced evaluations. As the common theme of these various sources is parents' concern

[16]Having both dimensions gives significant flexibility to the shape of the fertility-income relationship and allows for both directions of causality. Parents with higher human capital may choose to have more children because of an income effect, or fewer because of a substitution effect. They may also react to the quantity-quality trade-off differently and thus choose to have different numbers of children. Finally, parents who value their own consumption less than leisure and utility from children may choose to work less and hence have less income. This last channel is less common in the literature, but it is considered in Jones, Schoonbroodt, and Tertilt (2010), among others.

[17]We adopt the same functional form of externality as in Ljungqvist and Uhlig (2000). This functional form features non-homotheticity that might have heterogeneous effects across agents. To gauge the extent to which our results are driven by non-homotheticity, we also conducted the same set of quantitative analyses with a functional form that does not feature non-homotheticity: $\phi(n)\log(h'^{1-\chi}\bar{h}'^{\chi})$, as in Abel (1990). Our quantitative results—including those in Section 4.1 on level effects—are quite robust with respect to this change, implying that our findings are not mainly driven by non-homotheticity per se.

about their children's relative performance, we assume that the benchmark in parents' utility is children's human capital.[18] Setting $\chi = 0$ eliminates the externality and allows a comparison with standard models. Meanwhile, $\phi(n)$ captures the utility along the quantity margin. We require this function to be increasing and concave.

We assume that children's human capital is formed through three different inputs: an idiosyncratic (genetic) ability component κ, parental human capital h and education investments x.[19] Ability is correlated across generations following a Markov chain and is thus drawn from a discrete distribution: $\kappa \in \{\kappa_k\}_{k=1}^{N_\kappa}$ with the transition probability of π_{jk}^{κ} where j denotes the ability index of a parent with ability κ_j^p. This seeks to capture the fact that high-ability parents are more likely to have children with high abilities. The interpretation of the role of h in the human capital production function is that children tend to learn by imitation, meaning that the offspring of parents with higher human capital can learn more.[20] Parents decide on the amount x to invest in each child after κ is realized. We assume that these components are complementary and choose the following functional form, which is similar to the specification used in de la Croix and Doepke (2003) and Cavalcanti, Kocharkov, and Santos (2020):

$$h' = \kappa \left(\theta + (xh)^{\alpha}\right) \tag{2}$$

where $\alpha \in (0,1)$. The parameter $\theta > 0$ guarantees that even if parents choose zero education for their children, the children will have some baseline human capital. This can be interpreted as raw intelligence but can also capture publicly provided education.

[18] It would be interesting, yet challenging, to identify the ultimate driver behind the externality empirically. Note though that for most of our results, such as the impact of various policies on parental choices, the underlying driving force behind the externality does not matter. At the same time, welfare implications might be sensitive to the deeper sources.

[19] We abstract from time investments into children. If one added it, then the externality would affect parental time investments as well – so that both monetary and time investments would be higher compared to an economy without status concerns. Since in Korea mostly mothers engage in educational child-care activities, such an augmented model could perhaps explain the low female labor force participation of Korean women.

[20] Thus, in our model children's earnings are related to their parents' earnings for two reasons: genetics and investments. This dual role of parents is supported by empirical evidence, e.g. in Fagereng, Mogstad, and Rønning (2021) who compare adopted and own children to understand intergenerational wealth transmission.

A parent's ex-post preferences (after all uncertainty is resolved) can then be summarized by the following utility function:

$$U(c,l,n,h',\bar{h}') = b\log\left(\frac{c}{\Lambda(n)}\right) + \nu\frac{l^{1-\gamma}}{1-\gamma} + \phi(n)\log(h' - \chi\bar{h}') \tag{3}$$

where b is the preference type mentioned above. Some people intrinsically prefer a "market-consumption lifestyle" (i.e. high b), whereas others intrinsically prefer a "family-leisure lifestyle" (i.e., low b).[21]. We assume that b can take N_b possible values: $b \in \{b_i\}_{i=1}^{N_b}$ with the corresponding probabilities $\{\pi_i^b\}_{i=1}^{N_b}$. As is standard in the literature, utility depends on household consumption expenditures c divided by the household equivalence scale, denoted by $\Lambda(n)$. Parents value leisure l, weighted by $\nu > 0$. The curvature of utility from leisure is captured by $\gamma > 0$.

The timing within a period is as follows. Parents start the period endowed with human capital h (shaped by their own parents) and their own ability κ^p (which was relevant during their childhood). The preference type b is then realized. Given b, agents choose fertility n, taking into account their expectations of their potential child's ability. Next, the ability shock κ is realized. Finally, parents make decisions regarding their own consumption, labor supply and the education investments they will make in their children.

Given the above timing assumptions, the parental decision problem can best be summarized in two steps. First, the parent makes a discrete fertility choice, not yet knowing the ability of her children. For simplicity, we assume that all children of a given parent will have the same ability. The choice problem in the first step is thus to choose the number of children:

$$\max_{n=0,1,\ldots,N_n} \left\{\mathbb{E}_{\kappa|\kappa^p} V(h,b,\kappa,n;\bar{h})\right\}. \tag{4}$$

where $V(h,b,\kappa,n;\bar{h})$ is the household's value of having n children of type κ. The second step optimization gives a value function defined by the following choice

[21] This formulation was first suggested in Jones, Schoonbroodt, and Tertilt (2010) as a possible driver for differential fertility.

problem:

$$V(h,b,\kappa,n;\bar{h}) = \max_{c,x,l}\left\{b\log\left(\frac{c}{\Lambda(n)}\right)+\nu\frac{l^{1-\gamma}}{1-\gamma}+\phi(n)\log(h'-\chi\bar{h}')\right\} \quad (5)$$

$$s.t. \quad c+xn\leq wh\,(1-\lambda n-l) \quad (6)$$

$$h'=\kappa\,(\theta+(xh)^{\alpha}) \quad (7)$$

$$l\in[0,1-\lambda n] \quad (8)$$

$$\bar{h}'=\Gamma(\bar{h})\;. \quad (9)$$

This optimization problem shows that parents invest in their children's education given the number of children and the realized ability shock. The household's time endowment is normalized to one. The total time available for labor supply and leisure decreases with the number of children, since each child requires λ units of time. Denoting leisure as l, labor supply is then given by $1-\lambda n-l$. Finally, the Γ function describes the law of motion for $\bar{h}$ as perceived by households. In equilibrium, this should be consistent with the actual law of motion for $\bar{h}$.[22]

Note that parents care about children in a warm-glow fashion. That is, parents are not fully altruistic in the Barro-Becker sense (Becker and Barro 1988; Barro and Becker 1989). This assumption is made for tractability.[23] As we will see later, this assumption is not fully innocuous when it comes to policy implications, since only parents (i.e., and not children) face the externality originating from children themselves.

It should be clear by now that children are costly in our model for several reasons. First, each child requires an education investment in form of the endogenous education expenditure x. Second, a larger household with more children would reduce utility through the household equivalence scale channel $\Lambda(n)$, since the same consumption expenditure must be shared by more people in the household. Finally, children require time inputs, which reduces the time that can be used for leisure or

[22]In stationary equilibrium, this forecasting rule is trivial and can be abstracted. However, this is relevant when the economy is not in the steady state (e.g., transitional dynamics).

[23]A fully altruistic model with status externalities would lead to a complicated game across generations and dynasties, as parents would need to consider their own children's choices as well as those of all other children when choosing education inputs. See discussions in Section 4 of (Golosov, Jones, and Tertilt 2007) for more details.

labor supply.

Note that although parents would always have a positive expected utility when they choose to have a non-zero number of children (otherwise, they would prefer childlessness), we do not rule out the possibility of negative ex-post utility from children. This is possible when the children's ultimate level of human capital is low (specifically, $h' - \chi\bar{h}' < 1$ in our case), which would be the case among unlucky parents of modest means whose children's idiosyncratic ability is low.

Finally, to close the model, we assume that aggregate production is linear in aggregate labor. Specifically, letting L be average efficiency units of labor, output per capita is given by $Y = AL$ where A is total factor productivity.

The equilibrium definition is relatively standard and given in Appendix B.1. Given the status externality, the key object of the equilibrium both in the steady state an along the transition path is the endogenous distribution of human capital. This distribution is determined as a fixed point that agents take as given, while their expectations must be consistent with the actual evolution of the human capital distribution. Further details on how we compute the steady state equilibrium and equilibrium transitional dynamics are provided in Appendix B.1.

4 Calibration

We solve the model numerically and calibrate it to the cohort of Korean women born between 1970 and 1975 described in Section 2.[24] As is standard in the literature, the calibration proceeds in two steps. First, some parameter values are chosen externally based on direct data analogs, the literature, or simple normalization. Second, the remaining parameters are chosen to match relevant data moments. We also compare the model's predictions along several non-targeted dimensions.

We start with some preliminaries. We set the maximum number of children equal to three, $N_n = 3$, because the portion of households with more than three offspring is very small.[25] Because fertility is a discrete choice between 0 and 3, it is

[24] See Appendix B.1 for details on the numerical procedure.

[25] We include the small fraction of women with four and five children (1.59% in the data) in the

unnecessary to impose a parametric functional form on $\phi(n)$. Instead, we let it be non-parametric, and assume that $\phi(n) = \phi_n$ and $\phi(0) = 0$. This leads to three parameters, namely $\{\phi_n\}_{n=1}^3$.

We let the discrete distribution for b approximate a log-normal distribution:

$$\log b \;\sim\; \mathcal{N}(0, \sigma_b^2), \tag{10}$$

with $N_b = 20$. We let the Markov chain for κ approximate an AR(1) process:

$$\log \kappa = \rho_\kappa \log \kappa^p + \varepsilon_\kappa, \tag{11}$$

where $\varepsilon_\kappa \sim \mathcal{N}(0, \sigma_\kappa^2)$, using Tauchen (1986) with $N_\kappa = 20$. This leads to two parameters—ρ_κ and σ_κ—to be calibrated for the Markov chain. Our quantitative results are barely affected by this choice as long as N_κ is sufficiently large.

4.1 Externally Calibrated Parameters

The parameter γ governs the curvature of the utility function with respect to leisure. We choose $\gamma = 2$ so that the implied intertemporal elasticity of substitution is 0.5, as is standard in the literature. The parameter λ captures the time costs of children. In our samples from the KLIPS, the average time parents of children under 18 spend with each child is 5.7 hours per week.[26] Assuming a total weekly time endowment of 100 hours, this leads to $\lambda = 0.041$.[27] Note that we consider this time input to be constant. In contrast to the United States, where more educated parents spend substantially more time with their children (e.g. Guryan, Hurst, and Kearney (2008) and Yum (2018)), here we find that the educational gradient in parental

three children category. See Table A1 for the whole distribution in Appendix A.

[26] In the data, mothers spend more time with children (15.8 hours per week) than fathers (4.2 hours per week). This pattern is similar to the United States, where mothers and fathers spend 14.0 and 6.8 hours per week, respectively, with their children (Guryan, Hurst, and Kearney 2008), though the gender gap is slightly larger in Korea. Because our model does not address gender differences, we take an average of the time spent by mothers and fathers and divide it by the average number of children to obtain average time per child. See Appendix A.6 for more details.

[27] A disposable time endowment of 100 hours per week is fairly standard in the literature as it leaves 68 hours for sleep and personal care. We assume children are costly for parents until they become adults, i.e. for 18 years. Since a model period is 25 years, we further adjust the time cost by a factor of 18/25, which then gives $\lambda = 0.041$.

time is not significant.[28]

For the household equivalence scale $\Lambda(n)$, we use the OECD modified equivalence scale, which assigns 1 to the adult head, 0.5 to an additional adult, and 0.3 to each child. Finally, the TFP parameter A is normalized to one.

4.2 Internally Calibrated Parameters

The remaining ten parameters are $\phi_1, \phi_2, \phi_3, \sigma_\kappa, \nu, \sigma_b, \chi, \theta, \alpha$ and ρ_κ. We calibrate these parameters to match moments from the data described below, specifically by minimizing the sum of squared percentage differences between data and model moments. All data moments are based on the 1970-75 cohort of women in the KLIPS data, and are either directly taken from Section 2 or described in Appendix A.

The data moments include the fraction of families with one, two and three or more children. We further include the Gini coefficient of income as a measure of inequality. We compute this using long-term income from the KLIPS samples, as described in Appendix A.1. To capture the relationship between income and fertility, we include the income elasticity of fertility as well as the fraction of childless families in the poorest quintile. Since spending on education is a key object of interest in our model, we include both average education spending relative to income and the income elasticity of education spending. Finally, intergenerational income elasticity is included as a measure of income persistence across generations. Table 3 reports all data moments as well as the model analogues and the calibrated parameter values.

Since every moment that results from the model is a function of all parameters, there is no one-to-one link between parameters and moments. However, some moments are more informative for particular parameters than for others. It is thus instructive to explain these intuitive links, even though there is no formal identification procedure. The first three parameters in Table 3 govern the utility of having n children: $\phi(n)$. These utility parameters are directly related to the percentages of

[28] Specifically, we observe weak education gradients, especially for women, as reported in Table A4.

Table 3: Internally Calibrated Parameters and Moments: Model vs. Data

Parameter	Interpretation	Moment	Model	Data
$\phi_1 = 1.63$	Utility from number of children	$\Pr(\# \text{ child} = 1)$	.196	.196
$\phi_2 = 2.46$		$\Pr(\# \text{ child} = 2)$	.631	.631
$\phi_3 = 2.86$		$\Pr(\# \text{ child} \geq 3)$	.143	.144
$\sigma_\kappa = .338$	Ability dispersion	Gini income	.252	.263
$\nu = 1.66$	Leisure constant	Avg total hours worked	.299	.302
$\sigma_b = .552$	Preference dispersion	Income elasticity of fertility	.083	.082
$\chi = .094$	Status externality	Childless in 1st income quintile	.053	.053
$\theta = 1.80$	HK production technology	Avg investment-income ratio	.091	.092
$\alpha = .346$		Income elasticity of investment	.703	.698
$\rho_\kappa = .346$	Ability persistence	Intergenerat. income elasticity	.337	.330

Notes: All data moments are based on KLIPS data as described in Section 2 and Appendix A.

people with 0, 1, 2 and 3 children in the data. The calibrated values of these parameters are $\phi_1 = 1.63$, $\phi_2 = 2.46$ and $\phi_3 = 2.86$, which are increasing at a decreasing rate with n.

The parameter σ_κ determines the variability of the idiosyncratic ability component in child human capital development. Because this ability shock is an important exogenous source of income heterogeneity in the model, it is largely determined by the Gini coefficient in the data (0.263). Note that our Gini coefficient is somewhat lower than what is typically reported (e.g., by the OECD) because our Gini is based on income averaged over several years rather than annual income, as described in Appendix A.1. The calibrated value of σ_κ is 0.338.

The value parents place on leisure relative to consumption and children is given by the parameter ν. It is pinned down by the average total hours worked in the data (30.2 hours per week or 0.302), as described in Appendix A.6. This value is based on both members of the household and includes both intensive and extensive margins. This leads to a calibrated value of $\nu = 1.66$.

Next, σ_b controls the degree of heterogeneity in preferences. An important reason for having preference heterogeneity is to allow for flexibility in shaping the model-generated relationship between income and fertility. Specifically, a greater variability of b makes the equilibrium fertility-income relationship less positive (or more negative). Our calibrated value of $\sigma_b = 0.552$ allows the model to match the positive income elasticity of fertility of 0.082 from the data.

The strength of the status externality is governed by χ, which, in turn, is relevant for childlessness. Intuitively, a strong status motivation leads parents to want high education expenditures, making children costly. For some parents, especially at the bottom of the income distribution, children become so costly that they prefer to have none. Thus, we include the childlessness rate for the bottom quintile as a relevant target moment. The calibrated value of $\chi = 0.094$ allows the model to exactly match the childlessness rate of 5.3%.

The human capital production function includes two parameters: θ and α. To pin down each of these parameters, we include two moments related to education spending: average education spending relative to income and the income elasticity of education spending as documented in Section 2. Since θ decreases the marginal return of additional education spending, it reduces the incentive to invest in education, and is therefore useful to match average education spending. The average education spending per child relative to income in the data is 9.2%. The calibrated value of θ is 1.80. In the data, the income elasticity of private education spending is 0.698. This moment is useful for pinning down α because it shapes the marginal product of education investments. As a result of the calibration, we find that α is equal to 0.346.

Finally, note that ρ_κ governs the strength of the exogenous ability transmission from parents to children. This strongly affects how income is correlated across generations. Therefore, our last target statistic is the intergenerational elasticity of income, which is 0.33 in the data. This target moment is the mean value of the estimates based on different age combinations from two generations, as reported in Appendix A.3. This value is very close to its counterpart in the United States, 0.341 (Chetty et al. 2014). The calibrated value of ρ_κ is 0.346.

Table 4: Fertility across Income Quintiles

		Income quintile				
	All	1st	2nd	3rd	4th	5th
Completed fertility						
Data (KLIPS)	1.91	1.80	1.91	1.87	1.93	2.03
Model	1.89	1.74	1.89	1.94	1.91	1.95
Childlessness rate (%)						
Data (KLIPS)	2.9	5.3	4.0	2.0	1.3	2.0
Model	3.0	5.3	3.0	2.3	2.3	2.0

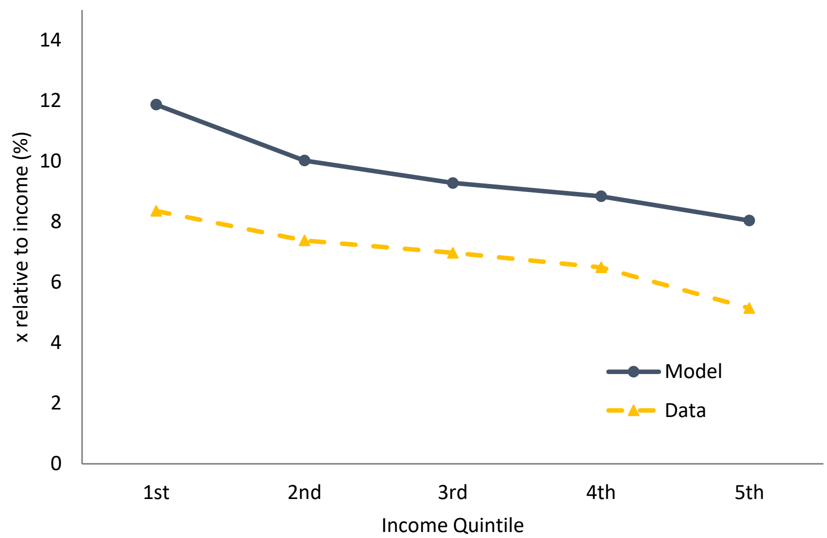
4.3 Non-Targeted Moments

Our calibration strategy targets the overall income gradient of fertility (i.e., via the income elasticity of fertility) and a select moment related to the extensive margin (i.e., the childlessness rate of the bottom income quintile). But how well does the model match fertility, childlessness, and education spending across the income distribution?

Table 4 shows that the model does a good job of matching the fertility rate across income quintiles in terms of both completed fertility and childlessness rates. Specifically, the model matches the data quite closely in reproducing the average number of children born to parents in the bottom income quintile (around 1.8), as well as the higher average number of children in the richest income quintile, at 2.0. Because we had included the fraction of families with a given number of children as targets, the model unsurprisingly matches the average fertility rate of 1.9. The childlessness rate across income quintiles in the model is also very similar to the data. Specifically, both in the model and the data, the childlessness rates in the third to fifth income quintiles are basically flat at around 2 percent, whereas they are higher among the first and second quintiles. Overall, the average childlessness rates are both around 3 percent both in the model and in the data.

Figure 2 plots education expenditures per child relative to income in the model and in the data, the latter of which are reported in the last column of Table 2. Since we target the overall slope (i.e., the income elasticity of investment), the model

Figure 2: Private Education Expenditure per Child Relative to Income



successfully generates the decreasing pattern of private education expenditures across income quintiles, as observed in the data.[29] In the model, parents in the top income quintile spend 8% of their income on education per child, whereas those in the bottom quintile spend a substantially larger fraction of their income on education (nearly 12% of total income per child).

5 The Role of the Externality for Fertility Choices

We now use our model to assess how the presence of the status externality affects fertility decisions and educational investment. In the first exercise, we put emphasis on its heterogeneous effects in addition to its role for aggregate fertility. In the second exercise, we explore the implications for the recent fertility decline over

[29]Note that the differences in the expenditure levels in the data and the model arise since the income gradient of education expenditure is estimated using individual-level survey data that contain a subset of total household expenditures on education (e.g., excluding household expenditures on school tuition), whereas the model is calibrated to match a more comprehensive measure of total education expenditures (9.2% of income) based on household-level survey data. Details are provided in Appendix A.2

time.

5.1 Fertility in Aggregate and along the Income Distribution

To understand the role of the status externality, we shut it down by setting $\chi = 0$. Figure 3 shows how fertility rates across income quintiles change in the absence of the externality. Two points are worth noting: first, the overall fertility rate increases across all income quintiles when the status externality is removed. The aggregate fertility rate, at 2.17 births per woman, is considerably higher than the value of 1.89 in the baseline model with the status externality. Second, the increase in fertility rates is relatively higher among low-income groups. The income elasticity of fertility takes a negative value in the model without the externality (-0.039), starkly contrasting with the elasticity of 0.083 in the baseline model. This finding is interesting, since the positive relationship between income and fertility disappears and becomes negative when the status externality is removed, bringing it more in line with countries such as the United States (Jones and Tertilt 2008). This result suggests that the status externality may be an important factor behind the positive income elasticity of fertility documented in Section 2.

Table 5 also reports the childlessness rates and educational investment per child across income quintiles for the two versions of the model. The top panel shows that childlessness rates are generally much lower—and increase with income—in the absence of the externality. This is in sharp contrast to the baseline model, which features relatively higher childlessness rates of around 5% among low-income households.

The bottom panel of Table 5 shows what parental expenditures on private education across income quintile would be in an economy without the externality. First, households would generally spend less on private education in the absence of a motivation to catch up with the children of others. This effect is relatively stronger among low-income households, reducing investment per child by 34%. The reason is that low-income parents in our baseline model invest more aggressively (measured by the percentage of family income spent on private education), in the hope

Figure 3: Status Externality and Fertility across Income Quintiles

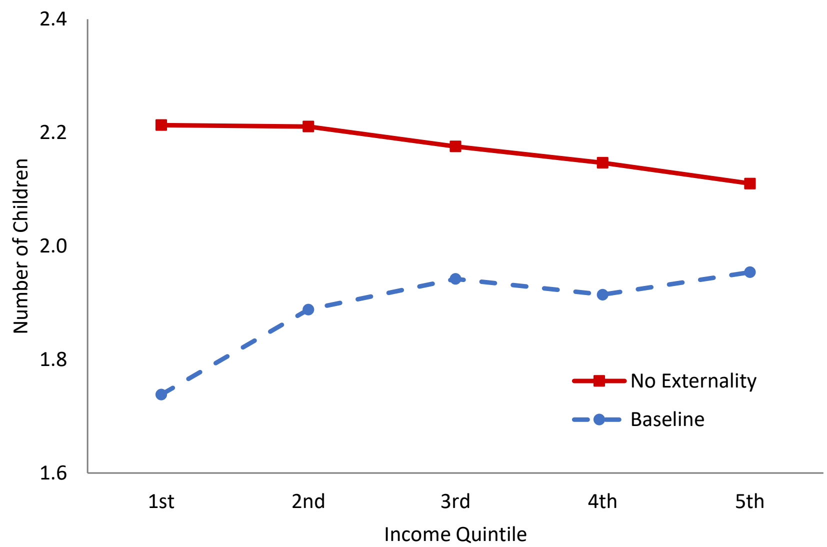


Table 5: Effects of the Status Externality across Income Quintiles

	Income quintile				
	1st	2nd	3rd	4th	5th
Childlessness rate (%)					
Baseline Model	5.3	3.0	2.3	2.3	2.0
No Externality	0.7	0.6	0.8	0.9	0.9
Investment per child, x					
Baseline Model	.046	.056	.067	.081	.111
No Externality	.028	.038	.048	.059	.087
Change relative to baseline (%)	-33.7	-27.1	-22.6	-20.8	-15.9

of bringing their children's status closer to that of the other children.[30] When it comes to the choice of having a child, we can see that the status externality also puts more pressure on low-income households in our baseline model, since childlessness falls substantially in the first income quintile when the externality is removed. This mechanism explains the effect of the status externality on the level of fertility as well as the income gradient of fertility.

An alternative reason for the extremely low fertility rate in Korea has been recently suggested by Myong, Park, and Yi (2020). They propose Confucian social norms—in particular the unequal division of childcare responsibilities within couples—as a mechanism that can explain the low marital fertility rates. We view this as a plausible complementary mechanism. Two differences should be stressed: in contrast to ours, their mechanism works through marriage, while marriage rates in Korea and other East Asian countries have declined substantially in recent decades (Raymo et al. 2015). Secondly, we focus on a somewhat more recent time period.[31]

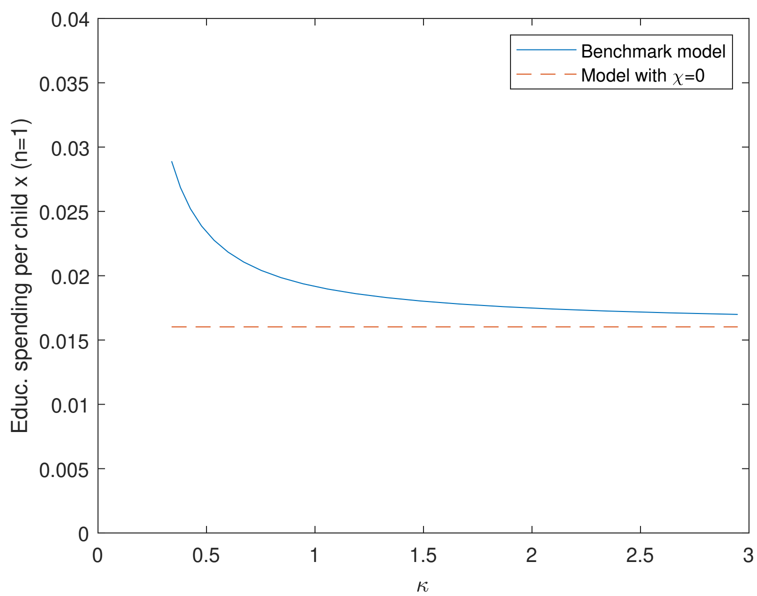
The externality is also relevant for spending on education by child ability, κ. Holding parental human capital constant, in our model without the externality, parental education spending is flat in κ. Yet when status concerns are operative, it is precisely parents of low ability children who aim to make up for their offsprings' low ability by investing in extra education. Figure 4 shows that parents who have a child with low κ would invest substantially more in our baseline model, as compared to the model without the externality. Thus, all else equal, the lowest ability children end up being the most educated, and even more so when their parents are wealthy. In the example displayed in the figure parents of low ability children (κ=0.5) invest almost twice as much in them as parents of high ability children (κ=3).[32] This effect is reminiscent of a point made in Fershtman, Murphy, and Weiss (1996) who argued that status concerns could lead the "wrong" individuals to acquire schooling which would depress growth.

[30] This effect echoes the empirical findings of Bertrand and Morse (2016), who find that poor people spend a larger share of income on housing, especially when they live close to high earners, which the authors interpret as status-seeking behavior.

[31] Recall that our baseline sample are cohorts born in the 1970s, while the sample in Myong, Park, and Yi (2020) includes cohorts born in between the 1940s and 1960s.

[32] The size of the gap does depend on the values of the other state variables.

Figure 4: Education Spending by Child Ability (κ)



Notes: The figure plots the decision rule of education spending per child, x, for a particular level of human capital, preference type, and number of children (specifically, $h = 0.89$, $b = 0.92$, and $n = 1$) both for the baseline model and the model without the status externality. Note that $n = 1$ is optimal for those parents with κ_p being between 0.48 and 0.67. The overall shapes do not depend on the values of the state variables.

5.2 Fertility Decline over Time

Like many other countries, Korea has been experiencing a decline in fertility over time. The consensus in the literature is that the quantity-quality trade-off was a key factor behind this decline.[33] Specifically, the main driving force behind the demographic transitions is considered technological progress which increases the return to human capital leading parents to want more educated, and hence fewer, children. Given that the mechanism works through the quantity-quality trade-off, it seems quite plausible that it would interact with the status concerns highlighted in our paper. To explore the extent of such interaction, we now conduct a simple exercise. We change parameters in the human capital production function so that

[33] For some classic contributions see Becker and Lewis (1973), Becker, Murphy, and Tamura (1990) and Galor and Weil (2000).

our benchmark model can account for the fertility decline and GDP increase observed in the data. We then ask the counterfactual question: how different would the time trends have been in a world without status concerns?

More precisely, we compare our benchmark cohort of women born between 1970–75 to women born in 1961–66. The average number of children born was 2.04 in the earlier cohort, i.e. about 7% higher than 1.91 in our benchmark cohort. At the same time, average long-term income in the earlier cohort is 16.1% lower then its counterpart in the baseline cohort. In terms of the model parameters, we allow two parameters to change over time: ς and α in the following human capital production:

$$h' = \kappa(\theta + \varsigma(xh)^{\alpha}), \tag{12}$$

where the baseline value of ς is one. We find that lowering α from 0.346 to 0.194 and lowering ς from 1 to 0.62 for the earlier cohort delivers the observed changes in fertility and average income simultaneously. We then turn the externality off by keeping $\bar{h}$ in the utility function constant at the equilibrium level for the earlier time period. When feeding in the same parameter changes into this modified model, we find a somewhat smaller fertility decline (to 1.91 instead of 1.89). More importantly, we find that childlessness would have not changed at all, compared to an increase from 2.7% to 3% for the baseline model with status concerns. This effect is most pronounced for the poorest households. Childlessness in the first income quintile rises from 3.1% to 5.3% in our baseline model, while without the status externality it would have risen by noticeably less (to 4.5%). In other words, the externality amplified the fertility decline over time, both at the intensive and the extensive margin, but particularly so at the extensive margin among the poor. At the same time, it also amplified increases in education investments and human capital, and thereby led to a larger increase in average income, as shown in Table 6.

6 Education Taxes and Pro-natal Transfers

Like many other countries, the Korean government has implemented various policies to fight falling birth rates. In particular, it initiated "The First Basic Plan on

Table 6: Model Implications for Changes across Cohorts

Externality feedback	Cohorts	Y	$\mathbb{E}(h)$	$\mathbb{E}(x)$/inc.	All $\mathbb{E}(n)$	All $\Pr(n=0)$	1st Quintile $\mathbb{E}(n)$	1st Quintile $\Pr(n=0)$
Yes	1961–66	.665	2.395	.039	2.01	2.7%	2.06	3.1%
	1970–75	.793	2.653	.091	1.89	3.0%	1.74	5.3%
No	1961–66	.665	2.395	.039	2.01	2.7%	2.06	3.1%
	1970–75	.788	2.644	.089	1.91	2.7%	1.78	4.5%

Notes: $\mathbb{E}(n)$ is average fertility and $\Pr(n=0)$ refers to the childlessness rate. The final two columns report conditional means for the first income quintile.

Low Fertility and Aging Society" in 2006 (Hong et al. 2016). In this section, we use our model to investigate the effect of policies aimed at stimulating the birth rate. Given the status externality in the model, two policies seem especially relevant. First, fertility could be directly stimulated by giving parents cash or in-kind transfers tied to a birth (i.e., a pro-natal transfer). Second, since the status externality appears to lead parents to overinvest in their children's education, a tax on private education could reduce equilibrium education spending, making children cheaper and thus stimulating fertility. In what follows, we investigate the effects of these policies, before turning to the more complicated question of an optimal policy in Section 7.

While we begin by conducting steady-state comparisons, useful for gauging the long-run implications, we also analyze transitional dynamics. More precisely, we compute the perfect-foresight transition path under the following timing assumptions: Until period 0 (i.e., $t = ..., -2, -1, 0$), the economy is in the initial steady state. This pre-reform economy is exactly the baseline specification. Then, at the beginning of period 1 ($t = 1$), a certain policy reform is introduced unexpectedly and permanently. The economy then moves into a new steady state.

Table 7: Policy Experiments: Long-run Effects of Pro-natal Transfers

	Benchmark	$\psi = 0.01$	$\psi = 0.02$	$\psi = 0.03$
Fertility rate n	1.887	1.923	2.010	2.104
(% change relative to benchmark)		(1.9%)	(6.5%)	(11.5%)
Childlessness rate	3.0%	2.7%	2.0%	1.7%
Avg x per child/income	9.08%	8.94%	8.61%	8.28%
Income elasticity of n	.083	.070	.013	-.036
Income elasticity of x	.703	.703	.738	.766
Avg labor supply	.299	.298	.297	.295
Avg human capital	2.653	2.645	2.616	2.590
Output per capita	.793	.788	.776	.763
Gini income	.252	.252	.254	.256
IGE	.337	.333	.329	.323
Tax/Y		2.4%	5.2%	8.3%

6.1 Pro-natal Transfers

Pro-natal transfers are an intuitive way to promote fertility, where a government (cash or in-kind) benefit is tied to the number of children a family has. A narrower definition of pro-natal transfers would be cash or in-kind benefits given to the parents of newborns. In Korea, some districts started introducing cash grants for births as early as 2003. By the end of 2011, most districts (229 out of 260) were providing such grants. Furthermore, starting in 2006, the "The First Basic Plan on Low Fertility and Aging Society" included pro-natal transfers on a national level.

Beyond newborns, many countries provide cash benefits for all children below a certain age. In Germany, for example, a cash transfer of about 200 euros per child (called *Kindergeld*) is given to parents until offspring reach the age of 18. This type of child allowance is also essentially a pro-natal transfer as it reduces the cost to parents of having children.

To capture such programs, we add two terms to the household budget constraints:

$$c + xn + T \leq wh(1 - \lambda n - l) + T_n(n) \tag{13}$$

where $T_n(n) > 0$ captures pro-natal transfers. In each period, the government budget is balanced by adjusting lump-sum taxes T to finance government spending:

$$\sum_{j}^{N_\kappa} \tilde{\pi}_j^\kappa \sum_{i}^{N_b} \pi_i^b \int \left(T_n(n) - T\right) F(dh, b_i, \kappa_j^p) = 0, \tag{14}$$

where $\tilde{\pi}_j^\kappa$ captures the probability mass of κ_j^p. We consider non-distortionary lump-sum taxes as we want to focus on the issue of the status externality while avoiding another distortion caused by taxes on income or consumption.

We consider a simple function for pro-natal transfers that increases linearly with the number of children: $T_n(n) = \psi n$. This type of transfer to households with children is very similar to the *Kindergeld* discussed above, and was recently introduced in Korea on a smaller scale. To investigate potential nonlinearity in the policy effects, we consider three different levels of ψ: 0.01, 0.02 and 0.03. A transfer of $\psi = 0.01$ corresponds to a monthly child allowance of 42 USD, or 1.8% of monthly income per child for 18 years.[34]

Table 7 shows the effects of different levels of pro-natal transfers in the model. First, pro-natal transfers clearly increase the fertility rate. For example, with a ψ of 0.01, the new steady-state fertility rate increases by 1.9%.[35] The positive impact on fertility is also clearly observed at the extensive margin: for example, when ψ increases to 0.01 and 0.02, the childlessness rate falls to 2.7% and 2.0%, respectively. Second, fertility increases at an accelerating rate with the size of cash payments. However, the required funding also increases rapidly, with 8.3% of output required to sustain a transfer of $\psi = 0.03$. The effects on fertility, meanwhile, are relatively small: the fertility rate only increases from 1.89 to 2.10 (or 11.5%).

The fertility effects of pro-natal transfers in our model are in line with recent empirical evidence from Korea. Cash transfers to the parents of newborns were in-

[34]This value is based on the assumption that annual GDP per capita is 28,732 USD (2015 estimate). Specifically, let M be the monthly payment to each child. The total transfer payments per child until the age of 18 in the data relative to GDP per capita over 25 years is $(M \times 12 \times 18)/(28732 \times 25)$. This corresponds to $\psi/0.793$ in the model. By equating these two, we obtain the relationship between M and ψ. Finally, M divided by 28,732/12 USD gives the fraction relative to monthly income.

[35]Interestingly, the effects of the policy are somewhat larger compared to a world without status externalities, as discussed in Appendix B.2.

troduced relatively recently in Korea. Hong et al. (2016) try to identify the causal effect of these transfers using regional and time variation, finding that a one-time cash bonus of 1,000 USD increases the crude birth rate by 4.4%. More recently, Kim (2020) exploits the same policy changes but uses birth outcomes over a longer time horizon, concluding that a 10% increase in cash transfers raised birth rates by approximately 0.4–0.6%, depending on birth order.[36] These estimates are similar in magnitude to our model. When we compare a transfer of $\psi = 0.02$ to $\psi = 0.03$, i.e., a 50% increase in pro-natal transfers, the completed fertility rate goes up by 4.6% (i.e., from 2.010 to 2.104).

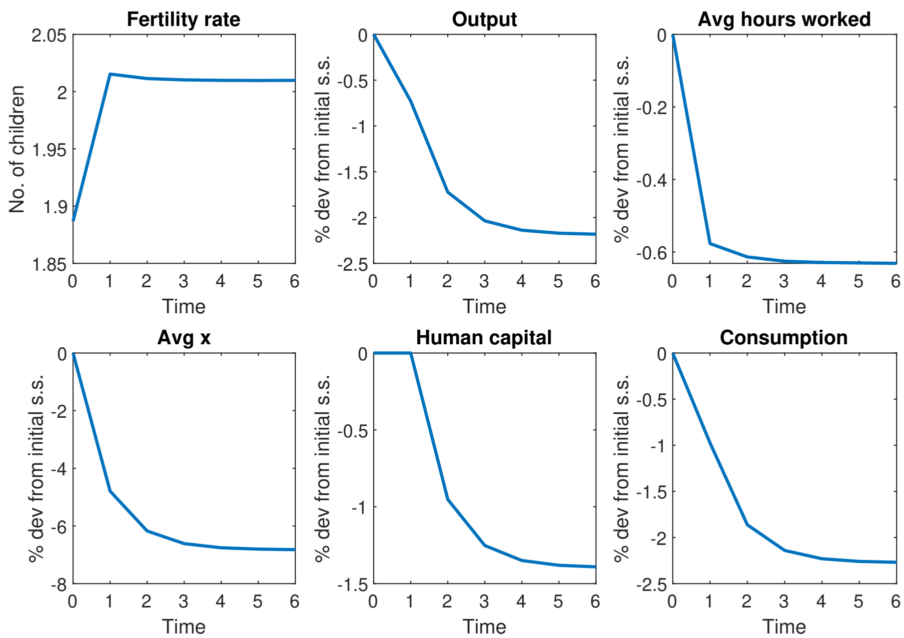
As fertility increases in response to pro-natal transfers, the average spending on private education per child decreases. For instance, with $\psi = 0.03$, parents have more children but invest less per child (education spending per child declines from 9.1% to 8.3% of income). It is also worth noting that the income elasticity of n decreases and that of x increases with larger pro-natal transfers. The rise in fertility among low-income families is greater than among high-income families in response to pro-natal transfers, leading to a larger decrease in per-child education spending in low-income families. Because parents invest less per child, the average human capital and output per capita are lower in the new steady states.

A pro-natal transfer also leads to a moderate long-run increase in inequality, reflected in a higher Gini coefficient. This is due to the reduced education spending by poorer parents relative to richer parents when a pro-natal transfer is introduced.

The results discussed thus far capture long-run changes. But how long would it take to reach the new steady state and what would be the effects on fertility during the transition? Figure 5 plots the transitional dynamics when a pro-natal transfer of $\psi = 0.02$ is introduced, unexpectedly and permanently, at the beginning of period 1. The fertility rate and labor supply respond immediately when the policy is introduced, while the other aggregate variables, such as output, consumption, and human capital, decline gradually towards the new steady state. Given that the change in labor supply is quite small, the decline in output per capita is driven by the decline in aggregate human capital due to reduced spending on education.

[36] The relatively large estimates of Hong et al. (2016) may partly be caused by a change in the timing of births, making the lower estimates of Kim (2020) more relevant here.

Figure 5: Policy Effects along the Transition: Pro-natal Transfers



Notes: A pro-natal transfer of $\psi = 0.02$ is introduced unexpectedly and permanently in period 1. A model period corresponds to 25 years.

Table 8: Policy Experiments: Long-run Effects of Private Education Taxes

	Benchmark	$\tau_x = 0.1$	$\tau_x = 0.2$	$\tau_x = 0.3$
Fertility rate n	1.887	1.886	1.884	1.882
(% change relative to benchmark)		(-0.1%)	(-0.1%)	(-0.2%)
Childlessness rate	3.0%	3.0%	3.1%	3.1%
Avg x per child/income	9.08%	8.14%	7.35%	6.68%
Income elasticity of n	.083	.073	.062	.052
Income elasticity of x	.703	.685	.672	.665
Avg labor supply	.299	.295	.291	.289
Avg human capital	2.653	2.620	2.591	2.566
Output per capita	.793	.774	.758	.744
Gini income	.252	.255	.257	.259
IGE	.337	.330	.323	.317
Transfers/Y		1.6%	3.0%	4.0%

6.2 Private Education Investment Tax

As shown in Section 4, low fertility is partially caused by the status externality, which leads to high education spending. One might therefore consider taxing education. While most countries subsidize rather than tax education, the Korean government has long struggled to reduce the high demand for private education. For instance, in 1980, the government entirely prohibited private education through hagwons and private tutoring, among others, although this was later declared unconstitutional by the Supreme Court. More recently, a 2016 law in Seoul forbid hagwons from operating between 10 p.m. and 5 a.m., the same was extended to private tutoring in 2017. Similarly, the Chinese government has recently introduced severe regulations in private education industries in an attempt to fight falling birth rates.[37] We thus investigate the effects of an education tax in our model.

To this end, we extend the budget constraint of the household as follows:

$$c + (1 + \tau_x)xn \leq wh(1 - \lambda n - l) + T \tag{15}$$

[37]See "China Bans For-Profit Tutoring In Reforms Aimed At Boosting The Birth Rate" (Forbes, July 24, 2021)

where $\tau_x \in [0,1]$ denotes tax on private education investment. Again, we require the government budget to balance in each period, which is achieved through lump-sum transfers T to consumers:

$$\sum_{j}^{N_\kappa} \tilde{\pi}_j^\kappa \sum_{i}^{N_b} \pi_i^b \int (\tau_x x n - T)\, F(dh, b_i, \kappa_j^p) = 0. \tag{16}$$

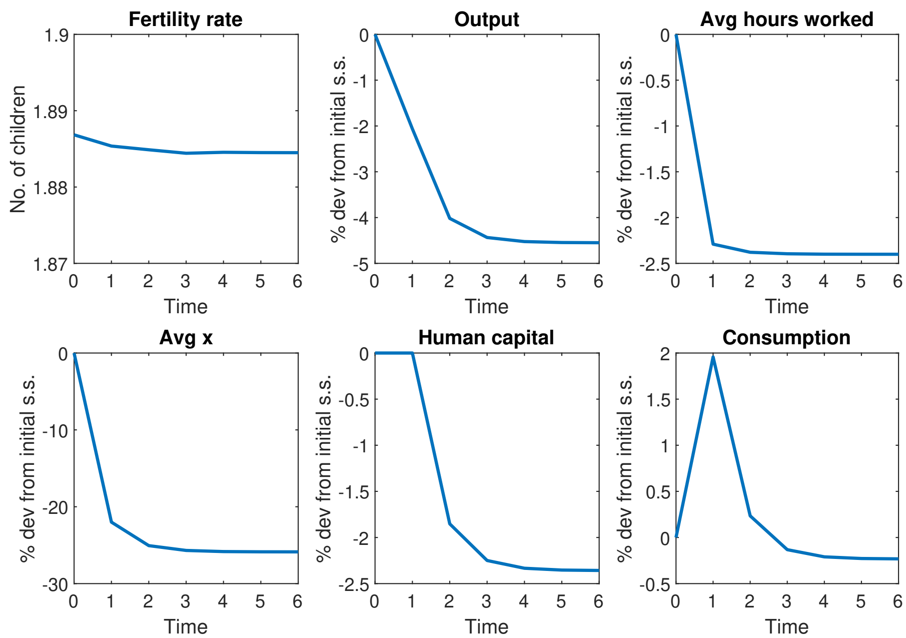
We consider three different levels of τ_x: 0.1, 0.2, and 0.3.

Table 8 summarizes the results for the steady-state comparison. We find that the education tax is quite powerful in reducing parental investment. Private education expenditures relative to income drop considerably, from 9.1% to 8.1%, 7.4%, and 6.7% for education tax rates of 10%, 20%, and 30%, respectively. However, a parent's lower demand for quality does not necessarily imply a higher demand for the quantity of children: in fact, the fertility rate falls slightly when an education tax is introduced. A higher education tax makes offspring more expensive, since it costs more to attain a certain child quality, thereby reducing the demand for both quantity and quality of children. Interestingly, we find that this effect is larger for high-income families, with the income elasticity of n and x decreasing as the tax rate increases. As parents choose to invest less per child, average human capital in the long run becomes lower, and parents work less. Together, these lead to lower output per capita in the long run.

Again, it is useful to look at the full transition dynamics to the new steady state with a higher tax on private education spending. Recall that the tax is introduced at the beginning of period 1 unexpectedly and permanently. Figure 6 shows how the key macroeconomic variables evolve during the transition to the new steady state.

In period 1, right after the introduction of the tax, we can see that education spending per child (x) drops quite significantly. Since the need for funds to spend on education decreases, parents work less. The level of human capital of adults entering the period 1 (or the first generation) is not affected by policy changes because human capital is a state variable. However, the human capital of the following generations is affected as the first generation's endogenous investment decisions start to have intergenerational consequences. Because people have lower human

Figure 6: Policy Effects along the Transition: Private Education Taxes



Notes: The reform ($\tau_x = 0.2$) is introduced unexpectedly and permanently in period 1. A model period corresponds to 25 years.

capital and work less, output per capita falls over time. This demonstrates that taxing education spending to address the externality-driven distortions may not be desirable for future generations due to the adverse long-run implications for human capital accumulation.

The bottom right panel of Figure 6 shows that the first generation actually experiences an increase in consumption. Given that the quantity and quality of children both decrease, parents (the first generation) benefit from the education tax by enjoying more leisure and consumption, whereas future generations experience lower human capital and consumption relative to the initial steady state.

Thus far, we have investigated the effect of pro-natal transfers and education taxes. Pro-natal transfers appear to be effective at raising fertility, while taxes on private education reduce expenditures on private education. Yet we also find that these policies necessarily involve intergenerational conflicts, since future generations suffer from lower human capital, which in turn leads to lower output per capita.

7 Optimal Policy

In the previous section, we saw that pro-natal transfers and education taxes affect fertility and education decisions. We have also seen that policies that lower education naturally reduce the human capital of future generations and thus reduce output per capita in the future. But how do these policies impact welfare? What might be the optimal policy? Is there a policy that addresses the distortion caused by the status externality without any negative impact on future generations?

While these are obvious questions to ask, answering them is far from straightforward. In this section, we begin by discussing why this problem is non-trivial in our heterogeneous-agent model with endogenous fertility and then explain how we address this challenge. Then, we investigate the optimal mix of the two policy instruments considered in the previous section.

7.1 Welfare Concepts and the Planning Problem

Externalities generally lead to distortions. In our model, parents do not consider the effect of their education investment on other parents' children. Thus, the marginal private return from education is higher than the marginal social return and parents will overinvest in education. Loosely speaking, the equilibrium allocation will thus not be efficient. One would then like to know what the optimal allocation is that a planner would choose. While this logic seems clear and simple, it entails a number of complications. The notion of Pareto Efficiency is not well-defined in models with endogenous fertility which imply changing sets of people across allocations. Naturally, then, there is no planning problem that recovers the first-best allocation, or, as is often the case, all allocations on the contract curve.

A concept closely related to Pareto Efficiency defined for models with endogenous fertility is $\mathcal{A}$*-Efficiency*, first proposed by Golosov, Jones, and Tertilt (2007). An allocation is considered $\mathcal{A}$*-efficient* if there is no other allocation that is weakly preferred by all people alive in both allocations and strictly preferred by at least one person alive in both allocations. Thus, $\mathcal{A}$*-Efficiency* is a natural modification of Pareto Efficiency, which focuses only on those alive in both allocations whenever comparing two allocations.[38] Applying this concept to our context, note that the logic is similar to the pollution example given by Golosov, Jones, and Tertilt (2007). Their example shows that when consumption affects other people negatively, the equilibrium allocation features higher consumption and population levels than optimal. Furthermore, taxes on consumption and children can address the externality and implement the first-best allocation. But how does one find such $\mathcal{A}$*-efficient* allocations? Golosov, Jones, and Tertilt (2007) prove that if the solution to a planning problem that maximizes the weighted sum of utility of the first generation is unique (and all weights are strictly positive), then the allocation is $\mathcal{A}$*-efficient*. In other words, we can recover many different $\mathcal{A}$*-efficient* allocations that maximize the weighted sum of the first generation with varying weights.

[38]Note that this concept ignores the "views" of additional people whenever one allocation has a higher fertility rate and ignores the "views" of those that do not come into existence when considering an alternative allocation with a smaller population. Since preferences of non-existing people are hard to define and impossible to measure, simply ignoring them is a pragmatic way to move forward.

An additional consideration is that a large part of welfare gains across policies in heterogeneous agent models typically originate from redistributing resources from the rich to the poor. While welfare gains through redistribution are of interest in a general sense, they are not the focus of our paper. Rather, we are interested in the effects of a particular distortion.[39] To isolate the welfare implications of the distortion from redistributional concerns, we use Negishi weights in our baseline welfare analysis. Negishi weights put a greater weight on rich people (in our case, those with high human capital) so that in an economy without distortions the planning problem simply recovers the equilibrium allocation.[40] Finally, rather than computing the unconstrained optimum, we follow the Ramsey tradition and allow the planner to use only a certain set of instruments, namely education taxes and pro-natal transfers.[41]

With these preliminary considerations, let us now define a planning problem for our model. The planner chooses the Negishi-weighted sum of the utility of the initial generation subject to the allocation being implementable as an equilibrium with education taxes and pro-natal transfers.[42] In each period following the initial reform period, we constrain the planner to impose the same taxes and transfers on everyone, and to balance the government budget constraint. The planner considers utilities along the entire transition path, rather than comparing steady states.[43] The baseline exercise considers permanent policy changes as in the previous section, but we also consider temporary policy changes, as shown in Appendix B.6. For a

[39]Note that welfare gains through redistribution can be achieved in almost all heterogenous agent models. Yet these "welfare gains" are usually not Pareto-improving, since the rich are made worse off, unless one takes a veil-of-ignorance perspective. See Davila et al. (2012) for further discussion of this point.

[40]Negishi weights are frequently used in environmental economics (e.g., Nordhaus and Yang (1996)). In contrast, the quantitative macroeconomics literature typically uses equal weights, mixing welfare gains from redistribution and those arising from correcting distortions. Notable exceptions include Domeij and Klein (2013), Heathcote, Storesletten, and Violante (2017), and Guner, Kaygusuz, and Ventura (2020).

[41]The first-best planning allocations that maximize the welfare of the first generation are computationally difficult to compute in our model due to the discrete choice of fertility. Specifically, since no tractable optimality condition can be derived and human capital is a continuous variable, it requires the planner to compare an infinite number of combinations of fertility choices across heterogeneous agents.

[42]Appendix B.4 provides details about how we construct Negishi weights.

[43]Simply comparing steady states with different policies would give misleading results, since this would compare allocations with different initial levels of human capital, ignoring the transition to reach the new level of human capital.

formal definition of the planning problem, see Appendix B.3.

7.2 Optimal Policy Results

We find that the optimal policy—that maximizes the Ramsey planning problem as described above and defined in Appendix B.3—is a 12% education tax and moderate cash transfers with a ψ of 0.017.[44] This amounts to a monthly cash transfer of 3% of average income for 18 years, according to the conversion of ψ into the monthly child allowance in Section 6.1. To verify that the planner's problem is well-behaved, we plot the utility of the planner for each instrument separately in Appendix Figure B.5. The figure shows that the value varies smoothly in each policy instrument individually. Relative to the baseline economy, the optimal policy increases the Negishi-weighted average utility of the first generation by 0.0025. To interpret the size of the welfare gain, we compute the change in consumption of the average agent with two children in the baseline equilibrium that is needed to achieve the same utility gain. We find that this corresponds to a 0.24% increase in lifetime consumption.[45] In other words, substantial welfare gains for the first generation are possible through the introduction of a sizeable education tax and a moderate pro-natal transfer.

Table 9 shows the effect of the policy on the first generation and how the resulting allocation differs from the equilibrium. Under the optimal policy, average fertility is higher by 5.6%, the childlessness rate is reduced by more than half a percent, and educational investments decrease by 16%. The effects are heterogeneous along the income distribution. Fertility increases the most (13%) for the poorest quintile and the least (2%) for the richest quintile. This is intuitive, since it is the poor who are most affected by the externality. Similarly, the childlessness rate decreases

[44] To check whether our Negishi weights are constructed correctly, we also solved the same planning problem for a modified economy without externality feedback, as described in Appendix B.2. With the modified economy, we find that the optimal τ_x and ψ are zero. This shows that our attempt to separate the distortion from redistributional concerns was successful. Instead, the optimal policy with equal welfare weights leads to a higher tax rate of $\tau_x = 0.364$ in the presence of the externality and a somewhat lower but still substantial tax rate of $\tau_x = 0.232$ in the absence of externality feedback. In both cases, pro-natal transfers are optimally set to zero. Details are given in Appendix B.6.

[45] It is not straightforward to apply the standard notion of consumption-equivalent variations with unequal welfare weights.

Table 9: Heterogeneous Effects of the Optimal Policy on the First Generation

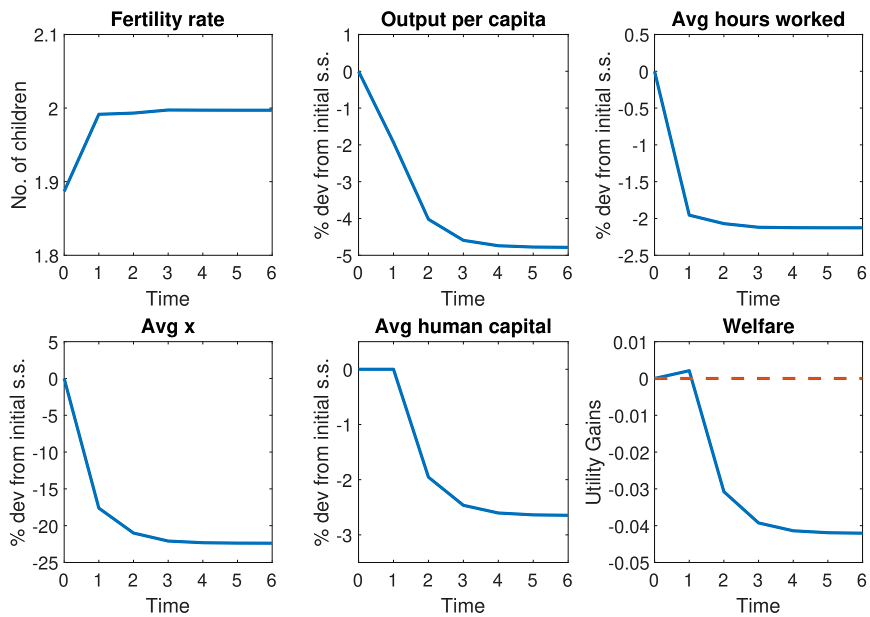
		Income quintile					
Average		1st	2nd	3rd	4th	5th	All
Fertility, n	Baseline	1.74	1.89	1.94	1.91	1.95	1.89
	Optimal	1.96	2.02	1.99	2.00	1.99	1.99
	% change	+13.0	+7.0	+2.4	+4.5	+1.7	+5.6
Childlessness rate (%)	Baseline	5.3	3.0	2.3	2.3	2.0	3.0
	Optimal	3.5	2.3	2.3	1.9	1.8	2.4
	p.p. change	-1.8	-0.7	-0.1	-0.3	-0.2	-0.6
Investment per child, x	Baseline	.046	.056	.067	.081	.111	.070
	Optimal	.037	.047	.056	.068	.094	.059
	% change	-19.4	-17.0	-16.0	-16.5	-15.7	-16.2

the most for the poorest quintile and very little for the top quintile. Education spending declines the most for the poorer quintiles, but the gradient is less steep here, falling between 16% and 19% for all quintiles.

How does the optimal policy affect future generations? Figure 7 shows the transitional dynamics of key variables to the new steady state with the optimal policy that is introduced unexpectedly and permanently in period 1. As discussed above, the optimal policy increases the fertility rate and decreases education spending. As a result, human capital and output decrease gradually over time. The last panel shows that the welfare of the first generation increases, while all future generations experience lower average utilities. Thus, future generations could be largely hurt by the introduction of the optimal policy, at least if measured by average utility.[46] The reason for this is twofold. First, in our model, only the parents—i.e., not the children—face the human capital externality. Secondly, our model considers education to be truly productive. If either of these two assumptions was changed, it should be possible to construct allocations that truly benefit every generation. To understand why, consider a status externality that leads parents to invest in improving signals for their children without truly educating them more. For ex-

[46]The same is still true, though less pronounced, in the case of a temporary policy, see Figure A5 in Appendix B.6.

Figure 7: Transition Equilibrium Under the Optimal Policy



Notes: The optimal policy reform is introduced unexpectedly and permanently in period 1. Welfare is measured by the change in Negishi-weighted average utility. A model period corresponds to 25 years.

ample, a large part of preparation for a national entrance exam might only improve test-taking skills without increasing children's knowledge and human capital. In such cases, less education spending will not lower the human capital of children and, accordingly, not harm the welfare of the children.[47] Alternatively, suppose children faced the same externality as the parents, i.e., they cared about consumption relative to their peers, similar to the original pollution example in Golosov, Jones, and Tertilt (2007) where parents and children suffered from pollution in the same way. In such a world, it should be possible to construct a dominating allocation with lower education and consumption for future generations that nev-

[47] The signaling value of education, which is costly but does not necessarily improve productivity, was first studied by Spence (1973). Several authors have formalized the point that investment into costly signals out of status concerns can lead to inefficient investments and even poverty; see, for example, Ireland (1998), Moav and Neeman (2010) and Moav and Neeman (2012). Recently, Macchi (2021) empirically identifies the wealth-signaling value of obesity—a harmful status symbol—in low-income countries.

ertheless improves their welfare. Finally, note that the population size of future generations becomes larger under the policy that maximizes the first generation's welfare. Alternative views of the optimal population problem could put weights on population size per se (something that our concept of $\mathcal{A}$-efficiency is silent on), which may offset the lower average utility of future generations.[48]

8 Conclusion

In this paper, we present a heterogeneous-agent model of endogenous fertility with a concern for the relative quality of children. Our model enriches the standard quality-quantity model and is able to account for various cross-sectional patterns of fertility and education investment in Korea. In our calibrated baseline model, the absence of a status externality leads to a 16.4% higher fertility rate, driven in particular by low-income households.

We investigate the transition of economic variables and the welfare of different generations after various policy reforms. In the literature, the effect of pro-natal policies on fertility is typically analyzed empirically. Our approach uses a dynamic equilibrium model framework, which allows us to study the dynamic effect on other variables over time and the welfare of different generations. This is important because the distortion from the status externality complicates the problem by affecting various decisions such as education investment and labor supply in addition to fertility decisions. This, in turn, leads to differential benefits for different generations following policy reforms. Indeed, we find that the optimal policy from the perspective of the current generation, which consist of pro-natal transfers and education taxes, may lead to undesirable outcomes for future generations and could cause a conflict between current and future generations. In addition, our policy experiments reveal the heterogeneous effects of policy reforms along the income distribution. For instance, the optimal policy disproportionately increases fertility and decreases education investment among low-income households.

In our paper, we have modeled status concerns as pure utility externality. An open question is whether there is a deeper cause behind such an externality. One

[48] See for example Dasgupta (1969) and de la Croix and Doepke (2021).

possibility is that status concerns capture capacity constraints in the national education system in a reduced form way. If children compete for a limited number of high-quality universities, parents will have a strong incentive to invest in private education to improve their children's chance to get in (e.g., a rug-rat race noted by Ramey and Ramey (2010)). In such a case, an optimal policy might be to simply expand the number of high-quality public universities. However, even then, as long as the ranking of universities matters, parents might compete for the very best ones and investments would still be inefficiently high (and reducing fertility). To investigate the role of the underlying sources of the externality for policy implications would be an interesting avenue to pursue in future research.

Our approach and findings should apply beyond Korea. Several other East Asian countries, most notably China, Singapore and Taiwan, similarly suffer from total fertility rates far below the replacement level along with high demands for private education. Concerns about relative education are often mentioned in those other countries as well. Even in the United States, with highly competitive university admissions (necessarily) based on relative achievements, our mechanism may apply to some extent.

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Online Appendix

A Data Appendix

A.1 Preliminaries

We use the Korean Labor and Income Panel Study (KLIPS) data to document fertility and educational spending across households. The KLIPS is a longitudinal survey of representative samples of Korean households and individuals. The survey has been conducted annually since 1998 on a sample of 5,000 households and members of the households. The data contains a rich variety of information including household demographics, education, labor market mobility, income, fertility, etc. We adjust income for inflation using CPI. The unit of income is 10,000 Korean Won (KRW), which is similar to 9 USD. We use the data up to the 20th survey which was conducted in 2017.

As in Jones and Tertilt (2008), we use a cohort-based approach. The baseline results focus on the women born in between 1970 and 1975.[49] Specifically, we include households in which the woman's age is between 40 to 43 and there are at least three observations within this age band. Also, we include only married or cohabiting couples in the analysis because single women are more likely to have lower fertility and lower family income than couples.The number of two-adult households satisfying all the required conditions is 756. We also provide the results including both singles and couples below for sensitivity.

Completed fertility is the number of children ever born to a woman, and includes both intensive and extensive margins of fertility. The extensive margin of fertility is whether to have any child or not. The intensive margin is about the number of children conditional on having at least one child. Table A1 shows the proportion of households, satisfying all the above requirements, with different numbers of children. The childlessness rate is 2.9%. Among parents who decide to have at

[49]The KLIPS used to represent the urban households in South Korea until 2008. In 2009, new households are added so that it can represent the whole population. Therefore, our empirical results are based on the data from 2009 which represent the whole population. We check robustness using the earlier cohort of 1961-1966 from the data before 2009.

Table A1: The Distribution of the Number of Children

Number of children	0	1	2	3	4	5
Proportion (%)	2.91	19.58	63.10	12.83	1.46	0.13

Notes: We calculate the proportion of households (married or cohabiting couples) using completed fertility of women born between 1970 and 1975.

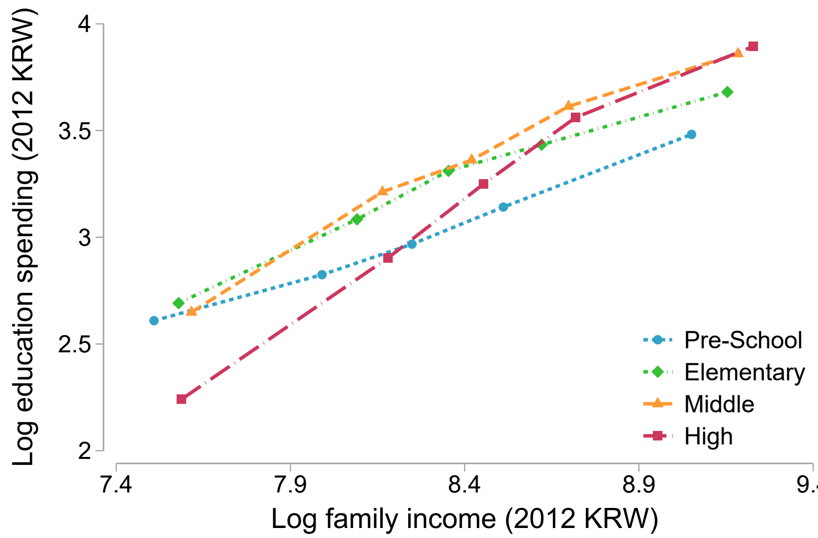
least one child, the proportion of parents with two children is the highest at 63.1%. There are few households, 1.6%, with at least four children. We further look into the relationship between income and fertility along both intensive and extensive margins.

To measure permanent household income, we utilize the longitudinal feature of the data by taking long term averages (Chetty et al. 2014). Specifically, we use the average income of households in which the woman's age is between 40 to 43. Our income measure is family income that combines labor income from both members of couples as well as capital income, but excludes income from social insurance and transfers. The Gini coefficient from our measure of long-term average income is 0.263.

A.2 Education Expenditures by Income and Child's Age

The KLIPS has two different types of questions regarding education in both the individual-level survey data and household-level survey data. First, the individual-level survey asks about per-child spending on private education, such as cram schools, for each child since Wave 3. Although this question excludes household expenditures on public education (e.g., tuitions), this is advantageous because we can observe the characteristics of the child which the money is spent on. We use this individual-level survey question to investigate the cross-sectional relationship between expenditures on private education and income. Second, the household-level survey also asks about total household-level expenditures on both private and public education since Wave 1. This gives a very comprehensive measure of out-of-pocket educational spending. However, it is hard to control for each child's

Figure A1: Expenditures on Private Education by Income and Education Stage

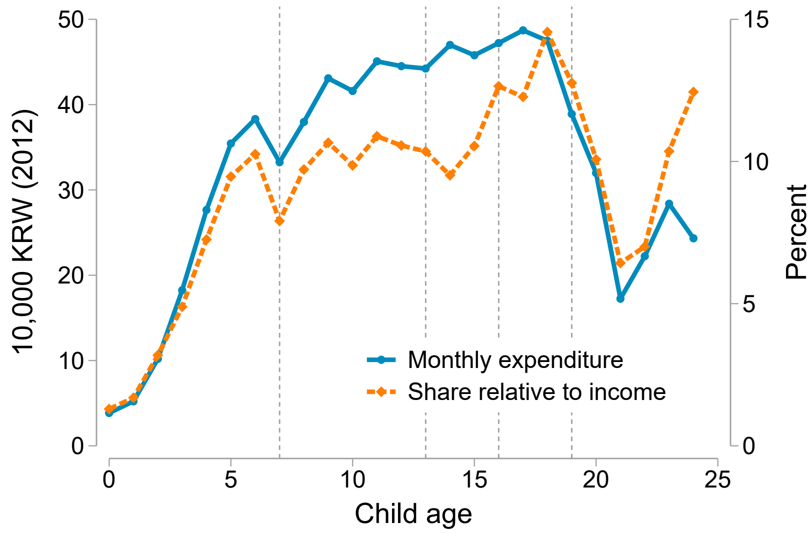


characteristics when there are multiple children in a household. Thus, we use this information to measure per-child spending on education relative to household income.

Figure A1 shows the relationship between the log of average education expenditures on private education per child and the log of average income for each income quintile and for each education stage. The slope implies the income elasticity of demand for private education and increases as children go to the next level of school: 0.57 for pre-school, 0.63 for elementary school, 0.77 for middle school, and 1.03 for high school. Because education costs also change as children go to the next level of school, we calculate the weighted average expenditures across different education stages using the number of years spent in each education stage as the weight. Specifically, we first calculate the education-stage-specific average spending for the given income quintile and education stage, and then averaging across different education stages weighted by the number of years spent in each education stage. The weighted average income elasticity of demand for private education is 0.698 and is used for calibration.

Figure A2 shows the average monthly education expenditures per child from birth

Figure A2: Private Education Expenditures by Child Age



Notes: This figure shows the total education expenditures on both private and public education per child for 25 years from age 0 to 24 and their shares in household income.

to age 24 and their shares relative to income. We use households with one child to plot this figure because we want to focus on the variation by child's age. The vertical lines indicate the typical ages at which children enter the next level of school in Korea. Note that the education expenditures increase rapidly, reaching 10% of income, before children enter elementary school. Then, it continually increases at a lower speed until children graduate from high school. The peak is at age 17 when children is in the second year in high school and the amount is around 500,000 KRW (similar to 450 USD) per month. The share of education expenditure in income has a similar shape but jumps when children enters high school. The share drops from the third year in college. Expenditures on private education for college students would be low but tuition is much higher. This implies that many Korean parents provide financial supports for their children's college tuition though their supports decrease rapidly from the third year in college.[50]

In Section 4, we use the fraction of total life-time education spending per child

[50]The rapid drop can be related to the conscription system in Korea. Many male students go to the army after finishing their second year in college.

in income to calibrate our model. The life-time spending per child for 25 years from birth to age 24 is 9.2% of income. To obtain this, we first sum the education expenditures and incomes separately across ages for 25 years and then divide the sum of expenditures with the sum of incomes (after-tax income).

A.3 Intergenerational Persistence

To estimate the intergenerational persistence of income between parents and children, we use our samples from the KLIPS data. Specifically, we first select households with information on labor earnings (including self-employed) for both parents and children in working ages. We focus on the average income of fathers aged 39 to 44 and that of children aged 30 to 35. We include households only when they have at least two observation for each person in the target ages. The number of matches increases as the gap in the target age bands for fathers and children getting apart. However, to get a better measure of the intergenerational earnings persistence, it is better to reduce the gap in target age bands. We select the current age bands for fathers and children to balance these two factors. Also, we can mitigate the life-cycle bias by focusing on the narrow target ages for parents and children (Haider and Solon 2006). Among parents, we use father's earnings because mother's working status is affected more by other factors than human capital, such as childbearing. This is standard in the empirical literature on intergenerational mobility (Solon 1999). Then, we regress the log income of children on the log income of father. The estimates depend on the target ages of fathers and children. Table A2 in Appendix shows that the estimated elasticity ranges from 0.2 to 0.5. We take the simple mean of the estimates, 0.33, for calibration. This value is quite close to the estimates from the United States (Chetty et al. 2014).

A.4 Fertility by Cohorts

Our baseline results focus on the women born between 1970 and 1975. Since the fertility rate has been decreasing quickly in Korea, we check how the relationship between fertility and income has been changed. Table A3 shows the number of children and childlessness rate for the recent cohorts (women born in 1970-75)

Table A2: Intergenerational Elasticity of Earnings

	Father's age		
	39-42	40-43	41-44
Child's age			
30-33	0.28	0.25	0.23
31-34	0.36	0.24	0.23
32-35	0.41	0.53	0.35

Notes: This table shows the estimated intergenerational earnings persistence when ages of fathers and children vary.

Table A3: Fertility and Income (Couples Only)

Income quintile	Number of children		Childlessness rate (%)	
	1970-75	1961-66	1970-75	1961-66
1th	1.80	1.99	5.26	3.17
2nd	1.91	1.97	3.97	0.79
3rd	1.87	2.06	1.99	0.79
4th	1.93	2.13	1.32	0.00
5th	2.03	2.08	1.99	0.01

Notes: This table shows the average fertility rate, the childlessness rate in each income quintile for each cohort group excluding single households.

and the earlier cohorts (women born in 1961-66). Overall, the number of children is higher and the childlessness rate is lower for the earlier cohorts. Next, we find that the positive slope between the number of children and income is slightly steeper for recent cohorts, as compared to the earlier cohorts. For example, the estimated income elasticity of fertility is 0.082 from our recent baseline cohort samples, whereas it is 0.041 in these earlier cohorts. Finally, the last two columns of Table A3 show that the relationship between fertility and childlessness rates is still negative also for the earlier cohorts although the overall childlessness rate was even lower at 1.1% compared to 2.9% for the recent cohorts.

A.5 Income and Fertility for Singles and Couples

As explained in Section 2, our main analysis focuses on the couples, excluding singles such as widowed, divorced, separated, and never married females. Among our target cohorts who answer the question about marriage status in KLIPS, there is no never-married women whose ages are in between 40 and 43. However, there are missing answers and we define these women as singles if they do not have information about spouse such as age. If they have information about spouses, we define they are couples. Among the target households, the portion of single women is around 8%. The portion of never-married women in Korea is in an increasing trend especially for young women in their 30s. These young women are not included in our analysis because they are still in their childbearing years and the completed fertility cannot be calculated for them.

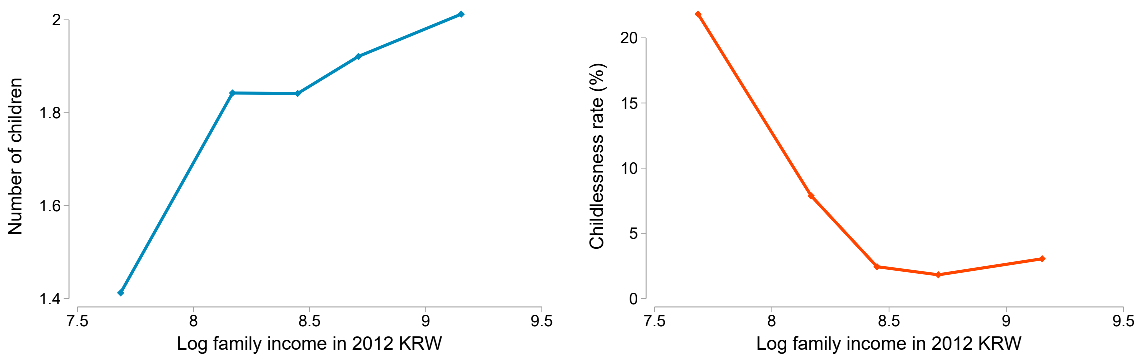
Note that there are several issues when it comes to the relationship between fertility and income if we include singles. First, the completed fertility, the number of children a woman ever had, and income are somewhat systematically influenced by being single. Single families tend to have lower income than couples and are more likely to have lower fertility. Therefore, the positive relationship between the completed fertility and income and the negative relationship between the childlessness rate and income become stronger when we include single households (See Figure A3). The changes mostly come from the childlessness rate and from the lowest-income quintile as this group includes most of the single women.

A.6 Time Use of Parents

We calculate the average weekly working hours and the average parental time per child using the KLIPS data. We focus on adults aged between 26 and 50 (inclusive). We use *regular* working hours for wage workers and *average* working hours for non-wage workers. The total average working hours include both intensive and extensive margins. As our model does not take into account gender differences, we take the equal-weight average of both members of households. As a result, we get the total average working hours of 30.2 hours per week.

To calculate the average parental time per child, we use the supplementary survey

Figure A3: Fertility by Income Quintile (including Singles)



Panel A. Completed fertility

Panel B. Childlessness rate

Notes: We group all households including singles into quintiles based on their long-term income and calculate the average completed fertility and the childlessness rate in each quintile for cohorts born in between 1970 and 1975.

of KLIPS on the use of time conducted in 2014. The survey respondents recorded what they did for 24 hours by a 30-minute interval. Thus, we take the total hours used for childcare and multiply 7 to calculate weekly parental time. We focus on parents whose children's ages are below 18 years old. On average, mothers spend 15.8 hours and fathers spend 4.2 hours per week for childcare. Mothers spend more time than fathers, as is typically the case in many countries. We take the equal-weight average of parents' childcare times and divide it by the average number of children. Finally, we get 5.7 hours of weekly parental time per child.

Table A4 reports the average childcare time by education. We consider two cases: (1) if the minimum age of children is less than or equal to five (i.e., with young children); and (2) if the maximum age of children is less than 18 (i.e., with any children). It is not clear that more educated parents spend more time with children in Korea. This is in contrast to the robust positive educational gradients in parental time observed in the United States (e.g., Guryan, Hurst, and Kearney (2008) and Yum (2018)).

Table A4: Average weekly childcare time by education

	(1) Young children		(2) Any children	
	COL	HS	COL	HS
Mothers	23.2	25.0	15.1	14.6
No. obs.	(288)	(433)	(539)	(990)
Fathers	6.3	6.0	4.5	3.8
No. obs.	(351)	(370)	(685)	(884)

Notes: This table reports the average weekly childcare time by education (1) if the minimum age of children is less than or equal to five, or (2) if the maximum age of children is less than 18. COL refers to college-educated, and HS refers to high school or below. Numbers in parentheses are the number of observations.

B Theoretical and Computational Appendix

B.1 Equilibrium Definition and Computation

The key object of the stationary general equilibrium is the endogenous distribution of human capital. In stationary equilibrium, $\bar{h}$ is constant, thus not an aggregate state variable.

A stationary equilibrium is a set of decision rules $n(h, b, \kappa^p)$, $l(h, b, \kappa, n)$, $c(h, b, \kappa, n)$, $x(h, b, \kappa, n)$, aggregate quantity L, and the distribution $F(h, b, \kappa^p)$ such that

- Given prices, households' decision problem leads to $n(h, b, \kappa^p)$, $l(h, b, \kappa, n)$, $c(h, b, \kappa, n)$, and $x(h, b, \kappa, n)$.
- Prices are competitively determined: $w = A$.
- Markets clear:

$$L = \sum_{j}^{N_\kappa} \tilde{\pi}_j^\kappa \sum_{i}^{N_b} \pi_i^b \int_h \sum_{k}^{N_\kappa} \pi_{jk}^\kappa \left(h \left(1 - \lambda n(h, b_i, \kappa_j^p) - l(h, b_i, \kappa_k, n(h, b_i, \kappa_j^p))\right)\right) F(dh, b_i, \kappa_j^p). \tag{A1}$$

- The stationary distribution of human capital is a fixed point:

$$\int_0^{h_c} F(dh, b_m, \kappa_k) = \frac{\sum_j^{N_\kappa} \tilde{\pi}_j^\kappa \sum_i^{N_b} \pi_i^b \int_{\{h|\mathfrak{h}(h,b_i,\kappa_k)\leq h_c\}} \pi_m^b \pi_{jk}^\kappa n(h, b_i, \kappa_j^p) F(dh, b_i, \kappa_j^p)}{2(1+g)} \tag{A2}$$

where $\mathfrak{h}(h, b_i, \kappa_k)$ is the human capital implied by the decision rules—$n(h, b_i, \kappa^p)$ and $x(h, b_i, \kappa_k, n(h, b_i, \kappa^p))$—and κ_k, and the population growth rate is given by

$$1 + g = \frac{\sum_j^{N_\kappa} \tilde{\pi}_j^\kappa \sum_i^{N_b} \pi_i^b \int_h n(h, b_i, \kappa_j^p) F(dh, b_i, \kappa_j^p)}{2}. \tag{A3}$$

Theoretically, $\bar{h}$ is also a key object but is immediately found as a by-product once we obtain the distribution. The key restriction of the equilibrium distribution is that it should be stable over time when implied by the policy functions given $\bar{h}$, which is implied by the distribution. The below algorithm uses an iterative method to find the policy tool that clears the government budget.

1. Make an initial guess for government lump-sum taxes (or transfers) T.
2. Make an initial guess for the distribution $F(h, b, \kappa_j^p)$ (which also gives $\bar{h}$).
3. Given $\bar{h}$ and T, compute $V(h, b, \kappa_k, n)$ and the (conditional) policy functions for consumption $c(h, b, \kappa_k, n)$, investment $x(h, b, \kappa_k, n)$ and leisure $l(h, b, \kappa_k, n)$.
4. Compute the expected value function $\sum \pi_{jk}^\kappa V(h, b, \kappa_k, n)$ and based on it, obtain the policy function for fertility $n(h, b, \kappa_j^p)$.
5. Obtain the time invariant distribution $F(h, b, \kappa^p)$, based on the policy functions for fertility $n(h, b, \kappa^p)$ and $x(h, b, \kappa, n)$. obtained above.
6. Iterate from 2 to 5 until $F(h, b, \kappa^p)$ converges.
7. Compute T by checking government budget based on the policy functions and the distribution obtained above.
8. Obtain Iterate from 1 to 7 until T converges.

The stationary equilibrium definition should be generalized slightly for equilibrium along the transitional path. There are two key changes. First, the state vector additionally includes an aggregate state: $\bar{h}$. Second, the last condition for the fixed-point stationary distribution is replaced by the consistency condition stating that in each period, the agents' perceived law of motion, $\bar{h}' = \Gamma(\bar{h})$, is consistent with the actual evolution of $\bar{h}$ implied by the current distribution $F(h, b_i, \kappa^p)$ and the equilibrium decision rules.

Along the transition path, the key equilibrium object is the distribution of human capital in each period over time (or mean human capital over time given the stationary distributions at the end periods). As in steady state, the key properties of these distributions are that they should be consistent with both individual agents' expectations and the actual evolution implied by the policy functions that take into account the expectation. Below is an algorithm to find the equilibrium transition that clears the government budget in each period as well, but note that there can be alternative ways of obtaining the same equilibrium.

The economy is initially in steady state. In period $t = 1$, the economy is hit by the policy change. Let $\tilde{t}$ denote the time period sufficiently long enough so that the economy converges to the new steady state with new policy.

1. Compute the original steady state and the new steady state following the algorithms above. Store the information of the original steady state as $t = 0$ and that of the new steady state as $t = \tilde{t}$.

2. Make initial guesses for a sequence of government taxes (or transfers if negative) for each period $\{T_t\}_{t=1}^{\tilde{t}-1}$.

3. Make initial guesses for the evolution of aggregate human capital $\{\bar{h}_t\}_{t=2}^{\tilde{t}-1}$.

4. For each period $t = 1, ..., \tilde{t} - 1$, given $\bar{h}_{t+1}$, T_t and policy variables specified, compute the (conditional) policy functions for consumption $c_t(h, b, \kappa_j, n)$, investment $x_t(h, b, \kappa_j, n)$ and leisure $l(h, b, \kappa_j, n)_t$.

5. Compute the expected value function $\sum \pi^{\kappa}_{jk} V_t(h, b, \kappa_k, n; \bar{h}_t)$ and based on it, obtain the policy function for fertility $n_t(h, b, \kappa^p_j)$ for all $t = 1, ..., \tilde{t} - 1$.

6. Obtain the distribution $F_{t+1}(h, b, \kappa^p)$ for $t = 1, ..., \tilde{t} - 2$, based on the policy functions for fertility $n_t(h, b, \kappa^p)$ and $x_t(h, b, \kappa_j, n)$ obtained above. Compute $\bar{h}_t$ based on $F_t(h, b, \kappa^p)$ for $t = 2, ..., \tilde{t} - 1$.

7. For $t = 1, ..., \tilde{t} - 1$, compute T_t by checking government budget based on the policy functions and the distribution obtained above.

8. Iterate from 1 to 7 until $\{T_t\}_{t=1}^{\tilde{t}-1}$ and $\{\bar{h}_t\}_{t=2}^{\tilde{t}-1}$ converge.

B.2 Policy Effects without Externality Feedback

One might ask what role the externality plays in the policy experiments presented in Section 6. In particular, does the externality amplify or mitigate government policy? To assess this, one could set $\chi = 0$ and recompute the policy experiments. However, note that a positive χ has not only equilibrium feedback effects but also a level effect, as is investigated in Section 5. To isolate the role of equilibrium feedback channel, we thus hold the functional form constant while fixing the value of $\bar{h}$ at its steady state value of the no-policy economy (i.e., treating it like a parameter). In other words, we allow no feedback effects and thereby essentially shut off the externality while keeping the functional form the same.

Table A5 reports the policy effects when we shut down the externality feedback channel. We can see that in the model without externality feedback, fertility tends to increase less (with respect to pro-natal transfers) or decrease more (with respect to education taxes). By contrast, negative effects of the two policies on education expenditure become mitigated in the absence of externality feedback. Overall, although magnitudes are quantitatively small, the above results indicate that externality feedback helps the policy tools to better achieve their policy goals (i.e., raising fertility while reducing education expenditures).

B.3 Ramsey Planning Problem

We consider a Ramsey-style optimal policy problem. Let us consider a social planner who faces the steady state equilibrium with $\tau_x(t) = \psi(t) = T(t) = 0$ in period

Table A5: Long-run Policy Effects without Externality Feedback

	Benchmark	$\psi = .02$		$\tau_x = .20$	
Externality Feedback?	Yes	Yes	No	Yes	No
Fertility rate n	1.887	2.010	2.008	1.884	1.877
		(6.5%)	(6.4%)	(-0.1%)	(-0.5%)
Childlessness rate	3.0%	2.0%	2.1%	3.1%	3.1%
Avg x per kid/income	9.08%	8.61%	8.63%	7.35%	7.40%
Avg labor supply	.299	.297	.297	.291	.292
Avg human capital	2.653	2.616	2.617	2.591	2.594

$t = ..., -2, -1, 0$. In period $t = 1$, given the distribution $F_{t=1}(h, b, \kappa^p)$, the planner is given the optimal policy instruments considered in Section 6: $\tau_x(t)$ and $\psi(t)$. The optimal policy problem is to maximize the weighted social welfare by introducing $\tau_x(t)$ and $\psi(t)$, while satisfying the period budget constraint through $T(t)$ for $t = 1, 2, ..., \infty$. Specifically, the planner solves

$$\max_{\tau_x(t),\psi(t)} \sum_{j}^{N_\kappa} \tilde{\pi}_j^\kappa \sum_{i}^{N_b} \pi_i^b \int_h \varphi(\cdot) \left\{ \mathbb{E}_{\kappa|\kappa^p} V_{t=1}(h, b, \kappa, n_t; \bar{h}_t) \right\} F_{t=1}(dh, b_i, \kappa_j^p)$$

subject to government budget constraints in period $t = 1, 2, ..., \infty$:

$$\sum_{j}^{N_\kappa} \tilde{\pi}_j^\kappa \sum_{i}^{N_b} \pi_i^b \int_h \left[\psi(t) n_t - \tau_x(t) x_t n_t - T(t)\right] F_t(dh, b_i, \kappa_j^p) = 0, \tag{A4}$$

where $\tilde{\pi}_j^\kappa$ captures the probability mass of κ_j^p and n_t, c_t, x_t and l_t are the policy functions that solve each family's optimization problem of (4)–(9) in each period t. We consider two possible welfare weights $\varphi(\cdot)$: (i) Negishi weights and (ii) equal weights. Next, note that we present two cases depending on policy tools allowed for the planner. The *permanent* policy reform restricts $\tau_x(t)$ and $\psi(t)$ to be $\tau_x^* \in [0, 1]$ and $\psi^* \in \mathbb{R}_{\geq 0}$, respectively, for all $t = 1, 2, ..., \infty$. On the other hand, the *temporary* policy reform allows $\tau_x(t = 1)$ and $\psi(t = 1)$ to be $\tau_x^* \in [0, 1]$ and $\psi^* \in \mathbb{R}_{\geq 0}$, respectively, and $\tau_x(t) = 0$ and $\psi(t) = 0$ for all $t = 2, ..., \infty$.

B.4 Welfare Weights

To construct Negishi weights, we estimate consumption of each household using state variables, such as h, κ^p, b, and κ. As Negishi weights are proportional to the inverse of the marginal utility of consumption ($b/\hat{c}$ in our model), Negishi weights are constructed as follows. First, using the simulated cross-sectional data in steady state, estimate coefficients, $\left\{\hat{\beta}_0, \hat{\beta}_1, ..., \hat{\beta}_4\right\}$ from

$$\log c = \beta_0 + \beta_1 \log h + \beta_2 \log \kappa^p + \beta_3 \log b + \beta_4 \log \kappa + \epsilon. \tag{A5}$$

Then, along the transition path, for an individual with a state vector (h, κ^p, b, κ), we use the estimated $\left\{\hat{\beta}_0, \hat{\beta}_1, ..., \hat{\beta}_4\right\}$ to predict $\hat{c}$, which gives $\varphi = \hat{c}/b$. Finally, we re-scale φ in each period such that they sum up to one.

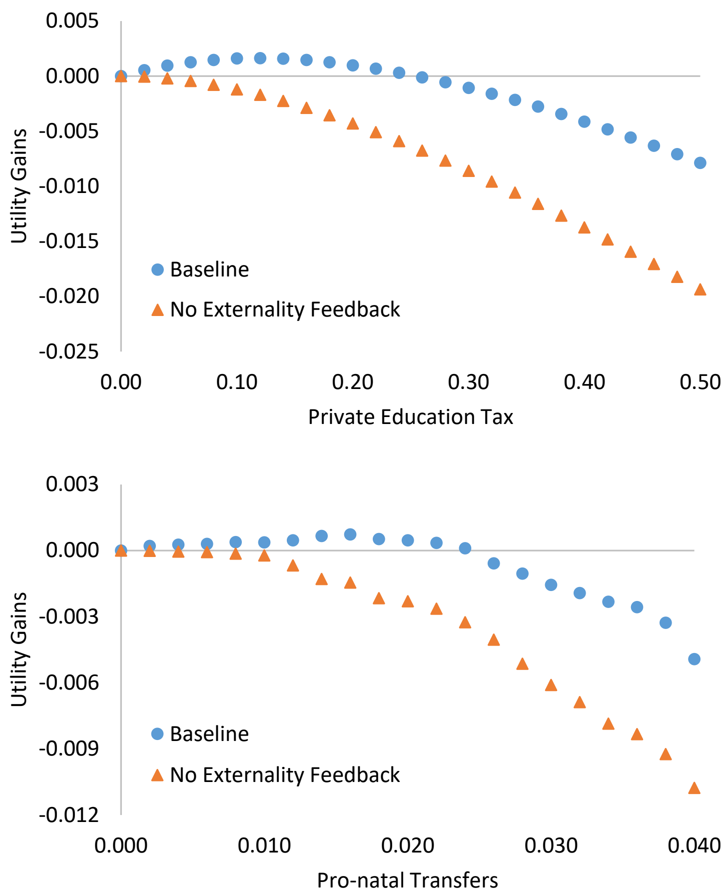
B.5 Marginal Effect of Each Policy

To better understand the importance of the two policy instruments, it is instructive to see the marginal effects of each policy instrument on the welfare of the first generation, which is portrayed in Figure A4. The change in the weighted average utilities of the first generation has a hump-shape for each policy individually in our baseline model with externality feedback. By contrast, it also shows that any positive education tax or positive pro-natal transfers would reduce the welfare of the first generation in an economy without such externality feedback.

B.6 Additional Optimal Policy Results

Note that policy reforms can also take place only temporarily on the first generation to focus on addressing distortions for the first generation that has the fixed pool of agents since their parents already made fertility decisions. The results are shown in Figure A5. The temporary policy reform has identical effects on the first generation. After the policy change is revoked, fertility, hours worked, and education spending go back to the initial level quite quickly while human capital and output move more slowly over time.

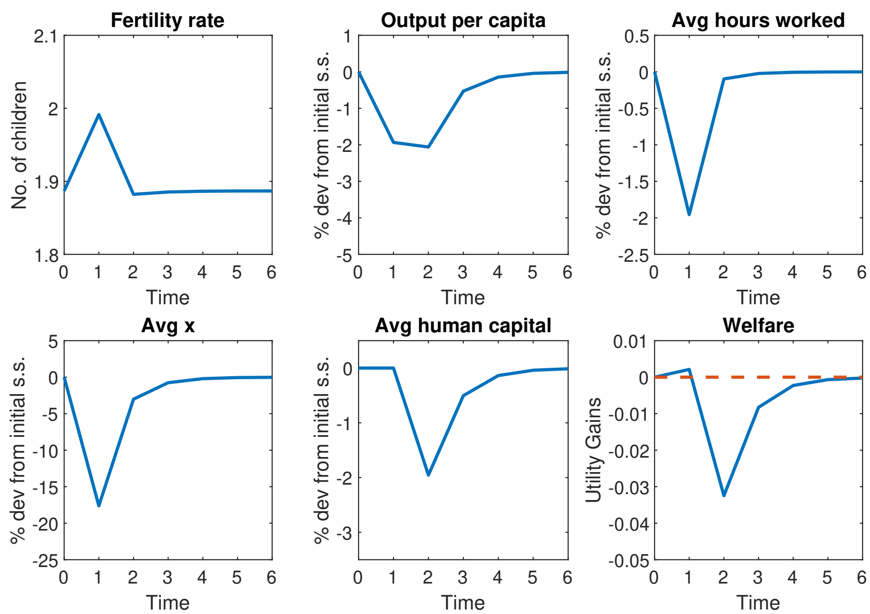
Figure A4: The Marginal Effects of Each Policy on the First Generation.



In the optimal policy exercise in the main text, we have mainly used unequal welfare weights that are designed to focus on the distortions generated by the status externality. In this subsection, we present optimal policy results when we use equal welfare weights, which are widely used in the quantitative macroeconomics literature. This exercise illustrates that these equal weights put substantial motives for redistribution, as compared to our baseline welfare weights.

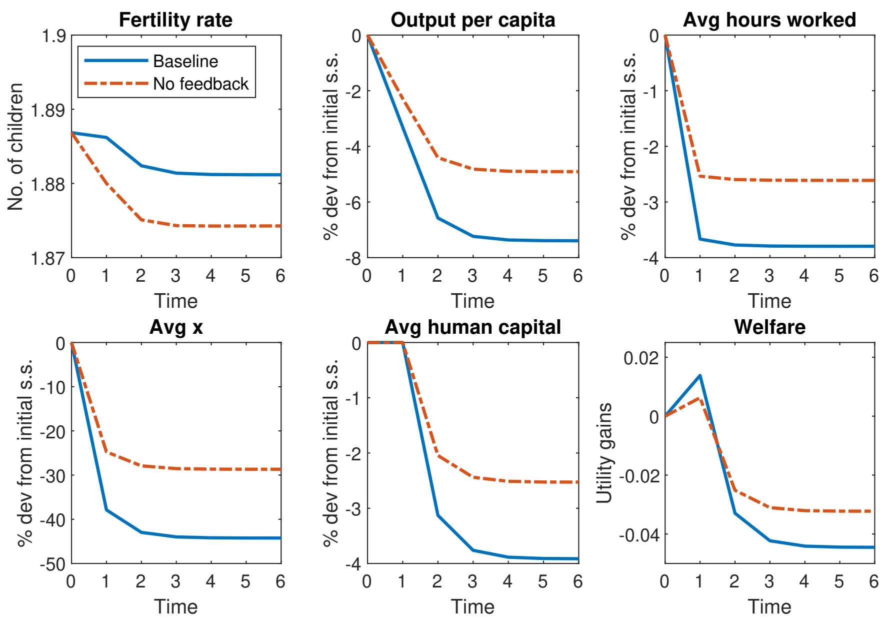
When we use the equal welfare weights, the optimal policy mix sets higher education tax rates while not using pro-natal transfers. Specifically, without externality feedback, optimal $\tau_x = 0.232$, which is smaller than the optimal tax $\tau_x = 0.364$ when we allow for the externality feedback. When the externality feedback is shut-

Figure A5: Optimal Policy: Unexpected and Temporary Policy Reform



Notes: The optimal policy reform is introduced unexpectedly and temporarily in period 1. Welfare is measured by the change in Negishi-weighted average utility. A model period corresponds to 25 years.

Figure A6: Optimal Policy: Unexpected and Permanent Policy Reform with Equal Welfare Weights

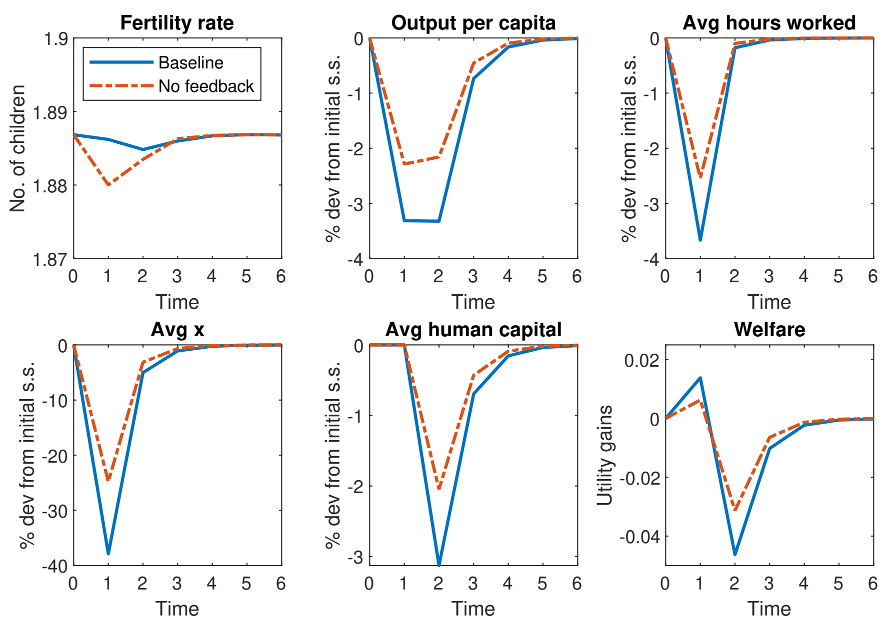


Notes: The optimal policy reform is introduced unexpectedly and permanently in period 1. Welfare is measured by the change in equally-weighted average utility. ”No feedback” shuts down externality feedback. A model period corresponds to 25 years.

down, the optimal policy leads to the welfare gain for the first generation that is smaller than its counterpart in the benchmark model with externality. Figure A6 shows the transition dynamics with respect to these two optimal policies for their corresponding economies (with/without externality feedback). Figure A7 shows the transition dynamics with respect to their counterparts when the policy change is temporary.

Finally, the top panel of Figure A8 plots the effects of education tax (marginal effects) on the first generation's welfare when equal welfare weights are used. Its bottom panel shows the counterparts of pro-natal transfers. It clearly shows that any positive pro-natal transfers would lead to the negative welfare impacts on the first generation, which is in line with the optimal policy that only uses education taxes.

Figure A7: Optimal Policy: Unexpected and Temporary Policy Reform with Equal Welfare Weights



Notes: The optimal policy reform is introduced unexpectedly and temporarily in period 1. Welfare is measured by the change in equally-weighted average utility. "No feedback" shuts down externality feedback. A model period corresponds to 25 years.

Figure A8: The Marginal Effects of Each Policy on the First Generation with Equal Welfare Weights

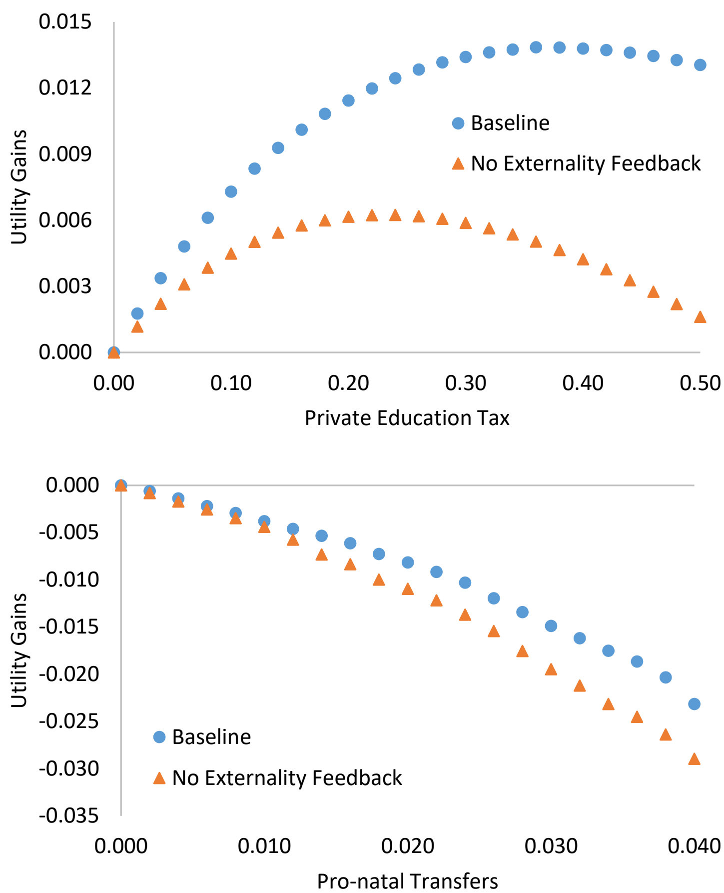


Table A6: Heterogeneous Effects of Optimal Policy on the First Generation with Equal Welfare Weights

		Income quintile					
Average		1st	2nd	3rd	4th	5th	All
Fertility, n	Baseline	1.74	1.89	1.94	1.91	1.95	1.89
	Optimal	1.78	1.89	1.93	1.91	1.92	1.89
	% change	+2.6	+0.1	-0.7	-0.3	-1.6	-0.0
Childlessness rate (%)	Baseline	5.3	3.0	2.3	2.3	2.0	3.0
	Optimal	5.1	3.0	2.5	2.7	2.1	3.1
	p.p. change	-0.2	+0.0	+0.2	+0.4	+0.1	+0.1
Investment per child, x	Baseline	0.046	0.056	0.067	0.081	0.111	0.070
	Optimal	0.031	0.039	0.046	0.055	0.076	0.048
	% change	-31.2	-31.1	-31.3	-31.6	-31.7	-31.5