Aggregate and Intergenerational Implications of School Closures: A Quantitative Assessment*

Youngsoo Jang Minchul Yum

September 2022

Abstract

This paper quantitatively investigates the medium- and long-term macroeconomic and distributional consequences of school closures through intergenerational channels. The model economy is a dynastic overlapping generations general equilibrium model in which schools, in the form of public education investments, complement parental investments in producing children's human capital. We find that unexpected school closure shocks have long-lasting adverse effects on macroeconomic aggregates and reduce intergenerational mobility, especially among older children. Higher substitutability between public and private investments induces smaller damages in the aggregate economy and the affected children's lifetime income, while exacerbating negative impacts on intergenerational mobility and inequality.

Keywords: Intergenerational Mobility, Lifetime Income, Parental Investments, Aggregate Loss, Substitutability, Covid-19

JEL codes: E24, I24, J22

*Jang: University of Queensland; e-mail: youngsoo.jang@uq.edu.au. Yum: University of Mannheim and CEPR; E-mail: minchul.yum@uni-mannheim.de. We thank the co-editor, Ayşegül Şahin, and two anonymous referees for their useful comments and suggestions. We are also grateful to Klaus Adam, Matthias Doepke, Hans-Martin von Gaudecker, Ingo Isphording, Matthias Meier, Krisztina Molnar, Kjetil Storesletten, Michèle Tertilt and participants at various conferences and seminars for helpful feedback. Financial supports from the German Science Foundation (DFG) through the CRC TR 224 Project A3 and Shanghai Municipal Education Commission through Gaofeng Chuangxin Project Grant are gratefully acknowledged.

1 Introduction

In early 2020, a majority of governments around the world unprecedentedly decided to close daycares, pre-schools, and primary and secondary schools nationwide in response to the COVID-19 pandemic. Interestingly, the extent to which governments engage in or maintain school closures varies significantly over time across countries.[1] The key to such decisions is understanding the benefits and costs of school closures during the pandemic. In this regard, there has been relatively active research on the short-run consequences of school closures, such as their implications for parents' economic activities (e.g., Alon, Doepke, Olmstead-Rumsey, and Tertilt 2020) and the epidemiological risk associated with the reopening of schools (e.g., Isphording, Lipfert, and Pestel 2021). However, there have been few studies that quantify and enhance the understanding of various factors behind the longer-term consequences of school closures. This line of research is important for policymakers who assess the relative costs and benefits of school closures, not only today but also as related to potential pandemics in the future.

In this paper, we quantitatively investigate the medium- and long-term aggregate and distributional consequences of school closures through intergenerational channels.[2] Specifically, we use a dynastic overlapping generations general equilibrium model where parents are linked to children through multiple transmission channels to study how school closures affect aggregate dynamics, inequality, and intergenerational mobility over time and across cohorts. The model economy combines a standard heterogeneous-agent incomplete-markets framework with production (Aiyagari 1994) with the model of altruistic dynasties in the tradition of Becker and Tomes (1986), while endogenizing several additional key ingredients relevant to our research questions. These include multi-stage human capital production technology for children (Cunha and Heckman 2007), where inputs include not only parental financial and time investments but also schools in the form of public investments that complement parental investments. Children become young adults with human capital and assets shaped by their parents and make their own college decisions that affect their future life-cycle wage profiles. Aggregate production combines skilled and unskilled workers along with capital to produce final outputs.

We calibrate the stationary equilibrium of the model to the U.S. economy in normal times. The stationary equilibrium of our model is consistent with various empirical features such as the increasing importance of parental financial investments over children's age, the income quintile transition matrix, and the rising income inequality over the life cycle, all of which are important for the main analysis of school closures effects. For the main quantitative analysis, we model the school closure shock as an unexpected temporary decline in the productivity of public investments

[1] The United Nations Educational, Scientific and Cultural Organization (UNESCO) provides a daily map showing the global status on school closures caused by COVID-19 at https://en.unesco.org/covid19/educationresponse.

[2] We focus on the consequences of school closures on the affected children through the intergenerational human capital production function. Our baseline analysis does not consider general pandemic effects on parents, which could in turn affect children indirectly. We illustrate how such COVID-19 induced income shocks on parents could change our baseline results in an extended model in Section 4.2.4.

in the child human capital production. We then investigate the economy over the full transition equilibrium paths.[3] In particular, our rich framework naturally enables us to answer how the effects of school closures differ across child cohorts of different ages at the time of the school closure and what role the substitutability between private and public investments plays in determining the consequences of school closures.

Our first finding on aggregate consequences is that school closures have long-lasting adverse effects on the aggregate economy. For instance, the year-long closure (including vacations) would lead to up to 0.8% decline in outputs over a number of decades to follow. When we sum up these persistent output losses over the next century, the accumulated output loss relative to annual output is around 42%.[4] In the short term, as parents' incentive to substitute for the reduced public inputs increases, aggregate capital accumulation is negatively affected, which in turn affects aggregate output negatively. More importantly, as the children directly affected by the school closure shocks enter the labor market, the decreased human capital accumulated during their childhood contributes negatively to outputs persistently in the following decades. On the other hand, we find that the adverse effects of school closures on college attainment and cross-sectional inequality exist to a lesser extent. We show that general equilibrium plays a very important quantitative role in mitigating the above aggregate effects. Specifically, when we fix the prices at the stationary equilibrium level, we find that college-educated labor inputs fall by more than twice as much, and output effects can be overstated substantially.

We then investigate the implications of school closures for intergenerational mobility. Unlike the modest effects on inequality, we find that the school closure shocks considerably strengthen the extent to which income distribution is associated between parents and children. Specifically, a 1-year school closure would lower the probability of children born into the bottom income quintile moving up to the top quintile by up to 8%. We also find significant losses (around 2% on average) in average lifetime income for the affected cohorts. In particular, these adverse effects are found to be generally larger among older children. This is due to the temporary nature of the school closure shock. We show that although younger school-aged children are more negatively affected on impact than older ones, the equalizing effect of public education (Fernandez and Rogerson 1998) enables those young children (especially more disadvantaged children) to recover over time in the following periods without school closures.[5] We further show that both the direct impact of the school closures on the child human capital production function as well as the endogenous parental responses, featuring positive income gradients especially in financial investments for older children, underlie the above findings.[6]

[3] Before conducting the main experiments, we confirm that our model-generated data following a short-term school closure shock are in line with the causal evidence of school closures on test scores in the Netherlands (Engzell, Frey, and Verhagen 2021) as well as time-use evidence in Germany (Grewenig, Lergetporer, Werner, Woessmann, and Zierow 2021).

[4] The half-year-long closure would lead to an accumulated aggregate output loss of 19% over the next 100 years.

[5] In fact, this is consistent with the empirical evidence by Kuhfeld et al. (2020) showing that students who lose more ground during summer break experience steeper growth during the following school year.

[6] We also explore how these school-closure effects would change in the presence of virtual schooling that dispropor-

Finally, we also systematically analyze the role of substitutability between public and parental investments in producing children's human capital. Motivated by the possibility that this elasticity of substitution could vary across countries, we consider an alternative model economy with a higher elasticity (6 versus 3 in the baseline economy).[7] We find that although the alternative economy is able to match the important target statistics equally well, it results in school closure effects that differ substantially as compared to the baseline economy. Specifically, it generates substantially smaller declines in aggregate output and the affected children' lifetime income, whereas it reduces intergenerational mobility more considerably. As public investments are easier to substitute, children experience smaller losses in human capital during childhood, which is mitigated by the stronger parental motive to compensate for the fall in human capital. This greater incentive to respond also implies a larger parental background role, thereby generating much stronger impacts on intergenerational mobility and inequality.

Following a seminal study by Restuccia and Urrutia (2004), the literature increasingly investigates intergenerational economic persistence in quantitative macroeconomic models with heterogeneous households where the distribution of income across generations is endogenously determined. The steady-state version of our general equilibrium model herein builds on the model in Yum (2021), which allows flexible substitutability between private and public investments—a departure from most existing papers in the literature that assume that public and parental investments are perfectly substitutable.[8] Unlike most existing studies that focus on steady-state comparisons, our quantitative exercise provides one of the few numerical implementations of the equilibrium paths over the perfect foresight transition in general equilibrium models with endogenous intergenerational human capital transmission (Daruich 2022).

A recent paper by Fuchs-Schündeln, Krueger, Ludwig, and Popova (2022) also studies the implications of school closures in a rich two-generations lifecycle model. Although both studies share similar emphasis on the importance of parental income and children's age, the focus is quite different. Specifically, while they focus on implications of school closures for affected children's welfare and inequality, we focus on the implications for macroeconomic aggregates and intergenerational mobility and on the role of substitutability between public and parental investments. Moreover, unlike theirs, our key interest of aggregate implications requires an overlapping-generations general equilibrium framework as a natural laboratory.[9] Agostinelli, Doepke, Sorrenti, and Zilibotti (2022) also provides another structural analysis on the implications of school closures. Although they

tionately benefits children from college-educated parents, capturing better-educated parents' advantages with better skills and network. We find that these would mitigate average lifetime income losses of the affected children at the expense of lower intergenerational mobility. In Section 4.2.4, we also provide how our baseline estimates might differ in the presence of recessionary effects of COVID-19 and a negative productivity shock in private monetary investments.

[7]For example, East Asian countries generally have large private education markets, which are believed to be very good substitutes for public education.

[8]For example, see Restuccia and Urrutia (2004), Herrington (2015), Holter (2015), Lee and Seshadri (2019), and Daruich (2022), among others.

[9]In Section 4.2.2, we indeed confirm that general equilibrium effects are quantitatively important for our research question.

focus solely on high school students' educational outcomes, their model highlights various channels through which school closures could affect skill formations, such as peer effects and parenting styles.

The empirical education and economics literature has shown that school interruptions can have negative consequences for children's learning and skills (e.g., Cooper, Nye, Charlton, Lindsay, and Greathouse 1996, Meyers and Thomasson 2017). A number of papers explore learning losses in terms of test scores during summer breaks, but the evidence is somewhat mixed in terms of magnitudes (see Atteberry and McEachin (2020) and references therein). Other papers exploit teacher strikes, weather-related school closures, and the German short school years in the 1960s, as summarized in Hanushek and Woessmann (2020) and Kuhfeld et al. (2020). There is a growing body of empirical literature that estimates how the COVID-19 pandemic has affected parental responses using real-time data (e.g., Adams-Prassl, Boneva, Golin, and Rauh 2020, Chetty, Friedman, Hendren, and Stepner 2020, Bacher-Hicks, Goodman, and Mulhern 2021). For example, Chetty et al. (2020) find that during the school closures, children, especially those who live in low-income areas, experienced reductions in math learning, measured by online Zearn Math participation. There are also empirical studies, such as Engzell et al. (2021) and Grewenig et al. (2021), which estimate these effects on learning losses and parental responses in European countries, which we discuss more extensively in Section 4.2.1. These empirical findings are broadly in line with the key mechanisms in our quantitative theory; that is, that school closures induce human capital losses, especially among children from low-income families, and that parents try to compensate for these losses. Our quantitative theoretical results could help better understand the underlying sources of these empirical observations.

To the best of our knowledge, our paper is the first to conduct analysis on aggregate effects of school closures in a dynamic general equilibrium model with endogenous parental decisions. Hanushek and Woessmann (2020) document the empirical literature on learning losses and suggest that such short-term evidence could potentially point to the sizeable long-term consequences of school closures.[10] Building on their insight, we bring various relevant factors, such as endogenous parental investment responses, dynamic effects on human capital, and general equilibrium considerations, into a structural framework. Our consequential estimates of the negative effects on the aggregate economy, based on the model that is broadly in line with the existing short-run empirical evidence, are somewhat conservative but are still highly relevant given that these output declines last for many decades to follow.

This paper is organized as follows. Section 2 presents the model economy and defines the equilibrium. Section 3 describes the calibration strategy and the properties of the stationary equilibrium of the calibrated model economy. Section 4 presents the main quantitative analysis of school closures along the full equilibrium transitional paths. Section 5 concludes the paper.

[10] In a recent study, Samaniego, Jedwab, Romer, and Islam (2022) consider a development accounting framework to quantify the macroeconomic effects of school disruptions for a wide set of countries, with a focus on gender differences.

2 Model Economy

We begin by describing the model economy used for the quantitative analysis. It is based on the model in Yum (2021), which builds on a standard incomplete-markets general equilibrium framework in a production economy (Aiyagari 1994) while following the tradition of Becker and Tomes (1986) for intergenerational transmissions. Parents face the identical multi-period human capital production technology but are heterogeneous in assets and productivity. To enrich the analysis of school closures, our model allows the elasticity of substitution between private and public investments to be less than perfect. In our equilibrium model with altruistic parents, parental choices such as parental investments and inter-vivos transfers take into account parents' expectations of the future paths of the economy following unexpected school closures today.

Time (t) is discrete, and a model period corresponds to five years. Our analysis not only considers steady states but also transitional dynamics across steady states. We now describe the model environments in more details.

2.1 Households

There is a continuum (measure one) of overlapping generations in the economy. A household always includes an adult, but it can also include a child. As summarized in Table 1, an adult lives for twelve model periods (age 20-79) as an active decision maker. Specifically, in the first model age $j = 1$, an agent chooses whether or not to obtain a college education. Once this higher education choice is made, the adult agent supplies labor from $j = 1$ until retirement at the beginning of $j = 10$ (age 65). The agent then lives for three more periods as a retiree and dies at the end of period $j = 12$ (age 79). In all periods, the agent makes a standard consumption-savings decision.

An important building block of our model is the intergenerational transmission. This initially happens at the beginning of $j = 3$ (age 30) when the adult is endowed with a child. In addition to the stochastic ability draw for the child, the parent invests time and money in their children in multiple periods $j = 3, 4, 5$ while taking into account the presence of public education. Before the child becomes independent, the parent decides the amount of inter-vivos transfers to give in $j = 6$. Then, the child, now an adult, forms a new household when the parent enters $j = 7$, and faces the same lifetime structure, described above.

All households share identical preferences over consumption c and hours worked n, represented by a standard separable utility function:

$$\frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi}, \tag{1}$$

where $\sigma > 0$ and $\chi > 0$ capture the curvatures and $b > 0$ is the disutility constant.

In all working-age periods ($j = 1, 2, ..., 9$), labor earnings y are subject to progressive taxation.

Table 1: Timeline of life-cycle events for a parent-child pair

	Parent												
Age	*20-24*	*25-29*	*30-34*	*35-39*	*40-44*	*45-49*	*50-54*	*55-59*	*60-64*	*65-69*	*70-74*	*75-79*	
$j =$	1	2	3	4	5	6	7	8	9	10	11	12	
	← – – Consumption-savings – – →												
	← – – Labor supply – – →									← Retirement →			
	College		← – Parental – → investments			Inter-vivos							
	Child												
Age			*0-4*	*5-9*	*10-14*	*15-19*	*20-24*	*25-29*	*30-34*	*35-39*	*40-44*	*45-49*	...
$j =$			← – – Childhood – – →				1	2	3	4	5	6	...
							← – – Consumption-savings – –						...
							← – – Labor supply – –						...
							College		← – Parental – → investments			Inter-vivos	

Specifically, after-tax earnings with respect to pre-tax earnings y are given by:

$$\lambda_j \left(y/\bar{y}\right)^{-\tau_j} y, \tag{2}$$

following a simple, yet widely used, parametric form (Benabou 2002; Heathcote, Storesletten and Violante 2014). Note that τ_j shapes the degree of progressivity, λ_j captures the scale of taxation and $\bar{y}$ denotes average earnings. We allow τ_j and λ_j to depend on age in order to capture differences in labor taxation across family structures (Guner, Kaygusuz and Ventura 2014; Holter, Krueger and Stepanchuk 2019).

In all periods, capital income is subject to a tax rate of τ_k if the capital income is positive. Households receive lump-sum transfers T and are allowed to borrow up to the borrowing limit $\underline{a} \leq 0$ (Aiyagari 1994).

We now present the household's decision problems sequentially starting with the first adult age $j = 1$. For notational simplicity, we will omit an individual index to represent cross-sectional heterogeneity.

Model Age 1 In period t, a child who forms a new household in the model age $j = 1$ (20 years old) begins their adult life with individual state variables such as age j, a human capital stock of h_t, a level of asset holdings a_t, the childhood learning ability ϕ, and the aggregate state variable of the distribution of households in the economy $\boldsymbol{\pi}_t$. The two individual state variables, h_t and a_t, are endogenously shaped by the parent of the agent during childhood. Although childhood ability does not enter adults' economic decisions directly, it is still a state variable because it determines the learning ability of their own child later in $j = 3$. The distribution of households in period t,

$\boldsymbol{\pi}_t$, is an aggregate state variable because equilibrium prices depend on it.

Given the state variables, the agent first decides whether or not to obtain a college education. The value of not completing college ($\kappa = 1$) is given by:

$$N(h_t, a_t, \phi; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_2(h_{t+1}, a_{t+1}, \kappa, \phi, \boldsymbol{\pi}_{t+1}) \right\} \tag{3}$$

subject to

$$\begin{aligned} c_t + a_{t+1} &\leq \lambda_1 \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_1} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\ &\quad + \left(1 + r_t(\boldsymbol{\pi}_t)\right) a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\ h_{t+1} &= \exp(z_{t+1}) \gamma_{1,\kappa} h_t \\ \kappa &= 1 \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t), \end{aligned}$$

where $w_{\kappa,t}(\boldsymbol{\pi}_t)$ is the rental price of human capital for skill type κ per unit hours of work, $r_t(\boldsymbol{\pi}_t)$ is the real interest rate, and a_t is the initial assets given by the parents (i.e., inter-vivos transfers). Human capital increases at the gross growth rate of $\gamma_{j,\kappa}$, which is allowed to depend on age j and education κ to capture the empirical age-profile of wage for each education type. Human capital is subject to the idiosyncratic shock z, which follows an independent and identically distributed (i.i.d.) normal distribution with mean zero and the standard deviation of σ_z. We assume a standard incomplete-markets structure by assuming that the idiosyncratic shock z is not fully insurable as a is not a state-contingent asset. $\Gamma(\boldsymbol{\pi}_t)$ captures the law of motion for the distribution of households as perceived by households, which should be consistent with the actual evolution of the distribution in equilibrium. Because h_{t+1} is uncertain in period t, households form expectation regarding the next period's value.

An alternative choice is to complete college and become a skilled worker. College education is costly and requires the agent to pay a stochastic fixed cost ξ, which follows an i.i.d. log normal distribution with a mean of μ_ξ and a standard deviation of σ_ξ. The value of completing college after the realization of ξ is given by:

$$C(h_t, a_t, \phi, \xi; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_2(h_{t+1}, a_{t+1}, \kappa, \phi; \boldsymbol{\pi}_{t+1}) \right\}$$

subject to

$$
\begin{aligned}
c_t + a_{t+1} + \xi \leq \lambda_1 \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_1} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\
+ \left(1 + r_t(\boldsymbol{\pi}_t)\right) a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\
h_{t+1} = \exp(z_{t+1}) \gamma_{1,\kappa} h_t \\
\kappa = 2 \\
\boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t).
\end{aligned}
\tag{4}
$$

The above conditional decision problem illustrates how college education could affect households in the model. First, college educated workers are in the skilled labor market ($\kappa = 2$), which gives $w_{\kappa,t}(\boldsymbol{\pi}_t)$. Second, college-educated workers experience a life cycle profile of wages that differs from that of their counterparts without a college degree through $\gamma_{j,\kappa}$.

Given the above two conditional value functions, households make a discrete college choice after observing a draw of ξ. The expected value at the beginning of $j = 1$ is:

$$
V_1(h_t, a_t, \phi; \boldsymbol{\pi}_t) = \mathbb{E}_\xi \max\left\{N(h_t, a_t, \phi; \boldsymbol{\pi}_t), C(h_t, a_t, \phi, \xi; \boldsymbol{\pi}_t)\right\}. \tag{5}
$$

Model Age 2 In $j = 2$, households face a standard life cycle problem with consumption-savings and labor supply decisions, represented by the following:

$$
V_2(h_t, a_t, \kappa, \phi; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}, \phi' | \phi} V_3(h_{t+1}, a_{t+1}, \kappa, \phi'; \boldsymbol{\pi}_{t+1}) \right\}
$$

subject to

$$
\begin{aligned}
c_t + a_{t+1} \leq \lambda_2 \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_2} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\
+ \left(1 + r_t(\boldsymbol{\pi}_t)\right) a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\
h_{t+1} = \exp(z_{t+1}) \gamma_{2,\kappa} h_t \\
\boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t).
\end{aligned}
$$

The higher education decision made in $j = 1$ shows up as a state variable κ. Because a child is going to be born in the next period, households take expectation over the ability of the new child to be born (ϕ'). We assume that it is correlated across generations, following an AR(1) process in logs

$$
\log \phi' = \rho_\phi \log \phi + \epsilon_\phi \tag{6}
$$

where $\epsilon_\phi \sim \mathcal{N}(0, \sigma_\phi^2)$. This form of the exogenous source of a positive correlation of human capital across generations is standard in the literature (e.g., Restuccia and Urrutia 2004; Lee and Seshadri 2019; Yum 2021), capturing any intergenerational persistence, such as genetic transmission, not endogenously explained by the model.

Model Ages 3–5 At the beginning of $j = 3$, a child is born with learning ability ϕ. Building on the childhood skill formation literature (Cunha and Heckman 2007, Caucutt and Lochner 2020), human capital formation is modeled as a multi-stage process that takes place in $j = 3, 4, 5$, featuring parental inputs in different periods that are complementary and parental investments that are more effective for those who have higher current human capital stock. In addition, we also introduce public investments in different stages, which are complementary inputs to parental investments, to capture the effects of schools (Fuchs-Schündeln et al. 2022).

The structure is similar to those in Lee and Seshadri (2019) and Yum (2021). Specifically, let I_j denote the total investment inputs in period j, aggregated following the two nested constant elasticity of substitution (CES) technology:

$$I_j = \left\{ \theta_j^p \left(\theta_j^x \left(\varsigma^x \frac{x_j}{\bar{x}} \right)^{\zeta_j} + (1 - \theta_j^x) \left(\varsigma^e \frac{e_j}{\bar{e}} \right)^{\zeta_j} \right)^{\frac{\psi}{\zeta_j}} + \left(1 - \theta_j^p \right) \left(\varsigma^g \frac{g_j}{\bar{g}} \right)^{\psi} \right\}^{\frac{1}{\psi}}, \tag{7}$$

where x_j denotes parental time investments, e_j is parental monetary investments, g_j denotes public education investment, $\{\varsigma^x, \varsigma^e, \varsigma^g\}$ capture the productivity of each corresponding input, $\theta_j^p \in (0, 1)$ denotes the share of private education inputs relative to the public inputs, and $\theta_j^x \in (0, 1)$ captures the relative share of time investments in period j.[11] Each input is entered after being normalized by its baseline unconditional mean.[12] The first CES aggregation is about parental time and money inputs. The elasticity of substitution between parental time and money investments depends on the stage j and is given by $1/(1 - \zeta_j)$, where $\zeta_j \leq 1$. The second CES aggregation is about the aggregated parental inputs and public investments. There, we allow the elasticity of substitution to be less than perfect, which is given by $1/(1 - \psi)$, where $\psi \leq 1$. Although this departure from perfect substitutability is relatively unexplored, we are going to show that this elasticity is highly relevant to the implications of school closures in various dimensions, as analyzed systematically in Section 5.

The aggregated inputs in different periods $j = 3, 4, 5$ shape the child's human capital at the end of $j = 5$. In other words, $h_{c,6}$, is given by the technology f :

$$h_{c,6} = \phi f(I_3, I_4, I_5). \tag{8}$$

[11] As discussed in Jones and Manuelli (1999), there can be another way of aggregating these three inputs. We think that our specification is reasonable given that schools involve the subsitution of not only parental monetary investments but also parental time. This is also in line with Lee and Seshadri (2019) where government investment is modeled as a mixture of time and goods investments. In Appendix B, we illustrate that parental time responses with respect to school closures could be qualitatively different depending on the aggregation order using a simple model.

[12] A change in parameters related to the elasticity of substitution has scale effects, and this normalization is useful for achieving computational stability in the presence of such scale effects. We provide more detailed discussions and simulations to illustrate this point in Appendix C.

As is standard in the literature, we assume unit elasticity of substitution across periods and constant returns to scale (e.g., Lee and Seshadri 2019, Fuchs-Schündeln et al. 2022, Yum 2021). This is captured by the following recursive formulation:

$$\begin{aligned} h_{c,j+1} &= \phi I_j^{\theta_j^I} h_{c,j}^{1-\theta_j^I}, \quad \text{if } j = 5; \\ &= I_j^{\theta_j^I} h_{c,j}^{1-\theta_j^I}, \quad \text{if } j = 3, 4, \end{aligned} \tag{9}$$

where $\theta_j^I \in (0,1)$. Note that this technology features two properties highlighted by Cunha and Heckman (2007) and Caucutt and Lochner (2020): (i) dynamic complementarity, meaning that a higher $h_{c,j}$ increases the productivity of investments in period j ($\frac{\partial^2 f}{\partial I_i \partial h_{c,j}} > 0$) and (ii) self-productivity, meaning that a higher $h_{c,j}$ increases human capital in the next period $h_{c,j+1}$. The initial human capital h_c in $j = 3$ when a child is just born is set to one as we allow for heterogeneity in learning ability ϕ (Lee and Seshadri 2019).

We now incorporate the above technology into the decision problem of parents. The following functional equation summarizes a parent's problem in $j = 3$:

$$V_3(h_t, a_t, \kappa, \phi; \boldsymbol{\pi}_t) = \max_{\substack{c_t, e_t \geq 0;\ a_{t+1} \geq \underline{a} \\ x_t, n_t \in [0,1]}} \left\{ \frac{(c_t/q)^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} - \varphi x_t + \beta \mathbb{E}_{z_{t+1}} V_4(h_{t+1}, a_{t+1}, \kappa, h_{c,t+1}, \phi; \boldsymbol{\pi}_{t+1}) \right\}$$

subject to

$$\begin{aligned} c_t + a_{t+1} + e_t &\leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\ &\quad + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\ x_t + n_t &\leq 1 \\ h_{t+1} &= \exp(z_{t+1}) \gamma_{3,\kappa} h_t \\ h_{c,t+1} &= \left\{ \theta_3^p \left(\theta_3^x \left(\frac{\varsigma_t^x x_t}{\bar{x}} \right)^{\zeta_3} + (1-\theta_3^x) \left(\frac{\varsigma_t^e e_t}{\bar{e}} \right)^{\zeta_3} \right)^{\frac{\psi}{\zeta_3}} + (1-\theta_3^p) \left(\frac{\varsigma_t^g g_3}{\bar{g}} \right)^{\psi} \right\}^{\frac{\theta_3^I}{\psi}} h_{c,t}^{1-\theta_3^I} \qquad (10) \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t). \end{aligned}$$

We assume that the child shares the household consumption c, captured by the household equivalence scale q. (10) is obtained by combining (7) and (9). Parents decide how much time and money to invest, while taking into account the returns to such investments, according to the production technology (8), the associated costs in terms of utility φ, and the reduced income available for consumption and savings.

The parent's decision problems in $j = 4, 5$ are similarly given by:

$$V_j(h_t, a_t, \kappa, h_{c,t}, \phi; \boldsymbol{\pi}_t) = \max_{\substack{c_t, e_t \geq 0;\ a_{t+1} \geq \underline{a} \\ x_t, n_t \in [0,1]}} \left\{ \frac{(c_t/q)^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} - \varphi x_t + \beta \mathbb{E}_{z_{t+1}} V_{j+1}(h_{t+1}, a_{t+1}, \kappa, h_{c,t+1}, \phi; \boldsymbol{\pi}_{t+1}) \right\}$$

subject to

$$\begin{aligned}
c_t + a_{t+1} + e_t &\leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\
&\quad + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\
x_t + n_t &\leq 1 \\
h_{t+1} &= \exp(z_{t+1}) \gamma_{j,\kappa} h_t \\
h_{c,t+1} &= \left\{ \theta_4^p \left(\theta_4^x \left(\frac{\varsigma_t^x x_t}{\bar{x}} \right)^{\zeta_4} + (1-\theta_4^x) \left(\frac{\varsigma_t^e e_t}{\bar{e}} \right)^{\zeta_4} \right)^{\frac{\psi}{\zeta_4}} + (1-\theta_4^p) \left(\frac{\varsigma_t^g g_4}{\bar{g}} \right)^{\psi} \right\}^{\frac{\theta_4^I}{\psi}} h_{c,t}^{1-\theta_4^I} \quad \text{if } j = 4 \qquad (11) \\
&= \phi \left\{ \theta_5^p \left(\theta_5^x \left(\frac{\varsigma_t^x x_t}{\bar{x}} \right)^{\zeta_5} + (1-\theta_5^x) \left(\frac{\varsigma_t^e e_t}{\bar{e}} \right)^{\zeta_5} \right)^{\frac{\psi}{\zeta_5}} + (1-\theta_5^p) \left(\frac{\varsigma_t^g g_5}{\bar{g}} \right)^{\psi} \right\}^{\frac{\theta_5^I}{\psi}} h_{c,t}^{1-\theta_5^I} \quad \text{if } j = 5 \qquad (12) \\
\boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t).
\end{aligned}$$

where state variables further include the child's human capital level at the beginning of the period h_c. Note that (11) and (12) are obtained by combining (7) and (9).

Model Age 6 At the end of $j = 6$, the child leaves the original household and forms a new household. The asset level of the newly formed household is shaped by the parents' decision on inter-vivos transfers a_c. Holding other things constant, this would facilitate the child's college decision indirectly by alleviating the financial burden of college and increase capital income flows over the child's lifecycle. At the beginning of $j = 6$, parents solve

$$V_6(h_t, a_t, \kappa, h_{c,t}, \phi; \boldsymbol{\pi}_t) = \max_{a_c \in [0, a_t]} \left\{ \tilde{V}_6(h_t, a_t - a_c, \kappa; \boldsymbol{\pi}_t) + \eta \beta V_1(h_c', (1 + r_t(1-\tau_k)) a_c, \phi; \boldsymbol{\pi}_{t+1}) \right\} \qquad (13)$$

$$h_c' = \gamma_c h_{c,t}$$
$$\boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t)$$

where they take into account the implications of their inter-vivos transfer choice on their child's life through the initial value function V_1, defined above in (5), discounted by the degree of altruism

$\eta > 0$. This continuation value clearly shows our dynastic set-up, where parents care about their child's utility, which in turn depends on the following generations' utilities in the spirit of Becker and Tomes (1986). Note also that parents cannot borrow from their child's future income since a_c cannot be negative.

Next, parents who hold the asset level net of the inter-vivos transfers then solve a standard consumption-savings and labor supply problem as follows:

$$\tilde{V}_6(h_t, a_t, \kappa; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{(c_t/q)^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_7(h_{t+1}, a_{t+1}, \kappa; \boldsymbol{\pi}_{t+1}) \right\} \tag{14}$$

subject to

$$\begin{aligned} c_t + a_{t+1} &\leq \lambda_6 \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_6} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\ &\quad + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\ h_{t+1} &= \exp(z_{t+1}) \gamma_{6,\kappa} h_t \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t). \end{aligned}$$

Model Ages 7–12 In periods $j = 7$ and onwards, the state variables do not include h_c and ϕ because there is no need to keep track of these after the child leaves the original household. Until they retire in $j = 10$, households make consumption-savings and labor supply decisions. Hence, the household's problems in $j = 7, 8, 9$ are standard:

$$V_j(h_t, a_t, \kappa; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_{j+1}(h_{t+1}, a_{t+1}, \kappa; \boldsymbol{\pi}_{t+1}) \right\}, \quad \text{if } j = 7, 8, 9 \tag{15}$$

subject to

$$\begin{aligned} c_t + a_{t+1} &\leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\ &\quad + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T \\ h_{t+1} &= \exp(z_{t+1}) \gamma_{j,\kappa} h_t \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t). \end{aligned}$$

After retirement, households receive social security pension benefits Ω_t. The value functions in the retirement periods ($j = 10, 11, 12$) are given by:

$$V_j(h_t, a_t, \kappa; \boldsymbol{\pi}_t) = \max_{c_t \geq 0;\ a_{t+1} \geq \underline{a}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta V_{j+1}(h_t, a_{t+1}, \kappa; \boldsymbol{\pi}_{t+1}) \right\} \tag{16}$$

subject to

$$
\begin{aligned}
c_t + a_{t+1} &\leq (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t + \Omega_t \\
\boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t),
\end{aligned}
$$

and $V_{j=13}(\cdot) = 0$.

2.2 Firm's Problem and Government

There is a representative firm that produces output with technology featuring constant returns to scale and nested CES specifications. Specifically, we assume that output is given by the Cobb-Douglas function:

$$Y_t = z_t K_t^{\alpha} H_t^{1-\alpha}, \tag{17}$$

where z_t is the total factor productivity, K_t is the aggregate capital stock, H_t is the aggregate labor input, and $\alpha \in (0, 1)$. The aggregate labor input H is then aggregated under the CES technology following:

$$H_t = \left[\nu H_{1,t}^{\rho} + (1-\nu) H_{2,t}^{\rho}\right]^{\frac{1}{\rho}}, \tag{18}$$

where $\rho < 1$ shapes the elasticity of substitution $(1/(1-\rho))$ between skilled workers H_2 and unskilled workers H_1.

Given the above production technology, the representative firm in competitive markets maximizes profits. One can easily show that the optimality conditions are given by:

$$\alpha z_t K_t^{\alpha-1} H_t^{1-\alpha} = r_t + \delta \tag{19}$$

$$(1-\alpha) z_t K_t^{\alpha} H_t^{-\alpha} \frac{1}{\rho} \left[\nu H_{1,t}^{\rho} + (1-\nu) H_{2,t}^{\rho}\right]^{\frac{1}{\rho}-1} \nu \rho H_{1,t}^{\rho-1} = w_{1,t} \tag{20}$$

$$(1-\alpha) z_t K_t^{\alpha} H_t^{-\alpha} \frac{1}{\rho} \left[\nu H_{1,t}^{\rho} + (1-\nu) H_{2,t}^{\rho}\right]^{\frac{1}{\rho}-1} (1-\nu) \rho H_{2,t}^{\rho-1} = w_{2,t}, \tag{21}$$

where δ is the capital depreciation rate.

The government collects taxes from households through (progressive) labor income taxation and capital income taxation. These tax revenues are spent on four categories: (i) social security pension Ω to retirees; (ii) lump-sum transfers T to all households, (iii) public education expenditures $\{g_j\}_{j=3}^{5}$; and (iv) government spending $G \geq 0$ that is not valued by households. We assume that the government balances its budget each period j.

2.3 Equilibrium

Let us denote by $x_{j,t} \in X_j$ a vector of individual state variables at age j in period t in the household's recursive problems described in the previous subsection. Given an initial distribution $\boldsymbol{\pi}_{-T} \equiv (\pi_{j,-T})_{j=1}^{12}$, a competitive general equilibrium is a sequence of factor prices $\{w_{1,t}(\boldsymbol{\pi}_t), w_{2,t}(\boldsymbol{\pi}_t), r_t(\boldsymbol{\pi}_t)\}_{t=-T}^{\infty}$, the household's decision rules, value functions $\left\{\{V_j(x_{j,t},\boldsymbol{\pi}_t)\}_{j=1}^{12}\right\}_{t=-T}^{\infty}$, government policies including $\{(g_{j,t})_{j=3}^{5}\}_{t=-T}^{\infty}$, and distributions $\{(\pi_{j,t}(\cdot))_{j=1}^{12}\}_{t=-T}^{\infty}$ over $x_{j,t}$ such that:

1. given the government policies and factor prices, household decision rules solve the associated household's life cycle problems in the previous subsection, and $V_j(x_{j,t},\boldsymbol{\pi}_t)$ are the associated value functions;

2. factor prices are competitively determined according to (19), (20), and (21);

3. market clears;

$$K_t = \sum_{j=1}^{12} \int a_{j,t} d\pi_{j,t}(x_{j,t})$$

$$H_{s,t} = \sum_{j=1}^{12} \int h_{j,t} n_{j,t}(x_{j,t},\boldsymbol{\pi}_t) d\pi_{j,t}(x_{j,t}|\kappa = s), s = 1, 2;$$

4. the government budget is balanced for each period: the sum of transfers payments, social security pension payments, public education spending, and government spending is equal to the sum of labor income tax revenues and capital income tax revenues for each period;

5. the evolution of the distribution $\boldsymbol{\pi}_t$ is given by $\boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t)$, which is consistent with the household optimal choices and the exogenous probability distributions.

Note that this competitive equilibrium nests its stationary version of equilibrium where market-clearing prices and aggregate quantities are constant over time.

3 Calibrating the Model Economy in Stationary Equilibrium

Before we access the aggregate and intergenerational implications of school closures using numerical experiments in the next section, we discuss how we calibrate the model economy. Our approach is to calibrate the model in stationary equilibrium to U.S. data in normal times.

We consider model economies in which the elasticity of substitution between public and parental investments differs. There is limited evidence of this elasticity in the literature. A number of papers assume perfect substitutability, while a few papers estimate that this elasticity of substitution is less than perfect.[13] Given that there is no clear consensus on this parameter that could be useful for understanding the theoretical mechanisms we study here, we consider different values.

[13] In the literature, it is common to assume that private and public investments are perfect substitutes. For example, see Restuccia and Urrutia (2004), Holter (2015), Lee and Seshadri (2019), Daruich (2022) among others. On the other hand, there are lower estimates of this elasticity of substitution, such as 1.92 by Blankenau and Youderian (2015) and 2.43 by Kotera and Seshadri (2017). The assumption on our imperfect substitutability might be more suitable for temporary changes in public education (as in our main experiment) because it could be more difficult for the private sector to replace public one in the short run. We thank a referee for bringing this last point.

Specifically, for the baseline economy we take the value of $\psi = 2/3$, implying that the elasticity of substitution is 3. This value means that public and parental investments are substitutable yet are less than perfect substitutes.[14] In addition, we will also consider an alternative model economy with $\psi = 5/6$, implying an elasticity of substitution of 6, which is twice as large as its counterpart in the benchmark economy.[15] This alternative model would enable us to investigate the role of the elasticity of substitution between private and public investments.

We first discuss the parameter values that are commonly set across the two model economies. Then, we explain the remaining parameters that are internally calibrated to match the relevant target statistics in the U.S. We then present the properties of the baseline model economy in stationary equilibrium before we conduct numerical experiments on school closures in the next section.

3.1 Common Parameters

We adopt a standard approach to match relevant U.S. statistics externally and internally. We first discuss the first set of parameters that are calibrated externally. These are also commonly set across the two model economies that vary in terms of the elasticity of substitution between public and parental investments.

First, for preference parameters, we set the value of σ equal to 1.5 such that the intertemporal elasticity of substitution for consumption is 2/3 and set the value of χ equal to 4/3 such that the Frisch elasticity is 0.75 (Chetty, Guren, Manoli, and Weber 2013). Because our model frequency is five years, the relevant margin of labor supply adjustments includes both intensive and extensive margins. The value of q, which determines how consumption enters into utility in the presence of a child in the household is set to 1.59, based on the OECD equivalence scale.

The life cycle wage profiles for high- and low-skilled workers are governed by the gross growth rates of human capital during adulthood $\{\gamma_{j,\kappa}\}_{j=1}^{8}$. These values are computed based on Rupert and Zanella's (2015) estimates from the Panel Study of Income Dynamics (PSID). As reported in Table A2, these estimates show two notable patterns: (i) for each education group, the growth rates are higher in the early adult periods and then decline with age, and (ii) college-educated workers experience much higher growth rates. The parameter γ_c that maps childhood human capital to adulthood human capital is calibrated to be 32.1 such that the steady-state annual output per capita is normalized to one.

There are several parameters in the childhood human capital production function that are externally calibrated. In doing so, we follow the calibration strategy in Fuchs-Schündeln, Krueger, Ludwig, and Popova (2022). Specifically, the parameter for the relative share of private investments θ_p^j is set to 0.324 for school periods ($j = 4, 5$) according to the estimate in Kotera and Seshadri (2017). Since θ_p^3 is relevant to kindergarten and pre-school, it is internally calibrated as discussed

[14] Our baseline elasticity of substitution is similar to the one in Fuchs-Schündeln et al. (2022).

[15] In Appendix F, we also present results from an economy with a lower elasticity of substitution (1.5).

below. There are productivity parameters for each input: $\varsigma_t^x, \varsigma_t^e$, and ς_t^g. In steady state, we assume that these values are normalized to one since these are not separately identified from the share parameters. In Section 5, we then consider our main experiment of school closures as an unexpected temporary decline in ς_t^g, as will be discussed in detail.

We now discuss parameters related to government. Recall that the degree of progressivity in labor taxation differs based on household structure in the model. As reported in Table A3, progressivity tends to be higher for households with a child. The capital income tax rate τ_k is set to 0.36. These taxation-related parameters are based on the estimates by Holter et al. (2019). For the next parameters about public investments $\{g_j\}_{j=3}^5$, we aggregate average public school expenses per student paid by the government over the corresponding age ranges using the PSID-CDS (Child Development Supplement) data set provided in the replication files of Lee and Seshadri (2019). This gives us the ratio of g_j in $j = 3, 4, 5$ to steady-state output per capita equal to 0.020, 0.092 and 0.109, respectively. A key feature of g_j is that it increases as a child progresses through education stages. Next, following Lee and Seshadri (2019), the value for government lump-sum transfers T is set to 2% of steady-state output per capita to capture welfare programs. The value of Ω is set to imply that the social security replacement rate is 33%.

Finally, we discuss parameters related to the production sector and others. We set $\alpha_K = 0.36$ to be consistent with the capital share in the aggregate US data. The total factor productivity z_t is assumed to be one. The five-year capital depreciation rate δ is based on 2.5% of the quarterly depreciation rate. These values are standard in the literature. We set $\rho = 1/3$, implying that the elasticity of substitution between skilled and unskilled workers is 1.5 (Ciccone and Peri 2005).

3.2 Internally Calibrated Parameters

We now discuss the parameters that are calibrated internally by matching the relevant target statistics in U.S. data, given the value of ψ. The discussion herein focuses on the baseline economy with $\psi = 2/3$, as summarized in Table 2, and Appendix F provides the calibrated parameters for the model economy with alternative values of ψ. These parameter values are determined as minimizers of the squared sum of the distance between the relevant statistics from the data and those from the model-generated data. Although there is a relatively large number of parameters and targets, each parameter is connected to its corresponding target quite well. We now explain these relationships. All target statistics reported in Table 2 are constructed and discussed in detail by Yum (2021).

The first parameter in Table 2 is β, which captures the household's discount factor. Its relevant target is chosen to be the annual interest rate of 4%. The next parameter b is the disutility constant for labor supply. Its relevant target is chosen to be the mean hours worked by those aged between 30 and 65 (or $j = 3, ..., 9$). Assuming that the weekly feasible time endowment is $105 (= 15 \times 7)$ hours, excluding sleeping time and basic personal care, this statistic in the data yields $30.16/105 = 0.287$ as a target. There is a disutility parameter φ, which affects parental time investment levels. Since

Table 2: Internally calibrated parameters and target statistics for the baseline model economy

Parameter		Target statistics	Data	Model
β	.939	Equilibrium real interest rate (annualized)	.04	.04
b	6.65	Mean hours of work in $j = 3, ..., 9$	.287	.302
φ	.467	Mean hours of work in $j = 3, 4, 5$	.299	.292
η	.274	Ratio of inter-vivos transfers over total savings	.30	.360
θ_3^x	.857	Mean parental time investments in $j = 3$	.061	.062
θ_4^x	.139	Mean parental time investments in $j = 4$	.036	.035
θ_5^x	.095	Mean parental time investments in $j = 5$	.020	.020
θ_3^p	.487	Rank corr. of parental income & child earnings	.282	.282
θ_3^I	.629	Mean parental monetary investments in $j = 3$	.056	.053
θ_4^I	.666	Mean parental monetary investments in $j = 4$	.136	.130
θ_5^I	.391	Mean parental monetary investments in $j = 5$	.160	.151
ζ_3	-1.91	Educational gradients in parental time in $j = 3$ (%)	20.9	19.4
ζ_4	0.30	Educational gradients in parental time in $j = 4$ (%)	14.8	14.3
ζ_5	0.27	Educational gradients in parental time in $j = 5$ (%)	20.2	20.1
ν	.551	Fraction with a college degree (%)	34.2	33.6
μ_ξ	.231	Average college expenses/GDP per-capita	.140	.141
δ_ξ	.619	Observed college wage gap (%)	75.0	67.8
ρ_ϕ	.064	Intergenerational corr. of percentile-rank income	.341	.386
σ_ϕ	.466	Gini wage	.37	.341
σ_z	.146	Slope of variance of log wage from $j = 2$ to $j = 8$	.18	.180
$\underline{a}$	$-.069$	Average unsecured debt rel. to annual disposable income	.010	.010

our calibration strategy controls average parental time investments in period j using human capital production technology as described below, we calibrate φ to match the mean hours worked by those who make time investments (age 30–44). Next, η governs the degree of altruism and is calibrated to match the mean inter-vivos transfers. Because inter-vivos transfers in the model are given only once, we choose the ratio of transfer wealth (including inter-vivos transfers and bequests) to total savings, which equals 0.3 (Lee and Seshadri 2019), as a target statistic.

We now discuss parameters related to the child human capital production functions. These parameters include three parameters—θ_j^x, θ_j^I and ζ_j—in each j and θ_3^p, as shown in (10), (11) and (12). We calibrate them by exploiting the clear linkages between each of these parameters and its corresponding target moment in the model economy. Specifically, θ_j^x captures the relative importance of parental time investments (vs. parental financial investments), and it clearly increases the mean parental time investments in period j, which are used as target statistics. Statistics on parental time investments are obtained from the 2003-2017 American Time Use Survey (ATUS) only with educational, interactive activities that require the presence of both a parent and a child in a common space.[16] A key feature of these moments is that the mean time investment is highest in the earliest period $j = 3$ (0.061 in the model or 6.4 hours per week) and it decreases with children's age.

Next, θ_j^I increases overall parental investments in each period j, and θ_3^p increases them in period 3. Furthermore, higher values of θ_j^I in all j strengthens intergenerational persistence of lifetime income through human capital transmission (i.e., labor income). Hence, we use the mean private education spending in each period (relative to steady-state output per capita) and the rank correlation of parents' income and their child's earnings—0.282 (Chetty et al., 2014)—as target moments. Parental monetary investment in the model is meant to capture various educational expenditures. Therefore, the mean private education expenditure is constructed as total private education expenditures (including child care, schooling tuition and supplies, and extracurricular activities) net of public school costs paid by parents using the PSID-CDS data set of Lee and Seshadri (2019). Consequently, we obtain the target statistics of $0.056, 0.136$ and 0.160 for $j = 3, 4$ and 5, respectively. Unlike the parental time inputs, parental financial inputs increase with children's education stage. Our calibration leads to a high value of θ_3^p, which implies that the importance of parental investments relative to public investments is higher for very young children compared to school-aged children.

Finally, ζ_j shapes the elasticity of substitution between time and money in period j. These are calibrated to match the salient facts in the U.S. that more educated parents spend more time with children (Guryan et al. 2008; Ramey and Ramey 2010). Specifically, we allow our model to replicate the fact that parents who are college-educated spend around 20 percent more time

[16] Such activities include reading to/with children, playing with children, doing arts and crafts with children, playing sports with children, talking with/listening to children, looking after children as a primary activity, caring for and helping children, doing homework, doing home schooling, and other related educational activities.

with their children than those without a college degree.[17] In particular, we allow the elasticity of substitution to be j-dependent since the same elasticity of substitution would lead to a lower educational gradient in early periods (Yum 2021). As a result, our calibration leads to a lower elasticity of substitution in $j = 3$ (0.34) than in later periods (1.42 and 1.37 in $j = 4$ and 5, respectively), implying that parental time and monetary investments are much more substitutable for school-aged children.

The next parameters are related to college education. In the aggregate production function (18), ν is calibrated to match the fraction of people with a college degree (34.2%). The mean of college costs is determined by μ_ξ, which naturally gives a target statistic: the equilibrium ratio of the mean (tuition and non-tuition) expenses after financial aid to per capita GDP. According to detailed procedures explained by Yum (2021), this statistic (relative to the five-year GDP) is 0.140. The next parameter is related to the variance of the college costs. Note that as σ_ξ increases, the observed wage premium would decline since college decisions are more strongly shaped by costs relative to pre-college human capital. Therefore, its relevant target is set to be the observed college wage premium of 75% (Heathcote, Perri, and Violante 2010).

Next, ρ_ϕ determines the persistence of exogenous ability across generations. We set its relevant target as the rank correlation of family income of 0.341 (Chetty et al. 2014). Note that Chetty et al. (2014) estimate intergenerational persistence using a proxy income variable instead of lifetime income due to the data limitation, as is common in the literature. Therefore, our target statistic from the model also uses proxy income.[18] The last two parameters in Table 2 govern the variability of wages in different ways. Although either would increase the overall wage inequality in the model, the variability of the idiosyncratic shocks to adult human capital σ_z also shapes the rising lifecycle inequality. Therefore, the two target statistics are the Gini coefficient of wage and the difference between the variance of log wage at age 55-59 ($j = 2$) and that of log wage at age 25-29 ($j = 8$), as reported in Table 2 (Heathcote et al. 2010).

Finally, the borrowing limit $\underline{a}$ is calibrated internally to match average debt in equilibrium. Livshits, MacGee, and Tertilt (2010) find that the average unsecured debt relative to annual disposable income is around 5–9% in the 1980s and 1990s. In line with this evidence, the target statistic is set to be 1% of the five-year GDP per capita.

The alternative models with different values of the elasticity of substitution between public and parental investments are calibrated using the same calibration strategy. The calibration results are reported in Appendix F.

[17] To be precise, the education gradient is defined as the percentage difference in mean parental time investments between education groups while controlling for parental observables. See Appendix A for details.

[18] Specifically, Chetty et al. (2014) measure a child's income at around 30 years old, averaged over two years. The parent's income is averaged over five years when parents' ages are around 45 years. Equivalently, our model-based proxy income is measured for parents in $j = 6$, and for children in $j = 3$.

Table 3: Intergenerational persistence estimates

	U.S. data	Model	
	Chetty et al. (2014)	Proxy income	Lifetime income
IGE: log-log slope	0.344	.349	.413
Rank corr: rank-rank slope	0.341	.386	.392

3.3 Properties of the Baseline Model in Stationary Equilibrium

In this subsection, we present the properties of the baseline model in stationary equilibrium before we conduct the main quantitative analysis on school closures.

We first evaluate the intergenerational mobility implied by the model. Specifically, we measure the model-implied intergenerational mobility in three ways and compared them to the data counterparts. The data counterparts are from Chetty et al. (2014) who use administrative data.[19] As mentioned above, income in the model is the five-year per parent sum of labor earnings, interest income, and social security benefits.

The first measure is the intergenerational elasticity (IGE), obtained from the following log-log equation:

$$\mathcal{Y}_{child} = \rho_0 + \rho_1 \mathcal{Y}_{parent} + \varepsilon, \tag{22}$$

where $\mathcal{Y}$ is log permanent income. This is a conventional way to measure the degree of intergenerational persistence in the empirical literature. Its interpretation is straightforward: a 1% increase in parental permanent income is associated with a ρ_1% increase in their children's permanent income. The second measure is to use a rank-rank specification instead of a log-log specification (Chetty et al. 2014). This can be estimated when $\mathcal{Y}$ is the percentile rank of income. This slope coefficient (or the rank correlation) tells us that a one percentage point increase in parent's percentile rank is associated with a ρ_1 percentage point increase in their children's percentile rank. In the model, we estimate these slopes using both proxy income, which is defined equivalently as its empirical counterpart, and the lifetime income, which is constructed as present-value lifetime income discounted according to the interest rate (Haider and Solon 2006) in stationary equilibrium.

Table 3 reports the two slope estimates from the data and the model. Recall that we directly targeted to match the rank correlation using proxy income. Although data limitation prevents researchers from investigating the lifetime income, it is possible to estimate the mobility measures using the lifetime income in the model. As is well known in the literature, we can see that the estimate of the IGE using lifetime income (0.413) is substantially larger than the counterpart using

[19] Specifically, parental income is defined as the average five-year pre-tax income per parent, which is either the sum of Adjusted Gross Income, tax-exempt interest income and the non-taxable portion of Social Security and Disability benefits (if a tax return is filed) or the sum of wage earnings, unemployment benefits, and gross social security and disability benefits. For children's income, they use a short horizon (2-year average) due to data availability.

Table 4: Income quintile transition matrices: data vs. model

Unit: %	U.S. data					Model									
	Chetty et al. (2014)					Proxy income					Lifetime income				
Parent	Child quintile					Child quintile					Child quintile				
quintile	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
1st	33.7	28.0	18.4	12.3	7.5	36.1	24.7	18.3	13.8	7.1	36.4	25.2	18.0	13.6	6.8
2nd	24.2	24.2	21.7	17.6	12.3	26.1	22.9	21.2	17.8	12.0	25.9	23.2	21.2	18.1	11.7
3rd	17.8	19.8	22.1	22.0	18.3	19.7	20.9	21.7	20.4	17.3	19.6	20.2	22.4	20.6	17.2
4th	13.4	16.0	20.9	24.4	25.4	13.6	17.9	20.8	23.3	24.4	13.6	17.5	20.9	23.2	24.8
5th	10.9	11.9	17.0	23.6	36.5	4.6	13.5	18.0	24.7	39.2	4.5	13.8	17.7	24.5	39.5

proxy income (0.349) because the short-term income may not represent the long-term lifetime income (Haider and Solon 2006). Interestingly, this attenuation bias is smaller in the rank correlation (0.392 versus 0.386).

The above slope estimates are easy to interpret and convenient, but they do not fully describe how income distribution persists across generations. The income quintile transition matrix provides a richer description of how economic status is transmitted across generations.[20] We now compare the quintile transition matrix from the model-generated data to the empirical quintile transition matrix (Chetty et al. 2014). Because calibration does not directly target any elements in the income quintile transition matrix, this is a natural way of evaluating how successful a model is as a quantitative theory of intergenerational mobility (Yum 2021).[21]

Table 4 reports the transition matrices, obtained from U.S. (Chetty et al. 2014) and model-generated data. The data shows that the probability of children remaining in the bottom quintile when their parents' income is also in the bottom quintile is 33.7%. Similarly, the probability of staying in the top income quintile is quite high at 36.5%. A particularly interesting one is the probability of moving up from the bottom quintile to the top quintile, namely upward mobility. In the data, the upward mobility rate is 7.5%. The middle panel of Table 4 displays the quintile transition matrix from the model when the equivalent measure of proxy income is used. The model successfully replicates the empirical patterns noted above. In particular, the upward mobility rate in the model is 7.1%, which is close to the data counterpart (7.5%).

Table 4 also reports the quintile transition matrix using lifetime income. Compared to the one with proxy income, we can see that the diagonal elements are generally higher, which is consistent with lower intergenerational mobility measured by the slope coefficients in Table 3. The upward mobility rate in terms of lifetime income is slightly lower at 6.8%. In the following numerical experiments, we use the intergenerational mobility measures based on lifetime income because the

[20] An income quintile transition matrix is a 5 by 5 matrix where the (a, b) element provides the conditional probability that a child's lifetime income is in the b-th quintile, conditional on the parent's income belonging to the a-th quintile. Quintiles are based on their own generation.

[21] Note that the same correlation of income across generations can be consistent with different quintile transition matrices. This is similar to the fact that the same Gini coefficient can be consistent with different shapes of income distributions.

mobility measures based on proxy income are subject to attenuation biases (Haider and Solon 2006) as also confirmed by the model-generated data in stationary equilibrium.

As is well known, cross-sectional inequality in labor market variables tends to increase over the lifecycle in the data (e.g., Heathcote et al. 2010). As Figure A7 shows, the model replicates the increasing dispersion in wages (left), earnings (middle), and income (right) quite well.[22] We note that these features are important because a higher dispersion in income among relatively older parents would be transmitted into the extent to which parents with different permanent incomes afford additional parental investments in response to school closures.

4 Quantitative Analysis of School Closures

We now move on to the main analysis of this paper on the implications of school closures. This requires us to compute the equilibrium away from the steady state. We first explain how we conduct the numerical experiments and then briefly discuss empirical consistency with the best existing evidence on the short-run effects of school closures. Afterwards, our main analyses on the medium- and long-run effects follow.

4.1 Computational Experiment Design

In this section, we analyze transitional dynamics following unexpected school closure shocks. In the simulation, in each period, the economy consists of 12 adult cohorts, and each is composed of 500,000 household units. Thus, the total number of households is 6,000,000 in each period t. We first simulate the model economy for sufficiently long periods until it reaches the stationary equilibrium.[23] The economy is in stationary equilibrium at $t = ..., -2, -1, 0$, and school closures unexpectedly take place at the beginning of $t = 1$. Our baseline exercise considers universal, nationwide school closures where all schools are closed for the same period of time.[24] We represent these school closures by reducing the productivity of public investments $\varsigma^g_{t=1}$ in (7) according to the closure length, similarly to Fuchs-Schündeln et al. (2022). For example, if a school closure lasts for one year, we reduce $\varsigma^g_{t=1}$ by 20%. We consider three different lengths of school closures: 0.5, 1 and 1.5 years. We note that our notion of school closure length should be interpreted in terms of academic years (AY), and should be mapped to the actual days of school closures with caution due to the presence of breaks, even in normal times.[25] In $t = 2, 3, ...$, there are no further shocks

[22] Note that this is disciplined mainly by the calibrated dispersion in idiosyncratic shocks to adult human capital.

[23] Specifically, we simulate 55 periods to reach the steady state from a given initial distribution and drop the first 50 periods. We keep the five periods of the steady state economy to keep information about parents whose children directly experience school closures.

[24] In the Appendix D, we also examine the effects of *partial* school closures where there is a stochastic difference in closure lengths across households. This exercise reflects the fact that there could be regional variations in the effective length of school closures, caused by the uncertain local pandemic progress and political factors not modeled herein.

[25] For example, as 4-5 months of vacation already exist in normal years, the school closure of 1-year length would correspond to the actual days of closure for 7-8 months (including weekends). We consider three possibilities since

and the economy returns to the original stationary equilibrium.[26] We compute the transitional equilibrium paths under perfect foresight.

In addition to the consequences of school closures on macro aggregates such as output, our analysis also focuses on heterogeneous impacts on children of different ages in which the school closure shock hits the economy. Therefore, we will also present the results for three child cohorts that directly experience the school closure in different ages: the cohort aged between 0 and 4 (*Cohort 1* or C1) at the school closure; that aged between 5 and 9 (*Cohort 2* or C2); and that aged between 10 and 14 (*Cohort 3* or C3). We also keep track of parents matched to these children to examine intergenerational implications.

4.2 Quantitative Results

4.2.1 Consistency with Short-run Evidence on School Closures

Since most governments (including the U.S. government) closed schools in early 2020 in response to the COVID-19 pandemic, there has been limited empirical evidence on the direct effects of such closures on the general child performance even in the short run.[27] Although there has been suggestive evidence to indicate significant drops in the amount of learning (Chetty et al. 2020), the lack of data prevents researchers from investigating the negative consequences of learning loss in a broader setting with causal interpretations. Ideally, we would need to have observations on a large number of representative students whose academic progress (e.g., in terms of test scores) in multiple points within a year is observed, not only in the regular year but also during the pandemic period when schools were almost universally closed.

An exception is Engzell et al. (2021) who use a rich nationally representative data set from the Netherlands. Their data set satisfies all of the ideal settings mentioned above, thereby allowing them to conduct a different-in-difference estimation. According to their estimates based on composite scores aggregating math, reading and spelling scores for the students aged 7-11, they estimate a learning loss of about 3.2 percentile points or 0.08 standard deviations during the lockdown which induced school closures of 2 to 2.5 months. Although child human capital in the model does not exactly correspond to the observed test scores, it is useful to compare how school closures affect human capital loss in the model.[28] In our model, we find that a 0.25-year closure leads to a human capital loss of 2.9 percentile points or 0.08 standard deviations among the corresponding children

they help us to investigate potential nonlinearity in the effects. In this regard, we also report results from a very long closure (4 years) in the appendix.

[26] Although shocks are temporary and relatively small, it is important to run the model economy long enough for several reasons. First, as our key variable is lifetime income, we need to generate the whole life-cycle for the youngest cohort that directly experienced the school closures. In addition, as we show below, school closure shocks have long-lasting effects. In our exercises, we use $t = 35$.

[27] The empirical literature on the learning loss during summer break (Cooper et al. 1996, Atteberry and McEachin 2020) could be useful, although it might be nontrivial to apply the summer break effects to the effects of closing during regular school periods, especially at longer horizons.

[28] Human capital in our model is supposed to be a broader concept than test scores on the selected subjects.

(C2). In addition, we also find a larger fall in children's human capital with lower parental permanent income (Figure 4), in line with their findings that parental education is the only significant factor shaping the negative impacts. This comparison shows that our model generates reasonable magnitudes of negative impacts on the children's outcomes.

As discussed below, parental responses to school closures are an important channel that not only mitigates the aggregate effects but also impacts intergenerational effects. A recent paper by Grewenig et al. (2021) provides interesting results related to our findings. They use a survey in Germany with detailed time use information and find that children reduced their daily learning time significantly during school closures. More interestingly, they also find that the reduction in learning time was not statistically different by parental education or income. This is in fact consistent with our finding below that the positive income gradients in parental responses materialize in terms of money, not in terms of time (see Figure 3).[29]

4.2.2 Aggregate Implications

We now present the main systematic results from the quantitative exercises. Figure 1 plots the dynamics of output, capital, efficiency units of labor for non-college and college graduates following unexpected school closures of different lengths in $t = 1$. Overall, the changes of these aggregate variables are quite persistent. The top-left panel shows that the aggregate output declines gradually over time, and this decline continues until $t = 11$. The top-right panel implies that the initial drop in output is due to dissaving to increase parental investments. This reduction in capital is amplified over time by lower human capital formations of those who experienced the school closures during their childhood. The bottom panels suggest that parents increase their labor supply to earn more income on impact, thus raising parental investments to counter school closures. The aggregate efficiency unit of labor for each skill type then starts to decrease when the cohorts, experiencing these school closures during childhood, enter the labor market with lower levels of human capital. This reduction in the aggregate labor continues to decline until $t = 11$ and gradually recovers afterward. However, there is another fall in college-educated labor in period 17. These lingering effects on future generations arise because when the initially affected children become parents, their own children will also suffer due to their lower income (and thus lower investments).

Another noticeable feature is that the responses of the aggregate variables are nonlinear to the length of school closures. The top-left panel of Figure 1 demonstrates that in period 11, while the 0.5-year-closure reduces output by 0.4%, the 1.5-year-closure decreases output by 1.3%. The top-right panel shows that the 1.5-year-closure reduces capital three times more than does the 0.5-year-closure. When we add up these persistent output losses over the next century, the accumulated output loss relative to annual output is around 19% for the 0.5-year-closure, which

[29] For example, richer parents could spend even more on better tablets or online resources of higher quality (Bacher-Hicks et al. 2021), which would increase the efficiency of learning further. In our model, this would be captured by disproportionately higher monetary investment e by richer parents. But they may not necessarily spend disproportionately more time on education-related activities.

Figure 1: Evolution of macroeconomic aggregates

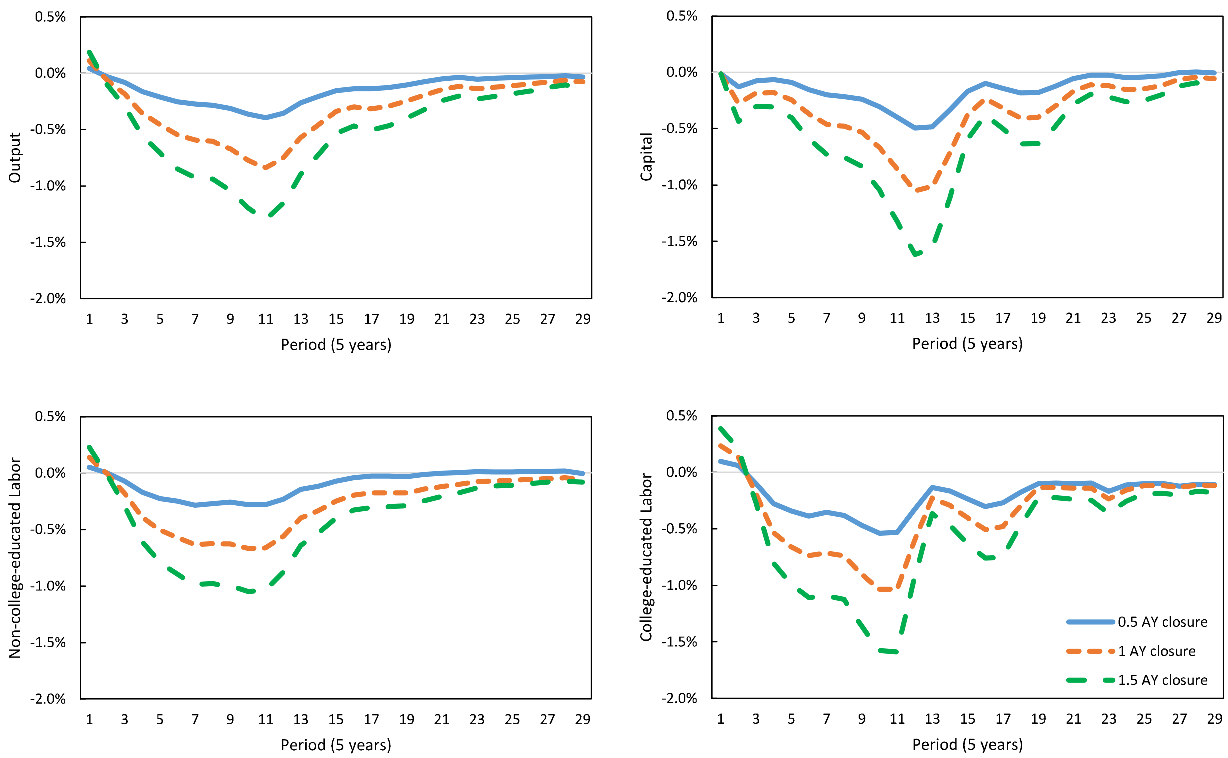
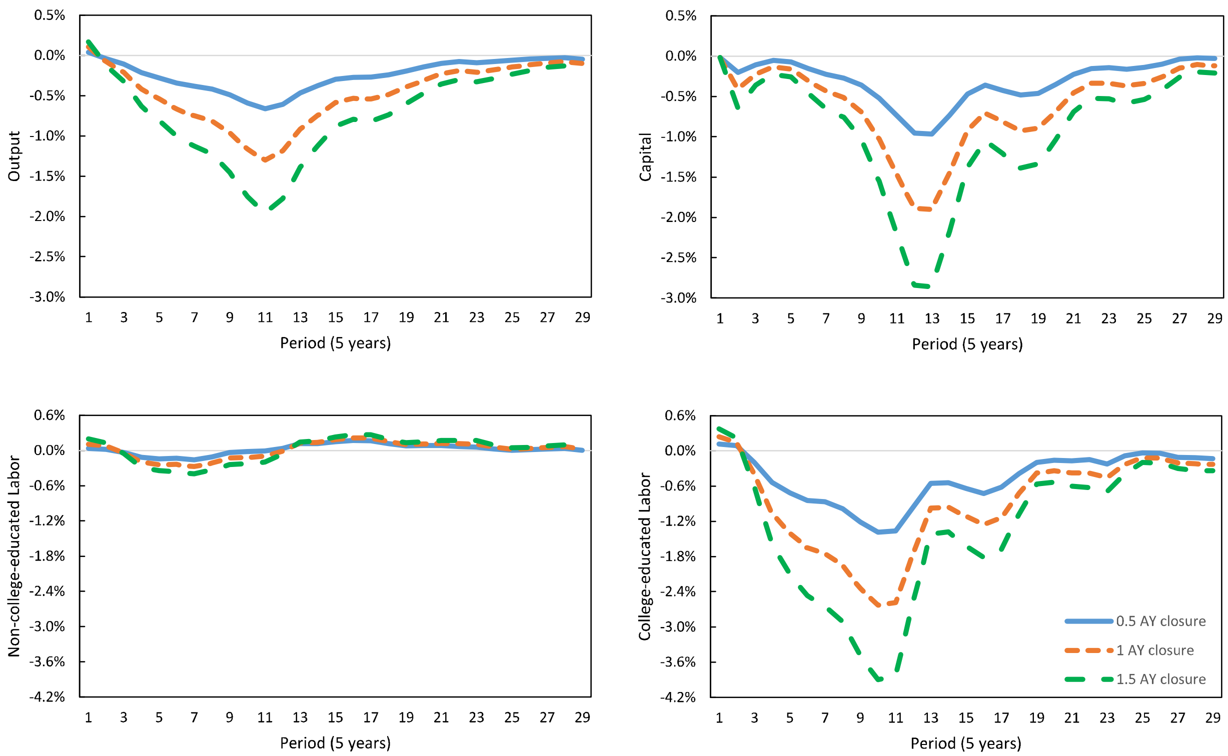


Figure 2: Evolution of macroeconomic aggregates without general equilibrium effects



Note: Factor prices are fixed at stationary equilibrium levels

increases nonlinearly with the closure length: 42% for the 1-year-closure and 66% for the 1.5-year-closure.

Another noticeable feature we highlight is the general equilibrium effects that play a role in adjusting the magnitude of the responses of these aggregate variables to the school closures. In particular, as revealed by a comparison between Figures 1 and 2, these general equilibrium effects tend to balance the responses of the efficiency units of labor between college and non-college graduates. Specifically, Figure 2 shows that when prices are fixed at their stationary equilibrium levels, aggregate labor for college graduates is more significantly reduced in response to these school closures. A change in efficiency units of labor in each education group can be driven by (i) the fraction of the skill group relative to population (extensive margin), (ii) hours worked conditional on working (intensive margin), and (iii) the quality of the work force (human capital). The large reduction in the efficiency units of labor for the college-educated individuals that materializes gradually over time is due to the direct loss of human capital and a relatively noticeable decrease in college attainment indirectly driven by lower child human capital, both of which were caused by school closures. By contrast, we see that labor efficiency for non-college graduates does not decline as much because the reduction in human capital is offset by an increased number of people who do

Table 5: Distributional changes over time

	Steady state	Time (1 period: 5 years) 1	2	3	4	5
		% change rel. to no school closure				
Closure length: 0.5 AY						
Gini income	.338	0.0	0.0	0.1	0.2	0.1
Bottom 20% inc (%)	8.1	-0.0	-0.0	-0.1	-0.1	-0.0
Share of college (%)	33.6	-0.0	0.0	-0.1	-0.2	-0.2
Closure length: 1 AY						
Gini income	.338	0.0	-0.1	0.3	0.4	0.3
Bottom 20% inc (%)	8.1	-0.1	0.0	-0.2	-0.3	-0.0
Share of college (%)	33.6	-0.0	0.1	-0.2	-0.4	-0.3
Closure length: 1.5 AY						
Gini income	.338	0.0	-0.1	0.4	0.7	0.4
Bottom 20% inc (%)	8.1	-0.1	0.1	-0.4	-0.5	-0.1
Share of college (%)	33.6	0.0	0.1	-0.3	-0.6	-0.4

not complete college in the case where prices are exogenously fixed. However, in general equilibrium, the decrease in college attainment tends to increase the relative premium of college graduates, thereby dampening the reductions in the efficiency units of labor for college graduates and amplifying those for non-college graduates. Similarly, general equilibrium effects mitigate the reductions in aggregate capital by higher equilibrium interest rates. Consequently, general equilibrium effects moderate the overall responses of output to these school closure shocks.

We now move on to the distributional changes over time. Table 5 reports the effects of school closures on three cross-sectional inequality measures, demonstrating that school closure shocks bring about relatively modest changes in cross-sectional inequalities. In the 0.5-year-closure scenario, there is almost no change in the Gini coefficient of current income for the first two periods and there is an increase of at most 0.2% in the last two periods. Just as the income share held by the lowest 20 percent shows no significant change for five periods, so does the share of college graduates. Longer school closures result in stronger impacts on cross-sectional inequalities. Compared to the steady state, the economy with the 1.5-year-closure increases the Gini income coefficient by 0.4% until $t = 3$ and by 0.7% in period 4. This pattern also appears in both the income share held by the lowest 20 percent and the share of college graduates. Overall, we also observe nonlinear effects of school closures when it comes to cross-sectional inequality.

Table 6: Effects on intergenerational mobility of lifetime income

	IGE			Rank cor.			Upward Mobility		
Steady state	.413			.392			6.8%		
	% change rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.1	2.2	2.4	0.1	2.0	2.2	-0.3	-3.2	-4.1
1.0 AY	0.3	4.5	5.0	0.2	4.1	4.6	-0.6	-6.8	-8.3
1.5 AY	0.4	7.0	7.8	0.3	6.3	7.1	-0.9	-10.9	-12.5

4.2.3 Intergenerational Implications

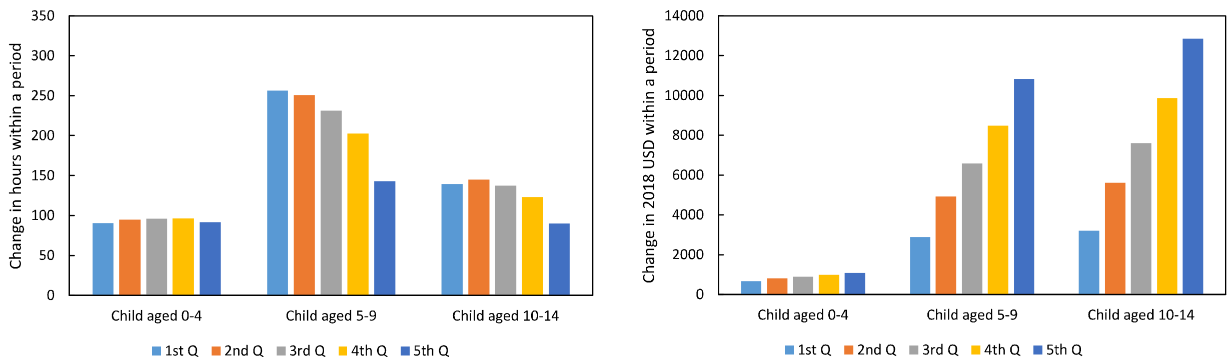
We now investigate how school closures affect lifetime income inequality across generations. Table 6 reports that the school closure shocks reduce intergenerational mobility quite substantially. Compared to the steady state, the 0.5-year-closure increases the IGE by 0.1–2.4% and the rank correlation by 0.1–2.2%, while decreasing the upward mobility by 0.3–4.1% across cohorts. These changes are amplified by the length of school closures. Across cohorts, the 1.5-year-closure generates increases in the IGE and rank correlation three times as large as the 0.5-year-closure. Likewise, the 1.5-year-closure reduces the upward mobility three times more than does the 0.5-year-closure.

Note that the school closure effects on intergenerational mobility are quantitatively heterogeneous across cohorts: the older cohorts are, the more reduced intergenerational mobility is. While the 1-year-closure increases the IGE by 0.3% for C1, it does so by 5.0% for C3. The rank correlation also has similar differences across cohorts. Similarly, given a school closure, older cohorts suffer from a greater reduction in upward mobility. The 1-year-closure decreases C1's upward mobility rate by 0.6% but C3's by 8.3%. These patterns are preserved regardless of the length of school closures. Both the 0.5-year-closure and the 1.5-year-closure lead older cohorts to experience greater reductions in upward mobility and larger increases in IGE and rank correlation.

To understand these intergenerational implications, it is useful to distinguish *direct* versus *indirect* effects of school closures on the human capital production function. Consider first the direct effects, the effects of changes in ς^g on the level of human capital produced while holding parental responses unchanged.[30] There are two key points worth noting regarding the direct effect. The first is about within-cohort differences: parents with low SES experience greater reductions in child human capital. Since the portion of public investment g_j is greater for lower SES parents, they are more adversely affected by school closures. The second regards cross-cohort differences: school-

[30] Figure A9 visualizes this relationship, and the arguments about heterogeneous direct effects of school closures in the next paragraph.

Figure 3: Parental responses by parental permanent income



Note: A set of five bars plots changes in parental investments by the quintile of parent's permanent income for each cohort, ordered by the child's age group during the 1-year school closure. The left shows time investment responses and the right shows monetary investment responses.

aged children face larger damages, compared to very young children. School disruptions matter more for school-aged children because the size of public investment g_j is higher at the outset and the relative importance of public investment is higher (i.e., a lower θ_p). The former channel is relevant to the implication of school closures for inequality, while the latter channel is important for the differential impacts of school closures on different cohorts.

In addition to the direct effects of school closures, the other important mechanism is related to endogenous parental responses: parents have incentives to respond to this reduced child human capital following school closures by increasing their parental investments. The indirect effect of school closures—governed by these parental investment behaviors—is different according to their children's age. As shown in Table 2, the importance of financial relative to time investments increases with children's age, in line with estimates by Del Boca, Flinn, and Wiswall (2014). These calibration results imply that parental time is more crucial in forming human capital in the very early childhood period (C1), but financial investments become more important in later periods (C2 and C3). In addition, the degree of substitutability between time and monetary investments is much stronger in C2 and C3 than in C1.

This age-dependent human capital production technology brings about differences in the composition of parental investments according to the child's age. Figure 3 presents the parental responses to the 1-year-closure by parental lifetime income (or permanent income). Clearly, the average monetary investment response is much stronger for older children (C2 and C3). Note that when children are aged between 0 and 4, parental responses in time are nearly flat across income distribution because time constraints are more equally distributed across parents than budget ones. The richer parents cannot easily compensate financially for the lack of time investments, as monetary investments are not as effective as or easily substitutable for time investments for children

Table 7: Effects on inequality and loss of lifetime income

	Lifetime income						Fraction of		
	Gini index			Average			College-educated		
Steady state	.282			4.2 (rel. to Y_s)			.336		
	% **change** rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.0	0.3	0.3	-0.1	-1.6	-1.7	0.7	-1.3	-1.6
1.0 AY	0.1	0.6	0.7	-0.1	-3.3	-3.4	1.4	-2.5	-3.2
1.5 AY	0.1	1.0	1.0	-0.2	-5.0	-5.1	2.1	-3.9	-4.9

Note: Y_s denotes steady-state output per capita.

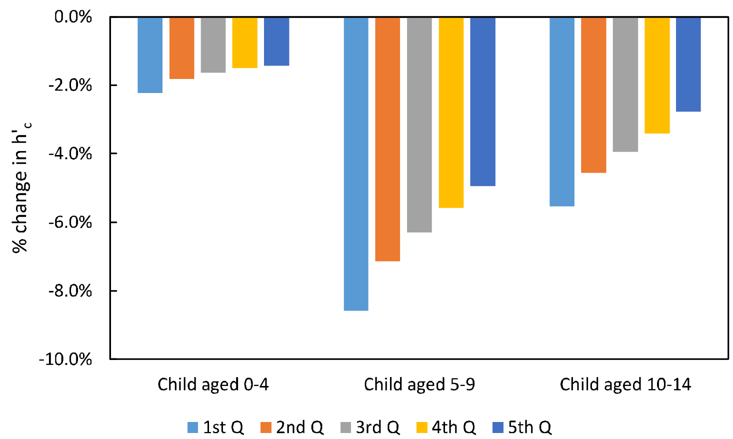
in the early period. In the later periods, as financial investments become more important, richer parents substitute time between the two more than poor parents do. Further, note that financial investments can better substitute time investments for older children (due to the higher elasticities of substitution) and that parents' income dispersion increases with age, which would show up as greater dispersions in financial investments for older children (Figure A7). These jointly result in substantial positive income gradients in monetary investment responses for the older cohorts (C2 and C3).

These heterogeneous parental investments play an important role in generating disparities in child human capital formation. Recall that children with low-income parents are disproportionately affected due to the direct effects of school closures. In addition, the heterogeneous parental investments discussed above amplify these differences. For the older cohorts (C2 and C3), larger differences in parental monetary investments lead to greater disparities in the changes of human capital across parental income groups, which in turn reduces intergenerational mobility. As a result, intergenerational persistence estimates increase more in the older cohorts.

Next, we investigate how school closures influence the overall economic status (or absolute mobility) by cohort and the dispersion of lifetime income within cohorts. Table 7 reports the effects of school closures on the average and inequality of lifetime income. While these school closures have relatively small adverse impacts on lifetime income inequality, the average reveals substantial losses. Specifically, the 0.5-year-closure increases the lifetime income Gini coefficient by up to 0.3% across cohorts, and the longer school closure of 1.5 years increases the Gini coefficient by up to 1.0%. The 0.5-year-closure reduces average lifetime income by around 1% on overage, and its magnitude increases with the length of school closures.

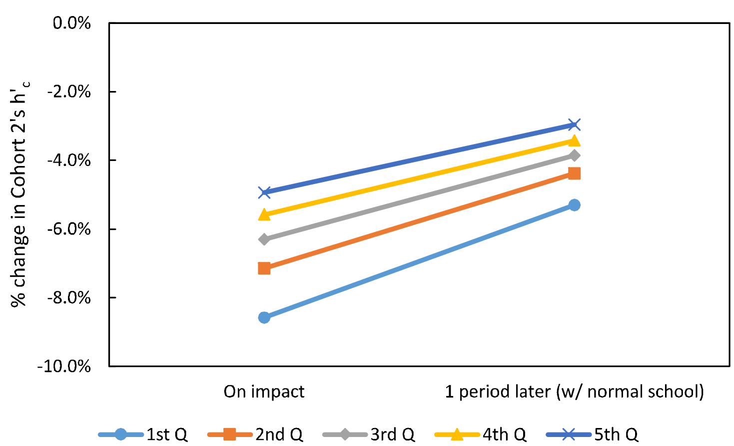
Note that school closures bring about more significant losses in lifetime income for older children (C2 and C3) than for the youngest cohort (C1). This result is mostly explained by the heterogeneous

Figure 4: Child human capital in the next period by parental permanent income



Note: A set of five bars plots percent changes in the next period human capital on impact by the quintile of parent's permanent income for each cohort, ordered by the child's age group during the 1-year school closure.

Figure 5: Effects of school closures on child human capital (initially aged 5-9) over time, by parental permanent income



Note: This figure plots percent changes in the next period human capital (relative to the case without school closure shocks) of Cohort 2 (C2) on impact and in the following period. A model period corresponds to five years.

direct effects of school closures across cohorts, as discussed above: public investments matter more for school-aged children due to higher government inputs at the outset and the greater relative importance of public inputs in the technology. Moreover, among the school-aged children, average lifetime income losses are nearly similar between C2 and C3, even though the former experience the strongest adverse impacts on child human capital on impact (Figure 4) in line with the literature highlighting the importance of earlier childhood in human capital formation (Heckman 2008). The key to understanding these seemingly contradictory results is the dynamic evolution of human capital. As can be seen in Figure 5, differences in human capital losses across different parental backgrounds for C2 become narrower in the next period (in the absence of school closure shocks), as the public investments normally play an equalizing role (Fernandez and Rogerson 1998). This is in line with the empirical evidence by Kuhfeld et al. (2020) who find that students who lose more ground during the summer of 2018 tend to experience steeper growth during the next school year. In fact, this narrowing gap in school closures' negative consequences is closely related to the property of the human capital production function. As the school closure shock reduces human capital more severely for children from lower SES parents, by the same token, the marginal productivity of investments in the following normal period is even higher for those children, due to the concavity of the production function.

4.2.4 Results from Extended Models

We now investigate how our benchmark results could change in several model extensions, where we introduce additional forces relevant to school closures. These extensions include virtual schooling, recessionary effects of COVID-19, and shocks to private monetary investments.

Virtual Schooling Although schools have been struggling in the beginning, they gradually adapt to online teaching during the school closures induced by the COVID-19 pandemic (Kuhfeld et al. 2020). In principle, virtual schooling could mitigate the negative consequences of school closures on child learning. The empirical evidence tends to suggest potential positive income gradients in online learning (Bacher-Hicks et al. 2021). Although it can be explained by financial investment responses for the quality of home learning environment such as laptops and tablets (Andrew et al. 2020), it could also capture the direct effects of parental skills and knowledge, which could enhance their children's virtual teaching experience given the same financial investments.

To quantitatively explore how much this effect can be relevant to the school-closure effects we have studied, we consider an alternative scenario where college-educated parents are able to fully mitigate the school closures through virtual schooling. More precisely, college-educated parents do not experience the fall in ς^g when the school closure shock hits the economy. Hence, this exercise is designed to provide an upper bound of the effects of such skill-gradients in virtual schooling.

As expected, we find that aggregate losses and the affected children's average income losses are mitigated substantially. As shown in Table 8, the year-long closure reduces the average lifetime

Figure 6: Evolution of macroeconomic aggregates following the 1-year closure in extended models

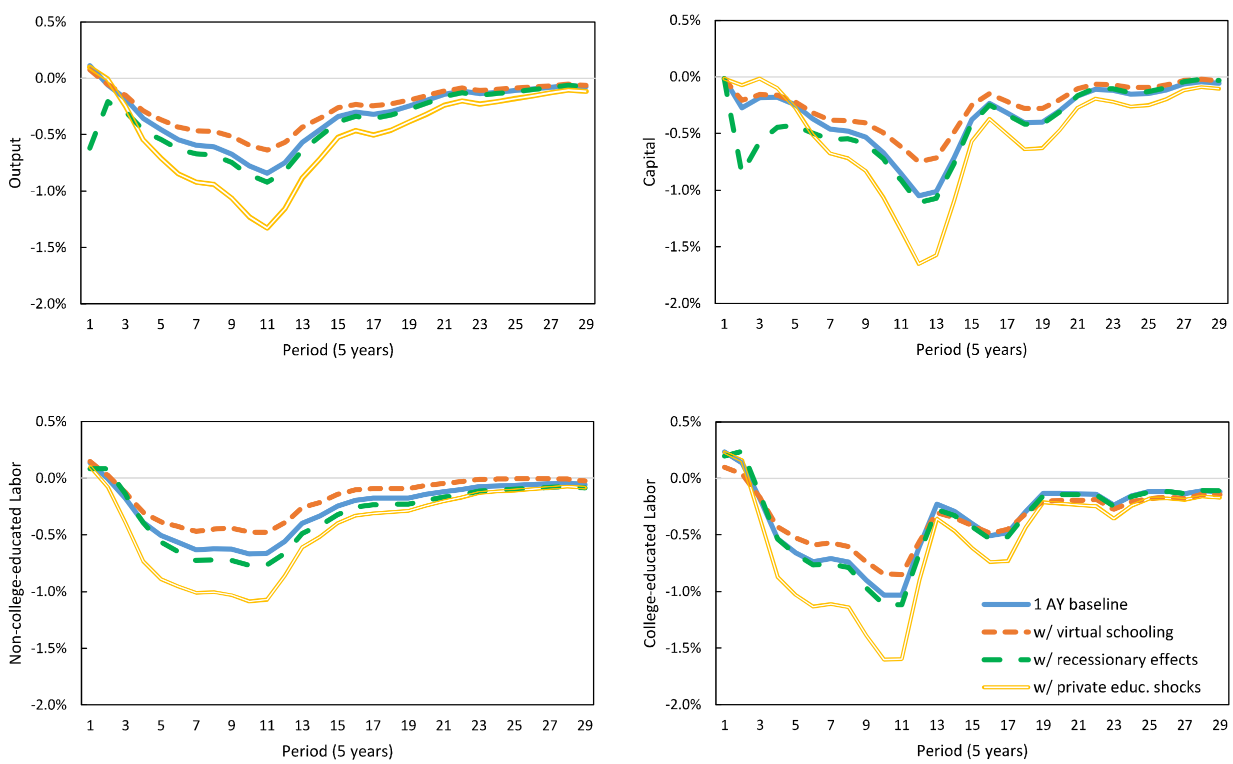


Table 8: Effects on lifetime income statistics in Extended Models

	IGE			Rank cor.			Upward Mobility		
Steady state	.413			.392			6.8%		
Closure length: 1 AY	**% change** rel. to no school closure, by cohort								
	C1	C2	C3	C1	C2	C3	C1	C2	C3
Baseline	0.3	4.5	5.0	0.2	4.1	4.6	-0.6	-6.8	-8.3
+ Virtual schooling	0.4	5.7	6.0	0.3	4.8	5.2	-0.7	-8.7	-10.1
+ Recession effects	0.3	4.4	5.0	0.2	4.0	4.6	-0.4	-6.6	-8.3
+ Private educ. shock	0.3	2.8	2.8	0.2	2.7	2.9	-0.9	-3.8	-3.8

	Lifetime income Gini index			Lifetime income Average			Fraction of College-educated		
Steady state	.282			4.2 (rel. to Y_s)			.336		
	% change rel. to no school closure, by cohort								
	C1	C2	C3	C1	C2	C3	C1	C2	C3
Baseline	0.1	0.6	0.7	-0.1	-3.3	-3.4	1.4	-2.5	-3.2
+ Virtual schooling	0.1	1.0	1.1	-0.0	-2.1	-2.2	0.8	-1.6	-2.1
+ Recession effects	0.1	0.6	0.6	-0.2	-3.4	-3.5	1.3	-2.5	-3.2
+ Private educ. shock	0.1	0.3	0.1	0.1	-4.9	-5.6	2.1	-3.1	-4.2

Note: Y_s denotes steady-state output per capita.

income very little for C1 and by around -2% for C2 and C3. These are much weaker than the baseline results (-0.1% for C1 and around -3% for C2 and C3). Figure 6 also shows that the responses of macroeconomic aggregates are much dampened. On the other hand, we find that the model generates stronger impacts on intergenerational mobility, as compared to the baseline experiment (e.g., IGE increases by 0.4%, 5.7% and 6.0% for C1, C2, and C3, respectively, as compared to 0.3%, 4.5% and 5.0%, respectively). The effects of school closures on lifetime income inequality almost doubles for C2 and C3, raising a Gini coefficient by nearly 1%. These results suggest that virtual schooling that disproportionately benefits children from more-educated parents could mitigate average income losses at the expense of lower intergenerational mobility and higher inequality.

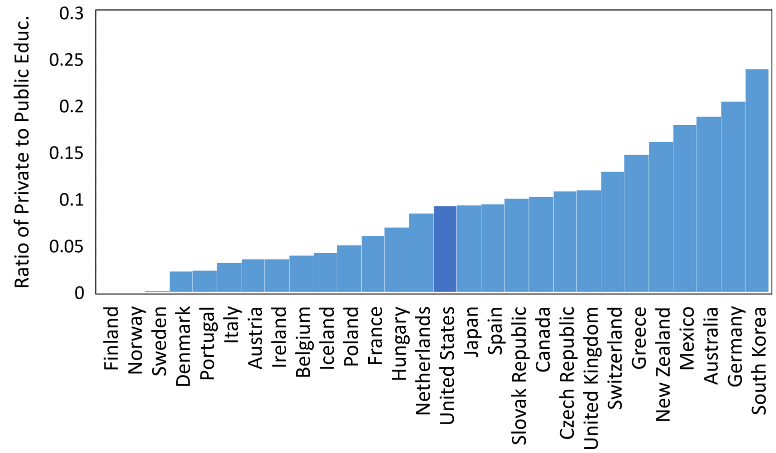
Recessionary Effects of COVID-19 COVID-19 not only brought about school closures. It also led to a severe recession with the GDP decline of 3.5% in 2020, according to the Congressional Budget Office. This negative effect on income could adversely affect parental monetary investments in children. The recessionary effect can be heterogeneous across different groups. Lee, Park, and Shin (2021) provide empirical evidence on unequal effects of COVID-19 on employment across various dimensions such as gender, race, and education. They find that although the initial employment impacts of COVID-19 in April 2020 were quite unequal (e.g., a greater fall in employment among the less educated), these differences disappeared by November 2020.

Given this empirical evidence and our focus on long-term implications, we now explore how a negative effect of COVID-19 on the GDP, along with school closures, could affect the long-term consequences obtained from our baseline experiment. Specifically, we assume that the year-long school closure shock is accompanied by a 3.5% year-long decline in the total factor productivity in $t = 1$, implying that z_1 becomes lower by 0.7%.

Figure 6 clearly shows that there is a strong recession in $t = 1$ caused by the fall in total factor productivity unlike the baseline case. However, medium- and long-run effects are relatively similar to the baseline case, implying that the baseline school closure effects on macroeconomic aggregates in the medium- and long-term are much heavily shaped by the factors present in our baseline experiment, compared to the short-run recessionary effects of COVID-19. On the other hand, Table 8 shows that the recessionary effects do not quantitatively influence the affected children's lifetime income losses, intergenerational mobility and inequality, in line with Fuchs-Schündeln et al. (2022).

Shocks to Productivity of Private Monetary Investment Our baseline school closure experiments consider a temporary shock to ς^g since public investments g capture public school expenditures. However, COVID-19 might also affect the productivity of private monetary investments since some private education activities such as private tutoring might be limited during a pandemic. Therefore, we now investigate how a temporary shock to productivity in private monetary investment ς^e, in addition to the shock to ς^g, will affect our benchmark results. As an illustrative

Figure 7: The ratio of private to public spending on education across OECD countries



Note: The ratio of private to public education spending is constructed using data from Education at a Glance, an annual OECD report, for the years 1995–2015. Public spending on education includes direct expenditure on educational institutions as well as educational-related public subsidies given to households and administered by educational institutions. Private spending on education refers to expenditure funded by private sources including households and other private entities. Only spending relevant to primary to post-secondary non-tertiary is taken into account.

example, we assume that $\varsigma^e_{t=1}$ is reduced by 49.6% and 64.6% of the fall in $\varsigma^g_{t=1}$ for C2 and C3, respectively. These values are based on the share of private school tuitions relative to total private monetary expenditures by parents (Lee and Seshadri 2019). Hence, these shocks to $\varsigma^e_{t=1}$ could capture a scenario where private schools are also equally closed. Since private schools were closed to a lesser extent in practice, it would provide an upper bound of the effects of private school closure shocks.

Figure 6 shows that the initial decrease in aggregate capital in the baseline case is substantially weakened when there is a shock to private monetary investment. This arises because these adverse private education productivity shocks essentially reduce parents' incentive to make up the learning loss caused by the school closure in period 1. Overall, the extent to which macroeconomic aggregates fall is larger. This is due to both the direct productivity effect through the lower values of $\varsigma^e_{t=1}$ and the indirect effect driven by smaller compensatory parental investments. As parents' endogenous responses in terms of parental investments are less effective, Table 8 shows that adverse effects on intergenerational mobility become much weaker in the presence of the shocks to private education investment.

4.3 Role of the Elasticity of Substitution between Private and Public Investments

To examine the role of the elasticity of substitution between private and public investments, we consider an alternative model economy with a higher elasticity of substitution (6 or $\psi = 5/6$) than the baseline economy (3 or $\psi = 2/3$) and recalibrate the model to match the same set of target statistics presented in Table 2.

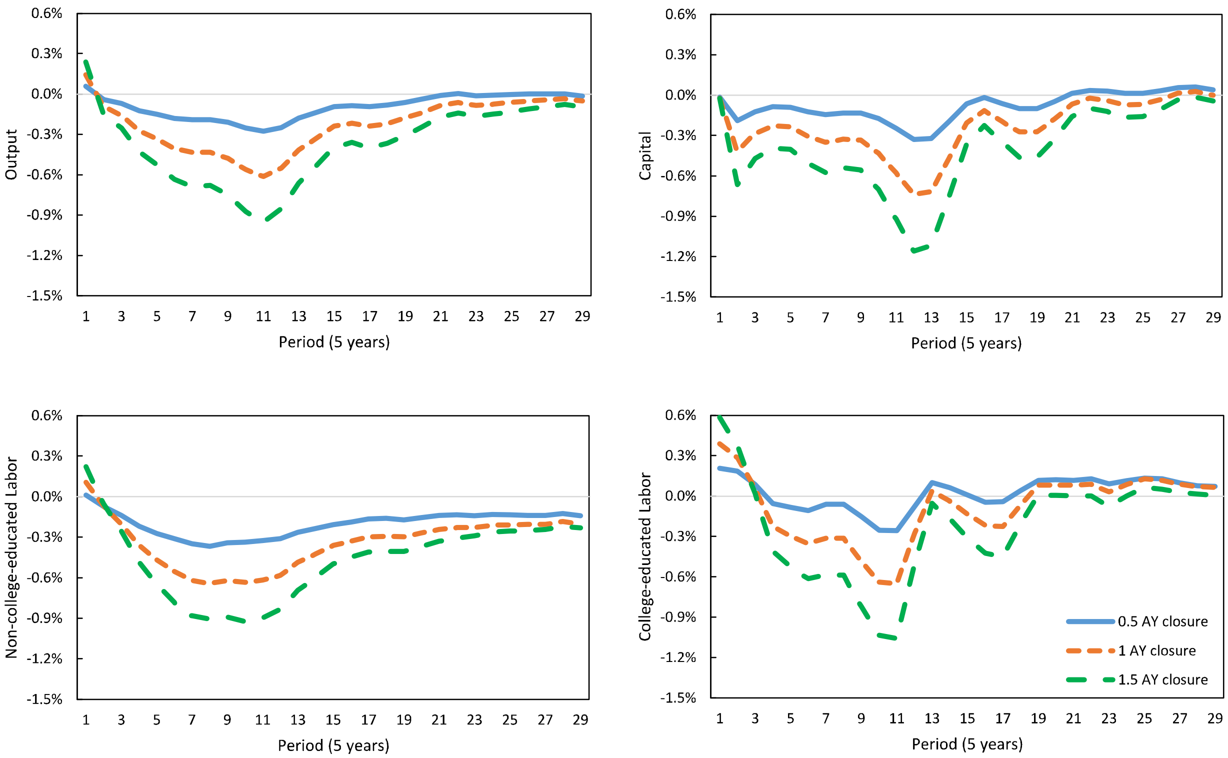
This elasticity of substitution, $1/(1-\psi)$, could vary across countries, and it could shape the observed patterns of the ratio of private to public education spending, which vary quite substantially as shown in Figure 7. Specifically, in Appendix E, we present a simple model that we use to characterize conditions under which the demand for private education relative to public education increases with substitutability between private and public education. This positive relationship holds when subsidies (or tax credits) related to private investment exist for children, or private investment is relatively more important than public investment. It is then possible that East Asian countries, such as South Korea (at the right corner of Figure 7), where private education is prevalent and large in market size (Kim, Tertilt, and Yum 2021), features a higher elasticity of substitution than Scandinavian countries (at the left corner of Figure 7) where the public sector plays a major role in education. Given this, our analysis herein could provide useful considerations for different countries with different approaches to public and private education.

Figure 8 shows the aggregate level evolution of output, capital, efficiency units of labor for non-college and college graduates in the case with $\psi = 5/6$. As shown in a comparison of Figure 1, although all these aggregate variables fall as in the baseline case, the magnitudes are smaller in the case with a higher elasticity of substitution. While the 1-year-closure decreases the aggregate output by up to 0.8% in the case with an elasticity of substitution of 3 (Figure 1), it does so by around 0.6% in the case with an elasticity of substitution of 6. The largest drop in aggregate capital in $t = 12$ in the case with a higher elasticity of substitution is also less pronounced than in the baseline economy. Effective labor units for both non-college and college graduates also displays smaller reductions. These results suggest that school closures bring smaller declines in aggregate variables for countries wherein public educational investment is easier to substitute with private educational investment, such as South Korea.

Table 9 shows intergenerational mobility of lifetime income in the case with a higher elasticity of substitution between public and parental investments. As revealed by a comparison with Table 6, as the degree of substitutability increases, the effects of school closures become stronger on intergenerational mobility. In all cases with three different closure lengths, increases in IGEs in the case with $\psi = 5/6$ are much larger than in the baseline model. Likewise, increases in the rank correlation in the case with a higher elasticity of substitution is noticeably larger. The upward mobility also displays similar patterns: declines in the upward mobility rate in the case with a higher elasticity of substitution are substantially more prominent than those in the baseline model.

As demonstrated previously with Figures 3 and 4, for C2 and C3, the substitution of the school-

Figure 8: Evolution of macroeconomic aggregates with a higher elasticity of substitution between private and public investments



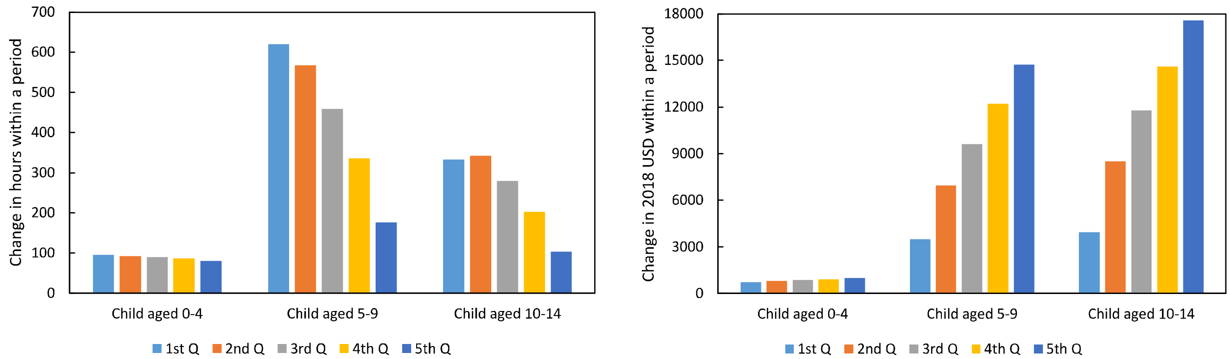
Note: The elasticity of substitution between private and public investments is equal to 6.

Table 9: Effects on intergenerational mobility of lifetime income with a higher elasticity of substitution between private and public investments

	IGE			Rank cor.			Upward Mobility		
Steady state	.425			.405			6.6%		
	% change rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.0	2.8	3.4	0.0	2.6	3.1	0.0	-5.7	-6.4
1.0 AY	0.1	5.5	6.7	0.1	5.0	6.1	0.0	-11.0	-12.7
1.5 AY	0.2	8.2	10.0	0.1	7.3	9.0	0.0	-16.7	-19.3

Note: The elasticity of substitution between private and public investments is equal to 6.

Figure 9: Parental responses by parental permanent income with a higher elasticity of substitution between public and parental investments



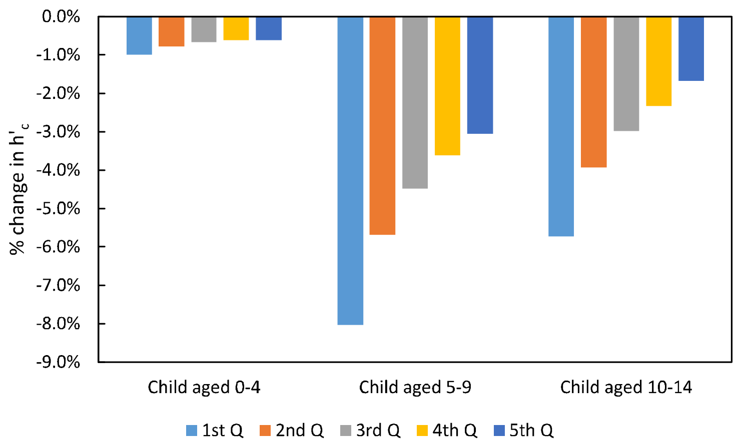
Note: A set of five bars plots changes in parental investments by the quintile of parent's permanent income for each cohort, ordered by the child's age group during the 1-year school closure. The left shows time investment responses and the right shows monetary investment responses.

closure-induced reductions in human capital with parental financial investments plays an important role in increasing differences in the responses of child human capital to school closures across parental permanent income groups, thereby reducing intergenerational mobility. These differences in the case with a higher elasticity of substitution are greater than those in the baseline case because this higher elasticity strengthens parents' incentive to compensate for school closures. As a result, Figure 9 shows that, on average, parental responses are substantially stronger in the model with $\psi = 5/6$ as compared to the baseline model. These more inflated responses in parental investments result in larger gaps in child human capital changes, as shown in a comparison between Figures 4 and 9. These findings imply that in countries where public investments are more easily replaceable with private ones, school closures could have more adverse impacts on inequality and intergenerational mobility.

Finally, Table 10 shows the responses of the average and inequality of lifetime income to school closures in the case with $\psi = 5/6$. As compared to the baseline model (Table 7), these school closures have stronger impacts on lifetime inequality, especially for older children (Cohort 3). However, school closures generally induce smaller losses in lifetime income in this model. As mentioned previously, under a higher elasticity of substitution between private and public investments, it is easier to compensate for the lack of public investments with parental financial investments, thus mitigating overall loss of child human capital, as shown in the comparison between Figures 4 and 10. Therefore, this muted reduction in child human capital leads to smaller decreases in overall college attainment and milder drops in average lifetime income.

To summarize, a higher elasticity of substitution between private and public investments leads to a smaller reduction in the aggregate variables and average lifetime income but a larger reduction in

Figure 10: Child human capital changes by parental permanent income with a higher elasticity of substitution between public and parental investments



Note: This figure plots percent changes in the next period human capital (relative to the case without school closure shocks) of Cohort 2 (C2) on impact and in the following period. A model period corresponds to five years.

Table 10: Effects on inequality and loss of lifetime income with a higher elasticity of substitution between public and parental investments

	Lifetime income						Fraction of		
	Gini			Average			College-educated		
Steady state	.281			4.2 (rel. to Y_s)			.342		
	% change rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.0	0.4	0.5	-0.0	-1.3	-1.4	0.4	-1.4	-1.8
1.0 AY	0.0	0.8	1.0	-0.0	-2.5	-2.7	1.1	-2.5	-3.3
1.5 AY	0.0	1.2	1.5	-0.1	-3.7	-4.0	1.8	-3.7	-5.0

Note: The elasticity of substitution between private and public investments is equal to 6. Y_s denotes steady-state output per capita.

intergenerational mobility. These results are driven by stronger substitutions by parental financial investments, generating overall dampened but more dispersed changes in child human capital across parental permanent income groups.

5 Conclusion

In this paper, we have investigated how school closures affect the aggregate economy, inequality, and intergenerational mobility through intergenerational human capital transmissions in the medium and long term. Using a dynastic overlapping generations general equilibrium model wherein altruistic parents invest in their children's human capital, which complements public schooling, we have found three main results. First, school closures bring about long-lasting adverse effects on the aggregate economy. General equilibrium effects play a substantial role in reshaping aggregate variables' dynamics. Second, school closures reduce the average lifetime income and intergenerational mobility of directly affected children, and these reductions are more severe for older children. These results are driven mainly by parental investment responses that differ by a child's age and parental income.

Moreover, we have shown that substitutability between private and public investment shapes school closure costs along different dimensions. While a higher elasticity of substitution induces less significant damages in the aggregate economy and overall lifetime incomes of the affected children, it exacerbates reductions in intergenerational mobility and raises inequality. Therefore, our result has interesting implications for the role of government. Depending on the degree of this substitutability and the social welfare function that puts different weights on aggregate efficiency and inequality or immobility aversion, the cost of school closures of the same length can vary substantially.[31]

Given these clear, interesting differences driven by substitutability between public and parental investments, we believe that school closure shocks might provide good opportunities to estimate the elasticity of substitution between private and public investments, which could vary across countries.[32] Likewise, our model framework would be useful for studying unexplored interesting research topics as data become more available and more accessible. For example, an interesting normative question is how to optimally make up for losses from school closures dynamically. We leave these interesting and important questions for future work.

[31] For example, the cost of school closures can be considered negligible if a country features high substitutability between private and public investment and cares little about its consequences for intergenerational mobility and inequality.

[32] Given the possibility that private education might take some time to adjust in its size, it seems necessary for such empirical analysis to address the presence of fixed costs in such private education businesses.

References

Adams-Prassl, Abi, Teodora Boneva, Marta Golin, and Christopher Rauh. 2020. "Inequality in the impact of the coronavirus shock: Evidence from real time surveys." *Journal of Public Economics*, 189: 104245.

Agostinelli, Francesco, Matthias Doepke, Giuseppe Sorrenti, and Fabrizio Zilibotti. 2022. "When the Great Equalizer Shuts Down: Schools, Peers, and Parents in Pandemic Times." *Journal of Public Economics* 206: 104574.

Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *The Quarterly Journal of Economics* 109 (3): 659-684.

Alon, Titan, Matthias Doepke, Jane Olmstead-Rumsey, and Michele Tertilt. 2020. "This Time it's Different: The Role of Women's Employment in a Pandemic Recession." CEPR DP No. 15149

Andrew, Alison, Sarah Cattan, Monica Costa-Dias, Christine Farquharson, Lucy Kraftman, Sonya Krutikova, Angus Phimister, and Almudena Sevilla. 2020. "Family time use and home learning during the Covid-19 lockdown." The Institute for Fiscal Studies.

Atteberry, Allison and Andrew McEachin. 2020. "School's Out: The Role of Summers in Understanding Achievement Disparities." *American Educational Research Journal.*

Bacher-Hicks, Andrew, Joshua Goodman, and Christine Mulhern. 2021. "Inequality in Household Adaptation to Schooling Shocks: Covid-Induced Online Learning Engagement in Real Time" *Journal of Public Economics*, 193: 104345.

Becker, Gary S. and Nigel Tomes. 1986. "Human Capital and the Rise and Fall of Families." *Journal of Labor Economics* 4 (3): S1-39.

Benabou, Roland. 2002. "Tax and Education Policy in a heterogeneous-agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?" *Econometrica* 70 (2): 481-517.

Blankenau, William and Xiaoyan Youderian. 2015. "Early Childhood Education Expenditures and the Intergenerational Persistence of Income." *Review of Economic Dynamics* 18 (2): 334-349.

Caucutt, Elizabeth M. and Lance Lochner. 2020. "Early and Late Human Capital Investments, Borrowing Constraints, and the Family." *Journal of Political Economy* 128 (3): 1065-1147.

Chetty, Raj, John N. Friedman, Nathaniel Hendren, and Michael Stepner. 2020. "How did Covid-19 and Stabilization Policies Affect Spending and Employment? a New Real-Time Economic Tracker Based on Private Sector Data." NBER Working Paper No. 27431

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. 2013. "Does Indivisible Labor Explain the Difference between Micro and Macro Elasticities? A Meta-Analysis of Extensive Margin Elasticities." In *NBER Macroeconomics Annual* 2012, Volume 27: University of Chicago Press.

Chetty, Raj, Nathaniel Hendren, Patrick Kline, and Emmanuel Saez. 2014. "Where is the land of opportunity? The geography of intergenerational mobility in the United States." *The Quarterly Journal of Economics* 129 (4): 1553-1623.

Ciccone, Antonio and Giovanni Peri. 2005. "Long-Run Substitutability between More and Less Educated Workers: Evidence from US States, 1950–1990." *Review of Economics and Statistics* 87 (4): 652-663.

Cooper, Harris, Barbara Nye, Kelly Charlton, James Lindsay, and Scott Greathouse. 1996. "The Effects of Summer Vacation on Achievement Test Scores: A Narrative and Meta-Analytic Review." *Review of Educational Research* 66 (3): 227-268.

Cunha, Flavio and James J. Heckman. 2007. "The Technology of Skill Formation." *American Economic Review* 97 (2): 31-47.

Daruich, Diego. 2022. "The Macroeconomic Consequences of Early Childhood Development Policies" Unpublished Manuscript.

Del Boca, Daniela, Christopher Flinn, and Matthew Wiswall. 2014. "Household Choices and Child Development." *The Review of Economic Studies* 81 (1): 137-185.

Engzell, P., A. Frey, and M. D. Verhagen. 2021. "Learning Loss due to School Closures during the COVID-19 Pandemic." *Proceedings of the National Academy of Sciences of the United States of America* 118 (17): 10.1073/pnas.2022376118.

Fernandez, Raquel and Richard Rogerson. 1998. "Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform." *American Economic Review* 88 (4): 813-833.

Fuchs-Schündeln, Nicola, Dirk Krueger, Alexander Ludwig, and Irina Popova. 2022. "The Long-Term Distributional and Welfare Effects of Covid-19 School Closures." *The Economic Journal* 132 (645): 1647-1683.

Guner, Nezih, Remzi Kaygusuz, and Gustavo Ventura. 2014. "Income Taxation of US Households: Facts and Parametric Estimates." *Review of Economic Dynamics* 17 (4): 559-581.

Grewenig, Elisabeth, Philipp Lergetporer, Katharina Werner, Ludger Woessmann, and Larissa Zierow. 2021. "COVID-19 and Educational Inequality: How School Closures Affect Low-and High-Achieving Students." *European Economic Review* 140: 103920.

Guryan, Jonathan, Erik Hurst, and Melissa Kearney. 2008. "Parental Education and Parental Time with Children." *The Journal of Economic Perspectives* 22 (3): 23-16A.

Haider, Steven J. and Gary Solon. 2006. "Life-Cycle Variation in the Association between Current and Lifetime Earnings." *American Economic Review* 96 (4): 1308-1320.

Hanushek, Eric A. and Ludger Woessmann. 2020. "The Economic Impacts of Learning Losses." *OECD Education Working Papers*, No. 225, OECD Publishing, Paris.

Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante. 2010. "Unequal we Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006." *Review of Economic Dynamics* 13 (1): 15-51.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante. 2014. "Consumption and Labor Supply with Partial Insurance: An Analytical Framework." *American Economic Review* 104 (7): 2075-2126.

Heckman, James J. 2008. "Schools, Skills, and Synapses." *Economic Inquiry* 46 (3): 289-324.

Herrington, Christopher M. 2015. "Public Education Financing, Earnings Inequality, and Intergenerational Mobility." *Review of Economic Dynamics* 18 (4): 822-842.

Holter, Hans A. 2015. "Accounting for cross-country Differences in Intergenerational Earnings Persistence: The Impact of Taxation and Public Education Expenditure." *Quantitative Economics* 6 (2): 385-428.

Holter, Hans A., Dirk Krueger, and Serhiy Stepanchuk. 2019. "How does Tax Progressivity and Household Heterogeneity Affect Laffer Curves?" *Quantitative Economics* 10 (4): 1317-1356.

Isphording, Ingo E., Marc Lipfert, and Nico Pestel. 2021. "Does re-opening schools contribute to the spread of SARS-CoV-2? Evidence from staggered summer breaks in Germany" *Journal of Public Economics* 198: 104426.

Jones, Larry E. and Rodolfo E. Manuelli. 1999. "On the Taxation of Human Capital." Unpublished Manuscript.

Kim, Seongeun, Michele Tertilt, and Minchul Yum. 2021. "Status Externalities in Education and Low Birth Rates in Korea." CEPR DP No. 16271.

Kotera, Tomoaki and Ananth Seshadri. 2017. "Educational Policy and Intergenerational Mobility." *Review of Economic Dynamics* 25: 187-207.

Kuhfeld, Megan, James Soland, Beth Tarasawa, Angela Johnson, Erik Ruzek, and Jing Liu. 2020. "Projecting the potential impacts of COVID-19 school closures on academic achievement." EdWorkingPaper No. 20-226

Lee, Sang Yoon Tim, Minsung Park, and Yongseok Shin. 2021. "Hit Harder, Recover Slower? Unequal Employment Effects of the Covid-19 Shock," NBER Working Papers No. 28354

Lee, Sang Yoon and Ananth Seshadri. 2019. "On the Intergenerational Transmission of Economic Status." *Journal of Political Economy* 127 (2): 855-921.

Livshits, Igor, James MacGee, and Michele Tertilt. 2010. "Accounting for the Rise in Consumer Bankruptcies." *American Economic Journal: Macroeconomics* 2 (2): 165-193.

Meyers, Keith, and Melissa A. Thomasson. 2017. "Paralyzed by panic: Measuring the effect of school closures during the 1916 polio pandemic on educational attainment." NBER Working Papers No. 23890

Ramey, Garey and Valerie A. Ramey. 2010. "The Rug Rat Race." *Brookings Papers on Economic Activity* 41 (1): 129-199.

Restuccia, Diego and Carlos Urrutia. 2004. "Intergenerational Persistence of Earnings: The Role of Early and College Education." *American Economic Review* 94 (5): 1354-1378.

Rupert, Peter and Giulio Zanella. 2015. "Revisiting Wage, Earnings, and Hours Profiles." *Journal of Monetary Economics* 72: 114-130.

Samaniego, Roberto, Remi Jedwab, Paul Romer, and Asif Islam. 2022. "Scars of Pandemics from Lost Schooling and Experience: Aggregate Implications and Gender Differences Through the Lens of COVID-19." Unpublished Manuscript.

Yum, Minchul. 2021. "Parental Time Investment and Intergenerational Mobility." Available at SSRN: `https://ssrn.com/abstract=3862378`.

Appendix

A Calibration Details

Most calibration targets are based on samples from the 2003-2017 waves of the ATUS, combined with the Current Population Survey (Yum 2021). Table A1 reports the estimation results that are used to compute the educational gradients in parental time investments. The sample is restricted to households who have any number of children and aged between 21 and 55 (inclusive), as in Guryan et al. (2008). The three periods in the model ($j = 3, 4, 5$) correspond to the youngest children's age bands: ages 0-4, ages 5-9, and ages 10-14, respectively. The coefficient on the dummy college variable, divided by the corresponding average, captures the educational gradient while controlling for parents' sex, age, and marital status. We note that the college coefficients are quite stable regardless of control variables, in line with the evidence in Guryan et al. (2008).

Table A1: Education gradients in parental time investments

	$j = 3$	$j = 4$	$j = 5$
College-educated	1.342	.561	.416
	(.133)	(.109)	(.091)
Sex	-2.62	-1.51	-1.20
	(.123)	(.101)	(.083)
Age	-.041	.016	.023
	(.009)	(.007)	(.006)
Married	-.911	-.318	-.102
	(.085)	(.064)	(.053)
R^2	.023	.014	.017
Average x	6.43	3.78	2.06

Notes: Numbers in parentheses are standard errors. The dependent variable is parental time investments (weekly hours). These estimates are from Yum (2021).

Table A2 reports the gross growth rates of human capital by age and education. These are computed based on the estimates from the PSID samples in Rupert and Zanella (2015).

Table A3 reports the estimates of τ_j and λ_j in labor taxation by age, obtained from Holter et al. (2019). We use the estimates for single households for $j = 1, 2$, and the estimates for married households for the later periods (either with a child for $j = 3, ..., 6$ or without children for $j = 7, 8, 9$).

Table A2: Gross growth rates of human capital by age and education

$j =$	1	2	3	4	5	6	7	8
$\gamma_{j,1}$	1.231	1.052	1.017	1.004	0.998	0.995	0.994	0.994
$\gamma_{j,2}$	1.317	1.152	1.101	1.063	1.032	1.004	0.975	0.942

Notes: The reported values are based on the estimates from the PSID samples in Rupert and Zanella (2015).

Table A3: Parameter values for progressive taxation

	τ_j	λ_j
$j = 1, 2$	.1106	.8177
$j = 3, ..., 6$	.1585	.9408
$j = 7, 8, 9$	.1080	.8740

Notes: The reported values are based on the estimates in Holter et al. (2019).

B Aggregation order in a Nested CES Technology and Parental Time Responses

In this section, we illustrate the implication of a different order of aggregation in a nested CES technology for parental time responses following school closures. Recall that our baseline technology aggregates parental time and monetary investments, which are then aggregated with public investment. As discussed in Jones and Manuelli (1999), one could consider an alternative order. Here, we consider a specification where parental monetary investment e and public investment g are aggregated first, and then the composite monetary investment is aggregated with parental time investment x.

Specifically, consider a simplified household's optimization problem:

$$\max_{c,x,e} \left\{ \log c - bx + \eta \log h' \right\}$$

subject to

$$c + e = m$$

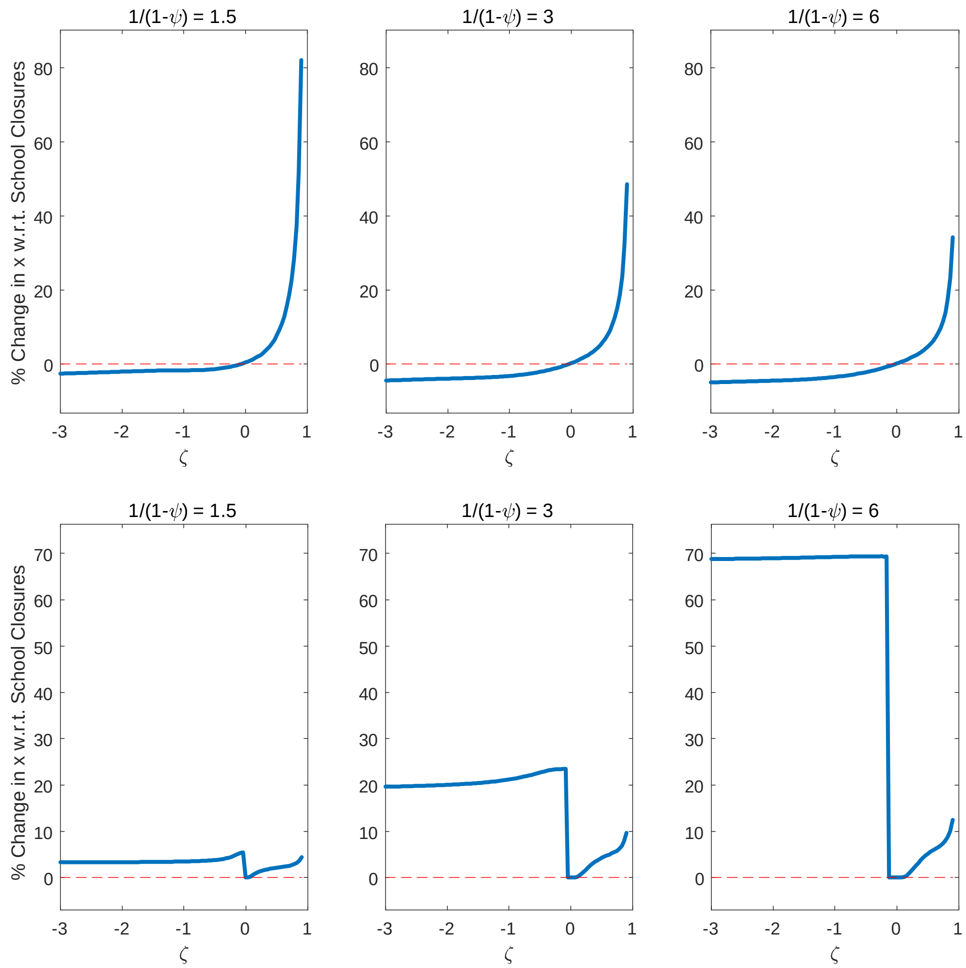
$$h' = \left\{ x^{\zeta} + \left(e^{\psi} + g^{\psi} \right)^{\frac{\zeta}{\psi}} \right\}^{\frac{1}{\zeta}}, \tag{A1}$$

where the human capital investment function (A1) features a nested CES technology with the alternative order of aggregation described above.[33] The structure is simple yet is similar to the

[33]We abstract from share parameters to focus on our key message about the order of aggregation.

quantitative model: b denotes the disutility of investing time, η captures altruism, m denotes disposable income, $\psi \leq 1$ shapes the elasticity of substitution between parental monetary investment and public investment, and $\zeta \leq 1$ governs the elasticity of substitution between parental time and the aggregated monetary investments.

Figure A1: Aggregation order of a nested CES technology and parental time responses to school closures



Note: The top panel shows percent changes in optimal x with respect to a 10% decline in g with a CES aggregation order which aggregates parental monetary investment and public investment first (A1). The bottom panel shows the counterparts with a CES aggregation order (A2), which aggregates parental time and monetary investments first in line with the one used in our quantitative model. We plot these effects for the three different values of ψ, as in the main text. The figures are based on the following values of parameters: $b = 1, \eta = 0.3, m = 10$ and $g = 1$.

The optimal parental time x is a function of g. The upper panel of Figure A1 shows how the optimal parental time investment responds with respect to a 10% decline in g with different values of ζ. The key takeaway is that the sign of changes in parental time responses depends on

ζ. Specifically, its sign is the same as the sign of ζ.[34] Intuitively, when parental time is strongly complementary to monetary investments (i.e., a more negative ζ), a decline in the latter would induce parents to reduce parental time investment as well. To see how this relationship might be affected by the elasticity of substitution between public and private investments, we plot these effects for the three different values of ψ, as in the main text. They all show the same qualitative effect of ζ on parental time responses despite their quantitative difference.

We now replace (A1) with

$$h' = \left\{ \left(x^{\zeta} + e^{\zeta} \right)^{\frac{\psi}{\zeta}} + g^{\psi} \right\}^{\frac{1}{\psi}}, \tag{A2}$$

which features the aggregation order used in our quantitative model. As shown in the bottom panel of Figure A1, this specification gives the response of parental time investment that is non-negative regardless of the value of ζ. As the value of ζ can be either positive or negative depending on the age group (Table 2), this implies that the aggregation order according to (A2) would imply that school closures lead to increases in parental time across all age groups. This is in contrast to the model with the aggregation order acording to (A1), which could lead to a decrease in parental time investment following school closures when parental time and monetary investments are strongly complementary to each other.

C Input Normalization in CES Technology

In our model, we divide inputs by their corresponding means in CES production functions: (10)–(12). This normalization helps us to achieve computational stability in our overlapping generations model by keeping human capital distributions within certain ranges while varying parameters related to the elasticity of substitution. The key source of the issue is the scale effects of changing the elasticity of substitution parameter in CES production functions.

To illustrate this, consider a CES production function:

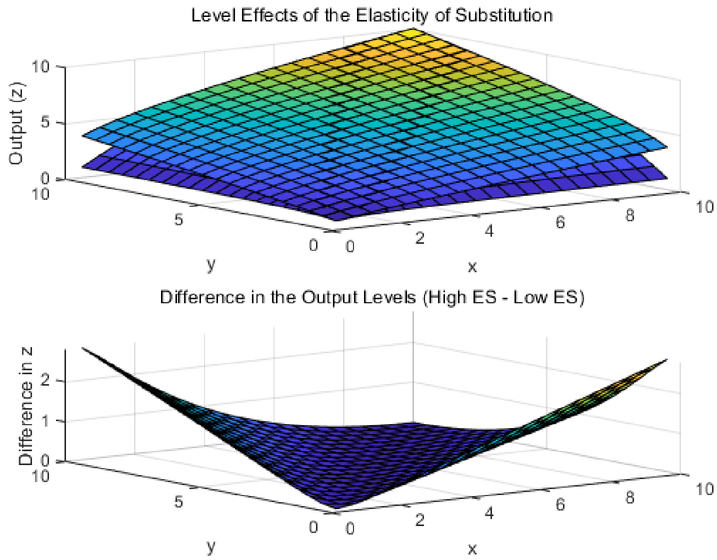
$$z = \left(0.5x^{\psi} + 0.5y^{\psi} \right)^{\frac{1}{\psi}} \tag{A3}$$

where $\psi \leq 1$, which determines the elasticity of substitution: $1/(1-\psi)$. The top panel of Figure A2 plots this function for two different values of ψ, -1 (below) and 0.5 (top), so that the corresponding elasticity of substitution becomes 0.5 and 2.0, respectively. It is clear to see that a higher ψ (or a higher elasticity of substitution) has positive level effects on the output especially when two input values (x and y) differ from each other.

To see its implications for output distributions more clearly, we simulate a sample of 100,000 where two inputs are randomly drawn from two normal distributions: $x \sim N(20, 1)$ and $y \sim N(10, 1)$. Then, we generate z according to (A3) again with two different values of $\psi : -1$ and 0.5.

[34] This result can be shown analytically in this simple model.

Figure A2: CES production functions with different elasticities of substitution



Note: The top panel shows the output level implied by the CES technology (A3) with a high elasticity of substitution (2.0, top) or a low elasticity of substitution (0.5, below). The bottom panel shows their difference in the output levels.

Figure A3 shows that the implied distribution of z is shifted to the right with its mean being 9.5% higher with the higher elasticity of substitution.

We also generate z according to the same technology with the two values of ψ *after* we divide each input by its corresponding mean:

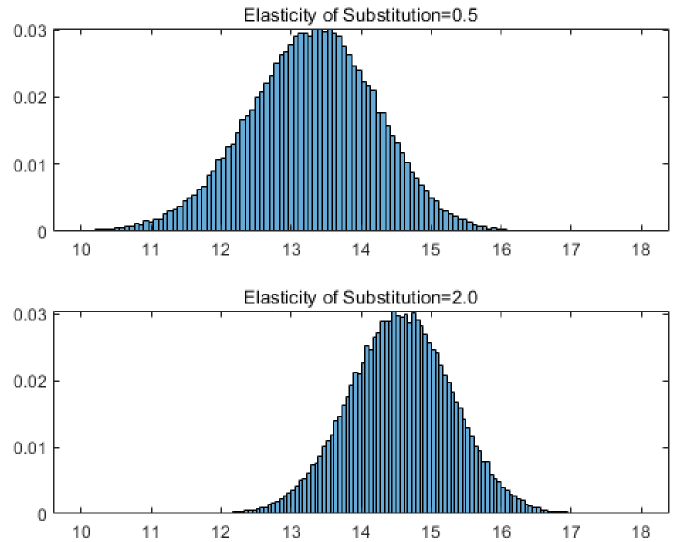
$$z = \left(0.5\,(x/\bar{x})^{\psi} + 0.5\,(y/\bar{y})^{\psi}\right)^{\frac{1}{\psi}} \tag{A4}$$

Figure A4 shows that the level effects are much mitigated: the mean difference is now very low at around 0.2%.

D Partial (Stochastic) Closures

We also consider additional experiments based on partial school closures. Specifically, we assume that school closures are still unexpected but there is another dimension of uncertainty: half of the agents still experience full closures, but the other half experience a school closure of limited intensity. This within-period variation could capture additional closures due to local outbreaks of COVID-19 cases even after re-opening nationwide. This could also capture the variability of effectiveness of online substitute teaching by schools. The results reported below are based on a partial intensity of 50%. As shown in Figure A5, and Tables A4, A5 and A6, our findings suggest that the main findings are generalizable in terms of the relationship between average school closure

Figure A3: Distributions of CES outputs with different elasticities of substitution



Note: The distribution of outputs implied by the CES technology (A3) is shown in the top panel for a low elasticity of substitution (0.5) and in the bottom panel for a high elasticity of substitution (2.0).

length and the corresponding aggregate effects. But they also suggest that partial closures induce additional variations that happen within each cohort, as shown in the bottom two panels of Tables A5 and A6.

E Determinants of the Relative Demand of Private to Public Education Investments

Motivated by Jones and Manuelli (1999), we present a simple model to demonstrate how the relative demand of private to public inputs for human capital formation can be shaped by different forces, which include substitutability between the two inputs in the human capital production function.

Specifically, the representative household with a child faces the following optimization problem:

$$\max_{c,e,g} \left\{ \log c + \eta \log h' \right\} \tag{A5}$$

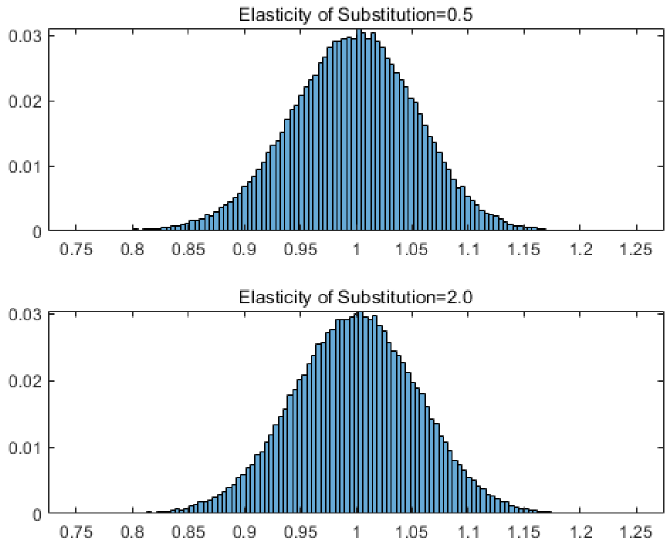
such that

$$c + (1-s)e = w - T \tag{A6}$$

$$h' = \left(\theta \left(\varsigma_e e \right)^{\psi} + (1-\theta) \left(\varsigma_g g \right)^{\psi} \right)^{\frac{1}{\psi}} \tag{A7}$$

where c is consumption, η captures the degree of altruism associated with the child's human capital

Figure A4: Distributions of CES outputs with different elasticities of substitution after input normalizations



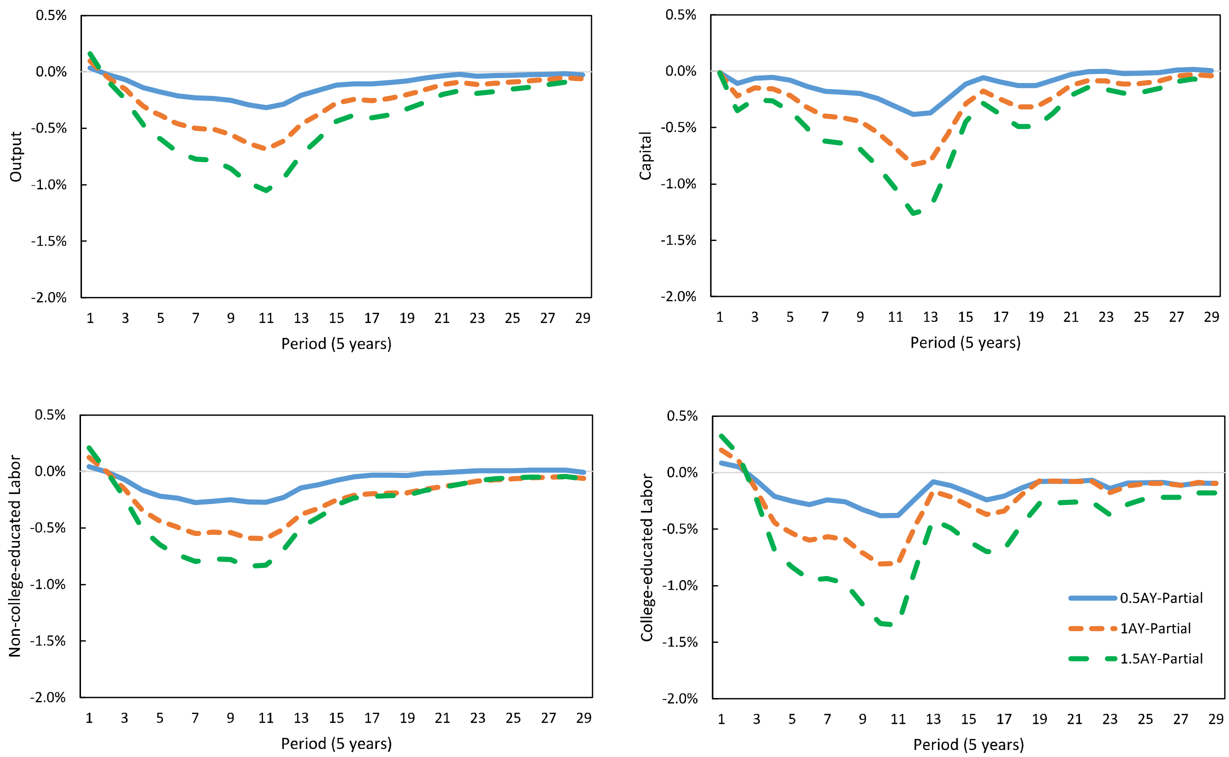
Note: The distribution of outputs implied by the CES technology after input normalizations (A4) is shown in the top panel for a low elasticity of substitution (0.5) and in the bottom panel for a high elasticity of substitution (2.0).

h', s is a subsidy rate for private human capital investment e, T is a lump-sum tax to finance the education subsidies and public education, and w is income. As shown in (A7), child human capital h' is shaped by two inputs—private investments e and public investments g—with a CES aggregator with an elasticity of substitution given by $1/(1-\psi)$. Each input in the production function is allowed to have different shares governed by $\theta \in [0,1]$ and different productivity levels, ς_e and ς_g. To make the illustration cleaner, we assume that both inputs are equally priced in the absence of subsidies.

We note that our goal is to analytically derive a mapping from the parameters related to human capital technology to the relative demand e/g by the representative household in a parsimonious way.[35] Therefore, we take the other government policies such as s and T as given. In the real world, various forms of subsidies to private education for children exist (e.g., income tax credits and childcare subsidies), set out by various factors (e.g., political reasons) other than optimal policy concerns.

[35] Therefore, our exercise herein differs from the Ramsey problem that seeks optimal tax/subsidy system, which is very interesting but is analytically less tractable.

Figure A5: Evolution of macroeconomic aggregates: Partial closures



Note: A half of agents experience full closures whereas the other agents experience partial closures, the intensity of which is given by 50%.

The first-order conditions are then given by:

$$[e] : \frac{-(1-s)}{w-T-(1-s)e} + \frac{\eta\psi\theta\left(\varsigma_e e\right)^{\psi-1}\varsigma_e}{\psi\left(\theta\left(\varsigma_e e\right)^{\psi}+(1-\theta)\left(\varsigma_g g\right)^{\psi}\right)} = 0,$$

$$[g] : \frac{-1}{w-T-(1-s)e} + \frac{\eta\psi(1-\theta)\left(\varsigma_g g\right)^{\psi-1}\varsigma_g}{\psi\left(\theta\left(\varsigma_e e\right)^{\psi}+(1-\theta)\left(\varsigma_g g\right)^{\psi}\right)} = 0.$$

Combining these two, we obtain

$$1-s = \left(\frac{\theta}{1-\theta}\right)\left(\frac{e}{g}\right)^{\psi-1}\left(\frac{\varsigma_e}{\varsigma_g}\right)^{\psi} \Rightarrow \frac{e}{g} = \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\psi}}\left(\frac{1}{1-s}\right)^{\frac{1}{1-\psi}}\left(\frac{\varsigma_e}{\varsigma_g}\right)^{\frac{\psi}{1-\psi}}.$$

This equation tells us that the relative demand of e to g can be shaped by three different forces (and their interactions). First, we begin with the effect of ψ or the elasticity of substitution $1/(1-\psi)$. The above equation implies that the effect of ψ on the ratio, e/g, would interact with the other human capital technology primitives. Specifically, it is positive if $\theta\varsigma_e > (1-s)(1-\theta)\varsigma_g$. This

Table A4: Distributional changes over time: Partial closures

		Time (1 period: 5 years)				
		1	2	3	4	5
	Steady state	**% change** rel. to no school closure				
Closure length: 0.5 AY						
Gini income	.338	0.0	-0.0	0.1	0.2	0.1
Bottom 20% inc (%)	8.1	-0.0	0.0	-0.1	-0.1	-0.0
Share of college (%)	33.6	-0.0	0.0	-0.1	-0.1	-0.1
Closure length: 1 AY						
Gini income	.338	-0.0	-0.1	0.2	0.4	0.2
Bottom 20% inc (%)	8.1	-0.0	0.0	-0.2	-0.2	-0.1
Share of college (%)	33.6	-0.0	0.0	-0.1	-0.3	-0.2
Closure length: 1.5 AY						
Gini income	.338	-0.0	-0.1	0.3	0.5	0.4
Bottom 20% inc (%)	8.1	-0.0	0.1	-0.3	-0.4	-0.1
Share of college (%)	33.6	-0.0	0.1	-0.2	-0.5	-0.3

condition is more likely to be satisfied if there are private education subsidies ($s > 0$) or private investments are relatively more important than public investments (i.e., a higher θ or a higher ratio of ς_e/ς_g). This implies that the representative agent would prefer to invest more through private education e instead of public education g if these two inputs are more substitutable in an economy where private education is subsidized or human capital technology puts more weight on private investments relative to public investments.[36]

The above equation suggests that this relative demand can also be affected by the other human capital technology primitives. Specifically, the relative demand, e/g, increases with θ, which captures the relative share of private investments in the technology. More interestingly, the relative demand increases with ς_e/ς_g, the ratio of productivity levels between private and public investments, provided that $\frac{\psi}{1-\psi} > 0$. This means that a higher productivity of private investments relative to public investments would increase the relative demand only if substitutability between the two inputs is strong enough (i.e., $\psi > 0$ or the elasticity of substitution being greater than one).

[36]In other words, a higher elasticity of substitution can play a role of amplifying the relative demand of private to public education in a society where private investment is relatively more important than public investment.

Table A5: Effects on intergenerational mobility of lifetime income: Partial closures

	IGE			Rank cor.			Upward Mobility		
Steady state	.413			.392			6.8%		
Closure length	**% change** rel. to no school closure, by cohort								
	C1	C2	C3	C1	C2	C3	C1	C2	C3
				All children					
0.5 AY	0.1	1.6	1.8	0.1	1.4	1.6	-0.3	-2.5	-3.0
1.0 AY	0.2	3.3	3.7	0.2	2.9	3.3	-0.4	-5.2	-6.0
1.5 AY	0.3	5.1	5.7	0.2	4.5	5.2	-0.7	-7.8	-8.9
				Children who experienced full closure					
0.5 AY	0.1	2.2	2.4	0.1	2.0	2.3	-0.3	-3.4	-4.1
1.0 AY	0.3	4.5	5.0	0.2	4.2	4.7	-0.6	-7.1	-8.3
1.5 AY	0.5	7.0	7.8	0.3	6.4	7.3	-1.0	-11.3	-12.3
				Children who experienced 50% closure					
0.5 AY	0.1	1.1	1.2	0.0	0.9	1.0	-0.0	-1.5	-2.1
1.0 AY	0.1	2.1	2.4	0.1	1.8	2.0	-0.1	-2.9	-3.9
1.5 AY	0.2	3.3	3.6	0.1	2.7	3.1	-0.4	-4.7	-5.7

F Models with Different Elasticities of Substitution between Private and Public Investments

The baseline model in the main text is calibrated with $\psi = 2/3$. We now report the calibration tables for the economies with a higher value ($\psi = 5/6$) in Table A7 and with a lower value ($\psi = 1/3$) in Table A8. We also report the key experiment results from the model with $\psi = 1/3$ in Figure A6, and Tables A9 and A10.

G Additional Figures and Tables

Table A6: Effects on inequality and loss of lifetime income: Partial closures

	Lifetime income						Fraction of		
	Gini			Average			College-educated		
Steady state	.282			4.2 (rel. to Y_s)			.336		
Closure length	**% change** rel. to no school closure, by cohort								
	C1	C2	C3	C1	C2	C3	C1	C2	C3
				All children					
0.5 AY	0.0	0.2	0.3	-0.0	-1.2	-1.2	0.7	-0.7	-0.9
1.0 AY	0.1	0.5	0.5	-0.0	-2.4	-2.5	1.2	-1.6	-2.1
1.5 AY	0.1	0.7	0.8	-0.1	-3.7	-3.8	1.5	-2.7	-3.5
				Children who experienced full closure					
0.5 AY	0.0	0.3	0.3	-0.1	-1.6	-1.7	0.7	-1.3	-1.6
1.0 AY	0.0	0.6	0.7	-0.1	-3.3	-3.4	1.1	-2.9	-3.4
1.5 AY	0.1	1.0	1.0	-0.2	-5.0	-5.1	1.4	-4.7	-5.5
				Children who experienced 50% closure					
0.5 AY	0.0	0.1	0.1	0.0	-0.8	-0.8	0.8	-0.1	-0.3
1.0 AY	0.0	0.3	0.3	0.0	-1.5	-1.5	1.3	-0.4	-0.8
1.5 AY	0.1	0.5	0.5	0.0	-2.3	-2.5	1.7	-0.8	-1.4

Note: Y_s denotes steady-state output per capita.

Table A7: Internally calibrated parameters and target statistics for the alternative model economy with a higher elasticity of substitution between public and parental investments

Parameter		Target statistics	Data	Model
$\psi = 5/6$		(*elasticity of substitution = 6*)		
β	.939	Equilibrium real interest rate (annualized)	.04	.04
b	6.76	Mean hours of work in $j = 3, ..., 9$	.287	.299
φ	.490	Mean hours of work in $j = 3, 4, 5$	.299	.290
η	.283	Ratio of inter-vivos transfers over total savings	.30	.364
θ_3^x	.819	Mean parental time investments in $j = 3$	.061	.062
θ_4^x	.158	Mean parental time investments in $j = 4$	.036	.036
θ_5^x	.126	Mean parental time investments in $j = 5$	.020	.020
θ_3^p	.517	Rank corr. of parental income & child earnings	.282	.294
θ_3^I	.597	Mean parental monetary investments in $j = 3$	.056	.056
θ_4^I	.665	Mean parental monetary investments in $j = 4$	.136	.130
θ_5^I	.397	Mean parental monetary investments in $j = 5$	.160	.157
ζ_3	-1.75	Educational gradients in parental time in $j = 3$ (%)	20.9	18.6
ζ_4	0.54	Educational gradients in parental time in $j = 4$ (%)	14.8	15.0
ζ_5	0.55	Educational gradients in parental time in $j = 5$ (%)	20.2	21.0
ν	.546	Fraction with a college degree (%)	34.2	34.2
μ_ξ	.226	Average college expenses/GDP per-capita	.140	.140
δ_ξ	.600	Observed college wage gap (%)	75.0	68.1
ρ_ϕ	.011	Intergenerational corr. of percentile-rank income	.341	.398
σ_ϕ	.445	Gini wage	.37	.340
σ_z	.148	Slope of variance of log wage from $j = 2$ to $j = 8$	.18	.184
$\underline{a}$	$-.070$	Average unsecured debt rel. to annual disposable income	.010	.010

Table A8: Internally calibrated parameters and target statistics for the alternative model economy with a lower elasticity of substitution between public and parental investments

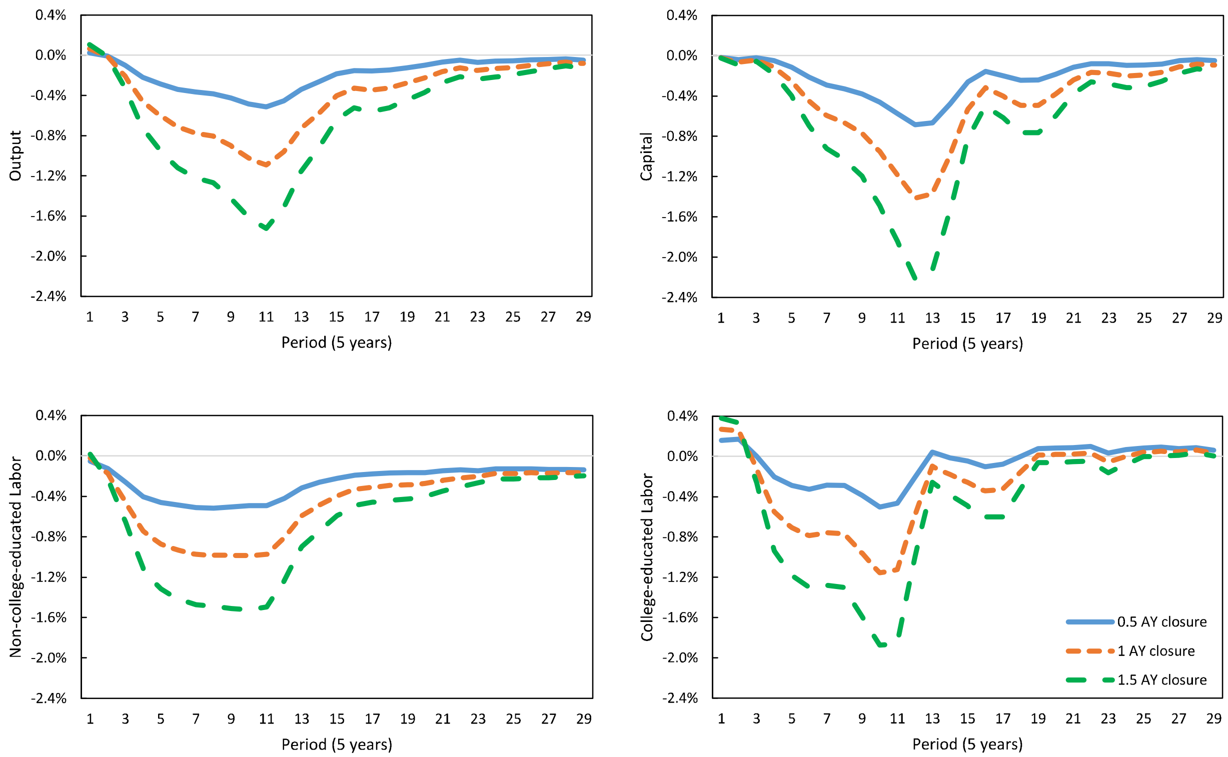
Parameter		Target statistics	Data	Model
$\psi = 1/3$		(*elasticity of substitution = 1.5*)		
β	.942	Equilibrium real interest rate (annualized)	.04	.04
b	6.72	Mean hours of work in $j = 3, ..., 9$	.287	.302
φ	.408	Mean hours of work in $j = 3, 4, 5$	.299	.292
η	.252	Ratio of inter-vivos transfers over total savings	.30	.342
θ_3^x	.869	Mean parental time investments in $j = 3$	.061	.058
θ_4^x	.115	Mean parental time investments in $j = 4$	.036	.036
θ_5^x	.049	Mean parental time investments in $j = 5$	.020	.020
θ_3^p	.621	Rank corr. of parental income & child earnings	.282	.268
θ_3^I	.672	Mean parental monetary investments in $j = 3$	.056	.056
θ_4^I	.684	Mean parental monetary investments in $j = 4$	.136	.125
θ_5^I	.398	Mean parental monetary investments in $j = 5$	.160	.150
ζ_3	-2.43	Educational gradients in parental time in $j = 3$ (%)	20.9	19.8
ζ_4	-0.19	Educational gradients in parental time in $j = 4$ (%)	14.8	14.0
ζ_5	-0.32	Educational gradients in parental time in $j = 5$ (%)	20.2	19.6
ν	.542	Fraction with a college degree (%)	34.2	35.0
μ_ξ	.227	Average college expenses/GDP per-capita	.140	.140
δ_ξ	.624	Observed college wage gap (%)	75.0	67.6
ρ_ϕ	.112	Intergenerational corr. of percentile-rank income	.341	.368
σ_ϕ	.487	Gini wage	.37	.343
σ_z	.148	Slope of variance of log wage from $j = 2$ to $j = 8$	.18	.185
$\underline{a}$	$-.068$	Average unsecured debt rel. to annual disposable income	.010	.010

Table A9: Effects on intergenerational mobility of lifetime income with a lower elasticity of substitution between public and parental investments

$\psi = 1/3$	IGE			Rank cor.			Upward Mobility		
Steady state	.394			.375			6.9%		
				% change rel. to no school closure, by cohort					
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.1	1.0	1.2	0.1	1.0	1.2	-0.3	-1.3	-1.6
1.0 AY	0.3	2.2	2.5	0.2	2.1	2.5	-0.6	-2.5	-3.1
1.5 AY	0.4	3.5	3.9	0.2	3.4	3.9	-0.8	-4.0	-5.0

Note: The elasticity of substitution between private and public investments is equal to 1.5.

Figure A6: Evolution of macroeconomic aggregates with a lower elasticity of substitution between public and parental investments



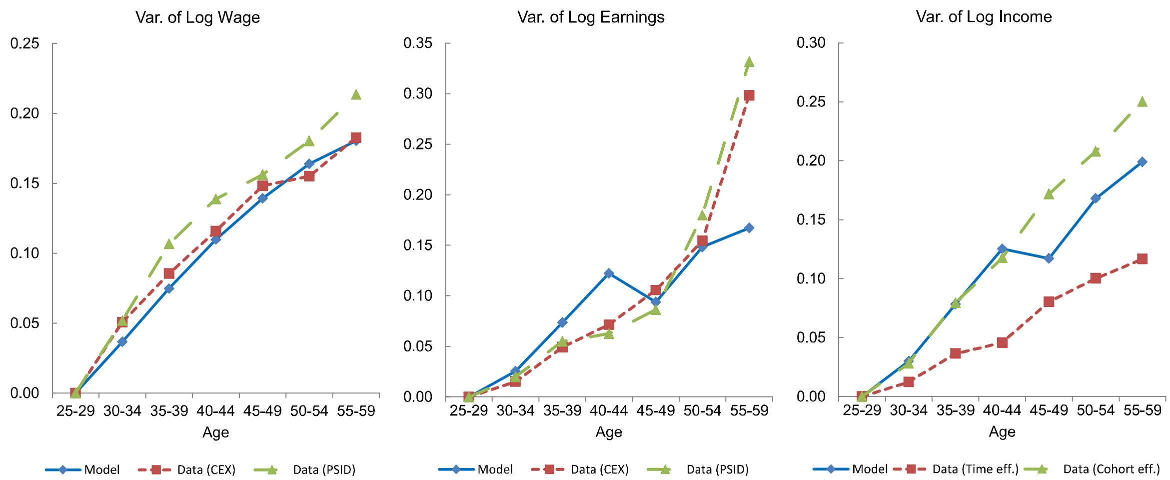
Note: The elasticity of substitution between private and public investments is equal to 1.5.

Table A10: Effects on inequality and loss of lifetime income with a lower elasticity of substitution between public and parental investments

$\psi = 1/3$	Lifetime income						Fraction of		
	Gini			Average			College-educated		
Steady state	.284			4.2 (rel. to Y_s)			.346		
				% change rel. to no school closure, by cohort					
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.0	0.0	0.1	-0.1	-2.1	-2.1	0.6	-1.3	-1.5
1.0 AY	0.0	0.1	0.1	-0.2	-4.3	-4.3	1.3	-2.7	-3.1
1.5 AY	0.1	0.2	0.1	-0.3	-6.7	-6.8	2.1	-4.3	-4.9

Note: The elasticity of substitution between private and public investments is equal to 1.5. Y_s denotes steady-state output per capita.

Figure A7: Inequality over the life cycle



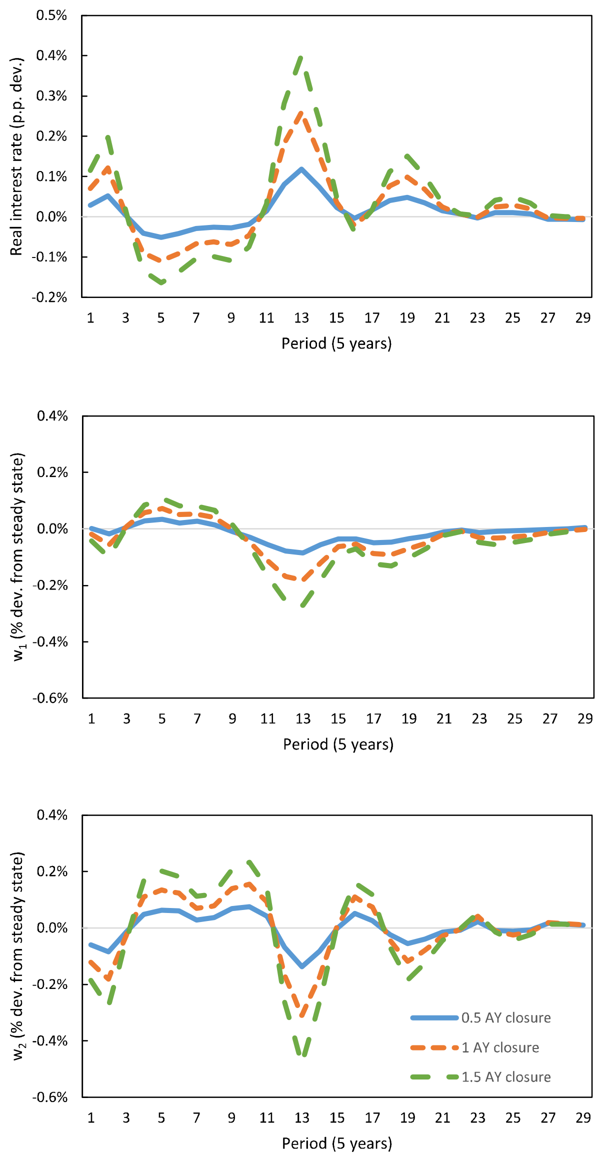
Note: The left figure shows the variance of log wage by age relative to age 25-29. The middle figure shows the variance of log earnings by age relative to age 25–29. The right figure plots the variance of log income by age relative to age 25–29. US data is from Heathcote et al. (2010).

Table A11: School closure effects on different cohorts with a very long closure

	IGE			Rank cor.			Upward Mobility		
Steady state	.413			.392			6.8%		
				% change rel. to no school closure, by cohort					
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
1 AY	0.3	4.5	5.0	0.2	4.1	4.6	-0.6	-6.8	-8.3
4 AY	0.9	22.6	26.1	0.4	19.6	23.0	-2.5	-32.6	-37.7
	Lifetime income Gini index			Lifetime income Average			Fraction of College-educated		
Steady state	.282			4.2 (rel. to Y_s)			.336		
				% change rel. to no school closure, by cohort					
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
1 AY	0.1	0.6	0.7	-0.1	-3.3	-3.4	1.4	-2.5	-3.2
4 AY	0.4	3.5	3.6	-0.6	-14.5	-15.4	6.4	-13.2	-16.7

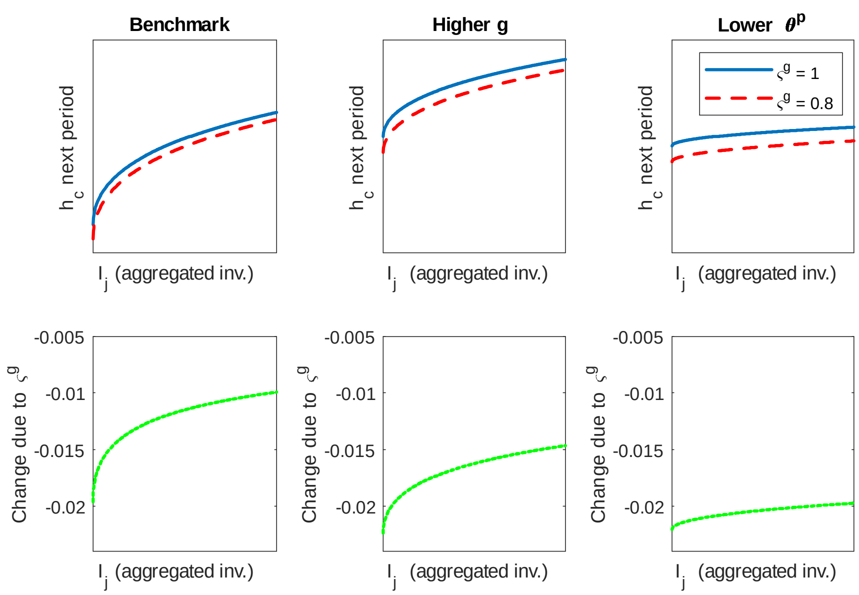
Note: Y_s denotes steady-state output per capita.

Figure A8: Evolution of equilibrium prices in the baseline model



Note: The top panel shows the equilibrium interests over the transition. The middle panel shows the equilibrium wages for non-college workers, and the bottom panel shows the equilibrium wages for college-educated workers over the transition

Figure A9: Illustration of direct effects of school closures on skill formation



Note: The figures visualize how children's human capital output h'_c is related to parental investments I_j aggregated from time and money depending on the presence of school closures (i.e., $\varsigma^g = 1$ or $\varsigma^g = 0.8 < 1$). Note that because parental investments are largely shaped by income, I_j can be interpreted as the parental socioeconomic status (SES). The middle panel raises the size of g and the right panel increases the relative importance of public schooling (with a lower θ_p).

Figure A10: Evolution of macroeconomic aggregates with a very long closure

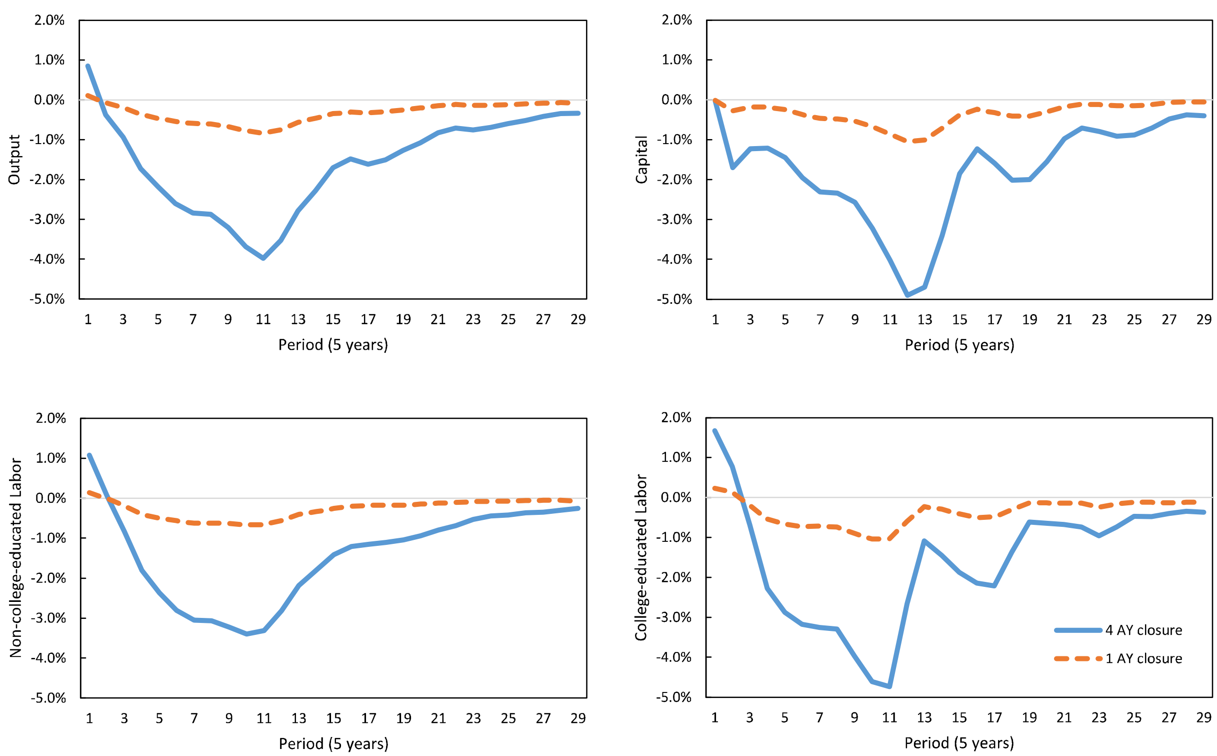


Figure A11: Evolution of macroeconomic aggregates with a very long closure: no general equilibrium effects