Aggregate fluctuations in a model of indivisible labor supply with endogenous workweek length

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Abstract

This paper studies aggregate fluctuations in a simple extension of the classical indivisible labor supply model of Rogerson (1988) and Hansen (1985). The model allows a firm to choose hours as well as employment in the presence of a nonlinear mapping from hours worked to labor services and employment adjustment costs. Households take as given state-dependent hours per worker, which are optimally chosen by the firm, and make intertemporal labor supply decisions along the extensive margin. Although the model does not explicitly allow households to choose desired hours worked, the preference parameter governing the intensive margin Frisch elasticity of households shapes aggregate labor market fluctuations along both intensive and extensive margins, in contrast to pure indivisible labor models.

Keywords: Indivisible labor, intensive margin, extensive margin, business cycles, Frisch elasticity

JEL codes: E32, J22

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1 Introduction

Models of indivisible labor supply following the aggregation theory of Rogerson (1988) can reconcile small micro-based individual labor supply elasticities with large aggregate labor supply elasticities. This feature is important since it can generate the large volatility of aggregate hours which we observe in the data, while being consistent with smaller estimates of the individual labor supply elasticities (Keane and Rogerson 2012). Despite this merit, pure indivisible labor supply models are subject to a disconnect between individuals and aggregate economy; that is, aggregate fluctuations are independent of the preference parameter governing individuals' labor supply elasticity. This is the case not only in stand-in household models of indivisible labor (Hansen 1985) but also in heterogeneous-agent models of indivisible labor supply with incomplete asset markets, pioneered by Chang and Kim (2006), in which aggregate labor market fluctuations depend on the endogenous wealth distribution but are still independent of the preference parameter governing the underlying individuals' Frisch elasticity.

This paper considers a simple extension of the classical indivisible labor supply model that circumvents this disconnect. The key is to note that the reason for the disconnect is not the indivisibility of labor per se, but the exogenously fixed workweek length (i.e., the intensive margin), which makes the variation of the aggregate hours occur only through changes in the fraction of workers employed (i.e., the extensive margin). I circumvent this issue by allowing equilibrium hours per worker to be state-dependent. The state-dependence of hours per worker is derived from the optimal choice of the firm solving a dynamic problem and is thus endogenously varying over the business cycle. The essential ingredients for this are (i) a nonlinear mapping from hours worked to labor services and (ii) the quasi-fixity of labor at the extensive margin.

The first element, the nonlinear labor services mapping, has been used in the literature to capture set-up costs and fatigue effects (Prescott, Rogerson and Wallenius 2009). When the production technology incorporates such nonlinearity, a firm would optimally select a fixed workweek for all identical workers in a setting where it can frictionlessly adjust labor input along both intensive and extensive margins. This fixed workweek is determined solely by the shape of the nonlinear labor services mapping, independent of the other factors such as market prices.[1] Thus, in a frictionless environment, firms will always adjust the employment level, in reaction to economic conditions, while maintaining a fixed workweek, as in pure indivisible labor models.

The other element, labor inputs being quasi-fixed from the perspective of firms, is key to creating interaction between the two margins of labor in a dynamic environment with aggregate uncertainty. Specifically, I assume that each period's employment level is predetermined, and that employment adjustments are subject to quadratic adjustment costs (Hansen and Sargent 1988).[2] When confronted with the employment frictions, the model distinguishes itself from pure indivisible labor models by providing firms with incentives to adjust both hours and employment over the business cycle. Specifically, since firms are unable to instantaneously adjust employment level, it gives rise to firm's incentive to temporarily deviate from the steady-state optimal workweek. In addition, if the required labor adjustment is large, it becomes too costly to adjust the employment level in the presence of employment adjustment costs, thereby giving rise to adjustment at the intensive margin.

I embed the above firm's technology into a standard dynamic general equilibrium business cycle model. In the model, both households and firms face dynamic decision problems. Households are assumed to take as given the state-dependent hours per worker, and the stand-in household makes intertemporal labor supply decisions along the extensive margin and a consumption-savings choice. I evaluate the calibrated model economy using a set of conventional business cycle statistics. Notably, the model does a good job of accounting for

[1] Card (1990) and Prescott et al. (2009) also show similar theoretical results.

[2] The idea that labor is a quasi-fixed input goes back to Oi (1962). The predetermined employment is often assumed in the business cycle literature, such as Burnside, Eichenbaum, and Rebelo (1993) and Shimer (2010). The quadratic employment adjustment costs approximate hiring and layoff costs.

the cyclical volatility of aggregate hours at both margins and reproduces the cyclicality and persistence of aggregate labor market variables remarkably well, conditional on the presence of non-extreme employment adjustment costs. For instance, the model can generate the fact that the extensive margin is considerably more volatile and more procyclical than the intensive margin. In addition, the model can reproduce both highly persistent employment and reasonably persistent hours per worker in the data.

The model economy is then used to quantify the relationship between individuals' intensive margin elasticity and its model-implied aggregate labor supply elasticities.[3] Despite the fact that the model does not explicitly allow households to choose desired hours worked due to the indivisibility, I find that the preference parameter governing the intensive-margin Frisch elasticity of households is precisely recovered. The estimated extensive-margin Frisch elasticity is found to be sizeable, broadly in line with the recent empirical evidence (e.g., Fiorito and Zanella 2012, Peterman 2016). Furthermore, it increases with the underlying households' Frisch intensive-margin elasticity, provided that the employment adjustment costs are not small. Overall, the aggregate labor supply elasticity strongly increases with individuals' Frisch elasticity in my extended indivisible labor model, in sharp contrast to the pure indivisible labor model.

Rogerson and Wallenius (2009) provide a theoretical reconciliation on the micro versus macro *steady-state* labor supply elasticities, which govern labor responses with respect to permanent tax changes. My paper focuses on a related yet different object (i.e., the intertemporal elasticity or Frisch elasticity) in a business-cycle environment. Although both models indicate the importance of the extensive margin, their model's prediction on the magnitude of the extensive margin elasticity as a function of that of the intensive margin elasticity is different. Specifically, my model implies that the extensive margin elasticity generally increases with the individual's intensive margin elasticity, whereas the extensive

[3] Specifically, given a parameter value for the underlying households' preference governing their Frisch labor supply elasticity, I simulate the model economy with aggregate productivity shocks. Then, I estimate various Frisch labor supply elasticities using the model-generated time-series data.

margin elasticity implied by the Rogerson and Wallenius model is essentially unrelated with or slightly decreases with the individual's intensive margin elasticity. Erosa, Fuster and Kambourov (2016) build a rich heterogeneous household environment with both extensive and intensive margins of labor supply and compute the aggregate labor supply elasticities with respect to transitory and permanent wage changes. While their framework emphasizes lifecycle aspects, this paper studies labor supply changes over the business cycle in general equilibrium. Despite the differences in model environments, the extensive margin elasticities recovered in my model is remarkably in line with their finding that the aggregate labor supply elasticity with respect to temporary wage changes is 1.75, of which 62% is due to the extensive margin.

This paper is also related to the business cycle literature that divides labor inputs into intensive and extensive margins, such as in Kydland and Prescott (1991), Bils and Cho (1994), Cho and Cooley (1994), Osuna and Rios-Rull (2003), and Chang, Kim, Kwon, and Rogerson (2019). In terms of key elements of the model, this paper is closely related to Kydland and Prescott (1991), which Osuna and Rios-Rull (2003) also builds upon. In Kydland and Prescott (1991), there is a nonconvexity in workweek length in the production side but a simple version of the nonlinear mapping from hours worked to labor services is in the household side. Bils and Cho (1994) and Cho and Cooley (1994) use alternative mechanisms to derive a stand-in household that disvalues labor along each separate margin separately. Chang et al. (2019) build a rich framework in which heterogeneous individuals can choose how many hours to work in the presence of a nonlinear labor services mapping, which enables them to study issues involving cross-sectional distributions of hours worked. Unlike these papers, the model in my paper has the firm solving a dynamic problem. Moreover, the key exercise of my paper concerns the relationship between individual and aggregate Frisch elasticities, which has not been explored.

The framework of endogenous indivisible labor studied in this paper resembles one in Rogerson (2011). There, he studies life cycle models including a coordinated working model where hours are indivisible, but the binary menu of hours is endogenous. In that model, although the fixed number of hours a worker faces is endogenous in that the level of fixed hours can be determined at the beginning of life once and for all, its microfoundation is left for a future study. The endogenous indivisibility of labor in this paper could be considered a dynamic version of a coordinated working time model with a micro-founded reason for indivisibility of labor, namely the nonlinear labor services mapping in the firm's technology. The model framework studied in this paper builds on Burnside et al. (1993), who present a labor hoarding model of indivisible labor where labor utilization is captured by employees' effort, which could be interpreted as hours per worker. Despite similarities in some model elements such as household aggregation and employment frictions, a key difference in the modeling assumption is that, in my model, workweeks of different lengths are not perfect substitutes (Prescott et al. 2009). In terms of the model performance, my model generates a sufficiently higher volatility of the intensive margin in line with the data, as compared to low volatilities of effort in their model.

The remainder of this paper is organized as follows. Section 2 introduces the firm's optimization problem in the presence of the nonlinear mapping in a simple static environment. In Section 3, I present the main dynamic general equilibrium business cycle model and its analytic properties. Section 4 conducts the main quantitative analysis. I first discuss how the model is calibrated and solved, and then present the main quantitative results about its business cycle performance and the relationship between the preference and the implied aggregate labor supply elasticities. Section 5 concludes the paper.

2 Optimal workweek length

In this section, we consider a simple static setting to illustrate how firms would optimally choose both margins of labor. A firm faces a continuum of households with measure one as potential employees and maximizes profit by choosing both the employment level n and the schedule of hours for each worker $h(i)$.[4] Assume that the production function f has a set of usual properties such as $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) < 0$. Taking as given the productivity z and hourly wage w, the firm solves

$$\max_{h(i), n \in [0,1]} zf(L) - w \left(\int_0^n h(i) di \right), \tag{1}$$

where L denotes the effective total labor input: $L = \int_0^n g(h(i))di$. The key element is a nonlinear labor services mapping $g(\cdot)$. I assume that the this function has the following properties:

$$\begin{aligned} g(h) &= 0 \quad \text{for } h \in [0, \phi] \\ &= \tilde{g}(\cdot) \quad \text{for } h \in [\phi, 1], \end{aligned} \tag{2}$$

where $\tilde{g}(\phi) = 0$, $\tilde{g}'(\cdot) > 0$ and $\tilde{g}''(\cdot) < 0$, as depicted in Figure 1. This nonlinear mapping reflects two important features in the relation between actual hours spent and effective labor input: the marginal returns are zero for the first several hours because of setup costs, and then are decreasing because of fatigue effects. Unlike the case where the labor services mapping is linear, the nonlinear mapping in the production function leads to the two theoretical properties of the labor demand decision, both of which characterize the optimal length of workweek. The first property is given by Lemma 1.

Lemma 1 *Assume that $g(\cdot)$ is nonlinear and satisfies (2). Then the firm, which solves (1), optimally chooses the same hours $h \in [\phi, 1]$ for all identical workers.*

The proof is straightforward. First, note that $h(i) \in [0, \phi]$ will never be chosen as it would give zero marginal services (as well as zero services) while incurring positive marginal costs w. Second, if one compares a constant schedule of hours to any other schedules having

[4] The key results in section do not change when I add capital as another production input. The business cycle model for the quantitative analysis in the following sections incorporates capital.

Figure 1: Nonlinear labor services mapping

the same $\int_0^n h(i)di$, it is always the case that the former (the identical hours) yields higher aggregate labor services (i.e., $\int_0^n g(h(i))di$) than the latter (Jensen's inequality). On the other hand, if the g function is linear, the firm would be indifferent between any schedules of hours for workers as long as the total labor input L is chosen optimally.

With the identical hours choice, the effective total labor input can be simply expressed as $L = g(h)n$. The firm's profit maximization problem can then be reduced to

$$\max_{h,n\in[0,1]} zf(g(h)n) - whn. \tag{3}$$

The first order conditions for h and n are

$$\begin{aligned} zf'(L)g'(h)n &= wn, \\ zf'(L)g(h) &= wh, \end{aligned}$$

implying that the optimal $\bar{h}$ is determined by

$$g'(\bar{h}) = \frac{g(\bar{h})}{\bar{h}}, \tag{4}$$

independent of other economic factors such as productivity z and market wage w.[5]

Lemma 2 *Assuming that $g(\cdot)$ satisfies (2), the schedule of optimal hours for the firm facing (3) is independently determined by $g(\cdot)$ such that (4) holds.*

In sum, a firm that can freely choose a schedule of hours for each employee and employment level will voluntarily choose the common workweek length, which is independent of other economic conditions. The firm would adjust its scale (employment level), while holding workweek hours at this optimal level. This is undoubtedly inconsistent with the U.S. data: hours per worker vary over the business cycle with approximately one third of volatility of output, and a serial correlation of 0.55. Ohanian and Raffo (2012) document that, in recent decades, the relative volatility of hours per worker to output has even increased in some OECD countries, including the U.S.

In a richer dynamic setup with aggregate uncertainty, this simple theoretical result can be easily extended. One way pursued in this paper is to introduce quasi-fixity of labor. With this friction, hours per worker are no longer independently determined by the labor services mapping but can vary depending on the aggregate states (e.g., z in the above problem). The next section describes the environments of the main dynamic stochastic general equilibrium model.

3 Equilibrium business cycle model

In this section, I present the main general equilibrium real business cycle (RBC) model, which extends the pure indivisible labor economy of Rogerson (1988) and Hansen (1985). A key feature is that the choice of indivisible hours each household faces depends endogenously on the aggregate state variables of the economy.

[5] Card (1990) also derives a similar independence result using the same effective total labor input which incorporates the nonlinear labor services mapping. Prescott et al. (2009) obtain the same independence result in an environment where the same functional form of the nonlinear labor services mapping is embedded in household's labor supply.

The process of the total factor productivity shock z is originally assumed as an AR(1) process in logs,

$$\log z_t = \rho \log z_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2),$$

and the expositions henceforth use the corresponding discretized shocks that follow a N_z-state Markov chain using Rouwenhorst's (1995) method.

3.1 Household

The economy is populated by a continuum of ex-ante identical and infinitely lived households on the unit interval. I assume that households have access to a complete set of Arrow securities. The period utility function for each household is assumed to be additively separable between consumption and leisure:

$$u(c_t, h_t) = \log c_t - \theta \frac{h_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}},$$

which is consistent with balanced growth. This utility function implies that individuals' intensive-margin labor supply elasticity is equal to γ.[6]

Following Rogerson (1988), assume that the budget set of individual households is non-convex and that there is perfect employment insurance through lotteries.[7] Thus, although each household can only choose either 0 or $\bar{h}_t$, the stand-in household, who chooses the fraction of working population n_t, has a convex constraint set. The period expected utility

[6] This is the Frisch elasticity (marginal utility of wealth constant). Theoretically, this is the largest among the other labor supply elasticities including the Marshallian elasticity (income constant) and Hicksian elasticity (utility constant). See e.g., Blundell and MaCurdy (1999) and Keane (2011) for more details.

[7] See also Burnside et al. (1993).

$U : R_+ \times [0,1] \rightarrow R$ for the stand-in household can be written as

$$\begin{aligned} U(c_t, n_t) &= n_t \left(\log c_t - \theta \frac{\bar{h}_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) + (1-n_t) \left(\log c_t - \theta \frac{0^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) \\ &= \log c_t - B(\bar{h}_t) n_t, \end{aligned}$$

where

$$B(\bar{h}_t) \equiv \theta \frac{\bar{h}_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}. \tag{5}$$

Note that the stand-in household takes as given $\bar{h}_t$, which may be state-dependent.[8] As can be seen above, the stand-in household's utility is linear in n_t, provided that $\bar{h}_t$ is taken as given. When $\bar{h}_t$ is exogenously fixed as in the pure indivisible model (i.e., $\bar{h}_t = \bar{h}$), aggregate fluctuations are independent of the value that we assign to the individual labor supply elasticity. This is because the marginal disutility of employment for the stand-in household, B, is an invariant constant not only in steady state but also over the business cycles.[9]

It is important to note that this disconnect between the individual's parameter and aggregate fluctuations is not due to the indivisibility of labor per se, but the exogenously fixed level of hours. It is easy to see from (5) that, when $\bar{h}_t$ varies, a different value of γ could matter for the stand-in household through the marginal disutility of employment $B(\bar{h}_t)$ that changes over the business cycle. For instance, when the stand-in household faces a high $\bar{h}_t$, the marginal disutility of increasing the fraction of workers becomes higher, affecting the optimal labor supply at the extensive margin n_t indirectly. Simply put, even in a simple indivisible labor framework, it is possible that individuals' intensive-margin elasticity could matter for aggregate fluctuations given that the hours choice changes over time.

[8] In a pure indivisible labor model, this assumption is unnecessary but could have been innocuously imposed, since the workweek is assumed to be fixed in any case.

[9] Recall that the only thing required to change for different γ's is the θ, which yields the same B, in order to match the same steady-state total hours in the calibration step.

The stand-in household's dynamic optimization problem can be written as the following functional equation:

$$W(a,k,z_i,s) = \max_{\substack{c\geq 0, k'\in\Gamma(k)\\ n\in[0,1]}} \left\{ \log c - B(\bar{h}(z_i,s))n + \beta \sum_{j=1}^{N_z} \pi_{ij} W(a'_j, k', z_j, s') \right\}, \tag{6}$$

subject to

$$\begin{aligned} c + \sum_{j=1}^{N_z} q_j(z_i,s)a'_j &= a + w(z_i,s)\bar{h}(z_i,s)n + r(z_i,s)k + \Pi(z_i,s) + (1-\delta)k, & (7)\\ N' &= M_1(z_i,s) \text{ and } K' = M_2(z_i,s), & (8) \end{aligned}$$

where z_i, is the today's total factor productivity shock and $s \equiv (N,K)$ denotes endogenous aggregate state variables, which consist of the aggregate employment level and aggregate capital, respectively. π_{ij} denotes the transition probability $\Pr(z' = z_j | z = z_i)$, β is the discount factor, a_j is the Arrow security that pays 1 in state j, q_j is its price, and $\Pi(z_i,s)$ denotes dividends. The household takes as given the last two perceived laws of motions for the endogenous aggregate state variables, N and K. The variables with a prime denote their values in the next period.

3.2 Firm

As discussed above, the key ingredient that is necessary to achieve the state dependence of the workweek length is employment frictions. To capture quasi-fixity of labor, assume that the employment level is predetermined for the next period before a next period productivity shock is realized, as in Burnside et al. (1993) and Shimer (2010), and that employment adjustment is subject to adjustment costs. The timing is as follows. At the beginning of period t, the employment level for period t, n_t, is fixed and the productivity shock z_t is realized. And then the firm chooses hours h_t, the amount of capital to rent k_t, and the employment level for the next period n_{t+1}. When choosing n_{t+1}, the firm knows how much adjustment costs, such as hiring costs or layoff costs, need to be paid. The firm discounts future profits using the prices of assets held by households and also perceives the aggregate laws of motion since the firm is a price-taker.

The production function $f(L,k)$ is assumed to be increasing, concave, and homogeneous of degree one in both arguments.[10] Specifically, I will assume that it is Cobb-Douglas. Hence, the output is given by

$$Y = z_i f(L,k) = z_i L^{\alpha} k^{1-\alpha},$$

where $\alpha \in (0,1)$ is used for labor, instead of capital, for algebraic convenience.

The firm's dynamic problem can be written as

$$v(n, z_i, s) = \max_{\substack{h,n' \in [0,1] \\ k \geq 0}} \left\{ z_i f(L,k) - w(z_i,s)hn - r(z_i,s)k - \Phi(n,n') + \sum_{j=1}^{N_z} q_j(z_i,s) v(n', z_j, s') \right\}, \tag{9}$$

subject to

$$L = g(h)n, \tag{10}$$

$$N' = M_1(z_i,s) \text{ and } K' = M_2(z_i,s), \tag{11}$$

where $\Phi(n,n')$ is the convex adjustment cost such that $\Phi(n,n)=0$ for all n.

3.3 Recursive competitive equilibrium

In equilibrium, the firm's choice on workweek length should be consistent with the stand-in household's optimality condition, given prices. More formally, a recursive equilibrium is a

[10] Note that when $g(h)$ is linear, this production exhibits standard constant returns technology with respect to total hours and capital. When h is held fixed as in the pure indivisible labor model, the production function has constant returns in employment and capital, which is also standard.

set of functions for prices, quantities, and values

$$\left\{w, r, (q_j)_{j=1}^{N_z}, n_h, k_h', (a_j)_{j=1}^{N_z}, h, n_f', k_f, M_1, M_2, \Pi, W, v\right\}$$

such that

1. W solves (6)-(8), and n_h, k_h' and $(a_j)_{j=1}^{N_z}$ are the associated policy functions for the household,

2. v solves (9)-(11), and h, n_f' and k_f are the associated policy functions for the firm,

3. prices of goods market, labor market, and asset markets are competitively determined; and

4. (*consistency*) the individual policy functions are consistent with the perceived aggregate laws of motion : i.e., $n_f'(N, z_i, N, K) = M_1(z_i, N, K)$ and $k_h'(K, z_i, N, K) = M_2(z_i, N, K)$ for all z_i, N, K.

3.4 Analytic optimality conditions

In this subsection, I derive some analytic results.

The stand-in household's first optimality condition is the labor-leisure condition:

$$\frac{1}{c} w(z_i, s)\bar{h}(z_i, s) = B(\bar{h}(z_i, s)). \tag{12}$$

Next, the Euler equation for consumption (or capital)

$$\frac{1}{c} = \beta \sum_{j=1}^{N_z} \pi_{ij} \frac{1 + r(z_j, s') - \delta}{c_j'}, \tag{13}$$

and the optimal portfolio choices satisfy

$$q_j(z_i, s) = \beta \pi_{ij} \frac{c}{c'_j}, \tag{14}$$

both of which are standard in RBC models.

As in Khan and Thomas (2003), using (14) and denoting by $p(z_i, s)$ the marginal utility of consumption, the firm's functional equation (9) can be rewritten as

$$V(n, z_i, s) = \max_{\substack{h, n' \in [0,1] \\ k \geq 0}} \left\{ p(z_i, s)[z_i (g(h)n)^{\alpha} k^{1-\alpha} - w(z_i, s)hn - r(z_i, s)k - \Phi(n, n')] + \beta \sum_{j=1}^{N_z} \pi_{ij} V(n', z_j, s') \right\},$$

subject to

$$N' = M_1(z_i, s) \text{ and } K' = M_2(z_i, s),$$

where the firm discounts future profits by β. From this functional equation, if one uses the first order conditions for the static choice variables, i.e., h and k, one can obtain the following optimality conditions with some algebra:

$$\begin{aligned} k(n, z_i, s) &= z_i^{\frac{1}{\alpha(1-\eta)}} \left(\frac{\alpha \eta}{w(z_i, s)} \right)^{\frac{\eta}{1-\eta}} \left(\frac{(1-\alpha)}{r(z_i, s)} \right)^{\frac{1-\alpha\eta}{\alpha(1-\eta)}} n, & (15) \\ h(z_i, s) &= \phi + z_i^{\frac{1}{\alpha(1-\eta)}} \left(\frac{\alpha \eta}{w(z_i, s)} \right)^{\frac{1}{1-\eta}} \left(\frac{(1-\alpha)}{r(z_i, s)} \right)^{\frac{(1-\alpha)}{\alpha(1-\eta)}} . & (16) \end{aligned}$$

First, the capital demand is proportional to the firm's current employment level as the firm would need a larger capital stock when more workers are employed, while the demand schedule of hours is independent of the firm's current employment level. With the employment frictions, the schedule of hours that is optimally chosen by the firm would exhibit independence only of its own employment level. Hours per worker now respond to the aggregate state variables, and thus can vary as overall economic conditions change. Since the employment level of the firm is predetermined, it may not be at the optimal level after the productivity shock is observed. Thus, the firm now has an incentive to deviate from the optimal workweek length, characterized by (4) in Lemma 2.

As the firm's decision on the employment level is dynamic in the presence of the frictions, we have an intertemporal optimality condition for the employment level:

$$\Phi_2(n, n') = \beta \sum_{j=1}^{N_z} q_j(z_i, s) \left[z_j f_1(L', k') g(h) - w'(z_j, s') h' - \Phi_1(n', n'') \right] \tag{17}$$

The left-hand side is the immediate marginal cost due to hiring costs or layoff costs when the firm plans to adjust its employment level next period. This immediate marginal cost must be equal to the expected discounted sum of marginal product of employment net of the two extra terms. The first extra term is the marginal cost of employment that increases the wage bill in the next period. Since the next period employment level becomes a state variable for the next period decision, the marginal reduction in adjustment costs next period should be also accounted for, which is reflected in the last term.

4 Quantitative analysis

For the quantitative exercises in this section, I impose the following functional forms. First, the nonlinear mapping from hours worked to labor services is assumed to be given by

$$\begin{aligned} g(h) &= (h - \phi)^{\eta} \quad \text{if } h \in (\phi, 1] \\ &= 0 \quad \text{if } h \in [0, \phi], \end{aligned}$$

where $\phi \in [0, 1)$ captures the range of unproductive hours at the workplace, $\eta \in (0, 1]$ captures the extent to which fatigue effects operate for a high h. Note that these satisfy the conditions described above in Section 2.

Second, the employment adjustment costs are assumed to be captured by a quadratic form:

$$\Phi(n, n') = \frac{\xi}{2}\left(\frac{n' - n}{n}\right)^2,$$

where $\xi \geq 0$ shapes the size of the adjustment costs. Note that when n does not change over time, as in steady state, no adjustment cost is incurred.

4.1 Calibration and solution method

I now explain how parameter values are chosen for the following quantitative exercises. The length of a period corresponds to a quarter. The first set of parameter values is chosen using the steady-state equilibrium (Cooley and Prescott 1995). Specifically, these parameter values are calibrated so that the model in steady state is consistent with the long-run averages of the US data from 1956Q1-2010Q4. To begin, imposing that all of the endogenous variables are constant over time without shocks, equations (12)-(17) characterize the analytic relationship between the variables in steady state. Then, these relations are used to map from the first moments in the data to the parameter values. The quarterly real interest rate of 1% gives $\beta = 0.99$. Next, the long-run investment-capital ratio implies the value of δ, and the long-run average capital-output ratio implies α. I choose $\delta = 0.025$ and $\alpha = 1 - 0.36$. These values are commonly used in the equilibrium business cycle literature, including Kydland and Prescott (1991), Cho and Cooley (1994) and Chang et al. (2019), all of which build a model with both intensive and extensive margins of labor.

Next, ϕ captures unproductive time yielding zero labor services while getting paid. Although it is not easy to precisely measure how much time is unproductive at the workplace, Burda, Genadek, and Hamermesh (2020) provide a useful estimate for this parameter. Specifically, using the American Time Use Survey, they define and estimate non-work at work. Their estimates show that the average fraction of time at the workplace that employees are not working is 6.9%. Accordingly, the baseline value of ϕ is chosen somewhat lower at 5%, and the value of ϕ equal to 10% of the steady state hours is also considered as a sensitivity check, as reported in Appendix. Next, note that the steady state workweek length is given by $h = \phi/(1-\eta)$. The average fraction of working hours of 39.4/84 in a week pins down $\eta = 0.95$, assuming that the weekly endowment of available hours for work and leisure is 84 hours. I set $\rho = 0.95$ and $\sigma_\epsilon = 0.007$, commonly used values in the business cycle literature (e.g., Chang et al. 2019).[11]

A large body of empirical literature often finds that individual labor supply elasticities are small. Nevertheless, there is still quite a bit of debate regarding its magnitude (see Blundell and MaCurdy (1999) and Keane (2011) for surveys). In observance of this wide range of estimates, I choose a range of parameter values for $\gamma = 0.5, 1.0$ and 1.5, each of which corresponds to different values of the underlying individuals' Frisch elasticity at the intensive margin. This is the approach also used by related papers such as Rogerson and Wallenius (2009) and Chang et al. (2019). Furthermore, a variation of this parameter is necessary for the main exercise in the following section. Given a parameter value for γ, θ is re-calibrated to match the long-run employment-population ratio of 59.6% in the US data. Specifically, θ is equal to 42.00, 13.15, and 8.50 for $\gamma = 0.5, 1$, and 1.5, respectively.

The last parameter to be calibrated is ξ, which determines the degree of the employment adjustment cost. A special feature of this parameter is that, in the model, ξ does not affect steady state prices and quantities, thereby requiring another approach rather than the traditional approach based on steady state. In fact, adjustment costs are known to be hard to estimate. For example, Hall (2004) shows that the estimates of the annual degree of quadratic labor adjustment costs for various industries are quite small with large standard errors. Given this, I consider a range of parameter values for the degree of employment adjustment costs: $\xi = 0.01, 0.1$ and 1.[12] As will be discussed, the economy with a low ξ would make the model behave like a pure indivisible labor model, whereas a large ξ would

[11] I note that this paper is not meant to study the source of business cycle fluctuations (see King and Rebelo 1999 for the extensive review).

[12] Alternatively, one can choose the calibration strategy to match the relative volatility of the intensive margin, as in Osuna and Rios-Rull (2003). In fact, the mid value, $\xi = 0.1$, approximately corresponds to the value calibrated in that way.

make the model behave like a divisible labor model.

To obtain the equilibrium business cycle data from the model with aggregate uncertainty, it is solved numerically. Although an easiest way might seem to solve the corresponding planning problem, note that households in this economy would have an incentive to affect hours if they were able to do so.[13] Thus, the social planner's problem would yield different allocations than the decentralized equilibrium, except for a special case where the individual supply elasticity is exactly equal to the aggregate elasticity, as shown in Appendix $A.1$. As for computing decentralized equilibria, in a setting where either households or firms have a static problem, we can embed the optimal choices of the static agent into the other agent's dynamic problem by substituting out market prices, and iterate a single value function without considering prices, as in Hansen and Prescott (1995). However, this method cannot be straightforwardly applied here because both the household and firm face dynamic problems. Therefore, I solve the decentralized equilibrium directly using a nonlinear method for the equilibrium value functions of both agents. In essence, the algorithm iteratively finds the equilibrium laws of motions that are equal to agents' perceived aggregate laws of motion, which are necessary to infer correct prices. In the meantime, I solve for dynamic decision rules of the household and firm while interpolating their expected future value function using the cubic spline interpolation and piecewise linear interpolation, respectively.

4.2 Business cycle results

In this subsection, I present some key business cycle statistics from the model-generated data with two objectives in mind: (i) to characterize how the individual parameter γ and aggregate fluctuations are connected and (ii) to investigate the importance of adjustment costs on the extensive margin for the relationship between γ and aggregate fluctuations. As is standard in the business cycle literature, statistics in this subsection are based on model-

[13]This problem would not arise in the pure indivisible labor economy because the workweek hours each household faces are assumed to be exogenously fixed.

Table 1: Cyclical volatilities relative to output

σ_x/σ_Y		$x =$						
		σ_Y	C	I	**h**	**N**	**h** $\times$ **N**	AC/Y
U.S. data		1.56	0.60	2.54	0.35	0.64	0.91	
ξ	γ							
0.01	0.5	1.40	0.35	3.01	0.12	0.62	0.63	6.5e-8
(low)	1.0	1.47	0.34	3.04	0.18	0.64	0.64	8.5e-8
	1.5	1.50	0.34	3.06	0.23	0.64	0.65	9.7e-8
0.1	0.5	1.19	0.37	2.91	0.17	0.40	0.44	1.2e-7
(mid)	1.0	1.30	0.36	2.96	0.24	0.44	0.51	1.8e-7
	1.5	1.36	0.35	2.99	0.29	0.45	0.55	2.2e-7
1.0	0.5	1.08	0.36	2.95	0.21	0.13	0.25	1.1e-7
(high)	1.0	1.18	0.36	2.98	0.32	0.16	0.37	1.9e-7
	1.5	1.25	0.35	3.00	0.38	0.18	0.43	2.5e-7

Note: Numbers are percentage standard deviations of HP filtered data; AC/Y refers to total adjustment costs over output in percentage. The last column reports the average adjustment costs relative to output.

generated data over long (10,000) periods, the first 1,000 periods of which are dropped. The logged variables are detrended using the HP-filter with the smoothing parameter equal to 1,600. U.S. data counterparts are computed using the aggregate data from 1956Q1 to 2010Q4 after applying the same procedures.

Table 1 summarizes cyclical volatilities, or percentage standard deviations, of the key macroeconomic variables relative to output.[14] I start with the results with a moderate size of employment adjustment costs ($\xi = 0.1$). We can clearly see the systematic relationship between the individual Frisch elasticity γ and the cyclical volatilities of the economy in contrast to the pure indivisible labor economy: that is, a higher γ increases the cyclical volatilities of overall aggregate variables.[15] More importantly, the positive relation is much

[14] It also reports the average adjustment costs relative to output. Note that they are rather small but shape labor market fluctuations quite substantially.

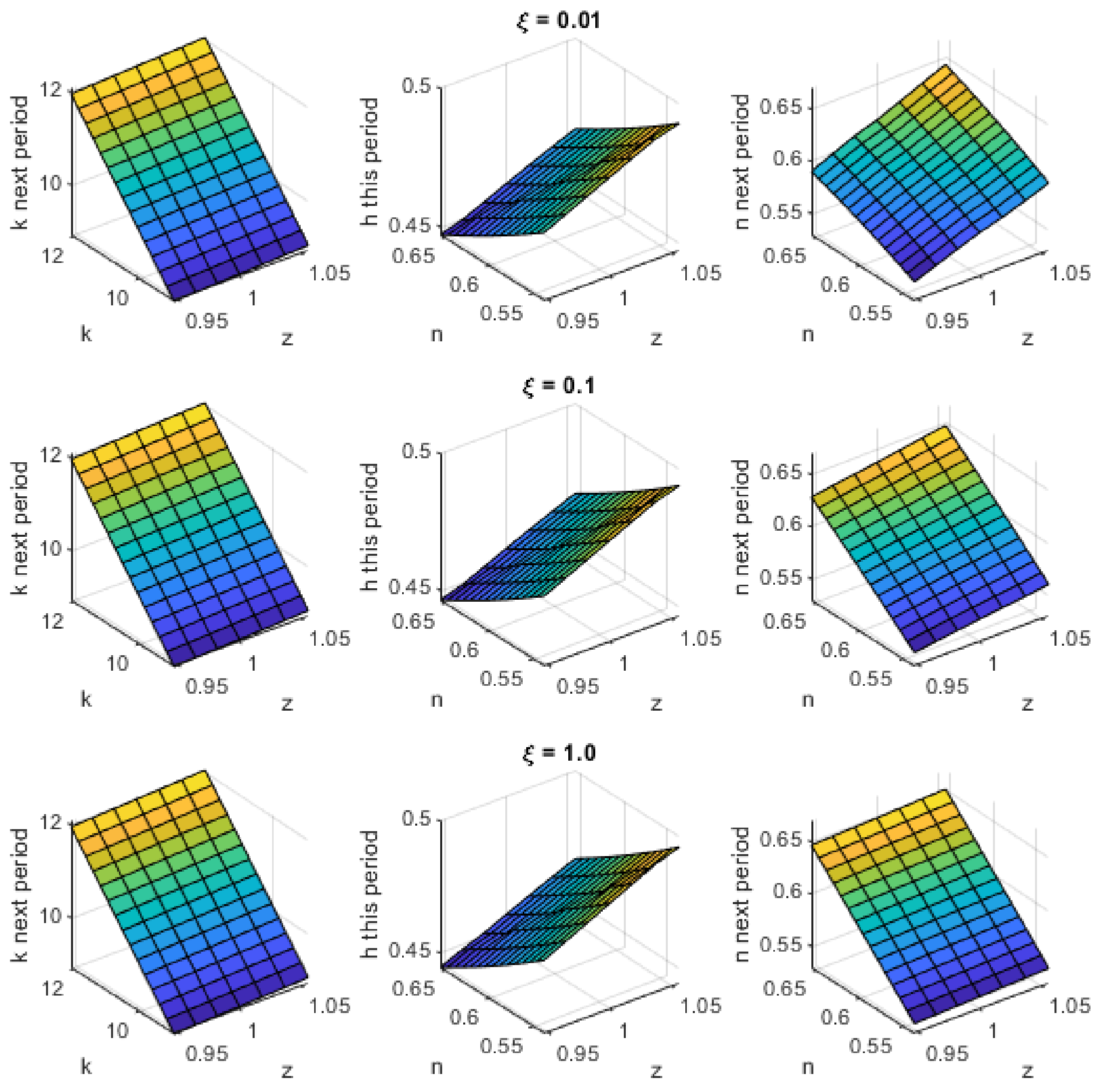
[15] Consumption also becomes more volatile but to a lesser extent, as compared to output. This is why the relative volatility of consumption becomes slightly reduced with a higher γ. Overall, note that the performance of the model in terms of cyclical volatility of non-labor market variables is very close to that of a standard RBC model.

stronger for aggregate labor variables, particularly along the intensive margin (h). When each individual is more willing to substitute labor intertemporally, the stand-in household who represents those individuals is more likely to accommodate the firm's need to deviate from the optimal workweek hours in the absence of aggregate shocks (i.e., steady-state h). On the other hand, if the underlying households' supply of labor is very inelastic (a lower γ), then the firm's temporary desire to adjust hours becomes more difficult to be met in the market, and it results in a lower volatility of equilibrium hours per worker.

It is instructive to look at the results with the other degrees of employment adjustment costs. First, with a low adjustment costs ($\xi = 0.01$), we see that labor adjustment takes more along the extensive margin. Figure 2 shows the equilibrium decisions by the stand-in household and firm. The right panels show the employment decision for the next period as a function of the current employment level and the productivity z. It clearly shows that a lower ξ enables the firm to rely more heavily on the extensive margin with respect to a higher z. Consequently, the relative volatility of the extensive margin is much larger, resembling the model properties of the pure indivisible labor model. Next, imposing a large degree of adjustment costs ($\xi = 1$), the performance of the model is similar to divisible labor models, as it causes adjustment of labor to occur more along the intensive margin. The cyclical volatilities exhibit a well-documented weakness of the divisible labor model; that is, the relationship between the individual and aggregate labor supply elasticities is tightly connected. Even a high labor supply elasticity of 1.5 cannot generate substantially lower volatility of total hours, which leads to weaker amplification of the productivity shocks through the model, as in traditional representative household RBC models.

Table 2 reports the cyclicality of aggregate variables. When it comes to correlations with output, the most noticeable fact from the data in Table 1 is that the intensive margin of labor (h) is procyclical but less so ($Cor(h, Y) = 0.71$), as compared to the extensive margin ($Cor(N, Y) = 0.80$). In the model, this pattern is largely shaped by the adjustment costs. Specifically, when ξ is not very high ($\xi = 0.01$ or 0.1), we see that the model correctly repro-

Figure 2: Equilibrium decision rules, by the size of employment adjustment costs



Note: The left panels show the household's equilibrium decision rule for k' when N is at the steady state level and $K = k$. The middle and right panels show the firm's equilibrium decision rules for h and N', respectively when K is at the steady state level and $n = N$. All figures are from the model with $\gamma = 1.0$.

Table 2: Cyclicality of aggregates

$Cor(x,Y)$		$x =$ C	I	$\mathbf{h}$	$\mathbf{N}$	$\mathbf{h} \times \mathbf{N}$
U.S. data		0.84	0.92	0.71	0.80	0.84
ξ	γ					
0.01	0.5	0.92	0.99	0.41	0.86	0.93
(low)	1.0	0.92	0.99	0.40	0.85	0.95
	1.5	0.91	0.99	0.40	0.84	0.96
0.1	0.5	0.95	0.99	0.73	0.70	0.91
(mid)	1.0	0.94	0.99	0.69	0.73	0.96
	1.5	0.93	0.99	0.68	0.73	0.97
1.0	0.5	0.94	0.99	0.93	0.39	0.98
(high)	1.0	0.93	0.99	0.92	0.44	0.99
	1.5	0.92	0.99	0.91	0.45	0.99

duces this pattern that the intensive margin is less procyclical than the extensive margin. As ξ increases to a quite high number ($\xi = 1$), which would become similar to a divisible labor model, the model not only produces counterfactual relative volatilities (Table 1) but also incorrect relative cyclicality of aggregate labor market variables: that is, the intensive margin is more procyclical than the extensive margin. Interestingly, the effect of γ on the cyclicality of aggregate variables is found to be quite limited.

As can be seen in Table 3, the model can reproduce the persistence of labor along the two margins remarkably well, given the intermediate size of employment adjustment costs ($\xi = 0.1$). Specifically, both in model-generated data and the US data, the intensive margin is quite persistent but is less persistent than the extensive margin, resulting in a very high (but lower than N) persistence of total hours. Figure 3 shows the impulse response functions, which can help understand these results. In the middle and right panels, I show how equilibrium labor along each margin moves over time from the steady state after the economy is hit by a -1% unexpected shock. Overall, it is clear that the response of the

Table 3: Persistence of aggregates

ρ_x		$x =$ Y	C	I	$\mathbf{h}$	$\mathbf{N}$	$\mathbf{h} \times \mathbf{N}$
U.S. data		0.85	0.85	0.89	0.55	0.91	0.86
ξ	γ						
0.01	0.5	0.84	0.82	0.86	0.36	0.87	0.91
(low)	1.0	0.83	0.83	0.84	0.28	0.85	0.90
	1.5	0.82	0.82	0.83	0.24	0.84	0.89
0.1	0.5	0.79	0.80	0.80	0.62	0.94	0.93
(mid)	1.0	0.79	0.80	0.80	0.58	0.93	0.91
	1.5	0.79	0.80	0.79	0.55	0.92	0.88
1.0	0.5	0.74	0.79	0.73	0.70	0.96	0.81
(high)	1.0	0.74	0.79	0.73	0.69	0.95	0.79
	1.5	0.74	0.80	0.73	0.68	0.95	0.77

intensive margin is quick and temporary, whereas the equilibrium employment response is sluggish and persistent. Notice also that the individual labor supply elasticity γ governs the magnitude of equilibrium labor responses at both intensive and extensive margins, which is consistent with the key cyclical volatility results reported in Table 1.

4.3 Aggregate labor supply elasticities

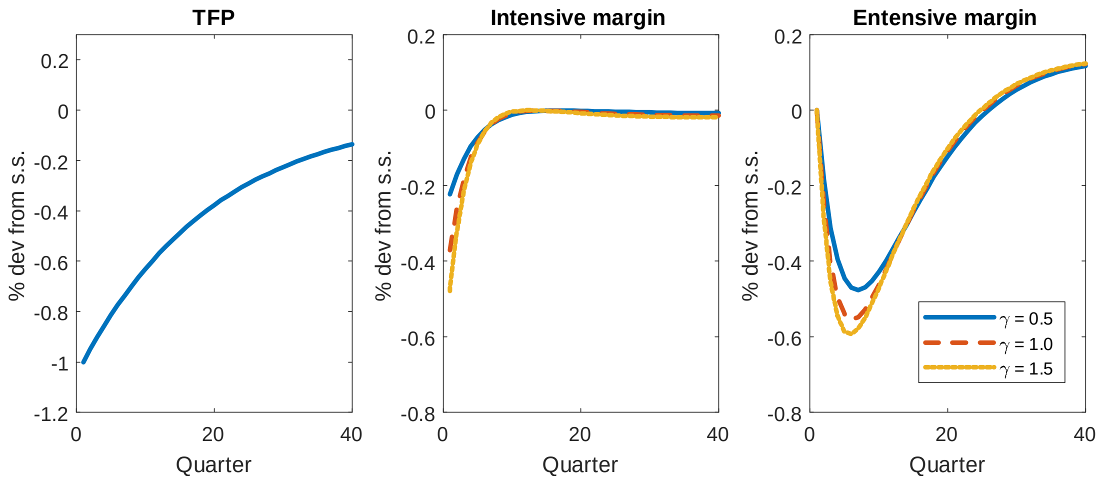
Given the finding that the individual preference parameter γ systematically shapes aggregate labor market fluctuations in the model economy, this subsection quantifies this relationship more carefully in the spirit of Rogerson and Wallenius (2009). Specifically, I simulate the model economy and estimate the following equations:

$$\log h_t = \alpha_0^h + \alpha_1^h \log w_t + \alpha_2^h C_t + \varepsilon_t^h \tag{18}$$

$$\log N_t = \alpha_0^N + \alpha_1^N \log w_t + \alpha_2^N C_t + \varepsilon_t^N \tag{19}$$

$$\log H_t = \alpha_0^H + \alpha_1^H \log w_t + \alpha_2^H C_t + \varepsilon_t^H \tag{20}$$

Figure 3: Impulse responses: labor along intensive and extensive margins



using the time-series aggregate data. This estimation equation gives the estimates of Frisch elasticity (Keane 2011) for the intensive margin (α_1^h), the extensive margin (α_1^N) and the aggregate hours (α_1^H).[16] The above equations are estimated using the data that are generated for different combinations of γ and ξ, as in the previous subsection.

Table 4 summarizes the result. The first noticeable finding is that the preference parameter γ, which governs the intensive-margin Frisch elasticity of households, is precisely recovered in all cases, especially with different values of ξ. Although the model does not explicitly allow households to choose desired hours worked due to the indivisibility, the stand-in household's employment decision implicitly takes into account the underlying households' desire to intertemporally substitute labor supply.

Next, a clear pattern arises in terms of aggregate labor supply elasticity: it is substantially larger than the assumed individual intensive margin elasticities. This is especially the case when ξ is moderate ($\xi = 0.1$). As noted in the literature, a straightforward reason why

[16]In the macroeconomics literature, γ is typically called the micro labor supply elasticity, whereas α_1^H would correspond to the macro labor supply elasticity (Keane and Rogerson 2012). On the other hand, Chetty, Guren, Manoli and Weber (2013) define micro vs. macro labor supply elasticities (at intensive and extensive margins), based on the source of data. According to their terminology, α_1^h, α_1^N and α_1^H are macro elasticities at different margins.

Table 4: Frisch labor supply elasticities

ξ	γ	Intensive margin elasticity, $\hat{\alpha}_1^h$	Extensive margin elasticity, $\hat{\alpha}_1^N$	Aggregate labor supply elasticity, $\hat{\alpha}_1^H$
0.01	0.5	0.50	0.73	1.23
(low)	1.0	1.00	0.72	1.72
	1.5	1.50	0.73	2.23
0.1	0.5	0.50	0.74	1.24
(mid)	1.0	1.00	0.97	1.97
	1.5	1.50	1.14	2.64
1.0	0.5	0.50	0.31	0.81
(high)	1.0	1.00	0.47	1.47
	1.5	1.50	0.61	2.11

the aggregate Frisch elasticities are larger than the micro (intensive margin) elasticities is because the representative agent model abstracts from extensive margin adjustment (Keane and Rogerson 2012). When we look at the extensive margin elasticity, their size is indeed quite significant. It is interesting to note that these model-implied elasticities are broadly in line with the recent empirical findings on the extensive margin Frisch elasticity. For instance, using the Panel Study of Income Dynamics (PSID), Fiorito and Zanella (2012) find quite substantial values of the extensive margin Frisch elasticity (0.8-1.4), and Peterman (2016) finds that contribution of the extensive margin to the aggregate labor supply elasticity is around 0.6-0.7.[17]

In addition, it is worth noting that the extensive margin elasticity increases with the individual's intensive margin elasticity γ, provided that ξ is not too low.[18] This result suggests that the individual's preference parameter, γ, governing labor supply elasticity along the intensive margin could also be an important determinant of the extensive margin elasticity. Overall, the aggregate labor supply elasticity increases with γ in all cases. This is

[17] Using the Survey of Income and Program Participation, Kimmel and Kniesner (1998) find that the extensive margin elasticity varies from 0.6 (for single men) to 2.4 (for single women).

[18] When the magnitude of employment adjustment costs is small, the extensive margin elasticity tends to be independent of individuals' Frisch elasticity. This is similar to the result in Rogerson and Wallenius (2009) for the steady-state elasticities.

in sharp contrast to the prediction of the pure indivisible models that the aggregate elasticity is independent of γ.

5 Conclusion

In this paper, I consider a simple extension of the canonical business cycle model of indivisible labor supply in which the length of workweek changes over the business cycle endogenously due to the firm's incentive. In contrast to pure indivisible labor models, which have a disconnect between the individual labor supply elasticity and aggregate fluctuations, this model relates the individual intensive margin Frisch elasticity to aggregate fluctuations, while maintaining the merit of the pure indivisible labor model that reconciles large aggregate labor supply elasticities with smaller individual labor supply elasticities. This difference is captured by sizeable extensive margin elasticities that also tend to be shaped by the individuals' preference parameter governing the intensive margin elasticity.

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A Appendix

A.1 Inefficiency of the decentralized equilibrium

Theorem 1 *The decentralized equilibrium yields the planner's allocations only when* $\gamma \to \infty$.

Proof. Let $\varepsilon = \frac{1}{\gamma}$. Consider a planner who maximizes the stand-in household with lotteries:

$$V(n,k,z_i) = \max_{\substack{k' \geq 0 \\ n',h \in [0,1]}} \left\{ \log c - \theta \frac{h^{1+\varepsilon}}{1+\varepsilon} n + \beta \sum_{j=1}^{N_z} \pi_{ij} V(n',k',z_j) \right\},$$

subject to

$$c + k' + \Phi(n,n') = z_i F(h,n,k) + (1-\delta)k.$$

The key is to note that $B(h) \equiv \frac{h^{\varepsilon}}{1+\varepsilon}$ is taken as given by households in the decentralized economy.

The three key optimality conditions for the planner are:

$$\begin{aligned}
\frac{1}{c} D_2 \Phi(n,n') &= \beta \sum_{j=1}^{N_z} \pi_{ij} \left\{ \frac{1}{c'_j} \left[z_j D_2 F(h',n',k') - D_1 \Phi(n',n'') \right] - \theta \frac{h'^{1+\varepsilon}}{1+\varepsilon} \right\}, && (21) \\
\frac{1}{c} &= \beta \sum_{j=1}^{N_z} \pi_{ij} \left\{ \frac{1}{c'_j} [z_j D_3 F(h',n',k') + 1 - \delta] \right\}, && (22) \\
\frac{1}{c} z_i D_1 F(h,n,k) &= \theta h^{\varepsilon} n. && (23)
\end{aligned}$$

On the other hand, recall the optimality condition from the decentralized problem with the labor-leisure conditions (12) gives

$$\begin{aligned}
p(z_i,s) D_2 \Phi(n,n') &= \beta \sum_{j=1}^{N_z} \pi_{ij} p(z_j,s') \left[z_j D_2 f(h',n',k') - w'(z_j,s') h' - D_1 \Phi(n',n'') \right] \\
&= \beta \sum_{j=1}^{N_z} \pi_{ij} \left\{ p(z_j,s') \left[z_j D_2 f(h',n',k') - D_1 \Phi(n',n'') \right] - \theta \frac{h'^{1+\varepsilon}}{1+\varepsilon} \right\} && (24)
\end{aligned}$$

which equals (21).

Next, the household's Euler equation (13) can be combined with the first order condition for

k from the firm's problem:

$$\frac{1}{c} = \beta \sum_{j=1}^{N_z} \pi_{ij} \frac{1}{c'_j} \left[z'_j D_3 F(h', n', k') + 1 - \delta \right], \tag{25}$$

which equals (22).

Finally, the first order condition for h from the firm's problem can be combined with the labor-leisure condition (12) by eliminating wage:

$$\frac{1}{c} z_i D_1 F(h, n, k) = \frac{1}{1+\varepsilon} \theta h^{\varepsilon} n. \tag{26}$$

Note that this last equation collapses to (23) if and only if $\varepsilon = 0$. Intuitively, the planner takes account of discrepancy between individual and aggregate labor elasticity. Thus, when the discrepancy collapses to zero, the planner has no margin to improve, and the decentralized equilibrium can produce socially efficient allocations. ■

A.2 Data

The aggregate labor data I use is based on Cociuba, Prescott, and Ueberfeldt (2018) that obtains data mostly from the U.S. Bureau of Labor Statistics. I define the intensive margin to be hours per worker, namely total hours divided by the number of the employed. The extensive margin is defined as the employment-population ratio. Output is the real GDP (chained 2005 dollars) from the U.S. Bureau of Economic Analysis. All the data series are quarterly and HP filtered using the smoothing parameter equal to 1,600. The sample periods are from 1956:I to 2010:IV, after eliminating the first and last four quarters of HP-filtered data.

Table A1: Cyclical volatilities relative to output

σ_x/σ_Y		$x =$						
		σ_Y	C	I	**h**	**N**	**h** $\times$ **N**	AC/Y
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ξ	γ							
0.01	0.5	1.45	0.34	3.04	0.10	0.66	0.66	9.5e-8
(low)	1.0	1.50	0.34	3.06	0.16	0.67	0.67	1.2e-7
	1.5	1.52	0.33	3.07	0.21	0.67	0.67	1.3e-7
0.1	0.5	1.26	0.36	2.94	0.14	0.49	0.51	2.2e-7
(mid)	1.0	1.35	0.35	2.98	0.21	0.52	0.56	3.2e-7
	1.5	1.40	0.35	3.00	0.25	0.53	0.58	3.7e-7
1.0	0.5	1.09	0.37	2.92	0.20	0.20	0.29	2.6e-7
(high)	1.0	1.19	0.36	2.96	0.29	0.24	0.39	4.3e-7
	1.5	1.26	0.35	2.98	0.35	0.26	0.44	5.4e-7

A.3 Sensitivity analysis

I also consider a model economy that is calibrated with a different value of ϕ. Specifically, instead of 5% of the steady-state hours per worker, I consider 10%. Although this is a relatively substantial change in terms of the value of ϕ (an increase of 100%), Tables $A1$-$A4$ show that the main results are quite robust.

Table A2: Cyclicality of aggregates

$Cor(x, Y)$		$x =$				
		C	I	**h**	**N**	**h** $\times$ **N**
U.S. data		0.84	0.92	0.71	0.80	0.84
ξ	γ					
0.01	0.5	0.92	0.99	0.33	0.88	0.93
(low)	1.0	0.91	0.99	0.33	0.87	0.95
	1.5	0.91	0.99	0.33	0.86	0.96
0.1	0.5	0.94	0.99	0.63	0.77	0.91
(mid)	1.0	0.93	0.99	0.59	0.79	0.95
	1.5	0.93	0.99	0.58	0.78	0.97
1.0	0.5	0.94	0.99	0.89	0.48	0.95
(high)	1.0	0.94	0.99	0.87	0.53	0.98
	1.5	0.93	0.99	0.86	0.55	0.99

Table A3: Persistence of aggregates

ρ_x		$x =$					
		Y	C	I	**h**	**N**	**h** $\times$ **N**
U.S. data		0.85	0.85	0.89	0.55	0.91	0.86
ξ	γ						
0.01	0.5	0.84	0.82	0.86	0.25	0.84	0.88
(low)	1.0	0.83	0.82	0.85	0.17	0.81	0.88
	1.5	0.82	0.82	0.84	0.14	0.80	0.88
0.1	0.5	0.79	0.80	0.83	0.56	0.92	0.94
(mid)	1.0	0.79	0.81	0.82	0.51	0.91	0.92
	1.5	0.79	0.81	0.81	0.47	0.90	0.90
1.0	0.5	0.74	0.79	0.74	0.69	0.95	0.87
(high)	1.0	0.74	0.79	0.75	0.67	0.95	0.84
	1.5	0.74	0.80	0.75	0.66	0.95	0.82

Table A4: Frisch labor supply elasticities

ξ	γ	Intensive margin elasticity, $\hat{\alpha}_1^h$	Extensive margin elasticity, $\hat{\alpha}_1^N$	Aggregate labor supply elasticity, $\hat{\alpha}_1^H$
0.01	0.5	0.50	0.56	1.06
(low)	1.0	1.00	0.49	1.49
	1.5	1.50	0.46	1.96
0.1	0.5	0.50	0.85	1.35
(mid)	1.0	1.00	1.05	2.05
	1.5	1.50	1.22	2.72
1.0	0.5	0.50	0.45	0.95
(high)	1.0	1.00	0.66	1.66
	1.5	1.50	0.84	2.34