Aggregate fluctuations in a model of indivisible labor supply with endogenous workweek length

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Abstract

This paper presents an extension of the classical indivisible labor supply model of Rogerson (1988) and Hansen (1985). The model allows a firm to choose hours as well as employment in the presence of a nonlinear mapping from hours worked to labor services and employment frictions. Households take as given state-dependent hours per worker and make intertemporal labor supply decisions along the extensive margin. The model is able to connect the micro elasticity to aggregate fluctuations and macro elasticities, in contrast to pure indivisible labor models, while generating empirically reasonable extensive-margin Frisch elasticities.

Keywords: Indivisible labor, intensive margin, extensive margin, business cycles, Frisch elasticity

JEL codes: E32, J22

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1 Introduction

Indivisible labor supply models (Rogerson 1988) can reconcile small micro-based individual labor supply elasticities with large aggregate labor supply elasticities (Keane and Rogerson 2012). Despite this merit, pure indivisible labor supply models are subject to a disconnect between the preference parameter governing the micro elasticity and aggregate fluctuations.\(^1\)

This paper presents its extension that circumvents this disconnect by allowing hours per worker to be state-dependent. Specifically, the firm solves a dynamic problem in the presence of a nonlinear labor services mapping—which captures set-up costs and fatigue effects (Prescott, Rogerson and Wallenius 2009)—and employment frictions (Hall 2004). The firm’s problem yields a state-dependent workweek length, which is taken as given by households. I evaluate the calibrated model using a set of conventional business cycle statistics.\(^2\)

I then examine the relationship between individuals’ intensive margin elasticity and its model-implied aggregate elasticities in the spirit of Rogerson and Wallenius (2009).\(^3\) Although the model does not explicitly allow households to choose desired hours worked due to the indivisibility, I find that the preference parameter governing the intensive-margin Frisch elasticity is precisely recovered. Furthermore, the estimated extensive-margin Frisch elasticities are found to be sizeable, broadly in line with the recent empirical evidence (Fiorito and Zanella 2012, Peterman 2016).

Notably, I find that the macro extensive margin elasticity increases with the micro intensive-margin elasticity, provided that the employment adjustment costs are not very small. Consequently, aggregate labor supply elasticities strongly increase with the micro elasticity in my extended indivisible labor model in contrast to the pure indivisible labor model.

2 Model

I first introduce the nonlinear labor services mapping in a simple setting, and then embed this technological setting into a real business cycle model.

2.1 Optimal workweek length

A firm faces a continuum of households with measure one and maximizes profit by choosing both the employment level \(n\) and the schedule of hours for each worker \(h(i)\).\(^4\) The production function

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\(^1\)This is the case not only in stand-in household models (Hansen 1985) but also in heterogeneous-agent models with incomplete asset markets, pioneered by Chang and Kim (2006), in which aggregate fluctuations depend on the endogenous wealth distribution but are still independent of the preference parameter governing the underlying individuals’ Frisch elasticity.


\(^3\)Rogerson and Wallenius (2009) consider steady-state micro and macro elasticities in a lifecycle model without aggregate uncertainty.

\(^4\)The following results hold with an capital input. The full business cycle model indeed incorporates capital as well.
f has a set of usual properties such as \( f(0) = 0 \), \( f'(\cdot) > 0 \) and \( f''(\cdot) < 0 \). Taking as given the productivity \( z \) and hourly wage \( w \), the firm solves

\[
\max_{h(i), w \in [0,1]} zf(L) - w \left( \int_0^n h(i)di \right), \tag{1}
\]

where \( L \) denotes the effective total labor input: \( L = \int_0^n g(h(i))di \). The key element is a nonlinear labor services mapping \( g(\cdot) \) (Prescott et al. 2009):

\[
g(h) = \begin{cases} 
0 & \text{for } h \in [0, \phi] \\
\tilde{g}(\cdot) & \text{for } h \in [\phi, 1],
\end{cases}
\tag{2}
\]

where \( \tilde{g}(\phi) = 0 \), \( \tilde{g}'(\cdot) > 0 \) and \( \tilde{g}''(\cdot) < 0 \). In contrast to the linear mapping, the nonlinear one leads to the two properties of the labor demand decision, both of which characterize the optimal workweek length.

**Lemma 1** Assume that \( g(\cdot) \) satisfies (2). Then the firm, which solves (1), optimally chooses the same hours \( h \in [\phi, 1] \) for all identical workers.

**Proposition 1 Lemma 2** Assuming that \( g(\cdot) \) satisfies (2), the schedule of optimal hours reduces to \( \tilde{h} \), independently determined by \( g(\cdot) \) such that

\[
g'(\tilde{h}) = \frac{g(\tilde{h})}{\tilde{h}}. \tag{3}
\]

The proofs are provided in Appendix A. In sum, the firm would adjust its scale (employment level), while holding workweek length at this optimal level independent of other economic conditions such as \( z \). This simple theoretical result echoes Rogerson (1988)’s exogenous workweek length.

### 2.2 Equilibrium business cycle model

I now present the full model economy. This dynamic model will provide the firm with incentives to adjust both hours and employment over the business cycle in the presence of employment frictions. Specifically, I assume that the employment level is predetermined for the next period before a next period productivity shock is realized (Shimer 2010), and that employment adjustment is subject to adjustment costs. The firm discounts future profits using the prices of assets held by households and also perceives the aggregate laws of motion since the firm is a price-taker.

The production function is Cobb-Douglas:

\[
Y = zf(L, k) = zL^\alpha k^{1-\alpha},
\]
where $\alpha \in (0, 1)$. The process of the total factor productivity (TFP) shock $z$ follows:

$$
\log z_t = \rho \log z_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2),
$$

and the expositions henceforth use the corresponding discretized $N_z$-state Markov chain.

The firm’s dynamic problem is given by:

$$
v(n, z_i, s) = \max_{h, n' \in [0, 1], k \geq 0} \left\{ z_i f(L, k) - w(z_i, s)hn - r(z_i, s)k - \Phi(n, n') + \sum_{j=1}^{N_z} q_j(z_i, s)v(n', z_j, s') \right\},
$$

subject to

$$
L = g(h)n, \quad N' = M_1(z_i, s) \quad \text{and} \quad K' = M_2(z_i, s),
$$

where $s \equiv (N, K)$ denotes endogenous aggregate state variables, which consist of the aggregate employment level and aggregate capital, respectively. Agents take as given the last two perceived laws of motions for the endogenous aggregate state variables, $N$ and $K$. The variables with a prime denote their values in the next period. $\Phi(n, n')$ is the convex adjustment cost such that $\Phi(n, n) = 0$ for all $n$.

The economy is populated by a continuum of ex-ante identical and infinitely lived households on the unit interval. Households have access to a complete set of Arrow securities. The period utility function for each household is given by:

$$
u(c_t, h_t) = \log c_t - \theta \frac{h_t^{1+\frac{1}{\gamma}}}{1 + \frac{1}{\gamma}},
$$

which implies that individuals’ intensive-margin elasticity is equal to $\gamma$. As in Rogerson (1988), households can choose either 0 or $\tilde{h}_t$, which is taken as given. As shown in Appendix A, aggregation gives rise to the stand-in household’s utility function:

$$
U(c_t, n_t) = \log c_t - \theta \frac{\tilde{h}_t^{1+\frac{1}{\gamma}}}{1 + \frac{1}{\gamma}} n_t,
$$

which is linear in $n_t$, provided that households take $\tilde{h}_t$ as given.

The stand-in household’s dynamic problem is given by:

$$
W(a, k, z_i, s) = \max_{c \geq 0, k' \in G(k), n \in [0, 1]} \left\{ \log c - B(\tilde{h}(z_i, s))n + \beta \sum_{j=1}^{N_z} \pi_{ij} W(a_j', k', z_j, s') \right\},
$$

subject to

$$
L = g(h)n, \quad N' = M_1(z_i, s) \quad \text{and} \quad K' = M_2(z_i, s),
$$

where $s \equiv (N, K)$ denotes endogenous aggregate state variables, which consist of the aggregate employment level and aggregate capital, respectively. Agents take as given the last two perceived laws of motions for the endogenous aggregate state variables, $N$ and $K$. The variables with a prime denote their values in the next period. $\Phi(n, n')$ is the convex adjustment cost such that $\Phi(n, n) = 0$ for all $n$.
subject to
\[ c + \sum_{j=1}^{N_S} q_j(z_i, s) a'_j = a + w(z_i, s) \bar{h}(z_i, s) n + r(z_i, s) k + \Pi(z_i, s) + (1 - \delta) k, \] (8)
\[ N' = M_1(z_i, s) \text{ and } K' = M_2(z_i, s), \] (9)

where \( \pi_{ij} \) denotes the transition probability \( \Pr(z' = z_j | z = z_i) \), \( \beta \) is the discount factor, \( a_j \) is the Arrow security that pays 1 in state \( j \), \( q_j \) is its price, and \( \Pi(z_i, s) \) denotes dividends.

In equilibrium, the firm’s choice on workweek length should be consistent with the stand-in household’s optimality conditions, given prices. The equilibrium definition is provided in Appendix A.

3 Quantitative analysis

3.1 Parameterization

The length of a period corresponds to a quarter. Following Cooley and Prescott (1995), the model in steady state is consistent with the long-run averages of the US data from 1956Q1-2010Q4. The quarterly real interest rate of 1% gives \( \beta = 0.99 \). The investment-capital ratio implies \( \delta = 0.025 \), and the capital-output ratio implies \( \alpha = 1 - 0.36 \).

The nonlinear mapping is assumed to be
\[ g(h) = (h - \phi)^\eta \text{ if } h \in (\phi, 1] \]
\[ = 0 \text{ if } h \in [0, \phi], \]
where \( \phi \in [0, 1) \) captures the range of unproductive hours at the workplace, \( \eta \in (0, 1] \) captures the extent to which fatigue effects operate for long hours. Burda, Genadek, and Hamermesh (2020) estimate that the average fraction of time at the workplace that employees are not working is 6.9%. Accordingly, the baseline value of \( \phi \) is set to 5%.\(^5\) The average weekly hours 39.4 pins down \( \eta = 0.95 \), assuming the weekly available hours of 84. I set \( \rho = 0.95 \) and \( \sigma_e = 0.007 \), commonly used values in the literature.

A variation of \( \gamma \) is necessary for the main exercises to investigate the mapping between individuals and aggregates (Rogerson and Wallenius 2009). I choose \( \gamma = 0.5, 1.0 \) and 1.5. Given a value of \( \gamma \), \( \theta \) is re-calibrated to match the employment-population ratio of 59.6%: \( \theta \) is equal to 42.00, 13.15, and 8.50 for \( \gamma = 0.5, 1, \) and 1.5, respectively.

The employment adjustment costs are assumed be quadratic:
\[ \Phi(n, n') = \frac{\xi}{2} \left( \frac{n' - n}{n} \right)^2, \]

\(^5\) Calibration with 10% is also considered in Appendix F.
Table 1: Cyclical volatilities relative to output

\[
\begin{array}{cccccccc}
\sigma_x / \sigma_Y & x = & \sigma_Y & C & I & h & N & h \times N & AC/Y \\
\hline
\text{U.S. data} & 1.56 & 0.60 & 2.54 & 0.35 & 0.64 & 0.91 \\
\xi & \gamma & 0.01 & 0.5 & 1.40 & 0.35 & 3.01 & 0.12 & 0.62 & 0.63 & 6.5e-8 \\
& & (low) & 1.0 & 1.47 & 0.34 & 3.04 & 0.18 & 0.64 & 0.64 & 8.5e-8 \\
& & & 1.5 & 1.50 & 0.34 & 3.06 & 0.23 & 0.64 & 0.65 & 9.7e-8 \\
& & 0.1 & 0.5 & 1.19 & 0.37 & 2.91 & 0.17 & 0.40 & 0.44 & 1.2e-7 \\
& & (mid) & 1.0 & 1.30 & 0.36 & 2.96 & 0.24 & 0.44 & 0.51 & 1.8e-7 \\
& & & 1.5 & 1.36 & 0.35 & 2.99 & 0.29 & 0.45 & 0.55 & 2.2e-7 \\
& & 1.0 & 0.5 & 1.08 & 0.36 & 2.95 & 0.21 & 0.13 & 0.25 & 1.1e-7 \\
& & (high) & 1.0 & 1.18 & 0.36 & 2.98 & 0.32 & 0.16 & 0.37 & 1.9e-7 \\
& & & 1.5 & 1.25 & 0.35 & 3.00 & 0.38 & 0.18 & 0.43 & 2.5e-7 \\
\end{array}
\]

Note: Numbers are percentage standard deviations of HP filtered data; AC/Y refers to total adjustment costs over output in percentage. The last column reports the average adjustment costs relative to output.

Where \( \xi \geq 0 \) captures intensity. Adjustment costs are difficult to estimate (Hall 2004). As in Kydland and Prescott (1991), I consider a range of values: \( \xi = 0.01, 0.1 \) and 1.

3.2 Business cycle results

As is standard in the literature, I use the model to generate data over long (10,000) periods (and drop the first 1,000 periods). The logged variables are detrended using the HP-filter with the smoothing parameter equal to 1,600. U.S. data counterparts are computed using the data (1956Q1 to 2010Q4) analogously.

Table 1 summarizes cyclical volatilities. With a moderate size of \( \xi = 0.1 \), we see systematic relationships between the micro elasticity and aggregate volatilities: that is, a higher \( \gamma \) increases the cyclical volatilities of aggregate variables (especially labor market variables). When each individual is more willing to substitute labor intertemporally, the stand-in household who represents those individuals is more likely to accommodate the firm’s need to deviate from the steady-state workweek length.

With \( \xi = 0.01 \), labor adjustment takes more along the extensive margin. A lower \( \xi \) enables the firm to rely more heavily on the extensive margin with respect to a higher \( z \), thereby generating larger relative volatilities at the extensive margin as in pure indivisible labor models. Meanwhile, with a higher value (\( \xi = 1 \)), the model becomes similar to divisible labor models, exhibiting its well-documented weakness: even a high \( \gamma \) of 1.5 generates substantially lower volatilities of hours (i.e., a weaker amplification of aggregate shocks).
Table 2: Frisch labor supply elasticities

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>( \gamma )</th>
<th>Intensive margin elasticity, ( \alpha^h_1 )</th>
<th>Extensive margin elasticity, ( \alpha^N_1 )</th>
<th>Aggregate labor supply elasticity, ( \alpha^H_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.5</td>
<td>0.50</td>
<td>0.73</td>
<td>1.23</td>
</tr>
<tr>
<td>(low)</td>
<td>1.0</td>
<td>1.00</td>
<td>0.72</td>
<td>1.72</td>
</tr>
<tr>
<td>1.5</td>
<td>1.50</td>
<td>0.73</td>
<td>2.23</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.50</td>
<td>0.74</td>
<td>1.24</td>
</tr>
<tr>
<td>(mid)</td>
<td>1.0</td>
<td>1.00</td>
<td>0.97</td>
<td>1.97</td>
</tr>
<tr>
<td>1.5</td>
<td>1.50</td>
<td>1.14</td>
<td>2.64</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>0.50</td>
<td>0.31</td>
<td>0.81</td>
</tr>
<tr>
<td>(high)</td>
<td>1.0</td>
<td>1.00</td>
<td>0.47</td>
<td>1.47</td>
</tr>
<tr>
<td>1.5</td>
<td>1.50</td>
<td>0.61</td>
<td>2.11</td>
<td></td>
</tr>
</tbody>
</table>

Appendix E also presents results on the cyclicality and persistence of macroeconomic aggregates. In essence, the model can generate the fact that the extensive margin is considerably more procyclical than the intensive margin, and reproduce both highly persistent employment and reasonably persistent hours per worker in the data.

### 3.3 Aggregate labor supply elasticities

To investigate the relationship between micro and macro elasticities (Rogerson and Wallenius 2009), I estimate the following equations:

\[
\begin{align*}
\log h_t &= \alpha^h_0 + \alpha^h_1 \log w_t + \alpha^h_2 C_t + \varepsilon^h_t \\
\log N_t &= \alpha^N_0 + \alpha^N_1 \log w_t + \alpha^N_2 C_t + \varepsilon^N_t \\
\log H_t &= \alpha^H_0 + \alpha^H_1 \log w_t + \alpha^H_2 C_t + \varepsilon^H_t
\end{align*}
\]

using the model-generated data with different specifications. These provide the estimates of Frisch elasticity for the intensive margin (\( \alpha^h_1 \)), the extensive margin (\( \alpha^N_1 \)), and aggregate hours (\( \alpha^H_1 \)).

Table 2 shows that \( \gamma \), which governs the intensive-margin Frisch elasticity, is precisely recovered in all cases. Although the model does not explicitly allow households to choose desired hours worked due to the indivisibility, the stand-in household’s employment decision implicitly takes into account the underlying households’ desire to substitute labor supply intertemporally.

Furthermore, aggregate labor supply elasticities are found to be substantially larger than \( \gamma \) due to the extensive margin (Keane and Rogerson 2012). Quantitatively, these model-implied extensive margin elasticities are broadly in line with the recent empirical findings. For instance, Fiorito and

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6 In the macroeconomics literature, \( \gamma \) is typically called the micro labor supply elasticity, whereas \( \alpha^H_1 \) would correspond to the macro labor supply elasticity (Keane and Rogerson 2012). By contrast, Chetty et al. (2013) define micro vs. macro labor supply elasticities based on the source of data.
Zanella (2012)'s estimates range between 0.8 and 1.4, and Peterman (2016) finds that contribution of the extensive margin to the aggregate labor supply elasticity is around 0.6-0.7.\footnote{These also align with Erosa, Fuster and Kambourov (2016) who find that the aggregate labor supply elasticity with respect to temporary wage changes is 1.75, of which 62\% is due to the extensive margin. Using the Survey of Income and Program Participation, Kimmel and Kniesner (1998) find that the extensive margin elasticity varies from 0.6 (for single men) to 2.4 (for single women).}

Notably, the extensive margin elasticity increases with the individual's intensive margin elasticity $\gamma$, provided that $\xi$ is not too low.\footnote{When $\xi$ is low, the extensive margin elasticity tends to be independent of $\gamma$. This is similar to the steady-state results in Rogerson and Wallenius (2009).} This result suggests that $\gamma$ (the intensive-margin elasticity) could also be an important determinant of the extensive margin elasticity. Lastly, the aggregate labor supply elasticity therefore increases with $\gamma$, showing that this model is not subject to the disconnect in pure indivisible models.

4 Conclusion

In this paper, I consider an extension of the canonical business cycle model of indivisible labor supply. In contrast to pure indivisible labor models, this model relates the individual micro elasticity to aggregate fluctuations, while maintaining the merit of the pure indivisible labor model that reconciles large macro elasticities with smaller micro elasticities. These results make it clear that the reason for the disconnect in pure indivisible labor models is not the indivisibility of labor per se, but the exogenously fixed intensive margin, which makes the variation of the aggregate hours occur only through changes along the extensive margin.

References


Online Appendix

A Proofs and results in Section 2

A.1 Lemma 1

First, note that \( h(i) \in [0, \phi] \) will never be chosen as it would give zero marginal services (as well as zero services) while incurring positive marginal costs \( w \). Second, if one compares a constant schedule of hours to any other schedules having the same \( \int_0^n h(i)di \), it is always the case that the former (the identical hours) yields higher aggregate labor services (i.e., \( \int_0^n g(h(i))di \)) than the latter (Jensen’s inequality). On the other hand, if the \( g \) function is linear, the firm would be indifferent between any schedules of hours for workers as long as the total labor input \( L \) is chosen optimally.

A.2 Lemma 2

With the identical hours choice due to Lemma 1, the effective total labor input can be simply expressed as \( L = g(h)n \). The firm’s profit maximization problem can then be reduced to

\[
\max_{h,n \in [0,1]} zf(g(h)n) - whn. \tag{A1}
\]

The first order conditions for \( h \) and \( n \) are

\[
zf'(L)g'(h)n = wn,
\]

\[
zf'(L)g(h) = wh,
\]

implying that the optimal \( \bar{h} \) is determined by

\[
g'(\bar{h}) = \frac{g(\bar{h})}{\bar{h}},
\]

independent of other economic factors such as productivity \( z \) and market wage \( w \).

A.3 Preference aggregation

Following Rogerson (1988), assume that the budget set of individual households is nonconvex and that there is perfect employment insurance through lotteries. Thus, although each household can only choose either 0 or \( \bar{h}_t \), the stand-in household, who chooses the fraction of working population \( n_t \), has a convex constraint set. The period expected utility \( U : R_+ \times [0,1] \rightarrow R \) for the stand-in

\[9\] Card (1990) also derives a similar independence result using the same effective total labor input which incorporates the nonlinear labor services mapping. Prescott et al. (2009) obtain the same independence result in an environment where the same functional form of the nonlinear labor services mapping is embedded in household’s labor supply.
household can be written as

\[
U(c_t, n_t) = n_t \left( \log c_t - \theta \frac{h_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right) + (1 - n_t) \left( \log c_t - \theta \left( \frac{1+\frac{1}{\eta}}{1+\frac{1}{\eta}} \right) \right)
\]

\[
= \log c_t - \theta \frac{h_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} n_t.
\]

A.4 Equilibrium definition

A recursive equilibrium is a set of functions for prices, quantities, and values

\[
\left\{ w, r, (q_j)_{j=1}^{N_z}, n_h, k_h, (a_j)_{j=1}^{N_z}, h, n_f, k_f, M_1, M_2, \Pi, W, v \right\}
\]

such that

1. \( W \) solves (7)-(9), and \( n_h, k_h' \) and \( (a_j)_{j=1}^{N_z} \) are the associated policy functions for the household,

2. \( v \) solves (4)-(6), and \( h, n_f' \) and \( k_f \) are the associated policy functions for the firm,

3. prices of goods market, labor market, and asset markets are competitively determined; and

4. (consistency) the individual policy functions are consistent with the perceived aggregate laws of motion: i.e., \( n_f'(N, z_i, N, K) = M_1(z_i, N, K) \) and \( k_h'(K, z_i, N, K) = M_2(z_i, N, K) \) for all \( z_i, N, K \).

A.5 Analytic optimality conditions

In this subsection, I derive some analytic results.

The stand-in household’s first optimality condition is the labor-leisure condition:

\[
\frac{1}{c} \frac{w(z_i, s) \bar{h}(z_i, s)}{h(z_i, s)} = B(\bar{h}(z_i, s)). \tag{A2}
\]

Next, the Euler equation for consumption (or capital)

\[
\frac{1}{c} = \beta \sum_{j=1}^{N_z} \pi_{ij} \left( 1 + r(z_j, s') - \delta \right) \frac{c}{e_j}, \tag{A3}
\]

and the optimal portfolio choices satisfy

\[
q_j(z_i, s) = \beta \pi_{ij} \frac{c}{e_j}, \tag{A4}
\]

both of which are standard in RBC models.
As in Khan and Thomas (2003), using (A4) and denoting by \( p(z_i, s) \) the marginal utility of consumption, the firm’s functional equation (4) can be rewritten as

\[
V(n, z_i, s) = \max_{h, n' \in [0, 1]} \left\{ p(z_i, s)[(g(h)n)^{1-\alpha} - w(z_i, s)hn - r(z_i, s)k - \Phi(n, n')] + \beta \sum_{j=1}^{N_z} \pi_{ij} V(n', z_j, s') \right\},
\]

subject to

\[
N' = M_1(z_i, s) \quad \text{and} \quad K' = M_2(z_i, s),
\]

where the firm discounts future profits by \( \beta \). From this functional equation, if one uses the first order conditions for the static choice variables, i.e., \( h \) and \( k \), one can obtain the following optimality conditions with some algebra:

\[
k(n, z_i, s) = \frac{1}{z_i^{\alpha(1-\eta)}} \left( \frac{\alpha \eta}{w(z_i, s)} \right)^{-\eta} \left( \frac{1-\alpha}{r(z_i, s)} \right)^{\frac{1-\alpha \eta}{\alpha(1-\eta)}} n, \quad (A5)
\]

\[
h(z_i, s) = \phi + \frac{1}{z_i^{\alpha(1-\eta)}} \left( \frac{\alpha \eta}{w(z_i, s)} \right)^{-\eta} \left( \frac{1-\alpha}{r(z_i, s)} \right)^{\frac{1-\alpha \eta}{\alpha(1-\eta)}} . \quad (A6)
\]

First, the capital demand is proportional to the firm’s current employment level as the firm would need a larger capital stock when more workers are employed, while the demand schedule of hours is independent of the firm’s current employment level. With the employment frictions, the schedule of hours that is optimally chosen by the firm would exhibit independence only of its own employment level. Hours per worker now respond to the aggregate state variables, and thus can vary as overall economic conditions change. Since the employment level of the firm is predetermined, it may not be at the optimal level after the productivity shock is observed. Thus, the firm now has an incentive to deviate from the optimal workweek length, characterized by (3) in Lemma 2.

As the firm’s decision on the employment level is dynamic in the presence of the frictions, we have an intertemporal optimality condition for the employment level:

\[
\Phi_2(n, n') = \beta \sum_{j=1}^{N_z} q_j(z_i, s) [z_j f_1(L', k') g(h) - w'(z_j, s') h' - \Phi_1(n', n'')] \quad (A7)
\]

The left-hand side is the immediate marginal cost due to hiring costs or layoff costs when the firm plans to adjust its employment level next period. This immediate marginal cost must be equal to the expected discounted sum of marginal product of employment net of the two extra terms. The first extra term is the marginal cost of employment that increases the wage bill in the next period. Since the next period employment level becomes a state variable for the next period decision, the marginal reduction in adjustment costs next period should be also accounted for, which is reflected in the last term.
B Solution method

To obtain the equilibrium business cycle data from the model with aggregate uncertainty, it is solved numerically. Although an easiest way might seem to solve the corresponding planning problem, note that households in this economy would have an incentive to affect hours if they were able to do so. Thus, the social planner’s problem would yield different allocations than the decentralized equilibrium, except for a special case where the individual supply elasticity is exactly equal to the aggregate elasticity, as shown below. As for computing decentralized equilibria, in a setting where either households or firms have a static problem, we can embed the optimal choices of the static agent into the other agent’s dynamic problem by substituting out market prices, and iterate a single value function without considering prices, as in Hansen and Prescott (1995). However, this method cannot be straightforwardly applied here because both the household and firm face dynamic problems. Therefore, I solve the decentralized equilibrium directly using a nonlinear method for the equilibrium value functions of both agents. In essence, the algorithm iteratively finds the equilibrium laws of motions that are equal to agents’ perceived aggregate laws of motion, which are necessary to infer correct prices. In the meantime, I solve for dynamic decision rules of the household and firm while interpolating their expected future value function using the cubic spline interpolation and piecewise linear interpolation, respectively.

C Inefficiency of the decentralized equilibrium

Theorem 2 The decentralized equilibrium yields the planner’s allocations only when $\gamma \to \infty$.

Proof. Let $\varepsilon = \frac{1}{\gamma}$. Consider a planner who maximizes the stand-in household with lotteries:

$$V(n, k, z_i) = \max_{k' \geq 0, n', h \in [0, 1]} \left\{ \log c - \theta \frac{h^{1+\varepsilon}}{1+\varepsilon} n + \beta \sum_{j=1}^{N_x} \pi_{ij} V(n', k', z_j) \right\},$$

subject to

$$c + k' + \Phi(n, n') = z_i F(h, n, k) + (1 - \delta)k.$$
The key is to note that \( B(h) \equiv \frac{h^\varepsilon}{1 + \varepsilon} \) is taken as given by households in the decentralized economy.

The three key optimality conditions for the planner are:

\[
\frac{1}{c} D_2 \Phi(n, n') = \beta \sum_{j=1}^{N_s} \pi_{ij} \left\{ \frac{1}{c_j} \left[ z_j D_2 F(h', n', k') - D_1 \Phi(n', n'') \right] - \theta \frac{h^{1+\varepsilon}}{1 + \varepsilon} \right\}, \quad (A8)
\]
\[\frac{1}{c} = \beta \sum_{j=1}^{N_s} \pi_{ij} \left\{ \frac{1}{c_j} [z_j D_3 F(h', n', k') + 1 - \delta] \right\}, \quad (A9)\]
\[\frac{1}{c} z_i D_1 F(h, n, k) = \theta h^\varepsilon n. \quad (A10)\]

On the other hand, recall the optimality condition from the decentralized problem with the labor-leisure conditions (A2) gives

\[
p(z_i, s) D_2 \Phi(n, n') = \beta \sum_{j=1}^{N_s} \pi_{ij} p(z_j, s')[z_j D_2 f(h', n', k') - w'(z_j, s')h' - D_1 \Phi(n', n'')] \]
\[= \beta \sum_{j=1}^{N_s} \pi_{ij} \left\{ p(z_j, s') [z_j D_2 F(h', n', k') - D_1 \Phi(n', n'')] - \theta \frac{h^{1+\varepsilon}}{1 + \varepsilon} \right\}. (A11)\]

which equals (A8).

Next, the household’s Euler equation (A3) can be combined with the first order condition for \( k \) from the firm’s problem:

\[
\frac{1}{c} = \beta \sum_{j=1}^{N_s} \pi_{ij} \frac{1}{c_j} \left[ z_i' D_3 F(h', n', k') + 1 - \delta \right], \quad (A12)\]

which equals (A9).

Finally, the first order condition for \( h \) from the firm’s problem can be combined with the labor-leisure condition (A2) by eliminating wage:

\[
\frac{1}{c} z_i D_1 F(h, n, k) = \frac{1}{1 + \varepsilon} \theta h^\varepsilon n. \quad (A13)\]

Note that this last equation collapses to (A10) if and only if \( \varepsilon = 0 \). Intuitively, the planner takes account of discrepancy between individual and aggregate labor elasticity. Thus, when the discrepancy collapses to zero, the planner has no margin to improve, and the decentralized equilibrium can produce socially efficient allocations. ■
Table A1: Cyclicality of aggregates

<table>
<thead>
<tr>
<th>Cor(x,Y)</th>
<th>x =</th>
<th>C</th>
<th>I</th>
<th>h</th>
<th>N</th>
<th>h × N</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td></td>
<td>0.84</td>
<td>0.92</td>
<td>0.71</td>
<td>0.80</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>ξ</td>
<td>γ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.5</td>
<td>0.92</td>
<td>0.99</td>
<td>0.41</td>
<td>0.86</td>
<td>0.93</td>
</tr>
<tr>
<td>(low)</td>
<td>1.0</td>
<td>0.92</td>
<td>0.99</td>
<td>0.40</td>
<td>0.85</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
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<td>0.91</td>
<td>0.99</td>
<td>0.40</td>
<td>0.84</td>
<td>0.96</td>
</tr>
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<td>0.5</td>
<td>0.95</td>
<td>0.99</td>
<td>0.73</td>
<td>0.70</td>
<td>0.91</td>
</tr>
<tr>
<td>(mid)</td>
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<td>0.99</td>
<td>0.69</td>
<td>0.73</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
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<td>0.68</td>
<td>0.73</td>
<td>0.97</td>
</tr>
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<td>0.99</td>
<td>0.93</td>
<td>0.39</td>
<td>0.98</td>
</tr>
<tr>
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<td>0.99</td>
<td>0.92</td>
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<td>0.99</td>
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<td>0.99</td>
<td>0.91</td>
<td>0.45</td>
<td>0.99</td>
</tr>
</tbody>
</table>

D Data

The aggregate labor data I use is based on Cociuba, Prescott, and Ueberfeldt (2018) that obtains data mostly from the U.S. Bureau of Labor Statistics. I define the intensive margin to be hours per worker, namely total hours divided by the number of the employed. The extensive margin is defined as the employment-population ratio. Output is the real GDP (chained 2005 dollars) from the U.S. Bureau of Economic Analysis. All the data series are quarterly and HP filtered using the smoothing parameter equal to 1,600. The sample periods are from 1956:I to 2010:IV, after eliminating the first and last four quarters of HP-filtered data.

E Additional business cycle results

Table A1 reports the cyclicality of aggregate variables. When it comes to correlations with output, the most noticeable fact from the data in Table A1 is that the intensive margin of labor (h) is procyclical but less so (Cor(h, Y) = 0.71), as compared to the extensive margin (Cor(N, Y) = 0.80). In the model, this pattern is largely shaped by the adjustment costs. Specifically, when ξ is not very high (ξ = 0.01 or 0.1), we see that the model correctly reproduces this pattern that the intensive margin is less procyclical than the extensive margin. As ξ increases to a quite high number (ξ = 1), which would become similar to a divisible labor model, the model not only produces counterfactual relative volatilities (Table 1) but also incorrect relative cyclicality of aggregate labor market variables: that is, the intensive margin is more procyclical than the extensive margin. Interestingly, the effect of γ on the cyclicality of aggregate variables is found to be quite limited.
As can be seen in Table A2, the model can reproduce the persistence of labor along the two margins remarkably well, given the intermediate size of employment adjustment costs ($\xi = 0.1$). Specifically, both in model-generated data and the US data, the intensive margin is quite persistent but is less persistent than the extensive margin, resulting in a very high (but lower than $N$) persistence of total hours. Figure A1 shows the impulse response functions, which can help understand these results. In the middle and right panels, I show how equilibrium labor along each margin moves over time from the steady state after the economy is hit by a -1% unexpected shock. Overall, it is clear that the response of the intensive margin is quick and temporary, whereas the equilibrium employment response is sluggish and persistent. Notice also that the individual labor supply elasticity $\gamma$ governs the magnitude of equilibrium labor responses at both intensive and extensive margins, which is consistent with the key cyclical volatility results reported in Table 1.

### F Sensitivity analysis

I also consider a model economy that is calibrated with a different value of $\phi$. Specifically, instead of 5% of the steady-state hours per worker, I consider 10%. Although this is a relatively substantial change in terms of the value of $\phi$ (an increase of 100%), Tables A1-A4 show that the main results are quite robust.
Figure A1: Impulse responses: labor along intensive and extensive margins

Table A3: Cyclical volatilities relative to output

<table>
<thead>
<tr>
<th>$\sigma_x/\sigma_Y$</th>
<th>$x = \frac{\sigma_Y}{C} I h N h \times N AC/Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>1.56 0.60 2.54 0.35 0.64 0.91</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>1.0</td>
<td>1.09 0.37 2.92 0.20 0.20 0.29 2.6e-7</td>
</tr>
<tr>
<td>(high)</td>
<td>1.19 0.36 2.96 0.29 0.24 0.39 4.3e-7</td>
</tr>
<tr>
<td>(mid)</td>
<td>1.40 0.35 3.00 0.25 0.53 0.58 3.7e-7</td>
</tr>
<tr>
<td>(low)</td>
<td>1.52 0.33 3.07 0.21 0.67 0.67 1.3e-7</td>
</tr>
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</table>
Table A4: Cyclicality of aggregates

<table>
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<th>$Cor(x,Y)$</th>
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<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
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<td>$C$</td>
<td>$I$</td>
<td>$h$</td>
<td>$N$</td>
<td>$h \times N$</td>
</tr>
<tr>
<td>U.S. data</td>
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<td>0.71</td>
<td>0.80</td>
<td>0.84</td>
</tr>
<tr>
<td>$\xi$</td>
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</tr>
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<td>0.91</td>
<td>0.99</td>
<td>0.33</td>
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<tr>
<td>$\gamma$</td>
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Table A5: Persistence of aggregates

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<tr>
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<tr>
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Table A6: Frisch labor supply elasticities

<table>
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<th>ξ</th>
<th>γ</th>
<th>Intensive margin elasticity, $\hat{\alpha}_1^h$</th>
<th>Extensive margin elasticity, $\hat{\alpha}_1^N$</th>
<th>Aggregate labor supply elasticity, $\hat{\alpha}_1^H$</th>
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<td>(high)</td>
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<td>1.50</td>
<td>0.84</td>
<td>2.34</td>
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References


