Asymmetric Frisch Elasticity
(Preliminary & Incomplete)

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Abstract

We propose a mechanism that generates an asymmetric Frisch elasticity of the aggregate labor supply. Based on Australian panel data, we document three stylized facts on the labor supply: (i) there are significant gaps between actual and preferred hours of work among workers, (ii) the distribution of the hours gap is asymmetric: workers are over-employed on average, and (iii) preferred hours decline steadily with age. We build a quantitative model that features a fixed cost of changing hours and increasing disutility of work over the life cycle. Our model successfully matches the above facts and, as a result, exhibits an asymmetric labor supply–compatible with a moderately elastic labor supply over the business cycle and at the same time an inelastic one found in studies based on tax holidays.

Keywords: Frisch Elasticity, Labor Supply, Adjustment Costs, Asymmetry

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1 Introduction

The labor supply is a cornerstone of many macroeconomic analyses. It is crucial for the equilibrium model's ability to match the short-run fluctuations of hours as well as evaluating the effect of fiscal policies. However, the fact that the labor supply elasticity at the individual level tends to be small has presented a challenge to macro models that rely on an elastic supply of labor. Previous attempts try to reconcile this tension between the micro and macro labor supply by recognizing the extensive margin (e.g., Hansen 1985; Rogerson 1988; Chang and Kim 2006; Rogerson and Wallenius 2009; Erosa, Fuster and Kambourov 2016).[1] For example, with a participation margin, the response of the labor supply at the aggregate level is determined mostly by the distribution of the reservation wages—more exactly, the *reservation raises*.[2]

As Chetty et al. (2013) pointed out, micro studies based on nationwide quasi-experiments, such as tax holidays in Iceland and Switzerland, find that Frisch elasticities at both the extensive and the intensive margins are close to zero (e.g., Bianchi, Gudmundsson and Zoega 2001; Sigurdsson 2019; Stefansson 2019; Martinez, Saez and Siegenthaler 2021).[3] However, the recent study by Mui and Schoefer (2021), based on the survey conducted in U.S. and Germany, reports a strong asymmetry in the extensive margin elasticity of labor supply–an elastic response of labor with respect to a small wage cut and, at the same time, an inelastic one with respect to a large wage gain. This evidence suggests that the small elasticities found from the tax holiday studies do not necessarily reject the use of sizeable aggregate Frisch elasticities for the business cycle analysis.

In this paper, we contribute to this literature by offering an economic mechanism that is compatible with asymmetric Frisch elasticity of aggregate labor supply, therefore consistent with the empirical findings in Mui and Schoefer (2021). Specifically, we propose a general equilibrium model which generates the asymmetric Frisch elasticities of aggregate hours by introducing two distinct elements: (i) a fixed cost of changing labor supply and (ii) an increasing age-profile of disutility

[1] See Section 4 in Keane and Rogerson (2015) for a more detailed discussion.

[2] Mui and Schoefer (2021) defines it as the percent wage change that renders a given individual indifferent between employment and nonemployment. It is equal to her reservation wage divided by her actual wage.

[3] There is also a strand of microeconometric literature in which occupation-specific labor supply decisions are used to estimate the Frisch elasticities (e.g., Fehr and Goette 2007; Farber 2015), but their identified moments are relatively inappropriate for evaluating macro moments because their samples are not representative of the whole economy.

from working.

To motivate the adjustment cost of changing hours, we first document the observed patterns of the gap between actual working hours and preferred hours in the data. Based on the Australian panel data, HILDA (Household, Income and Labour Dynamics in Australia), we found several interesting facts. First, there are significant gaps between actual and preferred hours of work among workers. Second, the cross-sectional distribution of the hours gap is asymmetric: workers are over-employed on average. Third, the larger the hours gap is, the more likely the worker changes the hours of work in the following period. Lastly, there is a pronounced age profile of desired work hours, characterized by a steady fall in preferred hours of work over the life cycle.

To match these features of the hours gap in the data, we introduce frictions in adjusting the labor supply, similar to Chetty et al. (2011). In our model, workers constantly face uninsurable idiosyncratic shocks to productivity. Along with an increasing age profile of disutility from working, these shocks generates a desire to change the labor supply over time. A fixed cost of adjustment creates an inaction region in labor supply decisions (i.e., sticky hours) and the gap between actual and preferred hours of work. We calibrate the size of the fixed cost of changing hours to match the cross-sectional distribution of the hours gap in the data. Our model reproduces the asymmetric cross-sectional distribution of the hours gap as in the data: workers are over-employed on average. Moreover, over-employed workers tend to show larger responses of hours because they anticipate a downward trend in (desired) hours over time. As a result, the aggregate labor supply exhibits asymmetric Frisch elasticities. This may justify a use of somewhat larger elasticities of labor supply in business cycle analyses than those found in quasi-experimental tax holiday studies (which capture a smaller side of asymmetric elasticity).

While we focus on labor supply decisions, the mechanism shares many similarities with the menu cost model in the sticky-price literature such as that in Golosov and Lucas (2007).[4] In our model, increasing disutility from working with age keeps hours of work close to the upper bound of (S, s), just as the underlying inflation keeps pushing firm's prices to the lower bound of (S, s) in the menu cost model. Thus, an asymmetric response of the aggregate labor supply in our model parallels the results in Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008): the frequency of price increases (relative to that of price decreases) increases with aggregate inflation.

[4] See Section 7 in Nakamura and Steinsson (2013) for a more detailed discussion.

This paper is organized as follows. Section 2 documents the stylized facts on labor-supply frictions based on the Australian panel data. Section 3 presents a quantitative model that features an adjustment cost in changing labor supply. In Section 4, we calibrate the model economy to match the stylized facts of hours gap in the data and then compare the arc Frisch elasticities of the aggregate labor supply between the model and data. Section 5 is conclusion.

2 Stylized Facts on the Hours Gap

This section briefly describes the data and reports several salient features of the labor supply that motivate our quantitative model.

2.1 Data

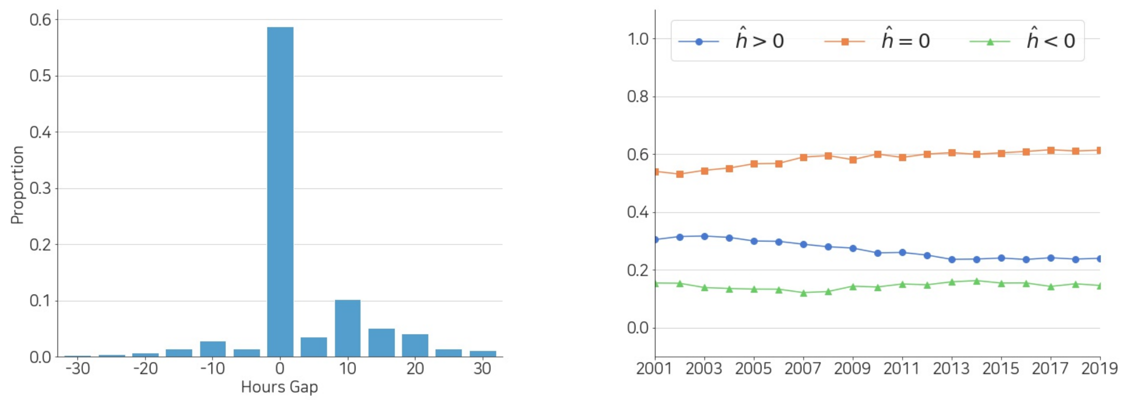
We use data from the Household, Income and Labour Dynamics in Australia (HILDA) survey from 2001 to 2018. These data contain unique questionnaires about workers' preferred hours, which we use to identify the adjustment frictions in changing hours. To be specific, those who work more than zero hours are asked the following question: *"If you could choose the number of hours you work each week, and taking into account how that would affect your income, would you prefer to work fewer? same? or more?"* If they choose the answers *"fewer"* or *"more,"* they are then asked *"In total, how many hours a week on average would you choose to work? Again, take into account how that would affect your income."* These two questions reveal the gap between the actual and preferred hours.

One caveat is the ambiguity of the phrase *"taking into account how that would affect your income."* We assume that workers understand this phrase as the change in income at a fixed wage rate, i.e., a linear income schedule in hours. This assumption might be appropriate for workers who are paid hourly and whose working hours are well below overtime. However, it is possible that salaried workers assume that their incomes do not vary significantly, even if they change their hours. Moreover, workers who work nearly full-time hours are likely to consider an overtime premium. Despite these limitations, we show later that small departures from this assumption do not significantly change the main conclusion of our analysis.

We restricted our data sample to prime male workers of ages 25-64 only because their labor supply is less sensitive to non-market/non-wage factors (e.g., childbearing, home production, etc.).

Figure 1: Distribution of Hours Gap

(a) Distribution of Hours Gap (b) Fraction of Workers over Time



Notes: In Figure 1a the size of the bin is 5 hours (except for $\hat{h} = 0$): i.e., $\hat{h} = (0,5]$, $(5,10]$, $(10,15]$,..., etc. In Figure 1b, each line represents the fraction of workers with $\hat{h} > 0$ (over-employed), $\hat{h} = 0$, and $\hat{h} < 0$ (under-employed), respectively.

2.2 Actual and Preferred Hours of Work

From the HILDA survey we define the hours gap as $\hat{h} = h - h_p$, where h is actual working hours and h_p is preferred hours. Note that if $\hat{h} > 0$, the worker is over-employed, whereas if $\hat{h} < 0$, the worker is under-employed.

Here are several facts regarding the hours gap. First, while most workers are working close to their preferred hours, a significant portion (40%) of workers are either over-employed or under-employed. Figure 1a shows the distribution hours gap $\hat{h}$. Nearly 60% of workers report that they do not feel the need to change their hours ($\hat{h}$=0). Among the other 40%, the distribution of the hours gap is not symmetric: the number of over-employed workers (26%) is much larger than that of the under-employed (14%). As Figure 1b shows, this asymmetry has been persistent over the last two decades in Australia.

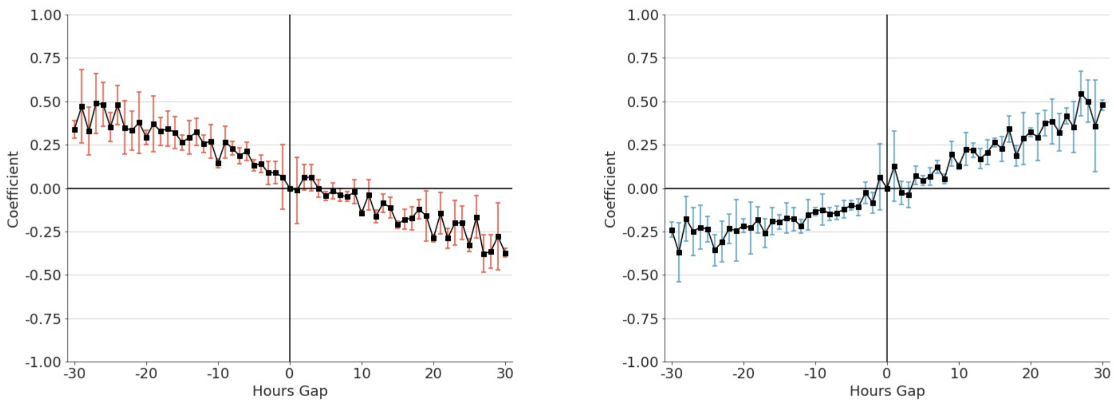
Second, workers *do* try to fill the hours gap over time. To quantify this adjustment of hours, we run the following panel regression of changes in hours on the previous period's hours gaps:

$$\text{Prob}(\triangle h_{j+1} \lesseqgtr 0) = \alpha_i + \lambda_j + \Sigma_{k\neq 0}\gamma_k \mathbb{I}\left(\hat{h} = k\right) + \epsilon_{ij} \tag{1}$$

where the subscript i indicates an individual worker, j denotes a worker's age, and

Figure 2: Probability to Change Hours

(a) Probability to Increase (b) Probability to Decrease



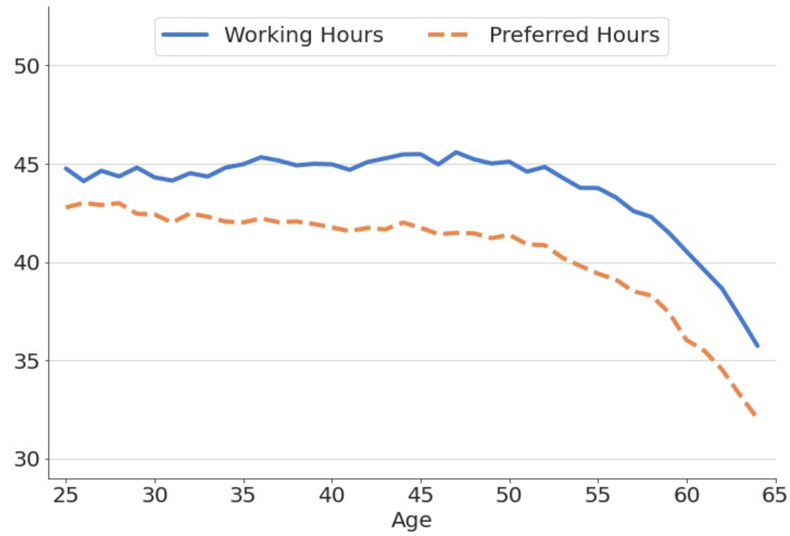
Notes: These figures report the estimated coefficients $\hat{\gamma}_k$'s in Equation 1 where the dependent variable is an increase of hours, $\mathbb{I}(\triangle h > 0)$ (Figure 2a), and a decrease of hours $\mathbb{I}(\triangle h < 0)$ (Figure 2b), respectively. The vertical line indicates the 95% confidence interval. The standard errors are clustered at the individual level.

k denotes the size of the hours gap. We control for individual and time fixed effects and cluster individual workers when calculating the standard errors. Of our primary interest is γ_k, a term that stands for the correlation between the hours gap and actual changes in hours in the following period. Since we exclude $k = 0$, these coefficients show the correlation relative to $k = 0$. Figure 2a and Figure 2b illustrate the estimation results of Equation (1): workers are more likely to change their working hours in the following period when there is a larger hours gap.

The above two facts suggest that labor supply responses at the aggregate level may be asymmetric because (i) the number of over-employed workers is larger than that of under-employed and (ii) workers are always trying to reduce the hours gap over time. While our estimates reflect correlation rather than causality, quasi-experimental evidence from Blundell, Brewer and Francesconi (2008) strongly supports our argument. Using British panel data (on labor-supply preference), they showed that over/under-employed workers were more likely to reduce the hours gap in response to tax reforms.

One might be concerned that the change in hours we use does not necessarily reflects the completely optimal choice of individual workers because the determination of working hours often reflect other components (such as a reduction in hours

Figure 3: Age Profiles of Actual and Preferred Hours



by firms due to slack in demand, coordination among co-workers, etc.) However, we will analyze the hours choice in the presence of adjustment frictions, and such adjustment costs in changing hours will reflect those frictions.

Finally, Figure 3 shows the life-cycle patterns of both working hours and preferred hours among the employed. Starting in age 25, actual hours always exceed preferred hours. Interestingly, preferred hours then steadily decrease, whereas actual hours remain relatively flat until age 50. At around age 50, actual hours begin to fall and preferred hours fall at a faster rate. In sum, preferred hours decrease steadily for the most of the life cycle, while actual hours exhibit a flat profile for the greater part of the working period.

We argue that actual hours cannot be drastically changed for various reasons (e.g., necessary coordination of work schedules among co-workers, complementary nature of labor inputs in production, costs of bargaining, etc.). In the model below we will incorporate a simple form of adjustment costs of changing hours to reflect such factors. Our modelling strategy on the adjustment cost of changing hours parallels that in sticky price models. We adopt a fixed cost of changing hours from the menu cost model of Golosov and Lucas (2007), in which individual firms have to pay a fixed cost to change the price of their products (in the face of idiosyncratic shocks to productivity). The graphs in Figure 2 suggest that worker's decisions to adjust their hours are intentional choices rather than random events. The fact that there is a continuous decline in preferred hours over time implies a constant push toward one side of the (s, S) boundaries.[5] This will generate an asymmetric distribution of the hours gap and, in turn, result in asymmetric responses of the aggregate labor supply.

3 Model

We extend the standard overlapping generations model of the labor supply where workers face uninsurable idiosyncratic productivity shocks. Two notable features are: (i) a fixed cost in changing working hours over time and (ii) increasing disutilities from working over the life cycle. In sum, our model is a dynamic version of Chetty et al. (2011), who also studied labor supply elasticities in the presence of adjustment costs in hours. Moreover, our primary focus is different from theirs in that we examine the Frisch elasticity, whereas they study Hicksian elasticity.

3.1 Environment

Demographics. The economy is populated by a continuum of workers with a total measure of one. A worker enters the labor market at age $j = 20$, retires at age $j_R = 64$ and lives until age $J = 80$. There is no population growth.

Preferences. Each worker maximizes the time-separable discounted lifetime utility over streams of consumption (c_{ij}) and working hours (h_{ij}). The utility from consumption and working hours in each period is also separable and given by

$$u(c_{ij}, h_{ij}) = \log c_{ij} - B_j \frac{h_{ij}^{1+1/\gamma}}{1+1/\gamma}$$

where $B_j > 0$ indicates age-specific disutility and $\gamma > 0$ stands for structural Frisch elasticity. Note that the structural parameter γ is different from the "aggregate Frisch elasticity," which corresponds to the observed elasticity in the data. However, they are strongly correlated in that if there are no frictions in the economy, the structural elasticity is identical to the observed elasticity. We also assume that workers have to pay a fixed cost ϕ when they change working hours.

[5]This is similar to the adjustment of prices under a positive inflation discussed in Nakamura and Steinsson (2008).

Income Profile. All workers are subject to idiosyncratic shocks to their productivity denoted by x_{ij}. The stochastic process of x_{ij} is described by the transition probability distribution function, $\pi_x(x'|x) = \text{Prob}(x_{i,j+1} \leq x'|x_{ij} = x)$. In our quantitative analysis below, we will assume that x_{ij} follows an AR(1) process in logs:

$$\log x_{i,j+1} = \rho_x \log x_{i,j} + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim N(0, \sigma_x^2)$$

While this is the only source of idiosyncratic productivity, our quantitative work will interpret this more broadly to represent all shocks to the relative return to market work and so influence the incentives to work. Individual labor income is $wx_{ij}h_{ij}$, where w is the aggregate wage rate per efficiency unit of labor.

Savings. We assume that capital markets are incomplete in two senses (e.g., Aiyagari (1994)): (i) there is no insurance for idiosyncratic productivity shocks and the only asset is a claim to physical capital, denoted by a and (ii) workers cannot borrow: $a_j \geq 0$.

Technology. There is an aggregate Cobb-Douglas production function that produces output using inputs of labor (H_t) and capital (K_t):

$$Y_t = H_t^{\alpha} K_t^{1-\alpha}$$

Output can be used for either consumption or investment, and capital depreciates at rate δ each period.

3.2 Equilibrium

We formulate equilibrium recursively. The individual state variables are age j, asset a, idiosyncratic productivity x, and hours worked in the previous period h^-. Prices are functions of the aggregate state: $w(\mu)$ and $r(\mu)$. The distribution of the economy is denoted by $\mu(j,x,a,h^-)$ and its law of motion is $\mu_{t+1} = T(\mu_t)$. The optimal decision rules are functions of consumption $c(j,a,x,h^-)$, working hours $h(j,a,x,h^-)$ and risk-free asset holdings for the next period, $a'(j,a,x,h^-)$ that solves the dynamic programming problem described below.

Market Entrants ($j = 20$). Workers enter the labor market with initial asset $a = 0$. Since they did not work in the previous year, their initial state variables are only

(j, a, x) and there are no adjustment costs. Thus,

$$V(j, a, x) = \max_{c, a', h} \left\{ \log c - B_j \frac{h^{1+1/\gamma}}{1 + 1/\gamma} + \beta \mathbb{E}_{x'|x} V(j+1, a', x', h) \right\}$$

subject to

$$\begin{aligned} c + a' &= wxh + (1+r)\, a \\ a' &\geq 0 \end{aligned}$$

Before Retirement (From $j = 21$ to $j = 64$). Workers now face adjustment costs ϕ every period. Thus,

$$V(j, a, x, h^-) = \max_{c, a', h} \left\{ \log c - B_j \frac{h^{1+1/\gamma}}{1 + 1/\gamma} - \phi \cdot \mathbb{I}(h \neq h^-) + \beta \mathbb{E}_{x'|x} V(j+1, a', x', h) \right\}$$

subject to

$$\begin{aligned} c + a' &= wxh + (1+r)\, a \\ a' &\geq 0 \end{aligned}$$

After Retirement (From $j = 66$ to $j = 79$). Workers retire from work so that they can only choose consumption $c(j, a)$ and the asset in the next period $a'(j+1, a)$. Thus,

$$V(j, a) = \max_{c, a'} \{\log c + \beta V(j+1, a')\}$$

subject to

$$\begin{aligned} c + a' &= (1+r)\, a \\ a' &\geq \bar{a} \end{aligned}$$

Finally, all workers die at age $j = 80$ so we set $V(80, a) = 0$ for all a. An equilibrium then consists of these value functions $V(j, a, x, h^-; \mu)$, individual decision rules $c(j, a, x, h^-; \mu)$, $a'(j, a, x, h^-; \mu)$, aggregate inputs $\{K(\mu), H(\mu)\}$, factor prices $\{w(\mu), r(\mu)\}$ and a law of motion $T(\mu)$. The other equilibrium conditions are as follows.

(i) The representative firm maximizes profits: For all μ,

$$w(\mu) = \alpha \left(\frac{K(\mu)}{H(\mu)} \right)^{1-\alpha},$$

$$r(\mu) = (1 - \alpha) \left(\frac{H(\mu)}{K(\mu)} \right)^{\alpha} - \delta.$$

(ii) The goods market clears: For all μ,

$$\int \{a' + c\}\, d\mu = Y + (1-\delta)K.$$

(iii) Factor markets clear: For all μ,

$$H(\mu) = \int x \cdot h d\mu,$$

$$K(\mu) = \int a d\mu.$$

(iv) Individual and aggregate behaviors are consistent:

$$\mu'(j^0, a^0, x^0, h^{-0}) = \int_{j^0, a^0, x^0, h^{-0}} \left\{ \int_{\mathcal{B}_j, \mathcal{B}_a, \mathcal{B}_x, \mathcal{B}_{h^-}} \mathbb{I}_{a'=a'(j,a,x,h^-)} d\pi_x(x'|x) d\mu \right\} dj' da' dx' dh.$$

3.3 Simulation of Survey

In order to capture the hours gap in the model, we compute preferred hours as the optimal choice of working hours without adjustment costs. Thus, the optimal choice (consumption, asset holdings, and hours worked) without adjustment costs $c^s(j, a, x, h^-), a'^s(j, a, x, h^-), h^s(j, a, x, h^-)$ is:

$$c^s, a'^s, h^s = \underset{c, a', h}{\operatorname{argmax}} \left\{ \log c - B_j \frac{h^{1+1/\gamma}}{1 + 1/\gamma} + \beta \mathbb{E}_{x'|x} V(j+1, a', x', h) \right\}$$

subject to

$$\begin{aligned} c + a' &= wxh + (1+r)\, a \\ a' &\geq \bar{a} \end{aligned}$$

4 Quantitative Analysis

In this section, we calibrate our model and present its quantitative results. First, we impose the (standard) values for some parameters and then calibrate the rest internally by matching some moments in the data. Next, we examine the role of frictions in generating heterogeneous responses of the labor supply at the individual level (i.e., Frisch elasticities in individual level). Finally, we show how the asymmetric cross-sectional distribution of hours gap maps into an asymmetric elasticity of labor supply at the aggregate level.

4.1 Calibration

Table 1: Parameter Values

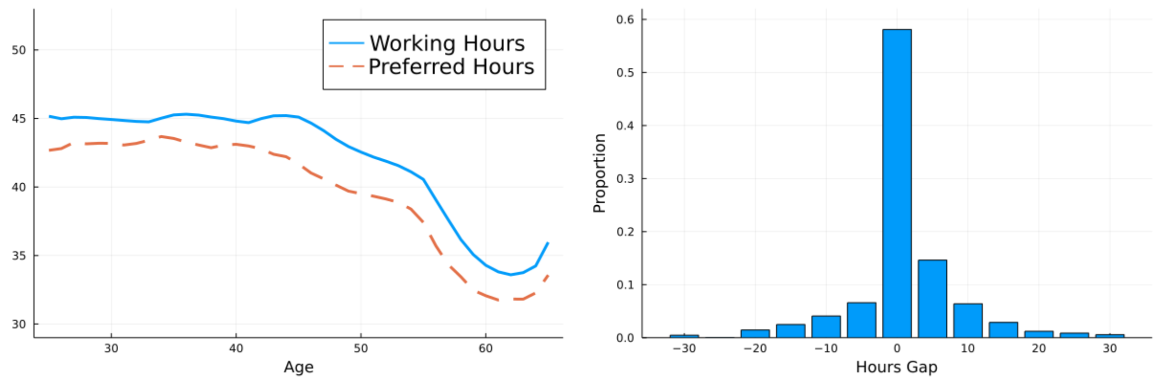
Parameter		Value	Moment to Match
Externally Calibrated:			
Cobb-Douglas Technology	α	0.64	Labor income share
Depreciation Rate	δ	0.10	Standard
Persistence of Productivity	ρ_x	0.822	Chang and Kim (2006)
Volatility of Productivity	σ_x	0.333	Chang and Kim (2006)
Internally Calibrated:			
Time Discount Factor	β	0.94	Annual Interest rate (4%)
Adjustment Costs	ϕ	0.02	Fraction of $\hat{h} = 0$ (58%)
Age-Specific Disutilities	$\left\{B_j\right\}$'s	see text	Age Profile of Hours Worked
Structural Frisch Elasticity	γ	1.2	Observed upward Frisch elasticity (0.3)

The labor share of the Cobb-Douglas production function, α, is 0.64 and the depreciation rate of capital, δ, is 10%. Regarding the stochastic process of idiosyncratic productivity shocks, there is a sizable literature (for example, Floden and Lindé (2001), Chang and Kim (2006), Heathcote, Storesletten and Violante (2008), Kaplan, Moll and Violante (2018), and Guvenen et al. (2021)). While the estimates vary widely across studies, the consensus is that these shocks are large and persistent. Guided by these empirical studies, we set $\rho_x = 0.822$ and $\sigma_x = 0.333$, which corresponds to the annual estimate in Chang and Kim (2006).

The other parameters are set internally to match the target moments as follows. We set the time discount factor $\beta = 0.94$ to match the annual interest rate of 4%. Based on hours worked and the hours gap from HILDA, we internally calibrate the adjustment cost parameters $\phi = 0.02$ to match the fraction of workers in the inaction region ($\hat{h} = 0$), the workers who are content with their current working hours. We choose the age profile of disutilities from working ($\left\{B_j\right\}$) to match the life-cycle pattern of working hours in the data. Lastly, we set the structural Frisch elasticity γ to match the estimates from the quasi-experimental studies based on tax holidays, which report small upward Frisch elasticities (usually less than 0.5). Surprisingly, we need a large structural Frisch elasity $\gamma = 1.2$ to match these. We will explain why this is the case in more detail in the next section. Table 1 summarizes the

Figure 4: Model Results

(a) Age Profiles of Hours (b) Distribution of Hours Gap



parameter values of the benchmark economy.

Figure 4 shows the age profile of hours worked and the distribution of the hours gap in our model. First, Figure 4a indicates that our model reasonably replicates the shape of the age profile of working hours in the data (Figure 3). Actual working hours always exceed preferred hours and preferred hours steadily decrease over the life cycle. Second, Figure 4b shows that our model successfully matches the empirical distribution of the hours gap. Similar to the data, the faction of over-employed workers (27%) is larger than that of under-employed (13%) in our model.

4.2 Microeconomic Mechanism

We show how frictions in our model induce heterogeneous responses of labor to temporary shocks in wages. To illustrate this in detail, we select the behavior of a 50-year-old worker who worked 40 hours in the previous year ($j = 50$ and $h^{-} = 40$) in our model. Figure 5 reports the labor supply and the hours gap of this type of worker.

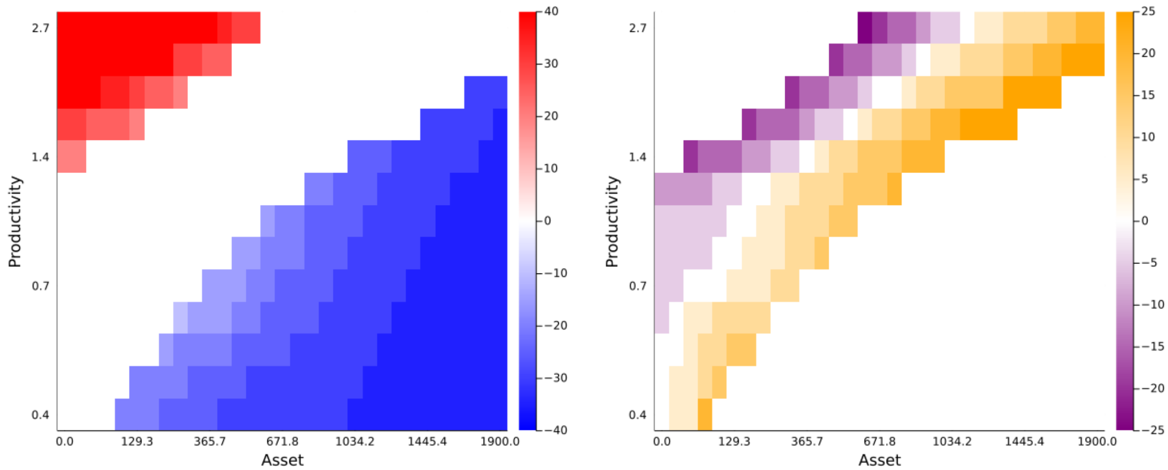
Figure 5a is the heat map of the (average) change in hours $\triangle h_{j,h^{-}}$ for each asset-productivity grid, (a, x):

$$\triangle h_{j,h^{-}}(a,x) = h_{j,h^{-}}(a,x) - h^{-}$$

The red contours (in the northwest region) represent the increase in working hours ($\Delta h > 0$) and the blue contours (in the southeast region) represent the decrease in hours ($\Delta h < 0$). As the color of the contour becomes darker, the larger is the absolute value of hours change. For example, at a given asset level (x-axis), a

Figure 5: Response of Labor Supply and Hours Gap

(a) Hours Change: $\triangle h_{j,h^-}(a,x)$ (b) Hours Gap: $\hat{h}_{j,h^-}(a,x)$



Notes: The figures are examples of the decisions of a 50-year-old worker ($j = 50$) who worked 40 hours in the previous year ($h^- = 40$) in our model. The color bars on the right axis indicates the value of the variable.

high productivity (y-axis) is associated with an increase in working hours (reflecting a Hicksian elasticity of the labor supply). Similarly, at a given productivity level, a larger amount of asset holding is associated with a decrease in hours worked (reflecting an income effect on the labor supply). The most notable feature in this graph is the white area between the two contours: the inaction region of labor supply ($h_{j,h^-}(a,x) = h^-$) caused by the adjustment cost in changing hours. The boundaries of this area represent the (S,s) boundaries of labor supply.

Figure 5b is the heat map of the hours gap $\hat{h}$:

$$\hat{h}_{j,h^-}(a,x) = h_{j,h^-}(a,x) - h^s_{j,h^-}(a,x)$$

The purple contours (close to the northwest region) represent workers who would have increased their working hours without adjustment costs (under-employed). The yellow contours (close to the southeast region) represent workers who would have decreased their working hours without adjustment costs (over-employed). A darker contour reflects a larger hours gap (in absolute value). Not surprisingly, under-employed workers (i.e. $\hat{h} < 0$) are closer to the red boundaries where workers actually increase their hours, whereas over-employed workers (i.e. $\hat{h} > 0$) are near the blue-contour boundaries where workers actually decrease their hours.

Combined with asymmetric distribution of hours gap in Figure 4b, we will show that aggregating these decision rules results in an asymmetric Frisch elasticity of labor supply at the aggregate level.

4.3 Measurement of Individual Elasticities

To compute the aggregate Frisch elasticity precisely, it is necessary to keep the marginal utilities of wealth of all individuals in the economy, which is hard to implement in practice, even in simulations. Thus, we instead follow the approach of Mui and Schoefer (2021), who elicit individual potential responses to temporary wages. Although marginal utilities are not exactly compensated in our model (so that income effects still remain), this approach is useful because it can capture a realistic intertemporal response in the real world.

Given a temporary productivity (wage) shock of $w \to (1+\Xi)w$ (where Ξ denotes the size of the wage increase), a worker at (j,a,x,h^-) grid chooses her optimal consumption, labor supply and savings as follows:[6]

$$c_\Xi^F, a_\Xi'^F, h_\Xi^F = \underset{c,a',h}{\operatorname{argmax}} \left\{ \log c - B_j \frac{h^{1+1/\gamma}}{1+1/\gamma} - \phi \cdot \mathbb{I}(h \neq h^-) + \beta \mathbb{E}_{x'|x} V(j+1, a', x', h) \right\}$$

subject to

$$\begin{aligned} c + a' &= (1+\Xi)wxh + (1+r)\,a \\ a' &\geq 0 \end{aligned}$$

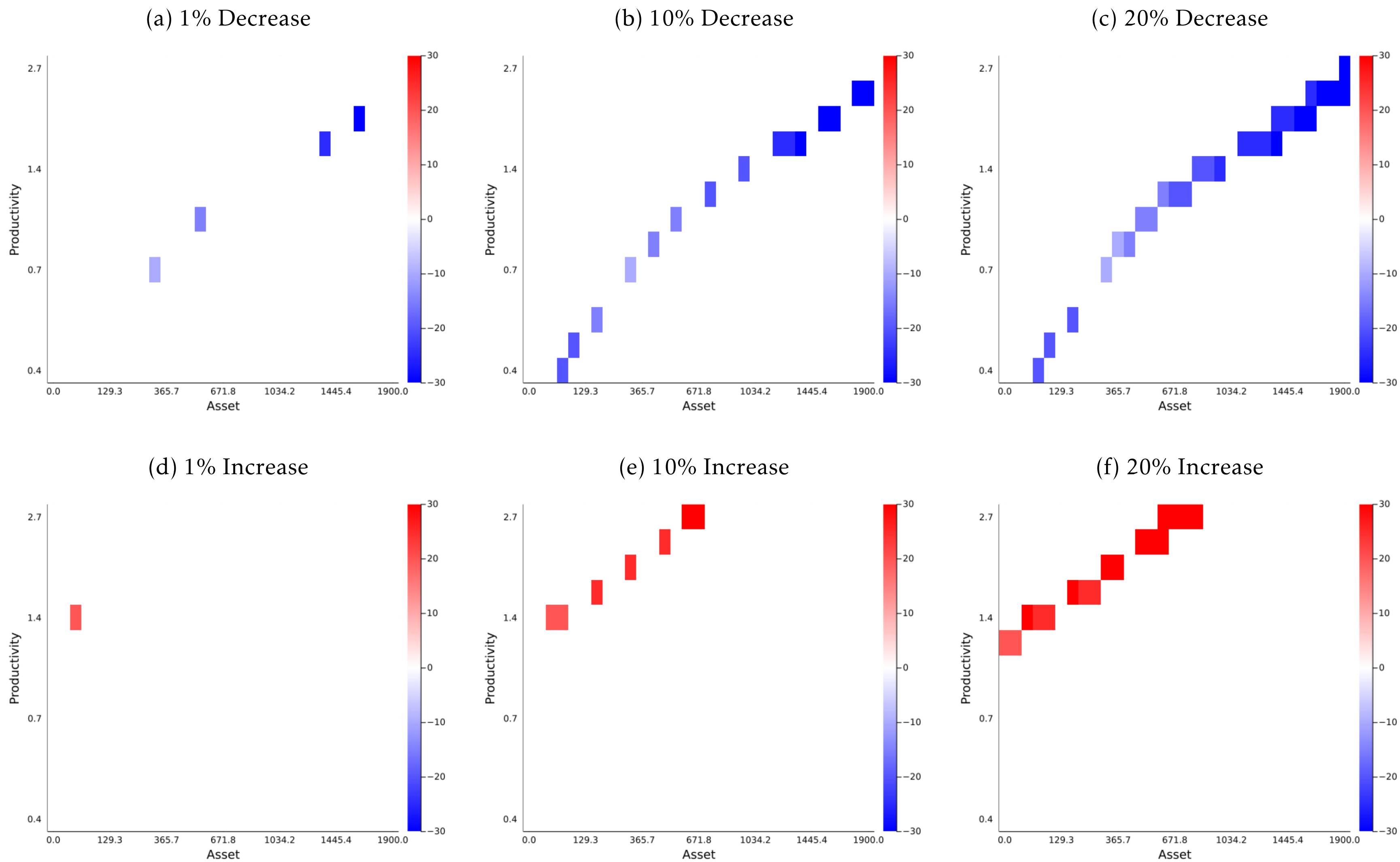
Then, we measure individual Frisch elasticities $\varepsilon_\Xi^F(j,a,x,h^-)$ to a temporary wage shock $w \to (1+\Xi)w$ by

$$\varepsilon_\Xi^F(j,a,x,h^-) = \left(\frac{h_\Xi^F(j,a,x,h^-) - h(j,a,x,h^-)}{h(j,a,x,h^-)} \right) / \;\Xi$$

Figure 6 shows the potential responses (of labor supply) to different sizes of temporary wage change for the same type of worker: 50 years old and worked 40 hours in the previous year ($j = 50$, $h^- = 40$). Figure 6a is the heat map of changes in hours with respect to a one percent decrease in wage: $\Xi = -0.01$. The x-axis is the asset holdings and y-axis is the productivity of a worker. The numbers in

[6] One caveat is that this is a partial equilibrium analysis since each worker in our model do not consider the changes in the interest rate and the economy-wide distribution.

Figure 6: Hours Responses to Temporary Wage Shocks



Notes: The decisions of a 50-year-old worker ($j = 50$) who worked 40 hours in the previous year ($h^{-} = 40$) in our model. The color bars on the right indicate the hours change.

the right axis indicates the magnitude of hours changes. The blue color represents the decrease in hours and the red represents the increase. The darker the bigger the change is. Those just above the (S,s) boundary reduce their hours in response to the wage decrease. Figure 6b ($\Xi = 0.1$) and Figure 6c ($\Xi = 0.2$) show that, as the size of the temporary shock increases to and, respectively, more workers reduce their hours. These figures imply that the aggregate elasticity will heavily depend on the density of such workers. A similar pattern is found in the case of wage increases (Figure 6d, Figure 6e, and Figure 6f). As we see more blue areas (than the red), workers are more likely to respond to the decrease (than increase) in wages, although this prediction depends on the distribution of workers at a time.

4.4 Asymmetry in Aggregate Elasticity

We now turn to the aggregation of micro elasticities. As we have shown above, the aggregate elasticity is not constant because aggregation involves the heterogeneous responses of individual workers as well as the density of potential respondents in the economy. We obtain the aggregate elasticity by aggregating individual responses as:

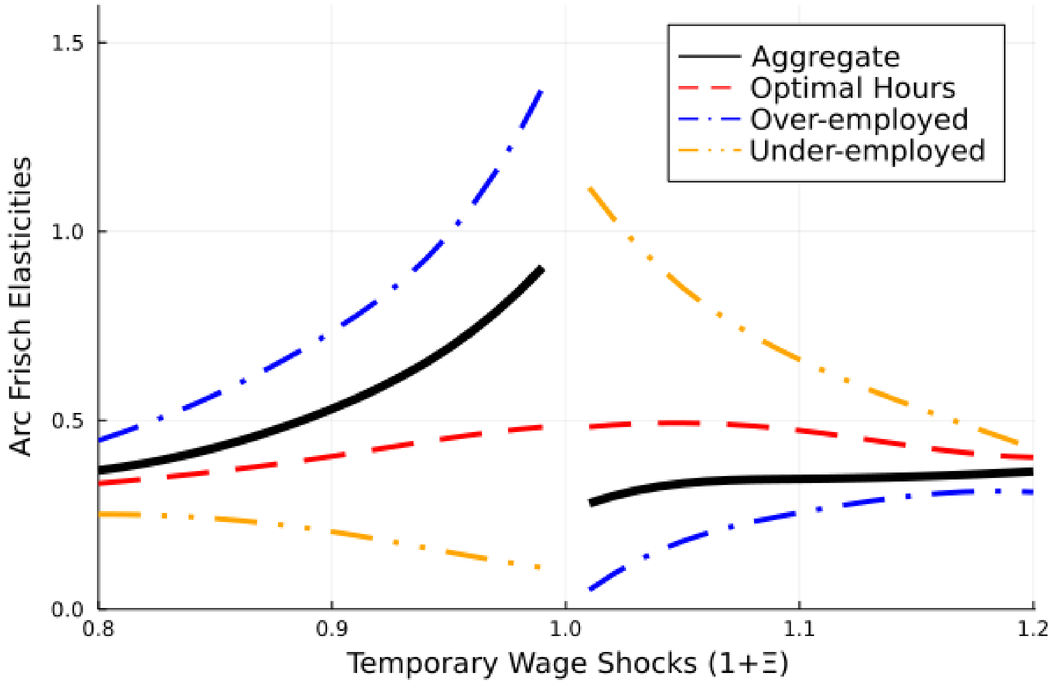
$$\varepsilon^{F}_{\Xi,\text{agg}} = \left(\frac{\int h^{F}_{\Xi}(j,a,x,h^{-})d\mu - H}{H} \right) / \ \Xi$$

Figure 7 shows the Frisch elasticities of the aggregate and the three groups: over-employed, under-employed, and those who are content with the current choice of hours in Figure 5b. The x-axis represents the size of wage shock, $1+\Xi$. In response to a wage *decrease*, the over-employed exhibit the largest response and the under-employed the smallest response. In response to a wage *increase*, the under-employed show the largest response and the over-employed the smallest. Given the asymmetric distribution of the hours gap, the response of over-employed dominates, resulting in an asymmetric aggregate elasticity. Note that there is also an asymmetry in the absolute size of the hours response to a decrease vs. an increase of wages. This is because workers anticipate a downward trend in hours over time (due to an increasing disutility from working over the life cycle). When they do react, they take a larger step in the event of decreasing hours. This also makes the aggregate Frisch elasticity larger for the wage decrease.[7]

The asymmetric nature helps us to resolve the tension between micro and macro

[7] This parallels the asymmetric price change in the presence of a positive (anticipated) average inflation rate in the menu-cost models of sticky price.

Figure 7: Arc Frisch Elasticities of Labor Supply



elasticities in the literature. Our finding illustrates that it is possible for the aggregate labor supply to exhibit *small* upward Frisch elasticities (from events such as tax holidays) and moderately *large* downward Frisch elasticities. Therefore, we argue that small elasticities found from the tax holiday studies do not necessarily reject the use of sizeable aggregate Frisch elasticities for the business cycle analysis, consistent with the empirical findings in Mui and Schoefer (2021).

5 Conclusion

The slope of the labor supply curve is at the heart of macroeconomic analyses. It is crucial in understanding economic fluctuations over the business cycle and in determining the effect of fiscal policies. Despite various attempts to reconcile the tension between micro and macro elasticities of the labor supply, the exact mechanism behind the mapping from individuals to the aggregate labor supply remains an important question to study. In this paper, we contribute to this literature by offering an economic mechanism that generates asymmetric elasticities of the aggregate labor supply.

Based on Australian panel data (HILDA), we find (i) large gaps between actual and preferred working hours, (ii) an asymmetric distribution of the hours gap (workers are over-employed on average), and (iii) a pronounced age-related decrease in desired hours.

By integrating (i) an increasing age profile of disutility from working and (ii) a fixed cost of changing the labor supply, our model can generate asymmetric responses of the labor supply at the aggregate level. The fixed cost of changing working hours, creates a region of inaction in the labor supply. With increasing age profile of disutility from working, the economy is persistently over-employed on average (i.e., there is an asymmetric cross-sectional distribution of the hours gap between actual and preferred hours). Moreover, over-employed workers tend to show larger responses of hours because they anticipate a downward trend in (preferred) hours over time. As a result, the aggregate labor supply exhibits asymmetric Frisch elasticities: larger one in response to a decrease in wage than that to an increase. This may partially justify a use of somewhat larger elasticities of labor supply in business cycle analyses than those found in quasi-experimental tax holiday studies.

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