Horizontal Differentiation of Tasks and Skills: Internal and External Labor Markets Part 1*

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June 30, 2025

Abstract

When workers' skills can only be revealed by matches into tasks, and then only via the productivity of their teams, productivity-improving internal re-matching should improve on skill information to the external market. We show that this is not necessarily so. Information generated in more productive firms may become garbled, even overturning productivity advantages generated in the internal market.

We derive this and other results within a parsimonious parametric model in which we transfer and adapt the horizontal differentiation model known from the IO literature to a labor market characterized by symmetric imperfect information about the typical worker's skills. Information about these skills is revealed only indirectly via the productivity of the teams in which the workers are active. Firms as teams vary in size, and technology reflected in their structure of tasks from specialized to diversified. Frictionless re-matching within the internal labor market improves on the team's productivity. Misalignment between the distribution of skills and that of tasks can be removed only by external labor market actions, constrained by informational frictions.

In part 1 of this paper, we analyze internal market equilibrium. For external market activities, we assume that firms can realize their imperfectly informed desired demand from a supply in which skills are revealed perfectly. In part 2, we realistically consider labor supply in the external market as in part furnished by separations, and structured by signals involving the typical worker's employment history.

We demonstrate generic opposite effects of internal on external market activities. In the internal market, constrained efficient re-matching yields the average productivity of firms to *increase* in size and diversity of structure. External market activity improves on firms' expected productivity, but the strength of improvement *decreases* in firm size and diversity of structure: The more internal re-matching leads to leftward skewness in the distribution of firms towards higher productivity, the less informative is the specification of desired demand (and, as we show in part 2: in the supply) in the external market, which may overturn internal market effects.

JEL-classification: J21, J23, J62, L23.
Key words: Internal Labor Markets, Horizontal Differentiation of Labor and Tasks, Imperfect Information, Employment History

*We are grateful to comments we have received from Gregorio Curello, Simon Jäger, Martin Peitz, and Peter Norman Sorensen as well as seminar participants at the University of Mannheim. Konrad Stahl gratefully acknowledges support by the Deutsche Forschungsgemeinschaft through CRC TR 224 (project B04).

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1 Introduction

In the labor market, the skills of a worker are often unknown before she is employed in a particular task, and only partially revealed after she has performed in that task for some time. Indeed, the worker's performance varies not only across tasks, but in the interaction with other workers within the firm. Sometimes, the productivity of a team member can be isolated; but often, only the aggregate outcome of all matches within the firm is revealed via the firm's operational result. Experimentation via re-matching workers by skills to tasks within the team or firm may improve on that productivity, but is constrained by the current stock of hires.

When becoming active in the external labor market to relax that constraint, the firm may intend to fill a vacated task by hiring a worker that optimally matches that very task. The above informational imperfections may hinder the maximization of the firm's productivity, however. Rather than pursuing *task-specific* optimality, the firm should anticipate productivity-increasing re-matching before the hire, i.e., pursue *firm-specific* optimality. To yield efficiency, the challenge in the firm's external labor market activity is then to minimize the mismatch between the *distribution of workers by skills*, and the *distribution of tasks* requiring particular skills.[1]

On the labor demand side, the challenge is that the observed team or firm productivity may be realized with different match patterns, hindering the precise inference of the worker skill in need. On the labor supply side, the challenge is to infer the precise skill of the typical worker from a noisy signal provided by her employment history. Intensified internal labor market activity should result in improvements in information on both sides of the external market, and thus the match quality in individual hires should improve. We show that while it does on average, it does not so in the way one might expect across firms with different size and structure. Indeed, firms that enjoy high productivity as a fruit of intense re-matching in the internal market may be outpaced in productivity by firms enjoying higher precision in their external market activity, that is due to less intense internal one.

We derive these results within a stylized parametric model involving a horizontally differentiated labor market akin to the horizontally differentiated product market that is a central workhorse in industrial economics. Firms, endowed with a simple technology that is additive in the number of workers, are differentiated by size, i.e., number of tasks; and structure, i.e., composition of tasks, from specialized to differentiated. We suggest a mapping of workers' skills into firms' tasks such that all workers are equally productive if matched into the task which they perform best. But skills and tasks differ, resulting in match patterns, and thus productivity to vary across workers and firms.

Our labor market is decomposed into many firm-specific internal markets and one external market. The typical firm-specific internal market involves the re-matching of workers the firm has hired from the external market with (initially) unknown skills to maximize the firm's pro-

[1] A typical example for productivity improving internal re-matches: large consulting firms, such as McKinsey, tend not to hire for particular tasks, but to re-match new hires with different tasks in order to find out about optimal matches.

ductivity. The external market is activated by random separations of workers from firms. Firms draw workers from the external market corresponding to the number and structure of vacant tasks. Towards that, the typical firm specifies its demand by inferring from the vacated task the desired skill of its hire. The typical worker signals the firm type and task in which she was employed previously.

To highlight the effects of internal labor market activity, we start by assuming initially that the pool of workers is not structured by employment history. The workers' individual productivity is assumed to be revealed only collectively by the firm's productivity. The resulting productivity wage reflects that aggregate productivity and cannot be individualized.

In the typical firm's internal labor market, re-matching of workers to tasks is considered frictionless. The intensity of internal labor market activity naturally varies, however, from none in fully specialized firms to maximal in firms fully diversified w.r.t. their tasks. While efficient, allocation decisions in the internal market are constrained by the workers in the firm being limited to the current draw. This constraint is relaxed by resorting to the external market.

To analyze the effects of internal labor market activity on the efficacy of the external market, we assume that a randomly selected worker leaves each firm and joins that market. When offering her services, she provides a signal about her former employment. Each firm then demands one worker in the external market to fill the vacated slot. Hiring from the external market is impaired by improved, but potentially still incomplete information about the skills required to maximize the firm's productivity, and about the skills offered by formerly employed workers that eventually constitute the dominant portion of supply. To isolate the information effect at the demand side, we assume in this first part of the paper that the firms formulate their desired demand by skill on the basis of the information available to it, but are confronted with a perfectly informative supply differentiated by skills, so that all imperfections in the resulting matches are due to the imperfections in the firms' specification of the desired demand. In the second part of the paper, we analyze the inverse of this construct, in which firms that are perfectly informed about their match pattern meet a supply of workers with skills noisily specified. Thereafter we analyze equilibrium based on imperfectly specified demand and imperfectly signaled supply.

We use several non-standard assumptions. First, our agents are indivisible: firm sizes and compositions by tasks are discrete, and matching is exclusive: each task can be occupied by one worker only. Furthermore, we consider three purely horizontally differentiated skills and three tasks, and use a particular mapping from skills into tasks. Second, match quality – the productivity of a worker in a particular task– is only indirectly revealed via the productivity of the entire team, here the firm. Internal re-matching may locally maximize that productivity. The revelation of productivity at the firm rather than the individual level necessitates payment of firm-specific uniform wages. Third, we summarize worker releases and quits by random separations. Fourth, we temporarily assume in part 1 of the paper that the firms' desired demands are satisfied without constraints.

Our first assumptions, the indivisibility of workers, and the differentiation of skills and tasks to more than two invite a critically much richer information environment than heretofore, allowing us to typify firms in a mode required to arrive at our key results. The analysis requires the employment of a parametric model.

With the second assumption, we highlight that an individual worker's remuneration based on productivity only indirectly depends on her individual skills, but directly on the match quality of her team –an externality that is often suppressed in the labor market literature. We ignore the possibility that information on individual productivity may be generated within internal labor market activities. While the employing firm may benefit from improved private information about its employees' skills, the current reduced-form setup allows us to focus on the information generated by the constrained efficient match to the external market via the typical separated worker's employment history.

Random separations – our third assumption highlighted here – directly reflect situations in which workers separate from their firm for exogenous reasons, e.g., retirement, or outmigration; beyond that, the revelation of quit vs. dismissal by the worker may be subject to strategic bias, and thus uninformative. By considering, with our fourth assumption, a perfectly informative supply side of the external labor market, we isolate the demand side effect of limited information on external labor market activity. Any constraints on the supply side would garble this effect. In the second part of our paper, we will first employ the opposite assumption and confront firms perfectly informed about their demand with supply based on noisy skill signals. With all this we decompose the complexities involved in matching imperfectly informed demand with imperfectly informative supply in the external market, the consequences of which we will discuss in the last part of the second paper, thus incorporating the information imperfections generic to the setup we are working in.

By our main result in part 1, the allocation effect generated by frictionless re-matching in the internal market, and the resulting information effect guiding hires in the external market work in opposite directions: On the one hand, re-matching in the internal labor market increases the firms' productivity the more, the more diversified their task structure. This results in a distribution of the firms by productivity that is the more left-skewed, the more intense the internal labor market activity.

On the other hand, the concentration of firms with a variety of match patterns that yield the same high productivity level has the effect that, except at maximal productivity that involves perfect matches in all tasks, the specification of demand by skill becomes more garbled.[2] The particularly strong productivity increase generated from internal labor market activity in diversified firms may be more than compensated by the (relative) decrease in match quality and thus firm productivity due to worsened signal quality in the external market. Thus, a relatively specialized firm type that did not perform particularly well in the internal market may overturn in

[2]We show in part 2 of the paper that the signals from released workers to the external market about their skills are garbled as well. The consequences on match quality spread over the entire external market, and further disadvantage relatively unproductive firms.

productivity the diversified firm type that did well in there.

In more detail: in the temporary equilibrium after optimal re-matching in all internal labor markets, firm-specific productivity and thus wage payments averaged over all realized productivity levels increase in firm size at a decreasing rate; furthermore, at given size, average equilibrium wages increase with increasing diversification in the firm structure. Indeed, the distributions of firms by productivity can be strictly ordered by the 1st order stochastic dominance criterion.

By the nature of the uncertainty considered here and the inferences that can be drawn from the matching decisions, it remains unclear which pattern of matches by skills into tasks has generated the observed productivity. Thus, uncertainty tends to remain on both sides of the market: on the demand side about the desired skill, and on the supply side on the skill offered by the typical worker; this even when characteristics of the most recent employment are revealed.

We show that nevertheless, the skill information provided to the external labor market improves. We show this in the present part 1 of the paper for the demand side – and in part 2 for the supply side. In contrast to what one might expect, however, the internal labor market activity that improves on the aggregate productivity of matches tends not to increase the quality of skill information provided from the internal to the external market, but to diminish it. And this down to the point that the productivity advantage from re-matching in the internal market enjoyed by firms with a diversified structure of tasks is more than counteracted by the information advantage enjoyed in the external market by less diversified firms.

The micro-detail we derive is revealing here: while intensified internal market activity increases aggregate productivity with increasing firm size and diversification in the firm's task structure, any separation from a then more productively matched worker increases the loss in the typical firm's productivity. At the same time, however, the information about the desired demand replacing any loss diminishes with firm size and diversification. In all, the increase in productivity to be expected even when the external labor market supplies workers with known skills becomes smaller with increasing firm size and diversification. We show by example that this may result in a switch in the rank order of firms by productivity, relative to the rank order generated by internal market activity.

To the best we know, we are the first to account for the effects of internal labor market activity on the productivity of the firm, and for the information generated to the external market on a critical element of the typical worker's employment history –her last job. In passing, that information creates a wedge between these workers and workers newly entering the labor market without any employment history, contributing to the empirically observed discrimination against the latter. In part 2 of the paper we will analyze these effects by comparing the effect on the typical firm's productivity, and thus its wage offer to hires as varying with the signaled information.

Central to our approach is that after constrained optimal internal re-matching and the ensuing loss of one worker, the firm's hire from the external market should be geared to minimize the mismatch between the *distribution of workers by skills*, and the *distribution of tasks*. This may imply that the skill sought by the firm in the external market does not match the vacated task, and constrained maximal productivity will be generated by the ensuing internal re-matching.

We arguably are also first to account for the hysteresis effects on the formation of labor demand generated by the remaining stock of workers, when a task is vacated; and correspondingly, the effects of the typical worker's employment history on the match quality attained in hires from the external market. Note that the more information available on released workers, the more the external market becomes differentiated on the supply side not only horizontally, but also vertically. Vertical differentiation is generated by variation in the quality of signals on the typical worker's previous match quality –and with it, on her skills. Quite intuitively, a high previous wage indicates high productivity, and thus a good match in the worker's previous employment.[3] We will analyze the effects of this differentiation in part 2 of the paper.

The remainder of this paper is structured as follows: In the next section 2, we report on the sparse literature we relate to. In Section 3 we specify the model, and in Section 4 we report on the results involving internal market equilibrium, and on external market equilibrium when firms can satisfy their desired demand from informative supply. We conclude with Section 5. In Part 2 of the paper, we first look at the effects of imperfect information when the firms satisfy perfectly specified demand from the imperfectly informative supply generated by the random separations, and finally close the model by solving for, and characterizing the equilibrium when both sides of the market send imperfectly informed signals.

2 Related Literature

We relate primarily to the literatures in organizational and personnel economics; in particular on matching, team productivity, and internal markets. We contribute by transferring and adapting the horizontal differentiation model known from the IO literature to a labor market characterized by symmetric imperfect information about the typical worker's skills, and by considering an indirect form of revealing information about these skills, via the productivity of the teams in which the workers are active. We focus on the interaction between firm-internal markets and the external market, via the information generated from firm-internal re-matching about employees' skills, and demonstrate that this interaction is strongly dependent on the size and internal structure of the firms demanding from, and supplying workers to that market.

In all these respects, our approach differs from those developed in the vast matching literature surveyed by Lazear and Oyer (2012) and Chade et al. (2017). By maintaining symmetric informational imperfection about unknown skills, we abstract from the consideration of effort and incentives that is in the focus of that literature, and indeed also in the literature on teams.

[3] As we consider purely horizontal differentiation, a very low wage is also informative, namely of a bad match, by which one predicts more productive employment in another task. This is a deliberately created artifact to isolate the effects of purely horizontal product differentiation.

Almost all matching models involve the vertical differentiation of labor. For recent contributions, see Pastorino (2015) and Pastorino (2024). Analyses involving horizontal differentiation of labor and tasks, and the firm's internal labor market activities and their relationship to its external ones are sparse. To the best of our knowledge, only Li and Tian (2013), Papageorgiou (2018), and Pastorino (2015) model an internal labor market in a form related to our approach, and only in Li and Tian (2013) the firm takes an active role in this. Team productivity is nowhere an issue. Furthermore, the external labor market remains undifferentiated, as any worker returning to that market is identical to a newly entering worker. This is not so in our model. Here the worker carries with her information about her previous employment relationship, with obvious implications on the firms' selection of workers. Accounting for the worker's employment history strikes us as an important aspect involving human capital allocation and the evaluation of the efficiency of the external labor market –also in the interplay with the internal labor market that potentially importantly contributes to its structure.

As to more detail, in Li and Tian (2013) the firm can create up to two occupations with differing skill requirements. By observing match qualities after an initial random assignment, the firm can potentially improve on those by re-assigning workers. Their matching quality is binary, so match quality is perfectly identifiable. We consider imperfectly productive matches as well, together with the revelation of only team productivity. By this, the same outcome may be generated by different match patterns. Furthermore, it yields an empirically relevant potential misalignment between the firm's and the typical worker's objectives: while the firm could match a worker to a task leading to her maximal *individual* productivity, it may prefer to arrange workers to achieve maximal *firm* productivity. With this we also highlight the team aspect involved in a firm's production process. Furthermore, we include generic noise on the demand side, and noisy information about previous employment on the supply side, both influenced by internal labor market activity. This yields our external labor market to be differentiated in a critical way, as influenced by the internal market.

Papageorgiou (2018)'s model is essentially single-agent –and essentially, single-firm. The worker takes the active role by moving between occupations within that firm. This way she sequentially learns her match-quality. By assumption the occupations sought by the worker are available at any time, and thus supplied fully elastically. The worker's aim is to find the match maximizing her productivity –and with it, her *individual* wage. In this setup, maximizing the worker's productivity is also in the firm's interest. By contrast, in our model, the occupations offered by the typical firm are limited. Thus, the worker's best match occupation may be filled with another worker, which introduces yet another realistic friction. Furthermore, the firm seeks to match workers to occupations in order to maximize the firm's, rather than the individual worker's productivity.

While Papageorgiou (2018) focuses on the internal labor market and ignores essential features of the external one, Papageorgiou (2014) focuses on the external labor market, but ignores the internal one. When the worker has learned that she is unproductive in one (single occupation) firm, she may direct her search towards other firms with occupations that likely require different skills. The model is rather similar to Papageorgiou (2018), and with this shares the differences to our model. In particular, the worker learns perfectly her match quality; the supply side of the external labor market remains unstructured; and firms remain inactive in the hiring process. Inasmuch relevant within our analysis, we replicate the stylized facts presented in Papageorgiou (2014) and Papageorgiou (2018) within a very different, but arguably more realistic modeling framework.

Pastorino (2015) considers learning in a vertically differentiated labor market, about workers' ability on jobs at different hierarchical levels (see also Pastorino (2024)). High-ability workers generate higher expected output in each job than low-ability worker. Furthermore, the high-ability worker perform best in high-ranking, and low-ability workers in low-ranking jobs. Jobs are differentiated not only according to how important high worker quality is for productivity but also how informative success or failure are about the worker ability level. Based on the information on the workers' performance in particular jobs, workers are reallocated to the jobs available in the economy. Our approach differs mainly by our horizontal differentiation, by our firm orientation distinguishing between firms by size and structure, and by our team productivity and remuneration approach. With our focus on horizontal differentiation, learning takes place not about workers' ability levels like in Pastorino (2015) but solely about his/her best-suited task. Horizontal differentiation also affects the structure of demands, by that firms recruit for an optimal distribution of workers rather than trying to find the best worker for a given vacancy.

A crucial feature of our paper is that only team outcomes are observed by the firm. Beginning with Holmstrom (1982), the focus of the sizable literature on teams has been on the provision of effort incentives in settings when team members' individual contributions to performance may or may not be difficult to ascertain. For example, team members' individual performance may have externalities on the performance of others or there are direct actions of helping, harming, or monitoring team members' performance, a recent example being Onuchic and Ramos (2023).

In contrast to those models, we abstract from incentive issues entirely. Instead, our concern is the dynamic reallocation of individuals to tasks *within* and *across* firms as well as firms' and workers' symmetric learning about matches between workers and task during that process. Few papers on incentives in teams include learning about workers. However, if learning plays a role, it is about overall ability, i.e., it has a vertical connotation (e.g., Auriol et al. (2002) and Chalioti (2016)) rather than the horizontal differentiation we focus on.

Our analysis of the internal market is reduced-form, and void of the institutional concepts described by Doeringer and Piore (2020) in their celebrated 1971 book. Rather, it is a black box, in which workers are re-shuffled across tasks to maximize the typical firm's productivity. Our focus is on the consequential precision on the skill signal to the external labor market.

In his survey of the analysis of internal markets, Waldman et al. (2012) focuses on vertical differentiation and agency. He does not consider horizontal differentiation and concomitant to that, productivity-improving re-matching –and thus, also not the information flow generated from internal market activity on the external market.

To the best we know, Tate and Yang (2015) are the only authors documenting the active re-matching of workers to jobs within firms. In their sample, diversified firms are especially likely to reallocate workers to lines of business in different industries within the firm in response to changing opportunities. Their evidence suggests that diversified firms use internal labor markets to allocate human capital more efficiently than the external labor market does: workers in diversified firms not only are on average more productive than in focused firms but also tend to be more highly sought-after externally. The transferability of human capital across industries within diversified firms may even facilitate diversifying acquisitions (Tate and Yang (2024)).

Their empirical results reflect the significant role of diversified firms for labor allocation in our model as well as the attractiveness of workers of workers from diversified firms in the external market. Our results are more nuanced, however, as we show that workers of moderately diversified firms tend to be even more attractive to other employers than those of highly diversified firms.

From a conceptual perspective, we feel that our horizontal differentiation of tasks and skills opens a new avenue for the analysis of matching processes. By the idea that a worker's skills are revealed only indirectly via the productivity of the team in which she is working in, we emphasize that the productivity of a worker may be critically dependent on her immediate working environment. By considering team-internal, or firm-internal re-matching, we account firstly for the possibility that re-matching may be productivity-improving –and that the potential for this improvement differs by the options open internally, i.e., differences in firm size and the firm-internal task structure. Secondly, we account for the informational effect of internal market outcomes on the external market: With improvements of match quality via internal re-matching, both firms and workers should become better informed about idiosyncratic abilities, which should improve on the efficiency of matches resulting from hires in the external market. We show, surprisingly, that this is not necessarily the case.

Finally, as the technologies of our firms are specified by the tasks to be performed therein, we see an analogy to the "task approach" discussed in Acemoglu and Autor (2011)'s survey. Indeed, one could interpret our approach as foundational to the analysis of labor market phenomena within that "task approach", as it allows, e.g., for the analysis of the consequences of separate shifts in either the task or the skill structure of firms.

3 Model

We model an island economy consisting of a large but finite number of risk neutral firms of given size, and a correspondingly large finite pool of workers. Firms, producing with labor as the only factor, are defined and vary exogenously by number and structure of tasks, and workers differ exogenously by skills. All differentiations are symmetric: a uniform distribution of tasks corresponds to a uniform distribution of skills, so that, if the economy were frictionless, the market would clear by having each task perfectly matched with a worker exhibiting the corresponding skill. We ignore the output market. Only firms are active, workers remain passive. The typical firm acts rationally, i.e., it knows the model. All labor market activities are costless. The market is perfectly competitive.

We limit firm size S to maximally $S = 3$ and call such a firm *large*, and *passim* extend our analysis to involve *medium-sized* two task firms, and *small* one task firms. There are three types of tasks $T, T \in \{A,B,C\}$. For instance, in a large firm, thus involving $S = 3$ tasks, T_n denotes task T at position $n \in \{1,2,3\}$. Let $k_S \equiv \{T_1^k,..,T_S^k\}$ denote the structure of a firm of size S. In all, our economy involves 10 large firm types with structure $k_3 \in \Gamma_3 = \{AAA,BBB,CCC,AAB,BBC,CCA,AAC,BBA,CCB,ABC\}$; 6 medium-sized firm types with structure $k_2 \in \Gamma_2 = \{AA,BB,CC,AB,BC,CA\}$; and 3 small firm types with structure $k_1 \in \Gamma_1 = \{A,B,C\}$. We drop the size subscript when we refer to the firm's structure in general.

A firm of size S needs S tasks to be filled with one worker each. Different tasks require workers with different skills to be matched with the tasks in a maximally productive mode. There are three horizontally differentiated skills. A worker is characterized by skill $t \in \{a,b,c\}$. *Ex ante* that skill is unknown to both the worker and the firms. It is revealed only indirectly via the productivity of the team involving all matches in the firm in which she is employed.

Workers with different skills are imperfect substitutes in fulfilling a given task. The degree of substitution decreases clockwise as in the circular Vickrey-Salop-model well known from the Industrial Organization literature. Thus, the match of a worker is denoted by $(T, t+i)$, $i \in \{0,1,2\}$. The productivity $P(T, t+i)$ of a match, observable only to the modeler, is quantified as follows: $P(T,t) = 1, P(T, t+mod\{1\}) = 2/3$, and $P(T, t+mod\{2\}) = 0$.[4] We call these matches perfect, imperfect, and bad, respectively. Figure 1 illustrates the notation.

We interpret this setup with an example: Let task A require technical; task B accounting, task C marketing activities; and a denote manual, b computational, c communicative skills. In manufacturing physically involving task A, skill a may fit task A best; skill b second best, and skill c third best, as good accounting requires knowledge of the technical flow of the production process, but marketing relates almost exclusively to the final product and not to the production process. Another example of the types of skills we have in mind involve optical vs. auditive vs. manual capabilities. Such skills may be productive independently of a particular educational background, and revealed only when employed in a particular working environment –a particular task and a particular team.

Prominently in our analysis figure variations in internal labor market activity and their effects on both the typical firm's external labor market activity and productivity, that are due to

[4]In more detail, let $T = B$. Then $(T,t) = (B,b)$ with productivity $P(B,b) = 1; (T,t+1) = (B,c)$ with productivity $P(B,c) = 2/3$; and $(T,t+2) = (B,a)$ with productivity $P(B,a) = 0$. Setting productivity to zero for bad matches is just a normalization.

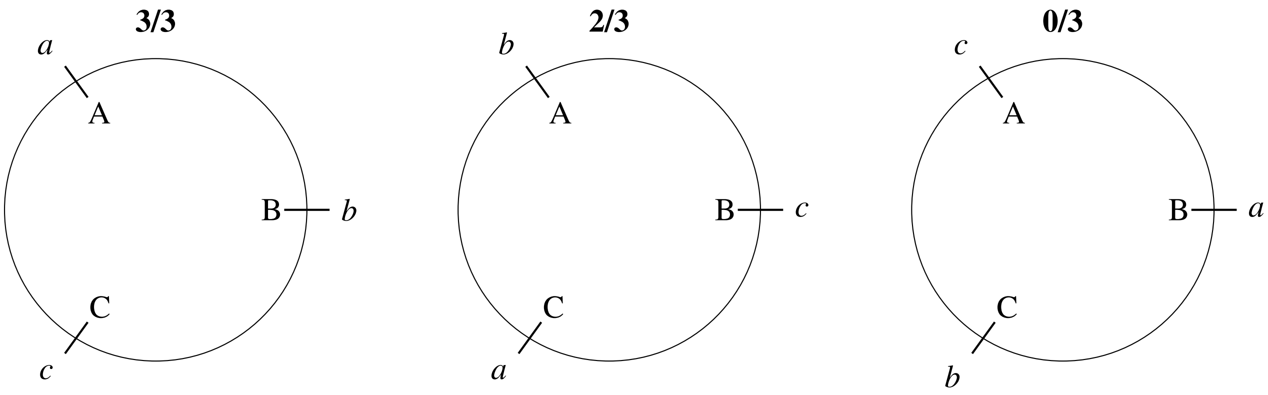

Figure 1: Productivity of Individual Matches

variations in the firm's (size and) structure. Towards that it is helpful to typify tasks and firms. Maximal variations arise in large firms. To begin, in a large firm structure, we call a task ***rare*** if it is contained only once, and ***common*** if otherwise.[5] Furthermore, as hires in the external market are uncertain w.r.t. skills, we typify firms by structure in view of their potential to productively accommodate any skill. We distinguish between fully and partially diversified, and fully and partially specialized firms, and denote these structure types by κ. Specifically, a firm is called

- ***fully diversified****, or FD-firm if it can match all available skills perfectly*
- ***partially diversified****, or PD-firm if one of the skills can be matched only imperfectly in a common task*
- ***partially specialized****, or PS-firm if one of the skills can be matched only imperfectly in a rare task*
- ***fully specialized****, or FS-firm if it can match one skill only badly.*[6]

Our firms' technologies are additive –that is, the typical firm's productivity is determined simply by the equally weighted sum of individual productivities. Let θ_S, $S = 1, 2, 3$, denote the set of workers hired by a firm with structure k_S. With match qualities known to the modeler, the productivity of that firm is specified by

$$P_S^k(\theta_S) \equiv 1/S \sum_{n=1}^{S} P(T_n^k, t + i_n), \tag{1}$$

if one worker is matched to every task, and 0 otherwise. We thus assume away economies of scale, in order to isolate the efficiency improving effect of larger internal labor markets in larger firms that involve a more differentiated structure of tasks. We assume the firm's productivity to shrink to zero, however, if one or more tasks remain unserved.

[5]Thus, in a firm with structure *AAB*, *B* is a rare, and *A* a common task.

[6]Hence *ABC* is fully diversified; *AAC*, *BBA*, or *CCB* are partially diversified; *AAB*, *BBC*, or *CCA* are partially specialized; *AAA*, *BBB* or *CCC* are fully specialized. While these definitions are specific to the limited size and structural composition of firms discussed in this model, they should indicate that within the context developed here, differences in the potential to productively accommodate skills may figure importantly.

When the firm structure under scrutiny is clear, we denote a match pattern simply by the sequence of the workers' skills. By example, for an *AAC*-firm, the alternative match patterns involving two perfect matches and one imperfect match are denoted by *aaa*, *abc*, and *bac*. With small modifications indicated below at the appropriate places, the above specifications translate into smaller firm sizes.

The typical firm hires from the external market the number of workers corresponding to the number of its vacant tasks, offering a wage conditional on its maximal productivity realized after internal re-matching. In equilibrium all firms pay a wage that is proportional to their productivity. Since only team productivity is revealed, the wage is uniform across all employees within the typical firm.

Uniformity of wages appears to be an extreme assumption, but is consistent with our (initial) information structure. Furthermore, it can be motivated by the following sequence of actions: The firm hires a worker, offering the conditional wage contract. Then, re-matching takes place. Based on the resulting productivity signal, the firm offers a corresponding uniform wage. Since productivity drops to zero whenever one worker refuses to accept the offer, all workers have identical outside options. Thus, the bargaining outcome yields the same wage for each of the firm's workers, e.g., their (identical) Shapley value.[7]

We consider two types of equilibria: An internal labor markets, or ILM-equilibrium, and an external market, or ELM-equilibrium. In an ILM-equilibrium, all firm maximize their productivity via frictionless internal re-matching, e.g., by trial and error, constrained by the set of workers as drawn from the external market, and pay a wage corresponding to the resulting firm productivity. In an ELM-equilibrium, all firms optimally hire from the external market to fill vacated tasks.

In principle, that external market is supplied by both newcomers without, and workers with employment history. For expositional purposes and to isolate phenomena, however, we consider, in part 1 of the paper, special versions of the two equilibria. First, to study the effects of internal market activities, all firms hire workers from an unstructured market, corresponding to a market involving newcomers only. Second, to study information consequences for external market operations, we wish to isolate the demand side and supply side effects. To isolate the demand side effect, we assume external market supply forthcoming so that the firms' demands are optimally satisfied. In part 2 of the paper, we will do first the opposite, namely confront firms that are perfectly informed about their demand with an imperfectly signalling supply. We then look at imperfect information on both sides of the external market, and study temporary equilibrium and stationary state.

Returning to the first point, let Θ_S the set of all permutations involving the set θ_S of workers hired by a firm. Each permutation involves a particular order of workers.[8] In its internal labor

[7]Empirically, firm-specific wages are supported by the observation that adding a firm specific productivity component to any Mincerian wage equation typically contributes significantly to wage determination.

[8]In firms of size $S = 1, 2$ and 3, these are 3, 9, and 27 permutations, respectively.

market, the firm varies that order by matching worker into tasks until its productivity is maximized. For our purposes, a permutation i relative to a firm's technology k is described by the fit $N_i^k \equiv \sum_{n=1}^{S} i_n$ that summarizes the "distance" between individual tasks and individual worker skills, i.e., the productivity of the individual match pattern. The firm with structure $k_S \in \{\Gamma_S\}$ chooses the permutation j that maximizes its productivity

$$\hat{P}_S^k(\theta_S) \equiv max_{j\in\{\Theta_S\}} 1/S \sum_{n=1}^{S} P(T_n^k, t+i_n). \tag{2}$$

We remain agnostic about the micro-structure of that re-matching process –and in particular about potential learning by the firm about the workers' skill. We assume, however, that after the re-matching process the firm learns its total productivity. That productivity may be reached by a variety of match patterns, in which case the firm associates equal probability to each. Since an increase in the firm's productivity is associated with an increase in its wage payments, any productivity-increasing re-matching process is beneficial to both the firm and its employees –in spite of the possibility that a worker may be re-matched from a perfect to an imperfect match to increase that productivity.

In an ILM-equilibrium, no firm has an incentive to change its match pattern. The equilibrium is characterized by densities $\phi(\hat{P}_S^\kappa(\theta_S))$, by productivity realized by firms of given structure type and size. The effect of internal re-matching can be expressed by moments involving these densities. Let $\hat{\bar{P}}_S^\kappa$ denote the average productivity of firms with structure type κ and $\Phi(\hat{P}_S^\kappa)$ the cumulative distribution function corresponding to density $\phi(\hat{P}_S^\kappa)$.

As to the second major assumption employed in part 1 of the paper: In the ELM equilibrium employed here, the desired demand generated to fill the vacated slot is assumed to be satisfied from a supply forthcoming optimally and without uncertainty, rather than from a market involving workers with imperfectly known skills. With this assumption, we tease out the information effect on the demand as based on internal market activities. The effect is garbled by matching that demand with noisy supply, which we will more realistically pursue in part 2 of the paper.

As to more detail on the external market activities, let a randomly selected worker vacate task T_n in a typical firm of size S and type k with constrained maximal productivity $\hat{P}_S^k$ realized before the worker had vacated that task. Knowing both the vacated task, its structure, and its productivity realized before vacation, the firm infers the set of workers $\{\theta_S|\neg T_n\}$ remaining in the firm, and the skill t of the worker to be hired so as to maximize the firm's expected productivity $\Pi_S^k(\theta_S|t)$. We call that hire ***firm-optimal***.

The desired skill may, or may not perfectly match the vacated task. If it does, so $P(T_n,t) = 1$ in expectation, we call the hire ***task-optimal***. The optimal hire may, however, anticipate internal market activity, in which case the hire will *not* be matched with the vacated task. Then the hire, while firm-optimal, will not be task-optimal. It will turn out that anticipating firm-internal re-matching may have drastic consequences on the typical firm's *ex post* performance.[9]

[9]With this general set-up we are at explicit variance with the labor market literature –in particular Papageorgiou

Finally, we introduce a ***precision*** index that indicates the information generated by market activities on the typical worker's skills. It is specified as the inverse of Shannon's entropy measure. For each level of productivity $\hat{P}_S^\kappa$ realized in the ILM-equilibrium involving a firm of size S with structure type κ, it is defined as

$$\pi(\hat{P}_S^\kappa) \equiv 1/E(\hat{P}_S^\kappa) \in [1,\infty) \tag{3}$$

where $E(\hat{P}_S^\kappa) \equiv \phi(\hat{P}_S^\kappa) \cdot \{pr_S^\kappa \cdot |ln(pr_S^\kappa)|\}$ is the entropy,[10] pr_S^κ the probability, and $\phi(\hat{P}_S^k)$ the frequency by which productivity $\hat{P}_S^k$ materializes.

The average precision involving firm structure type κ is simply

$$\pi^\kappa \equiv \frac{1}{\sum_{\hat{P}_S^\kappa} E(\hat{P}_S^\kappa)}. \tag{4}$$

4 Results

We will address first the internal, and then the external market activities; and within the respective subsections focus first on variations across the largest firms in our economy featuring the richest task structures, and then on variations in firm size.

4.1 Equilibrium re-matching in the internal labor market

We demonstrate and characterize the productivity effects of internal labor market activity by comparing the productivity distributions resulting from re-matching to those achieved with random matching, i.e., before re-matching activities. The typical firm's productivity naturally depends on the mode of selecting workers from the external market, which here is random as well. The effect of re-matching on firm productivity is the larger, the more intense the internal labor market activity, which depends on firm size and structure. The productivity of individual matches tends to be, but is not necessarily increased by internal market operations. If it is increased, it also increases the precision by which worker skills can be predicted from the task performed by the worker.

4.1.1 Variations across structures of large firms

In Table 1 we specify the match patterns for our guiding examples that characterize the four task structures, by productivity levels that arise first from random matches, and then from optimal internal re-matching.[11] By example, under the possible patterns arising in the FD-firm with task

(2014) and Papageorgiou (2018), in which hires are always considered task-optimal, no matter the decision maker is the firm or the worker.

[10] By continuity $0 \cdot ln(0) \equiv 0$.

[11] *AAC* also stands for *BBA* and *CCB*; *AAB* for *BBC* and *CCA*; *AAA* for *BBB* and *CCC*.

structure *ABC*, the 4th column indicates four alternative random assignments that yield firm productivity .67. None of these assignments survives optimal re-matching. Productivity-improving re-matching of *abb*, *aac* and *cbc* yields productivity .78; that of *bca* yields productivity 1.0. The match patterns in brackets under the relevant columns indicate all possible draws and initial random matches for which optimal re-matching yields that productivity.

As not infrequently observed in real life, the first three of the indicated patterns including two perfect matches and one bad one are not constrained optimal, but can be improved upon by re-matching so that two workers are imperfectly matched. Hence task-optimal matches are not necessarily firm-optimal. Overall, the distribution of matches that is almost perfectly symmetric under random matching becomes rightward skewed under re-matching.

In Table 2 we document the densities by productivity that arise in internal market equilibrium for large firms. The standard of comparison, the density from random matches, is shown in the first relevant row. It is identical across all firm structures. The overall mean productivity is .56. The densities below are organized by decreasing diversity of firm structure. By example, while random matching yields productivity .89 with probability 3/27, re-matching in FD-firms increases that probability to 9/27, in PD-firms to 7/27, and in PS-firms to 6/27. In FS-firms, the probabilities fall back to those involving random matching, as identical tasks do not allow for productivity increasing re-matching. The overall average productivity realized after re-matching shifts upwards by 50 percent for FD-firms, and monotonically decreases with decreasing diversity, while the range of the densities increases. It follows immediately that the relative importance of external labor market activity towards obtaining efficient matches decreases with increasing diversity.

We summarize our first insights in

Result 1: *In internal market equilibrium involving large firms,*

(i.) *the average productivity realized with internal re-matching strictly increases relative to that realized with random matching in all firms with diversified task structure*

(ii.) *realized average productivity increases with increasing diversity, i.e.,* $P^{FS} < P^{PS} < P^{PD} < P^{FD}$.

Proof: Inspection of Table 2.

Table 1: Large firms: match patterns with random and with re-match

FD: ***ABC***	Productivity	1.00	.89	.78	.67	.56	.44	.33	.22	.00
	Random Match	abc	aba	aca	abb	aaa	cca	aab	bab	cab
			acc	bba	aac	acb	baa	cbb	ccb	
			bbc	bcc	cbc	cba	bcb	cac	caa	
					bca	ccc				
						bac				
						bbb				
	Re-Match	abc	aba	aca		aaa				
		(acb)	(aab)	(aac)		bbb				
		(bac)	(baa)	(caa)		ccc				
		(bca)	acc	bba						
		(cab)	(cac)	(abb)						
		(cba)	(cca)	(bab)						
			bbc	bcc						
			(cbb)	(ccb)						
			(bcb)	(cbc)						
PD: ***AAC***	Productivity	1.00	.89	.78	.67	.56	.44	.33	.22	.00
	Random Match	aac	aaa	aba	aab	abb	cba	acb	cca	ccb
			abc	baa	acc	bab	bca	cab	cbb	
			bac	bbc	cac	aca	bbb	ccc	bcb	
					bba	caa				
						bcc				
						cbc				
	Re-Match	aac	aaa	aba	acc	cbc	bbb	ccc		
		(aca)	abc	baa	cac	bcc				
		(caa)	bac	(aab)	(cca)	(ccb)				
			(acb)	bbc	bba					
			(cab)	(cbb)	(abb)					
			(cba)	(bcb)	(bab)					
			(bca)							
PS: ***AAB***	Productivity	1.00	.89	.78	.67	.56	.44	.33	.22	.00
	Random Match	aab	aac	abc	aaa	aba	bba	aca	bca	cca
			abb	bac	acb	baa	bcc	caa	cba	
			bab	bbb	cab	acc	cbc	ccb	ccc	
					bbc	cac				
						bcb				
						cbb				
	Re-Match	aab	aac	abc	aaa	acc	cbc		ccc	
		(aba)	(aca)	bac	bbc	cac	bcc			
		(baa)	(caa)	(acb)	(cbb)	(cca)	(ccb)			
			abb	(cab)	(bcb)					
			bab	(cba)						
			(bba)	(bca)						
				bbb						
FS: ***AAA***	Productivity	1.00	.89	.78	.67	.56	.44	.33	.22	.00
	Random Match	aaa	aab	abb	aac	abc	bbc	acc	ccb	ccc
	=Re-Match		aba	bab	aca	acb	bcb	cac	cbc	
			baa	bba	caa	bac	cbb	cca	bcc	
					bbb	bca				
						cab				
						cba				

Beyond these results, we can show that the distributions by productivity, and thus, of firm wages structure obey a strict order according to the 1st order stochastic dominance criterion:

Table 2: Large firms: productivity with random and internal re-matching

		Productivity								
		1.00	.89	.78	.67	.56	.44	.33	.22	.00
No re-match	$\phi(P_3^K)$	1/27	3/27	3/27	4/27	6/27	3/27	3/27	3/27	1/27
	$\bar{P}_3^K$				.56					
FD-firms										
Re-match	$\phi(\hat{P}_3^{FD})$	6/27	9/27	9/27		3/27				
	$\hat{\bar{P}}_3^{FD}$				.84					
PD-firms										
Re-match	$\phi(\hat{P}_3^{PD})$	3/27	7/27	6/27	6/27	3/27	1/27	1/27		
	$\hat{\bar{P}}_3^{PD}$				.75					
PS-firms										
Re-match	$\phi(\hat{P}_3^{PS})$	3/27	6/27	7/27	4/27	3/27	3/27		1/27	
	$\hat{\bar{P}}_3^{PS}$				.73					
FS-firms										
Re-match	$\phi(\hat{P}_3^{FS})$	1/27	3/27	3/27	4/27	6/27	3/27	3/27	3/27	1/27
	$\hat{\bar{P}}_3^{FS}$				.56					

Result 2: *In internal market equilibrium,*

(i.) the cumulative productivity distributions of fully diversified large firms dominate first order stochastically those of the less diversified firms, i. e. $\Phi(\hat{P}_3^{FD}) \leq \Phi(\hat{P}_3^{PD}) \leq \Phi(\hat{P}_3^{PS}) \leq \Phi(\hat{P}_3^{FS})$*, with at least one strict inequality;*

(ii.) stochastic dominance is strict between fully and partially diversified large firms, $\Phi(\hat{P}_3^{FD}) < \Phi(\hat{P}_3^{PD})$*, as well as between partially and fully specialized firms,* $\Phi(\hat{P}_3^{PS}) < \Phi(\hat{P}_3^{FS})$*), and weak between partially diversified and partially specialized firms.*

Proof: Inspection of Table 2.

Result 2 is illustrated in Figure 2. Both Results 1 and 2 reflect the fact that firms with a more diverse task structure are better able to productively accommodate workers with unknown skills.

Table 3: Large firms: productivity gains from internal market activity relative to random assignment

	Common tasks		Rare task	
$\bar{P}_3^K$	PD-firms	PS-firms	PD-firms	PS-firms
.89	-.03	.03	.06	-.06
.78	-.04	.02	.04	-.08
.67	-.09	.08	.16	-.17
.56	-.23	-.06	.44	.12
.44	.23	-.11	-.44	.23
.33	-.33		.67	
.22		-.22		.45

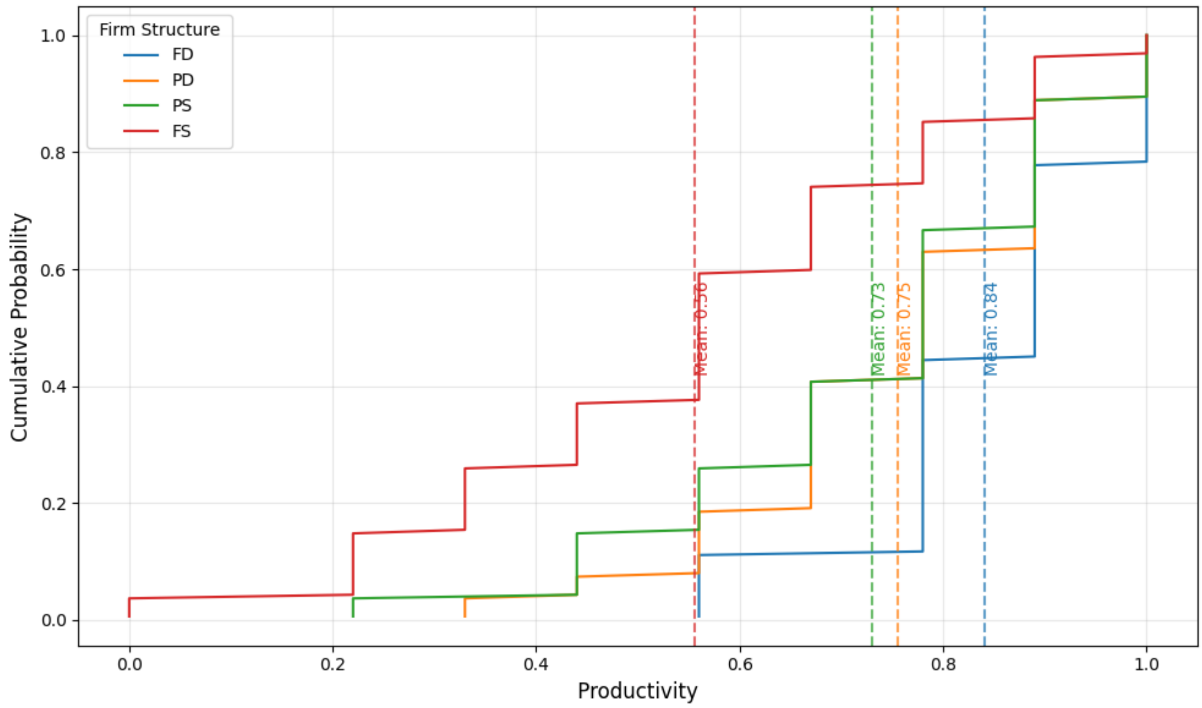

Figure 2: Large firms: cdf's by productivity, by structure type in IM-equilibrium

A comparison of the microstructure of internal market equilibrium involving firms with an asymmetric task structure, i.e., PD- and PS-firms, is particularly intriguing. Table 3 informs separately for common and rare tasks about the changes in the productivity of individual matches due to internal re-matching, relative to random matches. It is constructed by inferring from Table 1 the changes due to re-matching, in the frequency by which productivity levels arise.

Several patterns are striking. Focusing on above-mean realized productivity levels: First, in common tasks, productivity changes in PD-firms are strictly dominated by those in PS-firms. Indeed, productivity changes in the latter are positive, while in the former they are negative. The opposite holds for rare tasks. Second, the positive changes in PS-firms in common tasks are dominated by the positive changes in PS-firms in rare tasks; and the negative changes in PD-firms in common tasks are smaller than those in PS-firms in rare tasks. Before interpreting these regularities, we summarize them in

Result 3: *Consider internal market equilibrium in large PD- and PS-firms, by above-mean realized productivity levels.*

(i.) In common tasks, negative productivity changes in PD-firms are strictly dominated by positive productivity changes in PS-firms.

(ii.) In rare tasks, positive productivity changes in PD-firms strictly dominate negative productivity changes in PS-firms.

(iii.) Positive changes in PS-firms in common tasks are dominated by positive changes in PD-firms in rare tasks.

(iv.) Negative changes in PD-firms in common tasks are smaller than negative changes in PS-firms in rare tasks.

Proof: Inspection of Table 3.

These results can be explained by the combination of random draws and re-matching: random draws imply that for common tasks, the probability that good matches are drawn several times is lower than that for the rare task. By our very definitions of firm structures, PD-firms are more flexible in productively accommodating workers with unknown skills than PS-firms –yet, PS-firms can substitute away more easily from rare to common tasks. On average, this implies productivity-increasing matches into common tasks and productivity-decreasing ones into rare tasks in PS-firms, and vice versa in PD-firms. That the stark differences concentrate on the above mean productivity levels should not be of concern as, by Table 2, internal re-matching pushes weights into the higher productivity part of the distribution.[12]

Let us finally look at the information generated from rematching in internal markets (Table 4). For all structures, the level of precision is approximately u-shaped across realized productivity levels. The precision is infinite when workers are both perfectly well or perfectly badly matched. The productivity level at which the latter is the case decreases with decreasing diversification in firm structure. The precision is minimal close to mean productivity. In this region the number of alternative match patterns that generates the observed productivity is maximal, with the consequence that the inference firms and workers can draw from match patterns about skills is minimal.

In terms of total precision, the firm structures are ranked exactly opposite to the ranking by productivity realized after re-matching. Observe finally that the information generated from matches is minimal for fully specialized firms with precision 1.33, and close to minimal for the fully diversified firms with precision 1.82. We summarize in

Result 4: *In internal market equilibrium involving large firms,*

(i.) *the information (precision) realized with internal re-matching is approximately u-shaped in productivity for every task structure;*

(ii.) *zero re-matching generates minimal precision*

(iii.) *precision decreases with increasing diversity, so* $\pi^{PS} > \pi^{PD} > \pi^{FD}$.

4.1.2 Variation across firm sizes

Here we perform the same analysis for the smaller firms, as done above for large ones. The medium sized firm hires randomly two workers to be employed in two tasks, with $T_n \in \{A, B, C\}$, $n = 1, 2$. The smaller number of tasks allows the distinction only between diversified (D) and specialized (S) firms. Our guiding examples are AB and AA, respectively.[13] Matches and firm productivity before, and after re-matching are presented in Table 5 for a diversified and a specialized firm. The resulting frequencies are shown in Table 6. The average productivity before

[12] The share of firms with realized productivity at or below mean is between 7/27 and 5/27.

[13] Here, AB stands also for AC and BC; AA stands also for BB and CC.

Table 4: Large firms: Information generated from internal re-matching

		Productivity								
		1.00	.89	.78	.67	.56	.44	.33	.22	.00
No re-match	$\phi(P_3^K)$	1/27	3/27	3/27	4/27	6/27	3/27	3/27	3/27	1/27
	$\pi(P_3^K)$	∞	14.29	14.29	6.67	4.17	14.29	14.29	14.29	∞
	π^K				1.33					
FD-firm										
Re-match	$\phi(\hat{P}_3^{FD})$	6/27	9/27	9/27		3/27				
	$\pi(\hat{P}_3^{FD})$	∞	4.76	4.76		8.33				
	π^{FD}				1.82					
PD-firm										
Re-match	$\phi(\hat{P}_3^{PD})$	3/27	7/27	6/27	6/27	3/27	1/27	1/27		
	$\pi(\hat{P}_3^{PD})$	∞	6.67	7.69	4.76	20.00	∞	∞		
	π^{PD}				1.85					
PS-firm										
Re-match	$\phi(\hat{P}_3^{PS})$	3/27	6/27	7/27	4/27	3/27	3/27		1/27	
	$\pi(\hat{P}_3^{PS})$	∞	7.69	6.67	12.50	20.00	20.00		∞	
	π^{PS}				2.13					
FS-firm										
Re-match	$\phi(\hat{P}_3^{FS})$	1/27	3/27	3/27	4/27	6/27	3/27	3/27	3/27	1/27
	$\pi(\hat{P}_3^{FS})$	∞	14.29	14.29	6.67	4.17	14.29	14.29	14.29	∞
	π^{FS}				1.33					

re-matching remains as in the large firms. With re-matching, it increases in D-firms to .74, but stays strictly below the productivity of large FD-firms. The internal market obviously remains inoperative for the specialized firms.

Finally, turning to the small firms involving just a single employee, the typical firm, endowed with task $T \in \{A, B, C\}$, exhibits productivity $P_1 \in \{.00, .67, 1.0\}$, with an average realized productivity of .56 as before. The actual productivity obviously fully reveals the match quality.

Table 5: Medium-sized firms: match patterns with random and with re-match

D: ***AB***		1.00	.83	.67	.50	.33	.00
	Random Match	ab	ac	bc	aa	ba	ca
			bb		cb	cc	
	Re-Match	ab	ac	bc	aa	cc	
		(ba)	(ca)	(cb)			
			bb				
S: ***AA***		1.00	.83	.67	.50	.33	.00
	Random Match	aa	ab	bb	ac	bc	cc
	=Re-Match		ba		ca	cb	

Table 6: Medium-sized firms: productivity with random and internal re-matching

		Productivity					
		1.00	.83	.67	.50	.33	.00
No re-match	$\phi(P_2^\kappa)$	1/9	2/9	1/9	2/9	2/9	1/9
	$\bar{P}_2^\kappa$			.56			
D-firm							
Re-match	$\phi(\hat{P}_2^D)$	2/9	3/9	2/9	1/9	1/9	
	$\bar{\hat{P}}_2^D$			.74			
S- firm							
Re-match	$\phi(\hat{P}_2^S)$	1/9	2/9	1/9	2/9	2/9	1/9
	$\bar{\hat{P}}_2^S$			.56			

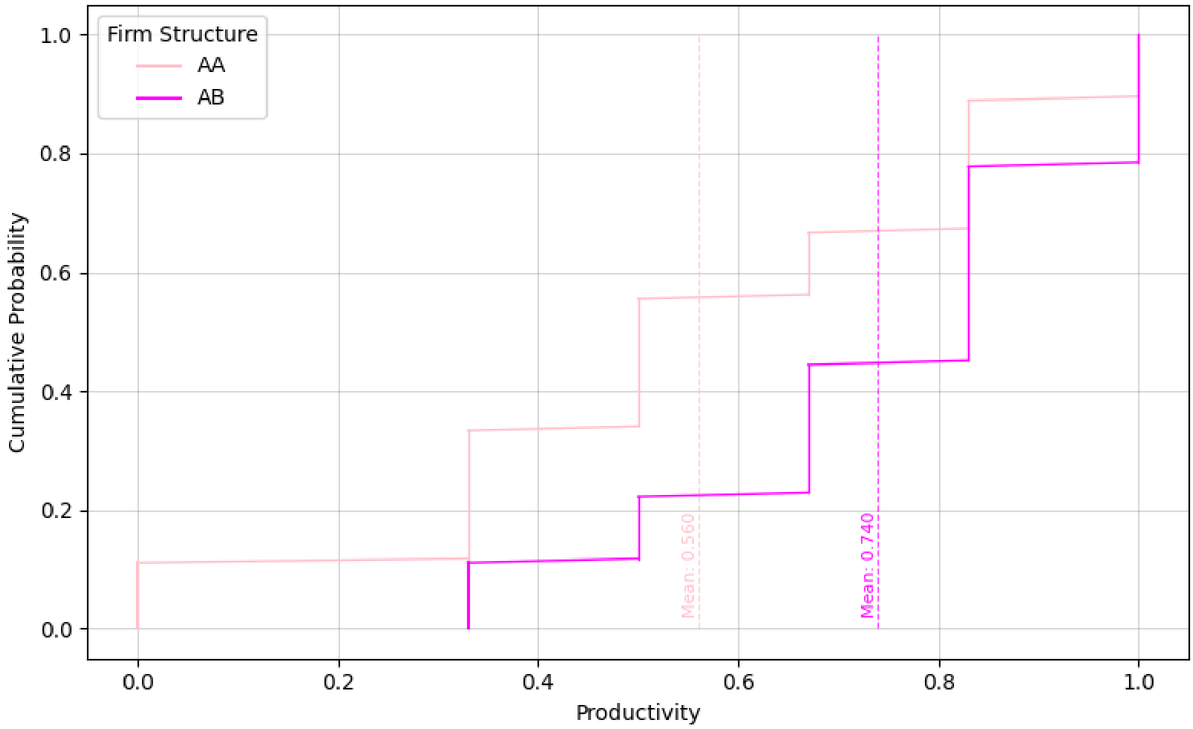

Figure 3: Medium-sized firms: cdf's by productivity, by structure type in IM-equilibrium

We compare firm size outcomes in

Result 5: *In internal market equilibrium involving medium-sized firms,*

(i.) the cumulative productivity distribution of diversified firms dominates first order stochastically that of specialized firms, i. e. $\Phi(\hat{P}_2^D) < \Phi(\hat{P}_2^S)$.

(ii.) the average firm productivity of diversified firms increases at decreasing rate with increasing firm size and increasing diversification, i.e., $P_1 < \hat{P}_2^D < \hat{P}_3^{FD}$.

(iii.) the cumulative productivity distribution of large diversified firms dominates first order stochastically that of the medium sized firms, and in turn that of medium sized that of small firms net of a small numbers effect, i. e. $\Phi(\hat{P}_3^{FD}) < \Phi(\hat{P}_2^D) \leq \Phi(P_1)$.

Proof: Comparison of Tables 2 and 6. See also Figures 3 and 4.

Not unsurprisingly, inspection of Tables 4 and 7, together with the immediate observation that precision is perfect in the small firm case, reveals that the information (precision) after rematching continues to decrease with increasing firm size.

4.2 External market with perfectly informed supply

We wish to isolate the effect of re-matching in the internal market on the typical firm's activity in the external market –and this separately on the specification of its demand, and on the supply generated from releases to the external market, which it confronts when hiring from it. We quantify the effect by specifying the difference between the firm's expected productivity when hiring a worker with the preferred skill into the vacated task, relative to its productivity realized

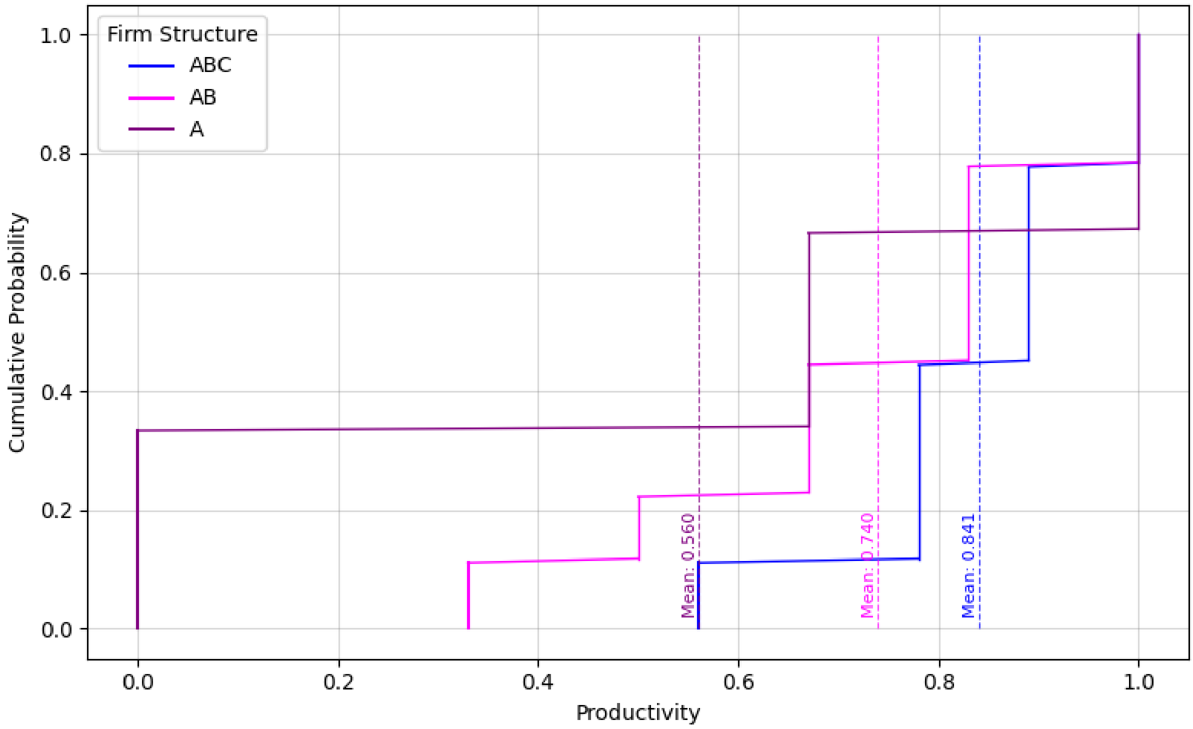

Figure 4: cdf's by productivity across firm sizes in IM-equilibrium

Table 7: Medium-sized firms: information generated from re-matching

		Productivity					
		1.00	.83	.67	.50	.33	.00
No re-match	$\phi(P_2^K)$	1/9	2/9	1/9	2/9	2/9	1/9
	$\pi(P_2^K)$	∞	4.76	∞	4.76	4.76	∞
	π			1.56			
D-firm							
Re-match	$\phi(\hat{P}_2^D)$	2/9	3/9	2/9	1/9	1/9	
	$\pi(\hat{P}_2^D)$	∞	4.76	∞	∞	∞	
	π^D			4.76			
S- firm							
Re-match	$\phi(\hat{P}_2^S)$	1/9	2/9	1/9	2/9	2/9	1/9
	$\pi(\hat{P}_2^S)$	∞	4.76	∞	4.76	4.76	∞
	π^S			1.56			

before the task was vacated. That difference depends on the quality of the match in the task before its vacation, and the precision by which the demand can be specified. The more productive the previous match, the larger the loss that needs to be covered with the hire. The higher the precision by which the demand for a skill can be specified, the higher the improvement in productivity. Both effects depend on the firm's size and structure.

The firm specifies its optimal demand by selecting the worker whose skill, combined with the skills of the retained workers, maximizes the firm's expected productivity, after contemplated internal re-matching. Expressed alternatively, the firm chooses a match pattern such that the difference between the distribution of its tasks and the distribution of the skills at disposal is minimized. Since the firm can typically realize the same productivity with different match patterns, the retained workers' skills can be inferred only imperfectly and with it, the firm's demand for the skill complementing these. To determine the hire's optimal skill and the wage offered to her (and with it, to the retained workers), we specify a vector involving the firm's expected productivity conditional on the hire's skill. The elements of this vector reflect the firm's willingness to pay for the given skill, i.e., its expected productivity. Once confronted with a supply of workers perfectly signaling their skills, the typical firm chooses the maximal element in this vector.

4.2.1 Large firms

To illustrate the derivation of the typical firm's equilibrium demand, consider the large FD-firm, exemplified by *ABC*. Before the worker leaves the firm, the firm observes its realized productivity $\hat{P}_3^{FD}$, and thereafter the task T_n vacated by the worker. Conditional on its size and structure, it infers the remaining workers' skills from these two items. By example, observing realized productivity 1.0 and vacated task $T_1 = A$, the firm infers that with probability 1 the retained workers exhibit skills b and c. Thus, the firm's desired demand involves skill a with probability 1.[14] The prediction in this example is perfect because the firm's productivity indicates a perfect match pattern. The prediction becomes garbled, however, with one or more imperfectly identifiable match.

In Table 8, the three leftward columns involve FD-firms with structure *ABC*. They contain the productivity expected when a worker with skill t complements the pool of remaining workers.[15] As one might predict, the expected average productivity generated from the best hire decreases monotonically with decreasing productivity of the typical firm realized before separation, while the difference between realized and expected maximal productivity increases. The decrease reflects the decreasing productivity loss due to the separation, and the increase the increasing opportunity to make up for it with the new hire.

[14] See the first row of Table 13 in the Appendix. Its two utmost left columns indicate the firm's productivity level and the vacated task; the next rightward column the alternative skill tuples of the retained workers, and in brackets the probabilities by which these occur; finally, the three most rightward columns exhibit the firm's productivity expected when it hires skill t.

[15] In Table 8 we condense the results of the more detailed tables 13 -16 in the Appendix.

The next three columns of Table 8 involve PD-firms exemplified by a firm with structure *AAC*. The productivity effect of hiring a worker with task-optimal skill into a common task (here *a* into *A*) is larger than into the rare task (here *c* into *C*). By contrast, around and below mean realized productivity, firm-optimal choices deviate from task-optimal choices in the rare task *C*.[16]

In contrast to the situation we identified for the PD-firms, the productivity of task-specific hires in PS-firms shown in the following three columns of Table 8 tends to be higher into the rare task (here *B*) as opposed to hires into the common task (here *A*). The exception is at the lowest realized productivity level (here .22). As for PD-firms, the firm-optimal choices deviate from task-optimal choices when hires replace separations from the rare task –with the exception of replacements at the two highest realized productivity levels.

The somewhat intricate reasoning is by example: In the *AAB*-firms, skill *a* is the preferred hire when task *B* is vacated –and this hire improves on productivity more than if task *A* were vacated. Indeed, in the internal market, arrivals of workers with skill *c* can only be productively accommodated in task *B*. A typical second best allocation thus involves a transfer of a skill *b* worker away from the optimal match to rare task *B* to one of the common tasks *A*. Consequently, once the rare task *B* is vacated, that *b*-worker is optimally transferred from task *A* into task *B*, inviting the specification of firm-optimal demand *a*, given the vacated task is *B*.

[16] The outlier draw throughout of skills *bbb* with realized productivity level .44 is known already from Result 3. With this, the firm prefers to hire on a vacated common task *A* a worker with skill optimally matching the rare task *C*.

We summarize in

Result 6: *Consider large firms. With perfectly informative supply,*

(i.) *expected maximal over realized productivity*

- is strictly positive at each but the highest productivity level

- increases with decreasing P.

(ii.) *separations from common tasks imply that*

- firm-optimal hires are almost always task-optimal

- productivity increases in PD-firms dominate those in PS-firms at all comparable levels of realized productivity

- productivity increases in PD-firms dominate those in FD- and FS-firms at all comparable levels of realized productivity

- productivity increases in FS-firms dominate those in PS-firms above mean realized productivity, and vice versa those at and below mean productivity

(iii.) *separations from rare tasks imply that*

- in PD-firms and PS-firms, firm-optimal hires dominate task-optimal ones in 3/4 of all cases

- PS-firms outperform both FD- and PD-firms at every productivity level

(iv.) *in PS-firms, separations from the rare tasks dominate separations from the common tasks in both PS- and PD-firms at every productivity level.*

Proof: (i.), (ii.) and (iv.): Inspection of Table 9.
(iii.): Calculations combining Tables 2 and 9.

We summarize the productivity gains in Table 9, in analogy to Table 3. Also here, the gains increase with decreasing realized productivity before separation (indicated in the first column), and this across all firm structures. Firm-optimal always correspond to task-optimal choices in FD- and FS-firms, and, as long as replacements involve common tasks, almost always in PD- and PS-firms.[17] That in FD- and FS-firms firm-optimal choices are always task-optimal is due to the fact that in FD-firms, there is no need, and in FS-firms, there is no possibility for productivity-improving re-matching. When separations involve rare tasks, however, firm-optimal choices dominate task-optimal choices in 75% of all cases. The reason are internal switches from the common to the rare task as discussed in the example just above.

Result 6 (i.) is not obvious because of three features: Internal labor market equilibrium delivers (constrained) best matches; separation was random rather than based on bad matches; and in all but the highest and the lowest realized productivity levels, the typical firm cannot perfectly identify the matches that had generated its productivity –which implies that it cannot precisely specify its desired demand. Nevertheless, the match pattern as anticipated by the firm dominates in productivity the match pattern abandoned with the random separation.[18]

[17]The outlier is once again the *bbb*-firm, in which *c* is the preferred hire no matter the separation.

[18]In the second part of our paper, we will study whether this remains to be the case with frictional supply.

Table 8: Demand typical FD-firms (*ABC*), PD-firms (*AAC*), PS-firms (*AAS*) and FS-firms (*AAA*) conditional on realized productivity $\hat{P}_3^{\kappa}$ and vacated task T_n

$\hat{P}_3^{\kappa}/T_n$	Exp. prod. *ABC*			Exp. prod. *AAC*			Exp. prod. *AAB*			Exp. prod. *AAA*		
1.0	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*
A	1.0	.78	.89	1.0	.89	.67	1.0	.89	.78	1.0	.89	.67
B	.89	1.0	.78				.67	1.0	.89			
C	.78	.89	1.0	.89	.78	1.0						
.89												
A	.93	.78	.81	.94	.83	.67	.92	.81	.64	.93	.81	.59
B	.81	.93	.78				.83	.94	.83			
C	.78	.81	.93	.79	.68	.91						
.78												
A	.85	.81	.85	.86	.75	.75	.84	.73	.52	.85	.74	.52
B	.85	.85	.81				.98	.87	.76			
C	.81	.85	.85	.72	.56	.83						
.67												
A				.81	.69	.69	.75	.75	.56	.78	.67	.44
B							.92	.83	.72			
C				.83	.67	.72						
.56												
A	.74	.74	.74	.78	.67	.44	.72	.61	.39	.70	.59	.37
B	.74	.74	.74				.89	.78	.56			
C	.74	.74	.74	.89	.78	.56						
.44												
A				.67	.44	.78	.67	.56	.33	.63	.52	.30
B							.78	.67	.44			
C				.67	.44	.78						
.33												
A				.67	.56	.33				.56	.44	.22
B												
C				.67	.56	.33						
.22												
A							.56	.44	.22	.48	.37	.15
B							.56	.44	.22			
.00												
A										.33	.22	.00

By Result 6 (ii.), when separations take place from common tasks, the PD-firms outperform all other firm types. When from rare tasks, however, the match pattern changes substantially, as specified in parts (iii.) and (iv.). Here, PS-firms outperform both FD- and PD-firms when we control for realized productivity. Almost all hires are not task-optimal, and thus productivity increases are associated with anticipated re-matching.

In PD-firms, the match productivity based on separation from the rare task is in almost all cases higher than from a common task. By Result 3 (i.), a worker separated from a common

Table 9: Large firms: Productivity gains from external market activity

Separation...	...from common task			...from rare task		
$\hat{P}_3^\kappa$	PD-firm	PS-firm	FS-firm	FD-firm	PD-firm	PS-firm
.89	.05	.03	.04	.04	.02	.05
.78	.08	.06	.07	.07	.05	.20$\Leftarrow a$
.67	.14	.08	.11		.16$\Leftarrow a$	.25$\Leftarrow a$
.56	.22	.16	.14	.18	.33$\Leftarrow a$	.33$\Leftarrow a$
.44	.34$\Leftarrow c$	.23	.19		.34	.34$\Leftarrow a$
.33	.34		.23		.34$\Leftarrow a$	
.22		.34	.26			.34$\Leftarrow a$
.00			.33			

task has a relatively low match quality –implying that the retained workers have a relatively high one. Thus, hiring task-specifically yields a relatively large productivity gain. By contrast, in the PS-firms, the average match quality is higher in the common tasks than in the rare tasks for high productivity levels, see Result 3 (ii.). For below mean productivity, the realized match quality is higher in the rare task than in the common tasks. Interestingly, on average over all productivity levels the match qualities on the common and rare tasks are identical –which is reflected in Result 3 (iii.). That all these outcomes can be explained by the outcomes generated from internal re-matching is based on the fact that with our assumption on the satisfaction of desired demand, we isolate for the uncertainty on the supply side of the external market.

That desired demands based on separations from the rare task exercised by PD- and PS-firms tend not to be task-specific is due to several factors. One could have expected that the rare task was on average matched better than the common one, in which case the vacated rare task should be filled with the same skill. Especially in PS-firms, however, re-matching in the internal market arises by substitution from optimal matches in the rare task to suboptimal matches in common tasks, with the outcome clearly reflected in Table 3. This induces a low match quality in the rare task, hence a small productivity loss that can be more than compensated by large productivity gains.

Against all this works uncertainty about the remaining workers' skills. Therefore, especially in the firm types with asymmetric task structures, i.e., PD- and PS-firms, the best hire is an optimal match for the common task. Here the PS-structure confers an advantage to the PD-firm, as the former admits substitution of that hire into the rare task. That this pattern concentrates on the firms with asymmetric task structures appears to be generated by the feature that as the firms don't know the pattern by which realized productivity is generated, they have difficulties inferring from separations what they have lost, and thus hedge against uncertainty by demanding a good match for the task represented more broadly in these structures.

Note finally that FD-firms cannot improve on expected productivity via firm-optimal hires relative to task-optimal ones. The reason is extreme noise in the specification of desired demand: the high realized productivity levels are achieved with equal probability by differing

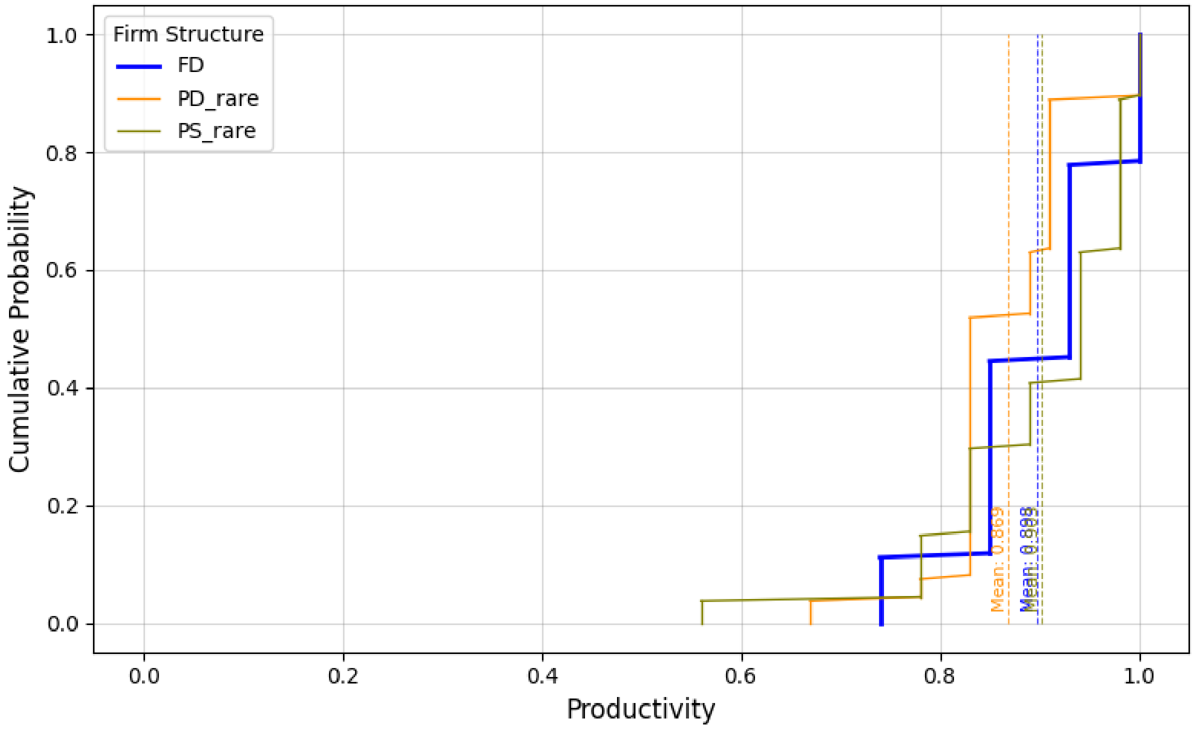

Figure 5: Cumulative distribution functions involving productivity for large firms in hypothetical external market equilibrium, by firm structure and separation on rare task

match patterns. This minimizes the precision by which these firms can specify their desired demand. By contrast, FS-firms can perfectly specify their demand, but improvements in productivity are constrained by the fact that the internal market is inoperative.

Separation from a less well matched task, together with improved specification of desired demand provides an opportunity to improve greatly expected productivity. This opportunity is illustrated in

Result 7: *With perfectly informative supply, PS-firms, when separating from rare tasks, achieve higher expected productivity than FD-firms.*

Proof: Calculations from Table 2 and Table 8. The resulting expected productivity of PS- and FD-firms is given by .92 and .90, respectively.

By this result, the PS-firm, a firm type that was relatively disadvantaged in achieving high productivity via internal re-matching, overturns in maximal productivity expected from external market activity the FD-firm, the firm type that was maximally benefiting from internal re-matching. Figure 5 illustrates that the cdf involving the PS-firms' expected productivity, while not stochastically dominating that of the FD-firms, dominates in the middle range of expected productivity. The figure also illustrates that the variances of the distributions involving expected productivity are substantively reduced, when comparing them to those characterizing the internal market equilibrium.

With this example we document our foundational insight: We have extracted two counteracting forces differentially influencing improvements in the productivity of firms by size and structure. On one hand, internal market activities favor large and diversified firms: by internal re-matching they achieve high productivity levels at higher frequency than small or specialized firms. On the other hand, productivity increasing hires in the external market require a precise specification of the firms' desired demand by skill, which necessitates precise knowledge of the firm-internal misalignment between its distribution of tasks and the distribution of its employee's skills. Here the concentration of alternative match patterns at high productivity levels turns into a disfavor, as it contaminates the precision by which the typical firm can specify its desired demand.

By Result 7, the disfavor may even more than compensate the favor, when it comes to a comparison of firms by structure at given size. On one hand, by Result 3 (ii.) and (iii.) –also illustrated in Figure 2, PS-firms gain minimally from re-matching into common tasks, so that they lose relatively little from separation. Yet, FD-firms gain maximally from re-matching, so they lose relatively much from separation. On the other hand, by Result 4, the precision generated from internal re-matching is maximal for PS-firms, implying that they can specify their desired demand with minimal noise, while FD-firms suffer maximal noise in the specification of their demand.

4.2.2 Variation across firm sizes

The demand of typical two-worker firms *AB* and *AA* is specified in Table 10. In analogy to Table 9, we summarize in Table 11 the gains in expected productivity when demand is optimally satisfied. Finally, in Table 12 we document the demand for the typical smallest, i.e., the one-person firm.

The arrows in Table 11 indicate that in 37.5% of all cases involving D-firms, firm-optimal demands do not correspond to task-optimal ones. By calculations similar to the ones showing Result 7, now involving Tables 6 and 11, we obtain that when separation takes place from the rare task, the expected maximal productivity .96 exceeds the maximal one observed for large firms. The maximal productivity expected for small firms is with 1.0 even larger. We summarize in

Result 8: *With perfectly informative supply, medium-sized D-firms dominate S-firms in expected productivity at every realized productivity level.*

Proof: See Table 11.

Result 9: *With perfectly informative supply, expected maximal productivity, given maximally differentiated firms, decreases in firm size for any realized level below maximal productivity.*

Proof: Calculations from Table 2 and the first three columns of Table 8 and from Table 6 and the first three columns of Table 10. Inspection of Table 12 immediately yields maximal expected productivity. The resulting expected maximal productivity of small, medium-sized and large firms is given by 1.0, .91, and .90, respectively.

The intuition for these results follows the intuition specified for Result 7. Roughly speaking, both match pattern alternatives and internal re-matching are constrained by firm size, but because of this, demand estimates are comparatively precise (for medium-sized firms, see Tables 6 and 7).

Table 10: Demand typical D-firms (*AB*) and S-firms (*AA*) conditional on realized productivity *P* and vacated task *T*

$\hat{P}_2^\kappa/T_n$	Exp. prod. *AB*			Exp. prod. *AA*		
$P = 1.0$	*a*	*b*	*c*	*a*	*b*	*c*
A	1.0	.83	.67	1.0	.83	.50
B	.50	1.0	.83			
$P = .83$						
A	.89	.72	.44	.92	.75	.42
B	.67	.94	.78			
$P = .67$						
A	.83	.67	.33	.83	.67	.33
B	1.0	.83	.67			
$P = .50$						
A	.50	1.0	.83	.75	.58	.25
B	.50	1.0	.83			
$P = .33$						
A	.83	.67	.33	.67	.50	.17
B	.83	.67	.33			
$P = .00$						
A				.50	.33	.00
B						

Table 11: Medium-sized firms: Productivity gains from external market activity

Separation...	..from common task		..from rare task
$\hat{P}_2^\kappa$	**A***B*	**AA**	*A***B**
.83	.06	.09	.11
.67	.16	.16	.33$\Leftarrow$ *a*
.50	.50$\Leftarrow$ *b*	.34	.50
.33	.50	.34	.50$\Leftarrow$ *a*

Table 12: Demand A-firms

$\hat{P}_1^\kappa/T$	Prod. after hiring		
1.0	*a*	*b*	*c*
A	1.0	.67	.00
.67			
A	1.0	.67	.00
.00			
A	1.0	.67	.00

5 Concluding remarks

We have developed a novel approach to analyze the impact of firm-internal labor markets on the external one and the joint impact of both on the productivity of firms when worker skills must be inferred from team productivity. In our parametric model, firms are differentiated by size and technology, where the technology is characterized by the composition of horizontally differentiated tasks to be performed.

Firm performance is shown to be critically influenced by two complementary forces. Increasing firm size and differentiation in the structure of tasks lead naturally to an increase in internal labor market activity, that via re-matching of workers by skill to tasks yields higher productivity. Concomitant to that, however, alternative match patterns that yield the same high productivity level culminate, and blur the specification of demand for the skills complementing the incumbent workers' skills. The jury is out as to which of the two effects dominates. We have shown just an example in which the latter effect dominates the former.

Within our model, wages increase in firm size based on internal labor market effects, but decrease when supply signals in the external market are perfect. We will see in part 2 of our paper, which pattern is sustained when imperfect supply signals of workers indicate their employment history. We will implement an equilibrium concept in which the firms will participate in a series of first-price auctions about the workers available in the external market, and show how a sequence of alternating external and internal market equilibria converges to a stationary one.

The framework developed here, while not elegant, allows for a number of interesting extensions. Examples include the re-specification of the space of workers' skills to include vertical differentiation; the consideration of endogenous quits and dismissals; the explicit consideration of firm-specific information generated by internal re-matching; or the consideration of demand, or supply shocks via, e.g., the business cycle or technological shocks.

Finally, our results allow for many empirical predictions. Central to our approach and results is that wages signal firm productivity, which in turn signals skill patterns. Inasmuch firm productivity is reflected in wages (there is ample evidence for this in Mincerian wage estimates), we have an extraordinary base for bringing our predictions to a test.

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Appendix: Additional tabulations

Table 13: Demand typical FD-firms (*ABC*) conditional on realized productivity *P* and vacated task *T*

P/T_n	remaining skills (pr's)	Exp. prod. *ABC*		
1.0		*a*	*b*	*c*
A	bc (1)	1.00	.89	.78
B	ac (1)	.89	1.00	.78
C	ab (1)	.78	.89	1.00
.89				
A	ab (1/3), bc (1/3), cc (1/3)	.93	.82	.78
B	aa (1/3), ac (1/3), bc (1/3)	.78	.93	.82
C	ab (1/3), ac (1/3), bb (1/3)	.82	.78	.93
.78				
A	ac (1/3), ab (1/3), cc (1/3)	.85	.85	.82
B	aa (1/3), ab (1/3), bc (1/3)	.82	.85	.85
C	ac (1/3), bb (1/3), bc (1/3)	.85	.82	.85
.56				
A	aa (1/3), bb (1/3), cc (1/3)	.74	.74	.74
B	aa (1/3), bb (1/3), cc (1/3)	.74	.74	.74
C	aa (1/3), bb (1/3), cc (1/3)	.74	.74	.74

Table 14: Demand typical PD-firms (*AAC*) conditional on realized productivity *P* and vacated task *T*

P/T_n	remaining skills (pr's)	Exp. prod. *AAC*		
1.0		*a*	*b*	*c*
A	ac (1)	1.00	.89	.67
C	aa (1)	.89	.67	1.00
.89				
A	aa (1/7), bc (3/7), ac (3/7)	.94	.83	.67
C	aa (1/7), ab (6/7)	.79	.68	.91
.78				
A	ba (1/4), aa (1/4), bc (1/2)	.86	.75	.75
C	ab (1/2), bb (1/2)	.72	.56	.83
.67				
A	cc (1/4), ac (1/4), ba (1/2)	.81	.69	.69
C	ac (1/2), bb (1/2)	.83	.67	.72
.56				
A	cc (1/2), bc (1/2)	.78	.67	.44
C	bc (1)	.89	.78	.56
.44				
A	bb (1)	.67	.44	.78
C	bb (1)	.67	.44	.78
.33				
A	cc (1)	.67	.56	.33
C	cc (1)	.67	.56	.33

Table 15: Demand typical PS-firms (*AAB*) conditional on realized productivity *P* and vacated task *T*

P/T_n	remaining skills (pr's)	Exp. prod. *AAB*		
1.0		*a*	*b*	*c*
A	ab (1)	1.00	.89	.78
B	aa (1)	.67	1.00	.89
.89				
A	ac (1/2), bb (1/4), ab (1/4)	.92	.81	.64
B	aa (1/2), ab (1/2)	.83	.94	.83
.78				
A	bc (3/7), ac (3/7), bb (1/7)	.84	.73	.52
B	ab (6/7), bb (1/7)	.98	.87	.76
.67				
A	aa (1/4), bc (3/4)	.75	.75	.56
B	aa (1/4), bb (3/4)	.83	.83	.72
.56				
A	cc (1/2), ac (1/2)	.72	.61	.39
B	ac (1)	.89	.78	.56
.44				
A	bc (1/2), cc (1/2)	.67	.56	.33
B	bc (1)	.78	.67	.44
.22				
A	cc (1)	.56	.44	.22
B	cc (1)	.56	.44	.22

Table 16: Demand typical FS-firms (*AAA*) conditional on realized productivity *P* and vacated task *T*

P/T_n	remaining skills (pr's)	Exp. prod. *AAA*		
1.0		*a*	*b*	*c*
A	aa (1)	1.00	.89	.67
.89				
A	ab (2/3), aa (1/3)	.93	.82	.59
.78				
A	bb (1/3), ab (2/3)	.85	.74	.52
.67				
A	ac (1/2), aa (1/4), bb (1/4)	.78	.67	.44
.56				
A	bc (1/3), ac (1/3), ab (1/3)	.70	.59	.37
.44				
A	bc (2/3), bb (1/3)	.63	.52	.30
.33				
A	cc (1/3), ac (2/3)	.56	.44	.22
.22				
A	bc (2/3), cc (1/3)	.48	.37	.15
.00				
A	cc (1)	.33	.22	.00

Table 17: Demand typical D-firms (*AB*) conditional on realized productivity *P* and vacated task *T*

P/T_n	remaining skills (pr's)	Exp. prod. *AB*		
1.0		*a*	*b*	*c*
A	b (1)	1.00	.83	.67
B	a (1)	.50	1.00	.83
.83				
A	c (2/3), b (1/3)	.89	.72	.44
B	a (2/3), b (1/3)	.67	.94	.78
.67				
A	c (1)	.83	.67	.33
B	b (1)	1.00	.83	.67
.50				
A	a (1)	.50	1.00	.83
B	a (1)	.50	1.00	.83
.33				
A	c (1)	.83	.67	.33
B	c (1)	.83	.67	.33

Table 18: Demand typical S-firms (*AA*) conditional on realized productivity *P* and vacated task *T*

P/T_n	remaining skills (pr's)	Exp. prod. *AA*		
1.0		*a*	*b*	*c*
A	a (1)	1.00	.83	.50
.83				
A	b (1/2), a (1/2)	.92	.75	.42
.67				
A	b (1)	.83	.67	.33
.50				
A	c (1/2), a (1/2)	.75	.58	.25
.33				
A	c (1/2), b (1/2)	.67	.50	.17
.00				
A	c (1)	.50	.33	.00