Appendix B to "Platform Ownership"

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In our analysis in the paper, we assume that buyers have free access to the platform. Indeed, it is often impracticable to charge consumers, especially in brick-and-mortar retail markets. In this web appendix, we consider two-sided pricing: as before, each seller needs to pay r to rent a platform slot, but now each buyer has to pay a "subscription fee" s (which can be positive or negative) to participate on the platform. We confine attention to the three basic ownership structures, O, C, and M. As we will show, the notions of weak and strong platform effects can be extended to the case of two-sided pricing. Most importantly, the ranking of ownership structures by induced platform size (proposition 1) carries over to this setting.

Since each buyer has to pay the subscription fee s to visit the platform, the mass of active buyers is now given by $z(m,s) \equiv g^{-1}(u(m)-s)$. Denote the sum of the profit of the marginal seller (gross of the rental price r) and the subscription revenue (from buyers) per seller/developer by

$$\Psi(m,s) \equiv z(m,s)\pi(m) - f(m) + sz(m,s)/m.$$

In analogy to our assumption on Φ, we assume now that Ψ is single-peaked in m, holding s fixed. In addition, Ψ is assumed to have a unique interior maximizer with respect to s for any given m.[1] For any given m, $\widehat{s}(m)$ denotes the unique solution to $\Psi_s(m,s)=0$.

Remark 1 *Assume $g''(z) \geq 0$. Then, for each m, $\Psi(m,s)$ is concave in s and has an interior maximizer.*

Proof. $\widehat{s}(m)$ satisfies

$$\Psi_s(m,\widehat{s}(m)) = z_s(m,\widehat{s}(m))\pi(m) + \frac{z(m,\widehat{s}(m)) + \widehat{s}(m)z_s(m,\widehat{s}(m))}{m} = 0. \tag{1}$$

We show that $\Psi_{ss}(m,\widehat{s}(m)) < 0$. That is, holding m fixed, $\Psi(m,s)$ has at most one extremum, and this extremum is necessarily a maximum. We have

$$\Psi_{ss}(m,\widehat{s}(m)) = z_{ss}(m,\widehat{s}(m))\left\{\frac{m\pi(m)+\widehat{s}(m)}{m}\right\} + \frac{2z_s(m,\widehat{s}(m))}{m}.$$

[1] We have verified that, for some parameter constellations, all assumptions made in this section are satisfied in our CES-example.

Note that

$$z_s(m,s) = -\frac{1}{g'(z(m,s))} < 0,$$

and

$$z_{ss}(m,s) = \frac{g''(z(m,s))z_s(m,s)}{[g'(z(m,s))]^2} \leq 0,$$

where the last inequality follows from our assumption that $g''(z) \geq 0$. From the first-order condition (1),

$$m\pi(m) + \widehat{s}(m) = -\frac{z(m,\widehat{s}(m))}{z_s(m,\widehat{s}(m))} > 0.$$

Hence, $\Psi_{ss}(m,\widehat{s}(m)) < 0$. It follows that, for any m, $\Psi(m,s)$ is single-peaked in s. If $\widehat{s}(m)$ exists, it is the unique maximizer of $\Psi(m,s)$.

Existence is guaranteed since, for any m, Ψ is increasing in s for s sufficiently small and decreasing in s for s sufficiently large. We first show that Ψ is decreasing in s for s sufficiently large. This can be seen as follows. Note that z is strictly decreasing and concave in s. Hence, for any m there exists some $\overline{s}(m)$ such that $z(m,\overline{s}(m)) = 0$. Recall that

$$\Psi_s(m,s) = z_s(m,s)\pi(m) + \frac{sz_s(m,s)}{m} + \frac{z(m,s)}{m}$$

Hence, $\Psi_s(m,\overline{s}(m)) < 0$ for any given m. We now show that Ψ is increasing in s for s sufficiently small. Fix any m. Then the sum of the first two terms on the r.h.s. of the above equation becomes positive for $s < 0$ sufficiently small (i.e. large in absolute value) because z_s is negative and $\pi(m) + s/m$ is also negative. Concerning the third term, for s sufficiently small z is positive. Hence, Ψ_s is positive for s sufficiently small. ■

Correspondingly, let for any given s,

$$\widehat{m}(s) = \arg\max_{m \geq 0} \Psi(m,s).$$

If $\Psi(m,s)$ is strictly increasing for all $m \geq 0$, then $\widehat{m}(s) = \infty$. Otherwise, $\widehat{m}(s)$ is implicitly defined by $\Psi_m(\widehat{m}(s),s) = 0$. Suppose that $\Psi(m,s) < c(m)$ for m sufficiently large, holding s fixed, and define $m^*(s)$ as the largest solution to $\Psi(m^*(s),s) - c(m^*(s)) = 0$. Hence, (generically) $\frac{\partial}{\partial m}\{\Psi(m^*(s),s) - c(m^*(s))\} < 0$. We now extend our previous definition of weak and strong platform effects.

Definition 1 *Platform effects are weak at s if $\widehat{m}(s) < m^*(s)$, and so $\partial\Psi(m^*(s),s)/\partial m < 0$. Platform effects are strong at s if $\widehat{m}(s) > m^*(s)$, and so $\partial\Psi(m^*(s),s)/\partial m > 0$.*

In line with standard results from oligopoly theory (e.g., Vives, 1999), we further assume that sellers' aggregate gross profits (per unit mass of buyers) are weakly decreasing in m, i.e., $d[m\pi(m)]/dm \leq 0$. Together with $g''(z) \geq 0$, this implies the following result.

Lemma 1 *Suppose $d[m\pi(m)]/dm \leq 0$ and $g''(z) \geq 0$. Then, $\Psi_{ms}(m,\widehat{s}(m)) > 0$, and so $\widehat{s}'(m) > 0$.*

Proof. Recall that $z_s(m,s) = -1/g'(z(m,s)) < 0$. We assumed that $u'(m) > 0$, and so $z_m(m,s) = u'(m)/g'(z(m,s)) > 0$. We also assume that $g''(z) \geq 0$. This implies that $z_{ms}(m,s) \geq 0$ since

$$z_{ms}(m,s) = \frac{g''(z(m,s))z_m(m,s)}{[g'(z(m,s))]^2}.$$

We have

$$\begin{aligned}
\Psi_{ms}(m,\widehat{s}(m)) &= z_{ms}(m,\widehat{s}(m))\pi(m) + z_s(m,\widehat{s}(m))\pi'(m) \\
&\quad + \frac{[z_m(m,\widehat{s}(m)) + \widehat{s}(m)z_{ms}(m,\widehat{s}(m))] - \frac{[z(m,\widehat{s}(m))+\widehat{s}(m)z_s(m,\widehat{s}(m))]}{m}}{m} \\
&= z_{ms}(m,\widehat{s}(m))\pi(m) + z_s(m,\widehat{s}(m))\pi'(m) \\
&\quad + \frac{[z_m(m,\widehat{s}(m)) + \widehat{s}(m)z_{ms}(m,\widehat{s}(m))] + z_s(m,\widehat{s}(m))\pi(m)}{m},
\end{aligned}$$

where the second equality follows from (1). We have $\Psi_{ms}(m,\widehat{s}(m)) > 0$ if

$$\begin{aligned}
& z_m(m,\widehat{s}(m)) + z_{ms}(m,\widehat{s}(m))\left\{m\pi(m) + \widehat{s}(m)\right\} \\
& + z_s(m,\widehat{s}(m))\left[\pi(m) + m\pi'(m)\right] \\
>\ & 0.
\end{aligned}$$

But this inequality is satisfied since $z_m(m,s) > 0$, $z_s(m,s) < 0$, $z_{ms}(m,s) \geq 0$, $\pi(m) + m\pi'(m) \leq 0$ by assumption, and since from (1),

$$m\pi(m) + \widehat{s}(m) = -z(m,\widehat{s}(m))/z_s(m,\widehat{s}(m)) > 0.$$

Applying the implicit function theorem,

$$\widehat{s}'(m) = -\frac{\Psi_{ms}(m,\widehat{s}(m))}{\Psi_{ss}(m,\widehat{s}(m))} > 0,$$

where the inequality follows since $\widehat{s}(m)$ maximizes $\Psi(m,s)$ and $\Psi_{ms}(m,\widehat{s}(m)) > 0$. ■

We are now equipped to discuss equilibrium allocations under open, closed, and monopoly platform ownership.

Open Platform Ownership. Suppose that if a subscription fee is charged to buyers, each active platform owner (intermediary) receives its fair share of the proceeds: $sz(m,s)/m$. In this case, any equilibrium is characterized by (i) free entry of platform developers, i.e., $r + sz(m,s)/m = c(m)$, and (ii) free entry of sellers, i.e, $r = z(m,s)\pi(m) - f(m)$. (Free entry of buyers is implicit in the definition of $z(m,s) = g^{-1}(u(m) - s)$.) In equilibrium, m and s must satisfy

$$m\left[z(m,s)\pi(m) - f(m) - c(m)\right] + sz(m,s) = 0,$$

or $\Psi(m,s) = c(m)$. Note that any tuple (m,s) satisfying this equation can be sustained as a competitive equilibrium: indeed, there is a continuum of equilibria in which all developed slots are rented out, $\mu_p = \mu_s = m$. Since there is free entry of platform developers, any equilibrium (m,s) on the open platform satisfies $m = m^*(s)$.

One natural equilibrium arises when platform developers charge a price equal to marginal cost on each side of the market, and so $s = 0$, and $m = m^O = m^*(0)$. This is the equilibrium that we analyzed before under one-sided pricing. Henceforth, we will focus on another natural equilibrium, namely the one that maximizes developers' joint profits $mr + sz - C(m)$, or $m\Psi(m,s) - C(m)$, subject to the constraint that each developer covers his cost. Hence, we can write the program as $\max_{m,s} m\Psi(m,s) - C(m)$, subject to $\Psi(m,s) = c(m)$. This program can be rewritten as $\max_{m,s} mc(m) - C(m)$ subject to $\Psi(m,s) = c(m)$. Since $d\{mc(m) - C(m)\}/dm = mc'(m) \geq 0$, the program is equivalent to maximizing platform size m subject to the free entry condition for developers, $\Psi(m,s) = c(m)$. The associated Lagrangian is given by $L = m + \lambda\{\Psi(m,s) - c(m)\}$. The first-order conditions are:

$$\frac{\partial L}{\partial m} = 1 + \lambda\{\Psi_m(m,s) - c'(m)\} = 0, \tag{2}$$

$$\frac{\partial L}{\partial s} = \lambda\Psi_s(m,s) = 0, \tag{3}$$

$$\frac{\partial L}{\partial \lambda} = \Psi(m,s) - c(m) = 0. \tag{4}$$

Let s^{O2} and m^{O2} denote the equilibrium subscription fee and platform size, respectively. From (3) and (4), it follows that, $s^{O2} = \widehat{s}(m^{O2})$ and $m^{O2} = m^*(s^{O2})$, which we assumed to be unique. The intersection of these functions is necessarily stable since

$$m^{*\prime}(s^{O2}) = -\frac{\Psi_s(m^{O2}, s^{O2})}{\Psi_m(m^{O2}, s^{O2}) - c'(m^{M2})} = 0,$$

where the second equality obtains since $\Psi_s(m^{O2}, s^{O2}) = 0$ and $\Psi_m(m^{O2}, s^{O2}) < c'(m^{M2})$. Note that the equilibrium subscription fee is given by the elasticity formula:

$$s^{O2} = -\frac{m^{O2}\pi(m^{O2})}{1 + \frac{1}{\varepsilon_{z,s}(m^{O2}, s^{O2})}}.$$

where $\varepsilon_{z,s}(m,s) \equiv z_s(m,s)s/z(m,s)$ is the elasticity of the number of active buyers with respect to the subscription fee s.

Closed Platform Ownership. Under closed ownership structure, r and s are chosen so as to maximize the profit of each active developer, $r + sz/m$. The program is thus given by $\max_{m,s} \Psi(m,s)$ subject to $\Psi(m,s) \geq c(m)$. If the constraint is non-binding, the first-order conditions are given by $\Psi_s(m,s) = 0$ and $\Psi_m(m,s) = 0$. Their solution is assumed to be unique, and it is given by $s^{C2} = \widehat{s}(m^{C2})$ and $m^{C2} = \widehat{m}(s^{C2})$. We assume that the intersection of $\widehat{s}(\cdot)$ and $\widehat{m}(\cdot)$ in (m,s)-space is "stable", i.e.,

$$\widehat{s}'(m^{C2}) < \frac{1}{\widehat{m}'(s^{C2})}. \tag{5}$$

Even if the constraint $\Psi(m,s) \geq c(m)$ is binding, it is optimal for platform owners to set s so as to maximize $\Psi(m,s)$ for any given m, and so $s^{C2} = \widehat{s}(m^{C2})$. However, in this case, the program becomes the same as that of the open platform, and so $m^{C2} = m^{O2}$ and $s^{C2} = s^{O2}$.

Independently of whether or not the constraint is binding, the equilibrium subscription fee is given by the same elasticity formula as under the open platform:

$$s^{C2} = -\frac{m^{C2}\pi(m^{C2})}{1+\frac{1}{\varepsilon_{z,s}(m^{C2},s^{C2})}}.$$

Monopoly Platform Ownership. The problem for a monopoly platform owner is to maximize $m\Psi(m,s)-C(m)$ with respect to m and s. For any given m, the profit-maximizing subscription fee $\widehat{s}(m)$ satisfies $\Psi_s(m,\widehat{s}(m))=0$. For any given s, the profit-maximizing platform size $\widetilde{m}(s)$ satisfies $\widetilde{m}(s)\Psi_m(\widetilde{m}(s),s)+\Psi(\widetilde{m}(s),s)-c(\widetilde{m}(s))=0$.

We assume that in (m,s)-space, the curves $\widetilde{m}(s)$ and $\widehat{s}(m)$ have a unique intersection at (m^{M2},s^{M2}) with $m^{M2}=\widetilde{m}(s^{M2})$ and $s^{M2}=\widehat{s}(m^{M2})$. Again, the intersection of these curves is assumed to be "stable", i.e.,

$$\widehat{s}'(m^{M2}) < \frac{1}{\widetilde{m}'(s^{M2})}. \tag{6}$$

From the first-order condition, the profit-maximizing subscription fee is given by the same elasticity formula as under the open platform

$$s^{M2} = -\frac{m^{M2}\pi(m^{M2})}{1+\frac{1}{\varepsilon_{z,s}(m^{M2},s^{M2})}}.$$

Comparison of Ownership Structures. As in section 3, we compare the equilibrium outcomes under the different ownership structures. In the remainder, we assume that sellers' aggregate gross profits (per unit mass of buyers) are weakly decreasing in platform size, whereas total gross surplus of buyers and sellers per unit mass of buyers are increasing in platform size, i.e., $d[m\pi(m)]/dm \leq 0 < d[m\pi(m)+u(m)]/dm$. In addition, we assume $g''(z)\geq 0$. We then obtain the following ranking of allocations with respect to ownership structures.

Proposition 1 *If platform effects are weak at s^{O2}, then $m^{C2}<m^{M2}<m^{O2}$, $z^{C2}<z^{M2}<z^{O2}$, and $s^{C2}<s^{M2}<s^{O2}$. If platform effects are strong at s^{O2}, then $m^{C2}=m^{O2}<m^{M2}$, $z^{C2}=z^{O2}<z^{M2}$, and $s^{C2}=s^{O2}<s^{M2}$.*

Proof. Recall that the monopolist's profit-maximizing platform size $\widetilde{m}(s)$ satisfies

$$[\Psi(\widetilde{m}(s),s)-c(\widetilde{m}(s))]+\widetilde{m}(s)\Psi_m(\widetilde{m}(s),s)=0. \tag{7}$$

(1) Suppose that platform effects are weak at s^{O2}. Then, $\widehat{m}(s^{O2})<m^*(s^{O2})=m^{O2}$. From the stability condition (5), and since $\widehat{s}'(m)>0$, it follows that, for any $s\geq s^{O2}$, $\widehat{m}(s)<\widehat{s}^{-1}(s)$. Hence, the unique intersection of $\widehat{s}(m)$ and $\widehat{m}(s)$, (m^{C2},s^{C2}), is such that $s^{C2}<s^{O2}$ and $m^{C2}<m^{O2}$. Since the intersection of $m^*(s)$ and $\widehat{s}(m^{O2})$ at (m^{O2},s^{O2}) is stable, it also follows that $m^{C2}=\widehat{m}(s^{C2})<m^*(s^{C2})$, i.e., the constraint $\Psi(m^{C2},s^{C2})\leq c(m^{C2})$ is nonbinding.

We will now show that $\widehat{m}(s^{O2})<\widetilde{m}(s^{O2})<m^*(s^{O2})$. (i) Note that $\widetilde{m}(s^{O2})<\widehat{m}(s^{O2})$ with $\Psi(\widetilde{m}(s^{O2}),s^{O2})<c(\widetilde{m}(s^{O2}))$ cannot be a profit-maximizing solution. To see this, let m' denote the smallest platform size larger than $\widetilde{m}(s^{O2})$ such that $\Psi(m',s^{O2})=c(m')$. Since

$\Psi(\widehat{m}(s^{O2}), s^{O2}) > c(\widehat{m}(s^{O2}))$, m' exists. But then, the monopolist's profit is larger when choosing platform size m' rather than $\widetilde{m}(s^{O2})$:

$$\begin{aligned} m'\Psi(m', s^{O2}) - C(m') &= m'c(m') - C(m') \\ &> \widetilde{m}(s^{O2})c(\widetilde{m}(s^{O2})) - C(\widetilde{m}(s^{O2})) \\ &> \widetilde{m}(s^{O2})\Psi(\widetilde{m}(s^{O2}), s^{O2}) - C(\widetilde{m}(s^{O2})), \end{aligned}$$

where the first inequality follows from the fact that $mc(m) - C(m)$ is increasing in m since $c'(m) \geq 0$. (ii) Next, note that $\widetilde{m}(s^{O2}) < \widehat{m}(s^{O2})$ with $\Psi(\widetilde{m}(s^{O2}), s^{O2}) \geq c(\widetilde{m}(s^{O2}))$ cannot be a profit-maximizing solution either since, in this case, the l.h.s. of (7) is strictly positive. (iii) For the same reason, $\widetilde{m}(s^{O2}) \neq \widehat{m}(s^{O2})$. (iv) Finally, we cannot have $\widetilde{m}(s^{O2}) \geq m^*(s^{O2})$ since, in this case, the l.h.s. of (7) is strictly negative. Hence, $\widehat{m}(s^{O2}) < \widetilde{m}(s^{O2}) < m^*(s^{O2})$. Using the same steps, one can show that $\widehat{m}(s^{C2}) < \widetilde{m}(s^{C2}) < m^*(s^{C2})$. It follows that $\widetilde{m}(s^{O2}) < \widehat{s}^{-1}(s^{O2})$ and $\widetilde{m}(s^{C2}) > \widehat{s}^{-1}(s^{C2})$. Hence, there exists a (unique) $s^{M2} \in (s^{C2}, s^{O2})$ such that $\widetilde{m}(s^{M2}) = \widehat{s}^{-1}(s^{M2})$. Since $\widehat{s}'(m) > 0$, it follows that the unique intersection of $\widetilde{m}(s)$ and $\widehat{s}(m)$, (m^{M2}, s^{M2}), is such that $\widehat{m}(s^{O2}) < m^{M2} = \widetilde{m}(m^{M2}) < m^*(s^{O2})$ and $s^{C2} < s^{M2} < s^{O2}$.

(2) Suppose that platform effects are strong at s^{O2}. Then, $\widehat{m}(s^{O2}) > m^*(s^{O2}) = m^{O2}$. From our stability condition (5), and since $\widehat{s}'(m) > 0$, it follows that, for any $s \leq s^{O2}$, $\widehat{m}(s) > \widehat{s}^{-1}(s)$. Hence, the unique intersection of $\widehat{s}(m)$ and $\widehat{m}(s)$, (m', s'), is such that $s' > s^{O2}$ and $m' > m^{O2}$. Since the intersection of $m^*(s)$ and $\widehat{s}(m^{O2})$ at (m^{O2}, s^{O2}) is stable, it also follows that $m' = \widehat{m}(s') > m^*(s')$, i.e., the constraint $\Psi(m', s') \leq c(m')$ is violated. Hence, at the solution (m^{C2}, s^{C2}), the constraint must be binding, and so $m^{C2} = m^*(s^{C2}) = m^{O2}$ and $s^{C2} = s^{O2}$.

We will now show that $m^*(s^{O2}) < \widetilde{m}(s^{O2})$. (i) Note first that $\widetilde{m}(s^{O2}) < m^*(s^{O2})$ with $\Psi(\widetilde{m}(s^{O2}), s^{O2}) < c(\widetilde{m}(s^{O2}))$ cannot be a profit-maximizing solution. The argument is the same as under (1), (i), above: the monopolist could increase its profit by choosing the smallest platform size $m' > \widetilde{m}(s^{O2})$ such that $\Psi(m', s^{O2}) = c(m')$. (ii) Next, note that $\widetilde{m}(s^{O2}) \leq m^*(s^{O2})$ with $\Psi(\widetilde{m}(s^{O2}), s^{O2}) \geq c(\widetilde{m}(s^{O2}))$ cannot be a profit-maximizing solution either since, in this case, the l.h.s. of (7) is strictly positive. Hence, $m^*(s^{O2}) < \widetilde{m}(s^{O2})$, and so $\widetilde{m}(s^{O2}) > \widehat{s}^{-1}(s^{O2})$. It follows that the unique intersection of $\widetilde{m}(s)$ and $\widehat{s}(m)$, (m^{M2}, s^{M2}), is such that $m^{M2} = \widetilde{m}(m^{M2}) > m^*(s^{O2})$ and $s^{M2} = \widehat{s}(m^{M2}) > s^{O2} = s^{C2}$.

It remains to be shown that there is a positive monotone relationship between m and z. Note that $\Psi_s(m, s) = z_s(m, s)\pi(m) + [z(m, s) + sz_s(m, s)]/m$.Since $s = u(m) - g(z(m, s))$ and $z_s(m, s) = -1/g'(z(m, s))$, we can rewrite the first-order condition $\Psi_s(m, s) = 0$ as $m\pi(m) + u(m) - zg'(z) - g(z) = 0$, which determines z as a function of m; denote it by $\widetilde{z}(m)$. We have

$$\widetilde{z}'(m) = -\frac{\pi(m) + m\pi'(m) + u'(m)}{-2g'(\widetilde{z}(m)) - \widetilde{z}(m)g''(\widetilde{z}(m))} > 0$$

since $\pi(m) + m\pi'(m) + u'(m) > 0$ and $g''(z) \geq 0$ by assumption. Hence, $z^\omega > z^{\omega'}$ if and only if $m^\omega > m^{\omega'}$, where $\omega, \omega' \in \{M2, C2, O2\}$. ■

The proposition shows that the ordering of ownership structures by induced platform size carries over to our setting with two-sided pricing. In spite of the fact that, in equilibrium, the subscription fee is larger for an ownership structure with a larger platform size, it continues to hold that an ownership structure that leads to a larger platform size also attracts more buyers. With respect to the prevalence of weak versus strong platform effects, there is an important

Figure 1: Weak Platform Effects at s^{O2}.

difference compared to our previous analysis. Here, $\partial\Psi(m^*(s), s)/\partial m$ is evaluated at $s = s^{O2}$, whereas under one-sided pricing it is evaluated at $s = 0$. The case of weak platform effects is illustrated in figure 2.

Welfare Properties. As regards buyer surplus, an ownership structure that attracts more consumers than another generates a larger buyer surplus. Since there is a positive monotone relationship between m and z across ownership structures (see proposition 1), buyer surplus is higher under one ownership structure than under another if and only if its induced size is larger. The ranking of ownership structures by consumer surplus then follows immediately from proposition 1.

Total surplus under two-sided pricing is given by

$$W(m, s) \equiv z(m, s)u(m) - \int_0^{z(m,s)} g(\xi)d\xi + mz(m, s)\pi(m) - \int_0^m f(\mu_s)\mu_s - C(m).$$

Let m^{W2} denote the welfare-maximizing platform size under two-sided pricing. In analogy to our welfare result under one-sided pricing, we state the following result.

Proposition 2 *The ranking of ownership structures by total surplus is:*

$$W(m^{C2}) < W(m^{M2}) < W(m^{W2}) \text{ if platform effects are weak at } s^{O2};$$

$$W(m^{C2}) = W(m^{O2}) < W(m^{M2}) < W(m^{W2}) \text{ if platform effects are strong at } s^{O2}.$$

As under one-sided pricing, a monopoly platform ownership may generate a larger total surplus than an open ownership structure, and necessarily does so if platform effects are strong at s^{O2}.[2]

References

[1] Vives, X. (1999), *Oligopoly Pricing: Old Ideas and New Tools*, MIT Press.

[2] In the CES-example, on a range of parameters there is excessive product diversity under open platform ownership, $m^{O2} > m^{W2}$.