Trade and the political economy of redistribution

G. Vannoorenberghe\textsuperscript{b}, E. Janeba\textsuperscript{b}

\textsuperscript{a}Tilburg University, Room K 420 PO Box 90153 5000 LE Tilburg, Netherlands.
Tel: +31 13 466 2511

\textsuperscript{b}University of Mannheim, Dept of Economics L 7, 3-5 68131 Mannheim, Germany.
Tel. +49 621 181 1795.

Abstract

This paper shows how international trade affects the support for policies which redistribute income between workers across sectors, and how the existence of such policies changes the support for trade liberalization. Workers, who are imperfectly mobile across sectors, vote on whether to subsidize ailing sectors, thereby redistributing income but also distorting the labor allocation. We present three main findings. First, redistributive policies are more “likely” to arise in a small open than in a closed economy for a broad range of parameters. Second, if a redistributive policy is adopted in both situations, income differences across sectors tend to be lower in the open economy. Third, the possibility to redistribute income across sectors raises the political support for trade liberalization in the first place.

Keywords: International trade, redistribution, political economy, factor mobility

Email addresses: G.C.L.Vannoorenberghe@uvt.nl (G. Vannoorenberghe), janeba@uni-mannheim.de (E. Janeba)
1. Introduction

The present paper shows how international trade affects the support for policies which redistribute income between workers across sectors, and how the existence of such policies changes the support for trade liberalization. Although cross-sectoral redistributive policies are generally considered inefficient (Acemoglu and Robinson (2001)), they remain an important channel through which governments across the world redistribute income or support employment. These typically take the form of bailouts, subsidies, or differential taxation across sectors and have gained importance during the recent crisis (OECD (2010)). Rickard (2012b) shows that their prevalence increased in developing countries in the 1980s and 1990s, and such policies are also widespread in developed economies, where they typically amount to well above 1% of GDP as shown\(^1\) in Figure 1.

The persistence of such policies may come as a surprise in light of the common belief that globalization imposes new constraints on governments’ ability to redistribute income or protect their citizens through the welfare state\(^2\) (see Brady, Beckfield, and Seeleib-Kaiser (2005)). The present paper however argues that opening up to trade reduces the inefficiency associated with cross-sectoral redistribution and makes such policies less costly to implement than in autarky. This translates into a stronger political support for

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\(^1\) Although the trend has been slightly declining in the 1990s, the various rules imposed by the EU or WTO have not made these policies disappear.

\(^2\) For example, Wilson (1987) shows that the higher mobility of the tax base in an open world limits the size of redistribution that a government can conduct, while Alesina and Perotti (1997) point to the negative effects of redistribution on a country’s competitiveness. Epifani and Gancia (2009) on the other hand argue that a terms of trade externality in the financing of public goods helps raise the size of governments in an open economy.
redistribution in open economies and raises the likelihood that redistribution arises in a voting equilibrium for a broad range of parameters. Anticipating this outcome, voters are more likely to accept trade liberalization - defined as a move from autarky to a small open economy - than in the absence of redistributive policies. Our theory therefore shows that (i) opening the economy to trade needs not undermine cross-sectoral redistribution and that (ii) the possibility to redistribute makes it more likely that voters favor trade liberalization.

Our economy consists of different sectors producing under perfect competition and using exclusively labor. The demand conditions for each sector differ, thus setting the stage for redistribution towards workers in sectors with low demand. To capture the inherent trade-off of cross-sectoral redistribution, we assume a Roy-type setup in which workers are heterogeneously productive across sectors. The dispersion of productivity draws captures in a tractable way the degree to which workers are specific to a sector (in the spirit of Grossman (1983)) It determines the extent to which interests are conflicting across sectors and to which redistributive policies distort the sectoral allocation of workers (the more specific, the smaller the distortion).

Within this framework we assume that workers determine the level of intersectoral redistribution by majority voting. This creates a conflict of interest between workers choosing to work in low-demand sectors, who benefit from redistribution, and those choosing sectors with high demand, who lose. Redistribution only arises in equilibrium if enough workers choose to work in low-demand sectors, an outcome which depends - among others things - on the number of low-demand sectors in the economy. The main conclusions of our model rest on the observation that a given degree of cross-sectoral redistribution causes less inefficiency in an open than in a closed economy.
Loosely speaking, the domestic distortion implied by redistributive policies is less costly when consumers can turn to foreign goods. If the world price of low-demand goods is not too low, these lower costs of redistribution translate into a stronger political support for redistribution, which manifests itself along two margins: (i) the median voter is “more likely”\(^3\) to vote for some redistribution in an open economy, and (ii) if redistribution is implemented, equilibrium wages in low demand sectors are relatively higher in an open economy. If the world price of the low-demand good is very low however, opening up to trade not only reduces the wage in low demand sectors, but also induces workers to move to high demand sectors, thereby eroding the political support for redistribution. If this causes redistributive policies to be abandoned, wages in low demand sectors further decrease. It is worth emphasizing that some degree of cross-sectoral worker mobility is needed for the political support of redistribution to be endogenous, and our results would not hold in a standard specific factors model\(^4\).

Finally, we allow workers to vote on whether to open the economy to trade before deciding on redistribution. We show that the possibility to implement cross-sectoral redistribution raises the set of parameters for which trade liberalization is chosen. In particular, when low demand sectors have

\(^3\)The term “most likely” refers to the fact that the minimum share of low demand sectors needed for redistribution to arise is lower in an open than in a closed economy.

\(^4\)In a specific factors model with two sectors and three types of workers (specific to low and high demand sectors, and mobile between them), the support for redistribution would be fixed and only workers specific to the low demand sectors would favor it. In that case, regardless of the costs of redistribution and of comparative advantage, the number of workers supporting redistribution would be the same in trade or autarky. Redistribution would only arise if more than 50% of workers are specific to low demand sectors.
a comparative disadvantage, we show that trade liberalization always wins.

The type or redistributive policies that we intend to capture is broad and well-known in the political science literature. Support to specific sectors is often direct through price subsidies, bailouts, guarantees (e.g. agriculture, coal mining, see Victor (2009)) or preferential tax rates. Subsidies can also target sectors indirectly when tied to characteristics of the production process (tax rebates on R&D, capital or energy). The exact form that these policies take varies across countries (Verdier, 1995), and has evolved over time (Aydin, 2007), but these remain widespread\(^5\) as shown in Figure 1.

A number of studies in political science link sectoral subsidies to globalization. Ford and Suyker (1990) argue that the emergence of industrial subsidies in the 1960s was a response to decreasing tariff levels. Rickard (2012b) shows that globalization proved instrumental in driving the rise of such subsidies in developing countries in the 1980s and 1990s and confirms a positive association between globalization and subsidies even for later periods for a large groups of countries (Rickard (2012a)). The results for developed economies are however mixed (Blais (1986), Zahariadis (2002), Aydin (2007)), and our model predicts an ambiguous link between globalization and the ratio of subsidies to GDP, depending on the patterns of comparative advantage. We rather view our theory as an explanation for why cross-sectoral redistribution is not receding, and sometimes progressing, in the face of globalization.

The present paper relates to the literature on the distributive effects of

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\(^5\)Tariffs are another widespread policy instrument for cross-sectoral redistribution in an open economy. We discuss how tariffs relate to our analysis, and in particular how they are inferior to production subsidies, in section 6.1.
international trade coming through a more elastic labor demand. Empirically, Slaughter (2001) finds evidence that the elasticity of labor demand has increased between the 1970s and 1990s in the U.S., although he cannot identify a strong effect of globalization on this pattern (see also Krishna, Mitra, and Chinoy (2001)). Spector (2001) shows how changes in elasticity matter for redistributive policies in an income taxation model à la Mirrlees. In contrast to this literature, a more elastic labor demand does not in itself affect the extent of redistribution in our approach, as voters can choose a policy which cancels the real effect of a higher elasticity. Much more central to our results is that consumer prices are not distorted by redistribution in a small open economy. On top of an increased elasticity of labor demand, Rodrik (1997) argues that globalization raises the exposure to external shocks, and thereby the demand for stabilization through government intervention. While Rodrik (1997) focuses on general government activity in a world where all citizens have similar interests, our framework makes predictions for policies which target some sectors at the expense of others, for which conflicts of interests are central. Finally, we also relate to the literature on international trade when factors are imperfectly mobile between sectors or occupations (Kambourov (2009), Artuc, Chaudhuri, and McLaren (2010), Ohnsorge and Trefler (2007)).

Section 2 describes the setup of the model. Section 3 solves the model for a given redistributive policy, and describes the key differences between the closed and open economy. Section 4 and 5 endogenize respectively the choice of redistributive policy and of trade policy (closed or open economy) by voters. Section 6 provides extensions of the model and section 7 concludes.
2. The setup

2.1. Demand

The country consists of a mass one of individuals who share the same Cobb-Douglas utility function over \( N \) goods:

\[
U = \prod_{n=1}^{N} q_n^{\alpha_n} \tag{1}
\]

where \( q_n \) is the consumption of good \( n \) and \( \sum_{n=1}^{N} \alpha_n = 1 \). Individuals, indexed by \( j \), maximize utility subject to their income. Defining the country-wide income as \( I \) and the price of good \( n \) as \( p_n \), the aggregate demand for \( n \) is:

\[
q_n^D = \alpha_n \frac{I}{p_n}. \tag{2}
\]

We assume that \( x_L \) of the \( N \) goods enter the utility with a weight \( \alpha_n = \alpha_L \) (the low demand or “L goods”) while \( x_H \) goods have a parameter \( \alpha_n = \alpha_H > \alpha_L \) (the high demand or “H goods”), where \( x_L \alpha_L + x_H \alpha_H = 1 \).

The country can be either in autarky or can open to trade as a small open economy, taking world prices as given. Each good is produced in a separate \textit{sector} (L- and H-sectors) using labor as the sole factor of production.

2.2. Workers

All individuals in the model are workers, who supply inelastically one unit of labor. Workers differ in their labor productivity, which is sector-
specific. Each worker is endowed with a productivity parameter $z$ for each sector, drawn independently from a Fréchet distribution:

$$F(z) = \exp(-z^{-\nu}).$$  \hspace{1cm} (3)

Worker $j$ observes his vector of productivity draws, $\{z_{jn}\}_{n=1}^{N}$. $z_{jn}$ denotes the number of efficiency units of labor that worker $j$ provides if he works in sector $n$. The parameter $\nu > 0$ affects the heterogeneity of productivity draws between sectors and, as will become clear in the next section, provides a parsimonious way of capturing the degree of sector-specificity of workers.

2.3. Production and redistributive policies

Each sector consists of a large number of perfectly competitive firms. Production in a sector equals the number of effective units of labor employed by the sector ($\Lambda$):

$$y_n = \Lambda_n.$$  \hspace{1cm} (4)

To redistribute income towards workers in particular sectors, the government uses a sector-specific ad-valorem production tax or subsidy. Profits in sector $n$ are:

$$\pi_n = [(1 - \tau_n)p_n - c_n]\Lambda_n$$  \hspace{1cm} (5)

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This interpretation is the counterpart to that of comparative advantage made by Eaton and Kortum (2002). Artuc, Chaudhuri, and McLaren (2010) use the Fréchet distribution to model idiosyncratic shocks to the benefits of working in a particular sector.

Our model is also a reduced form representation of subsidies tied to the production process. Consider sectors using two activities (R&D and assembly) with different intensities in a Leontieff production function. A differential taxation of these two activities would make the model isomorphic to the current model, and redistribute income towards the sectors intensive in the subsidized activities.
where $c_n$ denotes the wage paid per unit of effective labor in $n$. Anticipating the equilibrium solution of the model, the zero profit condition is:

$$c_n = (1 - \tau_n) p_n. \quad (6)$$

Sector specific taxes ($\tau_n > 0$) or subsidies ($\tau_n < 0$) thus affect the wage per unit of effective labor in $n$. For simplicity, we will refer to $c_n$ as the “wage” in sector $n$ in the rest of the analysis, which should be understood as the wage per unit of effective labor in $n$. $c_n$ also represents the equilibrium price obtained by producers (“producer price”) while $p_n$ is the price paid by consumers (“consumer price”, or “price”). We assume that the government applies the same tax to all low demand sectors ($\tau_L$) and similarly to all high demand sectors ($\tau_H$). To be feasible, the policy ($\tau_L, \tau_H$) satisfies the budget constraint:

$$\sum_{n=1}^{N} p_n y_n - c_n \Lambda_n = 0 \iff \sum_{n=1}^{N} \tau_n p_n y_n = 0 \quad (7)$$

where the second equation uses the zero profit condition (6).

### 2.4. Policy, voting and timing

At $t_0$, workers draw their productivity parameter for each sector. At $t_1$, they decide by majority voting whether to be a closed or open economy. At $t_2$, they decide by majority voting on a feasible subsidy vector ($\tau_L, \tau_H$). At $t_3$, workers decide in which sector to work, production and consumption take place. Two remarks are in order. First, the sequence of policy choices reflects the fact that international agreements are typically more constraining and less flexible than regular law, which governs redistribution. Second, we assume that workers choose their sector after the policy choices to capture in a static setup that policies affect the sectoral choice of workers.
3. Economic equilibrium

We now turn to the economic equilibrium at $t_3$ given any feasible vector of subsidies under autarky or under an open economy.

3.1. Sectoral choice of workers

At $t_3$, individuals decide in which sector to work. They observe their idiosyncratic vector of sector-specific productivity $\{z_{jn}\}_{n=1}^{N}$ and the vector of sectoral wages $\{c_n\}_{n=1}^{N}$. Worker $j$ chooses to work in the sector which gives him the highest income, which is the product of the wage in the sector ($c_n$) times the worker-sector specific productivity $z_{jn}$. As shown in the appendix 8.1, the supply of labor in sector $n$ is:

$$L_n = \frac{c_n^\nu}{\sum_{i=1}^{N} c_i^\nu}. \tag{8}$$

$L_n$ is increasing in the wage paid in sector $n$ and decreasing in the wage paid by other sectors. The parameter $\nu$ represents a measure of the sector-specificity of labor and determines the sensitivity of employment to relative differences in wages between sectors. The larger the $\nu$, the more similar the productivity draws across sectors and the more sensitive is an individual’s sectoral choice to relative wages. In a similar way, we show in the appendix 8.1 that the supply of good $n$ as given by (4) is equal to:

$$y_n = \Delta c_n^{\nu-1} \left( \sum_{i=1}^{N} c_i^\nu \right)^{\frac{1-\nu}{\nu}}. \tag{9}$$

where $\Delta \equiv \Gamma(1 - 1/\nu)$ and $\Gamma()$ denotes the gamma function. Sectors which pay higher wages have a higher supply curve since they attract more workers. For $y_n$ to be defined, we assume in the rest of the analysis that $\nu > 1$. Solving for $c_n$ in (9) shows that the wage in $n$ is increasing in $y_n$ and that the total
costs of production in sector \( n \) are convex. To expand, sector \( n \) needs to attract workers who may be relatively more productive in other sectors, and who therefore need to be paid a higher wage to accept working in sector \( n \).

3.2. The autarkic equilibrium

From (2) and (7), the government budget constraint in autarky is:

\[
\sum_{n=1}^{N} \alpha_n \tau_n = 0. \tag{10}
\]

In autarky, the market for each good must be in equilibrium, i.e. \( y_n = q_D^n \) for all sectors \( n \). Using (2) and (9), the goods market equilibrium implies\(^9\):

\[
\begin{align*}
    c_n^A & = (1 - \tau_n) p_n^A = (\alpha_n (1 - \tau_n))^{\frac{1}{\beta}} \\
    y_n^A & = \Delta (\alpha_n (1 - \tau_n))^{\frac{\nu - 1}{\nu}} \tag{11}
\end{align*}
\]

where \( p_n^A \) and \( y_n^A \) are the price and the production in sector \( n \) in the autarkic equilibrium and where we normalize\(^{10} \) \( I^A = \Delta \). Equation (11) shows the wage obtained by workers in sector \( n \). The wage and the production in a sector \( n \) are increasing in the demand parameter \( (\alpha_n) \) of a sector and in the redistributive policy towards it. The degree of worker mobility \( (\nu) \) indexes the extent to which these parameters affect wages or quantities produced.

3.3. The equilibrium in a small open economy

We now consider a small open economy facing a vector of exogenous prices \( \{p_T^T\}_{n=1}^{N} \) on the world market, where the wage (or producer price) in

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\(^9\)To obtain (11), we equate \( q_D^n \) of (2) and \( y_n \) of (9). We then solve for \( c_n^e \) and add up over all \( n \), which gives \( \sum c_n^e \). Plugging back in \( q_T^e = y_n \) and rearranging gives (11).

\(^{10}\)This choice of normalization only serves the purpose of simplifying notation. None of the results depends on how we normalize the model, which is irrelevant for real quantities.
sector $n$ is $c^T_n = (1 - \tau_n)p^T_n$. We assume that $p^T_H > p^T_L$, which guarantees that the high demand sector in autarky is also the high demand sector in the open economy.

In a small open economy, domestic supply and demand of a good are not necessarily equal. The equilibrium production of a sector is determined by the supply equation given prices:

$$y^T_n = \Delta((1 - \tau_n)p^T_n)^{\nu - 1} \left( \sum_{i=1}^{N} ((1 - \tau_i)p^T_i)^{\nu} \right)^{\frac{1-\nu}{\nu}}.$$  \hspace{1cm} (13)

Plugging (13) in the government budget constraint shows (7) that:

$$\sum_{n} \tau_n (1 - \tau_n)^{\nu - 1} (p^T_n)^{\nu} = 0.$$ \hspace{1cm} (14)

For simplicity, and without loss of generality, we define $\delta < (\alpha_H/\alpha_L)^{\frac{1}{\nu}}$ by:

$$\frac{p^T_L}{p^T_H} \equiv \delta \left( \frac{\alpha_L}{\alpha_H} \right)^{\frac{1}{\nu}},$$ \hspace{1cm} (15)

where $(\alpha_L/\alpha_H)^{\frac{1}{\nu}}$ is the relative price of the two goods in autarky in the absence of redistribution (see (11)). $\delta$ can thus be interpreted as the comparative advantage of low-demand sectors in the open economy.

### 3.4. Redistributive policies and distortion

The budget constraints (10) and (14) establish a link between $\tau_L$ and $\tau_H$ in autarky and in the open economy. Using this, all economic quantities of interest can be expressed as a function of the ratio:

$$\beta \equiv \frac{1 - \tau_L}{1 - \tau_H},$$ \hspace{1cm} (16)

which determines the extent to which low demand sectors are subsidized. From the equilibrium $c^A_n$ and $c^T_n$ derived above, the ratio of wages between
low and high demand sectors in autarky and in an open economy are:

\[ D_A(\beta) \equiv \frac{c^A_L(\beta)}{c^A_H(\beta)} = \left( \frac{\alpha_L}{\alpha_H} \right)^{\frac{1}{\nu}} \beta^{\frac{1}{\nu}} \quad \text{and} \quad D_T(\beta) \equiv \frac{c^T_L(\beta)}{c^T_H(\beta)} = \left( \frac{\alpha_L}{\alpha_H} \right)^{\frac{1}{\nu}} \delta \beta, \tag{17} \]

where \( L \) refers to any low demand and \( H \) to any high demand sector. If \( \beta = 1 \), the wage is higher in high demand sectors. The set of policies that we consider are those which redistribute income towards sectors with lower wages, i.e. we restrict attention to \( \beta \geq 1 \). The closer to one the ratio \( c_L/c_H \), the more equal the distribution of income between both types of sectors. Note that the ratios of wages is also the ratio of producer prices, which, from (12) and (13), is linked to relative supply by:

\[ \frac{y^S_L}{y^S_H} = (D_S)^{\nu - 1}, \quad S \in \{A, T\} \tag{18} \]

where \( S \in \{A, T\} \) indexes whether we are considering the autarkic or small open economy case (“trade”). Equation (18) makes explicit that redistribution occurs through a distortion of production patterns. The distribution of income depends on the ratio of production in both sectors in the same way in the open and in the closed economy.

From (1) and (2), we derive the indirect utility of a worker \( j \) in sector \( n \):

\[ V^S_{j\beta_n}(\beta) = z_{jn} u^S_{n}(\beta) = z_{jn} c^S_{n}(\beta) P^S(\beta), \tag{19} \]

where \( P^S = \prod_i \alpha_i^{-\alpha_i} p^S_i(\beta) \) is the price index in situation \( S \in \{A, T\} \). Since workers’ choices of sector or policy rest on a comparison of their indirect utility in different situations, this quantity will be at the core of our subsequent analysis. Equation (19) decomposes the indirect utility of worker \( j \) in sector \( n \) between an idiosyncratic parameter representing the productivity draw of \( j \) in \( n \), \( z_{jn} \) and the real wage per effective unit of labor (henceforth “real wage”) in sector \( n \) \( (u^S_{n}(\beta)) \), which is common to all workers in sector.
n. As shown in the appendix 8.2, the real wage of workers in low demand sectors in a closed and in an open economy are respectively:

\[
\begin{align*}
\ln u_A^L(\beta) &= \zeta \frac{(\alpha L \beta)^{\frac{1}{\nu}}}{\alpha L x_L \beta + \alpha H x_H} \beta^{\alpha L x_L \frac{\nu - 1}{\nu}} \\
\ln u_T^L(\beta, \delta) &= \zeta \frac{1}{\alpha L} \beta^{\delta \alpha H x_H} x_L \alpha L \delta^{\nu - 1} \beta^{\nu} + x_H \alpha H \frac{\nu}{\nu}.
\end{align*}
\]

(20) (21)

where \( \zeta \equiv (\alpha L x_L \alpha H x_H)^{\frac{\nu - 1}{\nu}} \). The autarkic and the open economy cases differ in three respects (appendix 8.2 shows a precise decomposition).

First, the elasticity of \( D_A \) with respect to \( \beta \) is \( 1/\nu \) while the elasticity of \( D_T \) with respect to \( \beta \) is one (see (17)). Income redistribution comes with a reallocation of workers towards L-sectors, causing a drop in the relative price of L-goods in autarky and dampening the redistributive effect of \( \beta \). This effect is however absent in a small open economy, where prices are given on the world market. If \( \delta = 1 \) (world prices are equal to autarky prices without redistribution), and for a given policy \( \beta \), the relative wage of workers in L-sectors is higher in the open than in the closed economy, i.e. a given nominal policy is more redistributive in a small open economy.

Second, for \( \delta = 1 \), a given increase in the relative wage of L-workers reduces real national income more in a closed than in an open economy. A given relative wage is associated with the same relative producer prices and production patterns (see (18)) in autarky and in an open economy. However, a given distortion in producer prices necessarily distorts consumer prices in autarky, while these are fixed at world prices in an open economy\(^{11}\).

\(^{11}\)This intuition relates to Corden (1957), who shows that subsidies in a small open economy are less costly than tariffs as they distort only production but not consumption patterns. In Bhagwati (1967), the government fixes the production of two sectors at a given level to attain non-economic objectives. Given this production, the small open economy
Loosely speaking, a given domestic output distortion is less costly in terms of welfare when consumers do not have to consume only domestic goods. This reasoning extends with one caveat to cases with \( \delta \neq 1 \): if \( \delta \) is very small, marginally raising the relative production of L-sectors is more costly in an open economy as it implies a reallocation of workers towards sectors with a very low price.

Finally, opening up to trade has additional effects if \( \delta \neq 1 \). If L-sectors have a comparative advantage (\( \delta > 1 \)), opening to trade mechanically raises the relative wage of L-workers (for a given \( \beta \)), while the opposite holds if \( \delta < 1 \). Furthermore, standard gains from trade arise due to patterns of comparative advantage. It can easily be seen that, with no redistribution, 
\[
\begin{align*}
 u_T^L(1, \delta) &\geq u_A^L(1) \\
 \text{with a strict inequality if } &\delta \neq 1.
\end{align*}
\]

4. The choice of redistributive policy

We now turn to the choice of redistributive policy by voters. As can be seen from (20) and (21), the indirect utility of workers solely depends on \( \beta \), so that workers choose one policy parameter only. The choice of \( \beta \), combined with the government budget constraints pins down \( \tau_L \) and \( \tau_H \). At time \( t_2 \), and given the trade regime (autarky or free trade) chosen at \( t_1 \), all individuals vote on the redistributive policy \( \beta \). The winning policy, chosen by majority voting, is the one that beats all the others in a pairwise comparison. We first sketch some technical results on the political equilibrium dominates autarky in terms of welfare as consumption prices are not distorted in the former. In contrast to Bhagwati (1967), however, we do not impose that the production of sectors be at the same level in autarky and trade but may differ to attain an explicit objective of redistribution.
before solving for the equilibrium policy $\beta$.

4.1. Pairwise policy comparison

When considering his preferred policy, each worker, knowing his own vector of sectoral productivity and the distribution of productivity in the population, correctly solves the economic equilibrium of the previous section. In other words, each worker correctly anticipates in which sector he would work given a policy $\beta$, as well as the prices which would obtain under that policy. When comparing two policies, each individual votes for the one giving him the highest utility, knowing that a deviation from the policy $\beta$ to some other policy $\beta'$ involves a change of worker allocation across sectors including his or her own choice. Formally, worker $j$, with productivity draw $z_{jn}$ in sector $n$ votes for policy $\beta$ over policy $\beta'$ if:

$$\max_n \{z_{jn} u_n^S(\beta)\} > \max_n \{z_{jn} u_n^S(\beta')\}$$

(22)

where $u_n^S(\beta)$, defined in (19), is the real wage in sector $n$ given policy $\beta$. Aggregating this condition shows when policy $\beta$ is preferred by half of the population against policy $\beta'$.

Proposition 1. Define $I_1$ as the set of sectors $n$ for which $u_n(\beta) < u_n(\beta')$ and $I_2$ as its complement. Policy $\beta$ wins over policy $\beta'$ if and only if:

$$\sum_{n \in I_2} (u_n^S(\beta))^{\nu} > \sum_{n \in I_1} (u_n^S(\beta'))^{\nu}$$

(23)

Proof: See Appendix

Proposition 1 reduces the problem of determining which of two policies wins a vote to a condition involving only the sector-wide real wages $(u_n^S(\beta))$. This considerably reduces the dimension of the problem, as we can abstract
from tracking the individual productivity draws \((z_{jn})\) and sectoral decisions. Intuitively, we only need to track the share of workers favoring one policy option over another and not the individual decisions of each worker.

Two elements play a role in determining which of two policies win. First, the number of sectors in which policy \(\beta\) is preferred to \(\beta'\) matters (the size of the sets \(I_1\) and \(I_2\)), as would be the case in models where voters are fully sector-specific. Second, since workers have some degree of mobility between sectors, the mass of workers deciding to work in a given sector depends relative real wages \((u_n(\beta))\). If the sectors in which \(\beta\) is preferred to \(\beta'\) offer a high real wage, more workers are likely to work in these sectors and to vote for \(\beta\). To further clarify the importance of mobility in (23), consider the workers who choose a sector \(i\) under \(\beta'\). When deciding to vote for \(\beta > \beta'\), these workers not only consider whether this sector would benefit from policy \(\beta\), but also whether they should switch to a sector \(n\), which benefits more from \(\beta\). If their net income in \(n\) under \(\beta\) is higher than in sector \(i\) under \(\beta'\), they favor \(\beta\) over \(\beta'\). Equation (23) aggregates these choices to determine which policy wins. It is worth noting that a worker choosing sector \(i\) under \(\beta'\) would only switch to \(n\) under \(\beta\) if his draw of \(z_{jn}\) is not too far from \(z_{ji}\). The lower the heterogeneity of \(z\) between sectors (the higher the \(\nu\)), the more sensitive is the choice of sectors to relative wages.

### 4.2. Voting on redistribution

In a next step, we ask what is the preferred policy of a worker given that he works in a low (respectively high) demand sector, that is, which policy \(\beta\) maximizes \(u^L_n(\beta)\) (respectively \(u^H_n(\beta))\). We then show that there exists a unique policy beating all others in a pairwise comparison.

Both in autarky and in a small open economy, workers in high demand
sectors favor policy $\beta = 1$ (no redistribution) over any other policy (formally: $\partial u^S_H(\beta)/\partial \beta < 0$ for all feasible $\beta$). Workers in high demand sectors are harmed from redistributive policies as they are net contributors to the government’s budget and lose from their distortive effect.

In contrast to high demand sectors, workers in L-sectors benefit from the redistributive effect of the policy, although they lose from its distortive effect. Setting $\partial u^S_L(\beta)/\partial \beta = 0$ characterizes the unique value of $\beta$, defined as $\beta_S$, that maximizes real income per efficiency unit in L-sectors under autarky and trade:\footnote{To show that this value is unique, take the second derivative of $u^S(\beta)$ with respect to $\beta$ and evaluate it at the first order condition (25). This expression is negative: if the first derivative is equal to zero, the second derivative is negative and $u^S(\beta)$ is single peaked. Since $\beta_S$ is unique, the equilibrium $\tau_L$ and $\tau_H$ are also unique by the government budget constraint (see (39) and (43) in the appendix).}

$$\begin{align*}
\beta_A &= 1 + \frac{1}{(\nu - 1)\alpha_L x_L} \\
 x_H \alpha_H &= x_L \alpha_L \delta^{\nu} \beta_T^{\nu - 1} (\beta_T (\nu - 1) - \nu). \tag{24} \tag{25}
\end{align*}$$

We assume for purposes of interpretation that redistributive policies do not make H-sectors worse off than L-sectors, i.e. $D_S(\beta_S) < 1$ for $S \in \{A, T\}$\footnote{The assumption on the parameters is: $\nu \alpha_L x_L < (\nu - 1)\alpha_H x_L - \alpha_H x_H$ and $\nu \alpha_L x_L \delta (\alpha_H / \alpha_L) < (\nu - 1)\alpha_H x_L - \alpha_H x_H$, i.e. the comparative advantage of L sectors cannot be too strong and $\alpha_H / \alpha_L$ should be large enough. In earlier versions of the paper, we did not restrict the parameter space but imposed that the policy is bounded above. None of the results change but the analysis becomes substantially more complicated.}.

Violating this assumption would have no effect on the structure of the model but would not fit our interpretation of a redistributive policy. Equations (24) and (25) show that $\beta_S$ is decreasing in $\alpha_L x_L$, as the fraction of net contributors to the policy decreases with $\alpha_L x_L$. As a measure of redistributive...
policies, we also compute the ratio of sector-specific subsidies to GDP in the economy, which is the typical measure used in empirical work. This ratio is:

$$\Theta_S = -\frac{x_L \tau_L p_L^S y_L^S}{I^S}, \quad S \in \{A, T\}$$  \hspace{1cm} (26)

where $\tau_L$ is the tax on L-sectors. The following Proposition offers a comparison of the preferred policy of workers in L-sectors between autarky and the open economy. For this comparison, we define the $\delta$ (comparative advantage of L-sectors) such that relative consumer prices in autarky with policy $\beta_A$ are equal to those under trade as $\delta^*$. From (11) and the definition of $\beta$, $p_L^A / p_H^A = (\alpha_L / \alpha_H)^{\frac{1}{\nu}} \beta_A^{\frac{1-\nu}{\nu}}$, so that:

$$\delta^* = \beta_A^{\frac{1-\nu}{\nu}} = \left(1 + \frac{1}{(\nu - 1)\alpha_L x_L}\right)^{\frac{1-\nu}{\nu}} < 1.$$  \hspace{1cm} (27)

**Proposition 2.** Preferred policy of L-workers in closed and open economy.

1. If $\delta > \delta^*$: $\beta_T < \beta_A$, $\Theta_T < \Theta_A$ but $D_T > D_A$.
2. If $\delta < \delta^*$, $\beta_T > \beta_A$, $\Theta_T > \Theta_A$ but $D_T < D_A$.
3. For any $\delta$, the bliss policy in L-sectors is such that $u_L^T (\beta_T, \delta) \geq u_L^A (\beta_A)$.

**Proof:** See Appendix \blacksquare

As long as the comparative disadvantage of L-sectors is not too strong ($\delta > \delta^*$), L-workers favor a policy such that their relative wage is higher under an open than under a closed economy. If $\delta > 1$, they benefit from a direct redistribution due to their comparative advantage and obtain a higher relative income with less distortion of production. Even with a moderate comparative disadvantage ($\delta^* < \delta < 1$), the fact that output distortions are less costly under an open than under a closed economy induces them to choose a higher level or redistribution under an open economy. If L-sectors
have a strong comparative disadvantage on the other hand \((\delta < \delta^*)\), the distortion of producer prices required to maintain relative wages at their autarky level becomes too costly (see appendix 8.2). Although L-workers choose a high level of redistributive policy \((\beta_T > \beta_A)\), wage inequality rises. In that case, however, since a very low \(\delta\) means strong patterns of comparative advantage, the traditional gains from trade are sufficiently large to make L-workers better off even with increased wage inequality. L-workers thus benefit from trade as long as their preferred policy prevails. It is worth noting that, even for \(\delta = 1\), the preferred policy of L-workers makes wages more equal in an open economy although the nominal level of the policy (e.g. \(\beta\) or subsidies to GDP ratio) is lower than in autarky. The reason is that, in autarky, the reallocation of workers across sectors affects prices, thereby dampening the redistributive effect of a given policy and inducing L-workers to vote for higher nominal policies.

We now turn to the proof that there is a unique equilibrium policy given the parameters of the model and that this policy must be either \(\beta = 1\) (the preferred policy of H-sectors) or \(\beta = \beta_S\), the preferred policy in L-sectors. The proof consists of two steps. First, consider the case where policy \(\beta_S\) beats policy \(\beta = 1\), i.e. \(x_L(u_L^S(\beta_S))^\nu - x_H(u_H^S(1))^\nu > 0\) by Proposition 1. Since \(x_H(u_H^S(1))^\nu > x_H(u_H^S(\beta))^\nu\) for any \(\beta > 1\), policy \(\beta_S\) strictly beats any other policy. A similar reasoning shows that if policy \(\beta = 1\) beats \(\beta_S\), it strictly beats any other policy. It remains to be checked under which condition \(\beta_S\) wins over \(\beta = 1\), a step conducted in the following Proposition, where \(\chi_S\) is the unique value of \(\alpha_L x_L\) for which \(x_L(u_L^S(\beta_S))^\nu = x_H(u_H^S(1))^\nu\).

**Proposition 3.** Equilibrium policy
For $S \in \{A, T\}$, the equilibrium policy is given by:

\[
\beta^*_S = \begin{cases} 
1, & \text{if } \alpha_L x_L < \chi_S \\
\beta_S, & \text{if } \alpha_L x_L \geq \chi_S 
\end{cases}
\tag{28}
\]

where $\beta_S$ is defined by (24) and (25).

**Proof:** See Appendix ■

The product $\alpha_L x_L$ is a direct determinant of the mass of workers in sectors with low demand and needs to be large enough for the redistributive policy to win by majority voting. If this is the case, the winning policy is $\beta_S$ while there is no redistribution otherwise. The following Proposition compares $\chi_A$ and $\chi_T$.

**Proposition 4.** Occurrence of redistribution in closed and open economy.

There exists a $\delta \in (\delta^*, 1)$ such that $\chi_T(\delta) < \chi_A$ if and only if $\delta > \bar{\delta}$. Redistribution is more “likely” to occur in the small open economy if and only if the comparative disadvantage of L-sectors is not too strong.

**Proof:** See Appendix ■

Two factors influence whether redistribution arises in equilibrium in an open compared to a closed economy. On the one hand, and as emphasized earlier, the inefficiency attached to redistribution is lower in a small open than in a closed economy, as consumer prices are not distorted. Redistribution therefore arises for a larger set of parameters in an open economy if $\delta$ is close to one. On the other hand, the strength of comparative advantage is a direct determinant of the number of workers in L-sectors, who favor redistribution in an open economy. The smaller the $\delta$, the less workers favor redistribution and the higher the required $\chi$ for redistribution to arise.
Note that such a result would not obtain in a standard model with specific factors as it requires some degree of labor mobility across sectors. If workers were immobile, the fraction of workers supporting redistribution would be exogenously given by the fraction of workers in low demand sectors.

Proposition 4 also points to a particularly interesting case. Consider a country with a strong comparative advantage in H-sectors ($\delta < \bar{\delta}$) but a relatively large number of L-sectors such that $\chi_T(\delta) > \alpha_L x_L > \chi_A$. In autarky, the equilibrium policy is $\beta_A$ and redistribution takes place. When opening to trade, such a country sees the relative wage in L-sectors drop for two reasons: (i) a low $\delta$ directly reduces the wage and employment in the L-sectors, and (ii) the decrease in employment from the direct effect implies that the median voter does not support redistributive policies anymore, creating a further decrease in wage and employment of L-sectors.

5. The choice of trade policy

At $t_1$ and before voting on the redistributive policies, the country decides by majority voting whether to open up to trade or to be autarkic. Opening up to trade gives rise to conflicts of interests as different groups are affected differently by international trade. An important factor determining whether high or low demand sectors benefit from trade is the parameter $\delta$, which determines the relative price of both goods in the world economy. If there is neither redistribution in autarky nor in the open economy ($\chi < \min\{\chi_T(\delta), \chi_A\}$), H-sectors benefit from trade if $\delta < 1$, since the relative price of the goods they produce is higher in an open than in a closed economy ($p_T^H(1)/p_T^L(1) > p_A^H(1)/p_A^L(1)$). L-sectors lose from trade in that case and the result is reversed for $\delta > 1$. To determine which policy (trade or
autarky) actually wins, we apply the result of Proposition 1 which requires that, for $\delta < 1$ and $\chi < \min\{\chi_T(\delta), \chi_A\}$, trade wins if and only if:

$$x_H(u_H^T(1, \delta))^\nu > x_L(u_L^A(1))^\nu. \quad (29)$$

The above inequality must hold since, in this particular case:

$$x_H(u_H^T(1, \delta))^\nu > x_H(u_H^A(1))^\nu > x_L(u_L^A(\beta_A))^\nu > x_L(u_L^A(1))^\nu, \quad (30)$$

where the first inequality uses that $\delta < 1$, the second that $\chi < \chi_A$ and the third relies on the definition of $\beta_A$ as the preferred policy of L-sectors in autarky. In words, since H-sectors are large enough to ensure that there is no redistribution in autarky, and since they attract even more voters when $\delta < 1$ with no redistribution, it must be that their best interest - opening up to trade - wins the majority. A similar reasoning shows that, if $\delta > 1$, the best interest of the H sectors wins and the country remains in autarky.

For larger values of $\chi$, on the other hand, we need to determine whether trade or autarky wins given that redistribution arises in some situations. We turn to the case where $\chi > \max\{\chi_A, \chi_T\}$, meaning that there is redistribution both in autarky and in the open economy. If this is the case, L-sectors always favor trade regardless of the value of $\delta$ (see part 3 of Proposition 2). If they know that redistribution will be implemented, workers in low demand sectors therefore support trade liberalization, even if their sector has a strong comparative disadvantage. Given that they are strong enough to impose their redistributive preferences, low demand sectors also win the vote on opening up to trade (if $\delta$ is small enough, both high and low sectors may even favor trade). We leave the discussion for the remaining values of $\chi$ to the appendix but summarize our results in Proposition 5 and in Figure 2, which gives a graphical account in the $(\delta, \chi)$ space.
We then compare the likelihood that the open economy wins the majority over autarky at \( t = 1 \) in two situations: (i) if workers can vote on redistributive policies at \( t = 2 \), and (ii) if there is no possibility to redistribute at \( t = 2 \). We show in the second part of Proposition 5 that the set of parameters for which trade wins at \( t = 1 \) is strictly larger if workers can vote on redistribution at \( t = 2 \). The reason is that workers in L-sectors prefer trade to autarky when their preferred redistribution is implemented.

**Proposition 5. Choice of trade policy**

- If \( \delta < 1 \), voters choose to open the economy to trade for any \( \chi \). If \( \delta > 1 \), there is a value \( \tilde{\chi}(\delta) \) such that \( \chi_T(\delta) < \tilde{\chi}(\delta) < \chi_A \) above which trade wins and below which autarky wins.

- The set of parameters for which trade wins over autarky is larger when workers vote on redistribution at \( t = 2 \) than if \( \beta \) is constrained to one.

**Proof:** See Appendix ■

6. Extensions

6.1. Tariffs versus subsidies

In an open economy, governments could also use import tariffs (or export subsidies) to redistribute income across sectors. Such a policy would impose a tariff on imports of L-sectors (or subsidy on their exports) to maintain the demand for Home labor in these industries. We here sketch how the analysis would change if we replace cross-sectoral subsidies by tariffs in an open economy. Specifically, a tariff \( t \) (\( t < 0 \) corresponds to an export subsidy) creates a wedge between the world price of L-goods (\( p^T_L \)) and the producer
and consumer prices perceived at Home \( (c_T^T = p_T^T (1 + t)) \). Given the Home demand (2) and supply (9), the tariff revenues equal in this case:

\[
\text{Tariff revenue} = x_L p_T^T \left( \frac{\alpha_L I_T}{c_L^T} - \Delta \frac{c_L'^{-1}}{(x_L c_L'^{\nu} + x_H c_H'^{\nu})^{\frac{1}{\nu}}} \right). \tag{31}
\]

We assume for simplicity that the tariff revenues are redistributed as a wage subsidy to each worker. Each worker receives a subsidy equal to \( \kappa \) times its wage. Total disposable income is therefore:

\[
I_T = \Delta(1 + \kappa) \left( x_L c_L'^{\nu} + x_H c_H'^{\nu} \right)^{\frac{1}{\nu}}, \tag{32}
\]

where the total amount transferred by the government to all workers is \( \kappa I_T / (1 + \kappa) \). Imposing a balanced budget for the government (Tariff revenue = \( \kappa I_T / (1 + \kappa) \)) and using (15) shows that:

\[
1 + \kappa = \frac{\delta^\nu (1 + t)^\nu \chi + (1 - \chi) (1 + t)}{\chi \delta^\nu (1 + t)^\nu + 1 - \chi (1 + (1 - \chi) (1 + t))} \tag{33}
\]

Rewriting \( 1 + t \) as \( \beta \) for ease of comparison with our previous results, we obtain the utility for L-workers in an open economy under tariffs as:

\[
u_{\text{T}}^L (\beta, \delta) = \frac{(1 + \kappa) c_L}{\alpha_L x_L \alpha_H x_H (p_L^T)^{\alpha_L x_L} (p_H^T)^{\alpha_H x_H} \delta^\delta} \left( \frac{1}{\alpha_L x_L + \alpha_H x_H \beta} \right) \nu_{\text{T}}^L (\beta, \delta), \tag{34}
\]

and where \( u_{\text{T}}^H (\beta, \delta) = (\alpha_H / \alpha_L)^{\frac{1}{\nu}} \beta^{-1} u_{\text{T}}^L (\beta, \delta) \). It is easy to show that the utility of any group is lower under a tariff than under a production subsidy with the same redistributive effect, a result in line with Corden (1957) in a small open economy. Tariffs, by affecting consumer prices in an open economy, come with a stronger welfare loss than ad-valorem production subsidies. Given an open economy, production subsidies dominate tariffs as a means of redistribution in this model and should be favored in the political game.
6.2. Lobbying models

Our setup assumes that redistributive polices are determined through voting by the population. Although voters exert control over redistributive policy through their votes, sectoral subsidies may also be affected by lobbying. We develop in the appendix a model à la Grossman and Helpman (1994) where low demand sectors lobby to obtain higher producer prices, thereby increasing the rent accruing to their specific factors. We show that the main intuition behind our results still applies in a lobbying model: if the comparative disadvantage of low demand sectors is not too strong, the relative rent of factors specific to the low demand sectors is higher in an open than in a closed economy. As in our baseline model, redistribution through a distortion of producer prices affects consumer prices in autarky, thereby imposing a higher welfare cost than in a small open economy and limiting the extent to which L-sectors wish to lobby in autarky. We also show that, in the open economy, L-sectors would lobby for an ad-valorem production subsidy and not for a tariff, in line with our intuition from section 6.1.

6.3. More than two types of sectors

An important assumption in the main model of sections 3 and 4 is that there are only two types of sectors. A large part of the analysis in section 3, however, directly applies to more general distributions of demand parameters \( \alpha_n \). Equations (1) to (21), which describe the setup and the economic equilibrium, hold for any distribution of \( \alpha_n \) when defining the policy as the vector of \( \beta_n = (1 - \tau_n) / (1 - \tau_0) \) and the vector of comparative advantage as \( \delta_n \), where we index the sector with the lowest demand with 0. The redistributive and distortive effects of cross sectoral subsidies as discussed at the
end of section 3 also extend naturally to a setup with\textsuperscript{14} any distribution of \(\alpha_n\). Therefore our parsimonious modeling approach to labor mobility can be implemented in more general economic environments.

Generalizing the results concerning the political equilibrium in section 4 is more challenging. Proposition 1 holds for any distribution of \(\alpha_n\) but proving existence of a unique political economy equilibrium with many different types of sectors requires much stronger assumptions, because the dimensionality of the policy vector becomes larger and may induce vote cycles under pairwise voting.

We can circumvent this problem, while maintaining the assumption of more than two types of sectors, if we restrict the form of redistribution. For example, if the sectoral policy \(\beta_n\) is of the form:

\[
\beta_n = \left(\frac{\alpha_0}{\alpha_n}\right)^b,
\]

the policy problem can be reduced to voting over the shape parameter \(b \in (0, 1)\). While this approach restricts the ability of voters to target specific sectors, it captures the general idea that redistribution is stronger towards sectors with weaker demand. Under additional assumptions on the distribution of \(\alpha\) (to guarantee existence and uniqueness of a political equilibrium) and imposing \(\delta = 1\), the main results from section 4 with two types of sectors extend to this more general setup (details, including proofs available upon request).

\textsuperscript{14}The concept of redistribution has to be made more precise when considering an arbitrary distribution of \(\alpha\), and we think of redistribution as a compression of the income distribution in the sense that the ratio of wages between any two sectors becomes closer to one. A precise description of how section 3 extends to any distribution of \(\alpha\) can be found in Vannooorenberghe and Janeba (2013).
7. Conclusion

This paper shows how opening up to trade affects the support for cross-sectoral redistributive policies in a median voter analysis. We consider an environment where workers are heterogeneously productive across sectors, giving rise to a convex labor supply curve per sector. By subsidizing the production of some sectors, the government raises their production (distortive effect) and the relative wage of workers in these sectors (redistributive effect). In a closed economy, the distortion of producer prices also affects consumer prices, making cross-sectoral redistribution relatively costly compared to a small open economy, where consumer prices are given. Because of this difference in distortion, and for moderate patterns of comparative advantage (i) more workers vote for these policies in a small open than in a closed economy, and, conditional on such policies being implemented (ii) wages are more equal in the open than in the closed economy. It is worth noting that, even if a policy is more redistributive in an open economy, its nominal level (e.g. subsidies to GDP) is typically lower due to the dampening effect of prices on redistribution in autarky.

In an open economy, comparative advantages also affect the likelihood and strength of redistributive policies. If sectors paying low wages in autarky have a comparative advantage, they obtain a higher income just by opening to trade, making the need for redistribution less strong but the support for such policies broader. If these sectors have a strong comparative disadvantage however, many workers choose to exit these sectors and the support for redistributive policies erodes. Finally, we show that the possibility to vote on cross-sectoral redistribution affects the choice of voters on trade liberalization in the first place. When voters know that they will vote
on cross-sectoral redistribution, trade liberalization wins the majority for a strictly larger set of parameters than in the absence of redistribution.

REFERENCES


8. Appendix

8.1. Derivation of \( L_n \) and \( y_n \) in (8) and (9)

If worker \( j \) receives a productivity draw \( z \) in sector \( n \), the probability that it is best to work in \( n \) is the probability that the draws of \( z_i \) in all others sectors are lower than \( c_n z / c_i \):

\[
G \left( \frac{c_n z}{c_i} \right) \equiv \prod_{i \neq n} F \left( \frac{c_n z}{c_i} \right) = \exp \left( -(c_n z)^{-\nu} \left( \sum_{i \neq n} (c_i)^\nu \right) \right). \tag{36}
\]

The supply of workers in sector \( n \) is given by the integral over all \( z \) of the probability that a draw of \( z \) makes it optimal to work in \( n \), i.e.:

\[
L_n = \int_0^\infty G \left( \frac{c_n z}{c_i} \right) dF(z) = \frac{c_n^\nu}{\sum_{n=1}^N c_i^\nu}. \tag{37}
\]

The supply of goods from sector \( n \) is the total effective labor in \( n \), i.e.:

\[
y_n = \int_0^\infty zG \left( \frac{c_n z}{c_i} \right) dF(z) = \Delta c_n^{\nu-1} \left( \sum_{i=1}^N c_i^\nu \right)^{-\nu} \tag{38}
\]

8.2. Derivation of \( u_A^L \) and \( u_T^L \) in (20) and (21)

Using the government budget constraint in autarky (10), the definition of \( \beta \), and \( x_L \alpha_L + x_H \alpha_H = 1 \) shows that:

\[
x_L \alpha_L \beta + x_H \alpha_H = \frac{1}{1 - \tau_L} = \frac{\beta}{1 - \tau_L} \tag{39}
\]

The price index in autarky is:

\[
P^A = \left( \frac{P^A_L}{\alpha_L} \right)^{\alpha_L x_L} \left( \frac{P^A_H}{\alpha_H} \right)^{\alpha_H x_H} = \zeta^{-1}(1 - \tau_L)^{\frac{1 - \nu}{\nu}} \beta^{\alpha_H x_H \frac{\nu - 1}{\nu}} \tag{40}
\]

where the second equality uses (6) and the definition of \( \beta \). The real income of a worker in the L-sectors is given by \( c_L^A / P^A \) which, from (6), (39) and (40), gives (20). We define the country’s total real income in autarky \( (R^A) \):

\[
R^A = \frac{I^A}{P^A} = \Delta \zeta \beta^{\alpha_L x_L \frac{\nu - 1}{\nu}} (x_L \alpha_L \beta + x_H \alpha_H)^{\frac{1 - \nu}{\nu}}. \tag{41}
\]
The real income of L-sectors in autarky can be rewritten as a function of the wage ratio between L and H sectors ($D_A(\beta)$ in (17)):

$$u^A_L(\beta) = \frac{D_A}{x_L D^\nu_A + x_H} \frac{D^{\alpha L x_L (\nu - 1)} (x_L D^\nu + x_H)^{\frac{1}{\nu}}}{R^A / \Delta}.$$  \hspace{1cm} (42)

Using the definition of $\beta$, the budget constraint of the government (14) in an open economy can be rewritten as:

$$1 - \tau_L = \beta (1 - \tau_H) = \frac{x_L \alpha_L \delta^\nu \beta^\nu + x_H \alpha_H \beta}{x_L \alpha_L \delta^\nu \beta^\nu + x_H \alpha_H}.$$  \hspace{1cm} (43)

Since $c^T_L = (1 - \tau_L) p^T_L$ and by the definition of the price index:

$$u^T_L(\beta, \delta) = \frac{c^T_L}{P^T} = (1 - \tau_L) \alpha_L^{\alpha L x_L} \alpha_H^{\alpha H x_H} \left( \frac{p^T_L}{p^T_H} \right)^{\alpha H x_H}.$$  \hspace{1cm} (44)

Plugging in (43) and the definition of trade prices (15) gives (21). Total nominal and real income in the open economy are:

$$I^T = \Delta \left( x_L (c^T_L)^\nu + x_H (c^H_T)^\nu \right)^{\frac{1}{\nu}} = \Delta (1 - \tau_H) p^T_H \alpha_H^{-\frac{2}{\nu}} (x_L \alpha_L \delta^\nu \beta^\nu + x_H \alpha_H)^{\frac{1}{\nu}}$$

$$R^T \equiv \frac{I^T}{P^T} = \Delta \zeta^{\alpha L x_L} \frac{x_L \alpha_L \delta^\nu \beta^\nu - 1 \alpha_H}{(x_L \alpha_L \delta^\nu \beta^\nu + x_H \alpha_H)^{\frac{1}{\nu}}}. \hspace{1cm} (45)$$

Using (17) allows rewriting ($D_T$ is a function of $\beta$):

$$u^T_L(\beta, \delta) = \frac{D_T}{(x_L D^\nu_T + x_H)^{\frac{1}{\nu}}} \frac{\zeta^{\alpha L x_L} \alpha_L^{\frac{1}{\nu}} x_L \delta D_T^{\nu - 1} + x_H^{\frac{1}{\nu}}}{(x_L D^\nu_T + x_H)^{\frac{1}{\nu}}} \hspace{1cm} (46)$$

The real income in autarky and in trade in (42) and (46) consist of two parts. The second part is the total real income in the economy while the first is the fraction of that total income that accrues to workers in a given L-sector. Relative wages affect the first part in the same way in autarky and in the open economy, while the real income depends differently on real wages in
both situations. The elasticity of total real income to relative wages is:

\[
\frac{\partial R^A D_A}{\partial D_A R^A} = (\nu - 1) \frac{x_L x_H}{x_L D_A^\nu + x_H} (\alpha_L - \alpha_H D_A^\nu)
\]  

(47)

\[
\frac{\partial R^T D_T}{\partial D_T R^T} = (\nu - 1) \frac{x_L x_H}{x_L D_T^\nu + x_H} \frac{\alpha_L^\frac{1}{\nu} \delta D_T^{\nu-1} - \alpha_H^\frac{1}{\nu} D_T^\nu}{\alpha_L^\frac{1}{\nu} x_L \delta D_T^{\nu-1} + x_H \alpha_H^\frac{1}{\nu}}
\]  

> (\nu - 1) \frac{x_L x_H}{x_L D_T^\nu + x_H} \frac{\alpha_L \delta - \alpha_H D_T^\nu}{\alpha_L^\frac{1}{\nu} x_L \delta + x_H \alpha_H^\frac{1}{\nu}}.
\]  

(48)

Equations (47) and (48) show that the marginal efficiency loss of higher relative wages above their equilibrium value is the same in the open and closed economy at \(\delta = \delta^*\). If \(\delta < \delta^*\), the optimal distribution from the perspective of L-workers entails stronger inefficiency costs than in autarky.

As described in section 3.4, if \(\delta = 1\) and supposing that \(D_T = D_A\), an increase in the relative wage of L-sectors has a stronger distortive effect in autarky than in an open economy.

In the discussion following Proposition 2, we claim that, if \(\delta < \delta^*\) (\(\delta^*\) is defined in (27)), the efficiency loss of maintaining the autarkic income distribution becomes large. At \(\delta = \delta^*\), \(D_T = D_A\) and, by the definition of \(\delta^*\), \(\delta^* D_T^{\nu-1} = (\alpha_L / \alpha_H)^{\frac{\nu-1}{\nu}}\). Equations (47) and (48) show that the marginal efficiency loss of higher relative wages above their equilibrium value is the same in the open and closed economy at \(\delta = \delta^*\). If \(\delta < \delta^*\), the optimal distribution from the perspective of L-workers entails stronger inefficiency costs than in autarky.

8.3. Proof of Proposition 1

The proof of Proposition 1 consists of three steps. In the first step, we derive the probability \((p_i(z_{ji}))\) that worker \(j\) prefers policy \(\beta\) to \(\beta'\) conditional on (i) the fact that sector \(i\) is his preferred sector under policy \(\beta'\) (ii) his idiosyncratic productivity draw in \(i\) is \(z_{ji}\). In a second step, we relax the second part of the conditional exercise above and ask what is the probability \((p_i)\) that a worker prefers policy \(\beta\) given that he works in sector \(i\) under policy \(\beta'\). Finally, the third step derives the unconditional probability \((p)\) that
a worker prefers policy $\beta$ to $\beta'$. Since the economy consists of a continuum of agents, this is also the fraction of workers who prefer $\beta$ to $\beta'$.

- Step 1: probability that worker $j$ prefers $\beta$ over $\beta'$ if $z_{ji}u_i(\beta') = \max_n \{z_{jn}u_n(\beta')\}$.

If $u_i(\beta) > u_i(\beta')$, worker $j$ prefers policy $\beta$ for sure (even if $i$ is not the best choice of sector under policy $\beta$). If $u_i(\beta') < u_i(\beta)$ however, he finds policy $\beta$ better than $\beta'$ if for at least one sector $n \neq i$: $z_{jn}u_n(\beta) > z_{ji}u_i(\beta')$. Conditional on $i$ being the best sector for worker $j$ under policy $\beta'$, the probability that $j$ prefers policy $\beta$ to $\beta'$ is:

$$p_i(z_{ji}) = \mathbb{I}\{u_i(\beta) < u_i(\beta')\} \left(1 - \prod_{n \neq i} \text{Prob}_c(u_n(\beta)z_{jn} < u_i(\beta')z_{ji})\right) + \mathbb{I}\{u_i(\beta) > u_i(\beta')\}$$

with $\mathbb{I}$ the indicator function and $\text{Prob}_c()$ the conditional probability operator $\text{Prob}(z_{jn}u_n(\beta') < z_{ji}u_i(\beta'))$. $\text{Prob}_c(u_n(\beta)z_{jn} < u_i(\beta')z_{ji})$ is the probability that $z_{jn} < z_{ji}u_i(\beta')/u_n(\beta)$ given that $z_{jn} < z_{ji}u_i(\beta')/u_n(\beta')$. It is equal to one for all sectors $n \in I_1$. For sectors $n \in I_2$ it is:

$$\text{Prob}_c(u_n(\beta)z_{jn} < u_i(\beta')z_{ji}) = \exp\left(-\left(z_{ji}u_i(\beta')\right)^{-\nu}\left(\sum_{n \in I_2} (u_n(\beta))^\nu - (u_n(\beta'))^\nu\right)\right),$$

which implies:

$$p_i(z_{ji}) = 1-\mathbb{I}\{u_i(\beta) < u_i(\beta')\}\exp\left(-\left(z_{ji}u_i(\beta')\right)^{-\nu}\left(\sum_{n \in I_2} (u_n(\beta))^\nu - (u_n(\beta'))^\nu\right)\right).$$

- Step 2: probability ($p_i$) that a worker prefers $\beta$ given that he works in sector $i$ under $\beta'$
For a given $z$ in $i$, the probability that $i$ is the best choice under $\beta'$ is the probability that for all $n \neq i$, $z_{jn}u_n(\beta') \leq z_{ni}(\beta')$, which is $\exp[-(zu_i(\beta'))^{-\nu} \sum_{n \neq i}(u_n(\beta'))^\nu]$. Integrating all $z$ gives the probability that $i$ is the best sectoral choice under $\beta'$. We define the density function $h_i(z)$ as the probability that a worker has productivity $z$ in $i$, conditional on $i$ being its choice of sector under $\beta'$:

$$h_i(z) = \frac{\nu z^{-\nu - 1} \exp(-z^{-\nu}) \exp\left(-\left((zu_i(\beta'))^{-\nu} \sum_{n \neq i}(u_n(\beta'))^\nu\right)\right)}{\int \nu \zeta^{-\nu - 1} \exp(-\zeta^{-\nu}) \exp\left(-\left((\zeta u_i(\beta'))^{-\nu} \sum_{n \neq i}(u_n(\beta'))^\nu\right)\right) d\zeta}.$$  \hspace{1cm} (53)

The conditional density $h_i(z)$ shows that, if $i$ is the best sector for a worker under $\beta'$, the likelihood that the worker has drawn a given $z$ depends on both (i) the unconditional likelihood to draw $z$ (given by $\nu z^{-\nu - 1} \exp(-z^{-\nu})$) and (ii) the likelihood that $z$ is sufficient to make $i$ the best choice under $\beta'$, which is larger the higher the $z$. The probability ($p_i$) that a worker prefers policy $\beta$ given that he chooses to work in sector $i$ under policy $\beta'$ is:

$$p_i = \int p_i(z) h_i(z) dz.$$  \hspace{1cm} (54)

Using (52) and integrating gives:

$$p_i = 1 - \mathbb{I}\{u_i(\beta') > u_i(\beta)\} \frac{\sum_{n=1}^{N}(u_n(\beta'))^\nu}{\sum_{n \in I_2}(u_n(\beta))^{\nu} + \sum_{n \in I_1}(u_n(\beta'))^{\nu}}.$$  \hspace{1cm} (55)

- Step 3: fraction ($p$) of workers who find policy $\beta$ better than $\beta'$

We take the sum of $p_i$ over all $i$ weighted by the likelihood that sector $i$ is the best choice of policy under $\beta'$\hspace{1cm}15. This yields:

$$p = \frac{\sum_{n \in I_2}(u_n(\beta))^{\nu}}{\sum_{n \in I_2}(u_n(\beta))^{\nu} + \sum_{n \in I_1}(u_n(\beta'))^{\nu}}.$$  \hspace{1cm} (56)

Setting $p \geq 1/2$ generates Proposition 1.

\hspace{1cm}15This likelihood is equal to $(u_i(\beta'))^{\nu}/(\sum_{n=1}^{N}(u_n(\beta'))^{\nu})$.  

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8.4. Proof of Proposition 2

8.4.1. Proof of parts 1 and 2

We first establish that $\beta_T$ is strictly decreasing in $\delta$ and that $\delta \beta_T$, which determines the relative wages across sectors, is weakly increasing in $\delta$. If $\beta_T$ is interior, i.e. if (25) holds:

$$\frac{\partial \beta_T}{\partial \delta} \frac{\delta}{\beta_T} = -\frac{(\nu - 1)\beta_T - \nu}{(\nu - 1)(\beta_T - 1)} < 0,$$

(57)

where the numerator is positive from (25). The elasticity of $\beta_T$ to $\delta$ is negative, and larger than -1, so that $\frac{\partial \delta \beta_T}{\partial \delta} > 0$.

We then show that $\beta_T(\delta^*) = \beta_A$ and $\delta^* \beta_T(\delta^*) = \beta_A^{1/\nu}$. Using $\delta^* = \beta_A^{1-\nu}$ (equation (27)) combined with (24) and plugging in (25) gives:

$$x_H \alpha_H = x_L \alpha_L \left( \frac{\beta_T(\delta^*)}{\beta_A} \right)^{\nu-1} (\beta_T(\delta^*)(\nu - 1) - \nu)$$

(58)

$$x_H \alpha_H = \alpha_L x_L ((\nu - 1)\beta_A - \nu)$$

(59)

so that $\beta_T(\delta^*) = \beta_A$ and $\delta^* \beta(\delta^*) = \beta_A^{1/\nu}$. As a corollary: $\delta^{*\nu}(\beta(\delta^*))^{\nu-1} = 1$.

Finally, we establish that $\Theta_T(\beta_T) > \Theta_A$ if and only if $\delta < \delta^*$. In autarky, due to the Cobb Douglas preferences, (26) becomes:

$$\Theta_A = -\alpha_L x_L \tau_L.$$  (60)

Plugging (39) in (60) gives:

$$\Theta^A(\beta) = \frac{\alpha_L x_L \alpha_H x_H (\beta - 1)}{\alpha_L x_L \beta + \alpha_H x_H}$$  (61)

Rearranging (13) shows that:

$$\frac{x_L p_T^L y_T^L}{I^T} = \frac{x_L \alpha_L \beta^{\nu-1} \delta^\nu}{x_L \alpha_L \beta^{\nu-1} \delta^\nu + x_H \alpha_H}.$$ (62)

Combining (43) and (62) in (26) gives:

$$\Theta^T(\beta) = \frac{\alpha_L x_L \alpha_H x_H \delta^\nu \beta^{\nu-1}(\beta - 1)}{(x_L \alpha_L \delta^\nu \beta^{\nu-1} + x_H \alpha_H)(x_L \alpha_L \delta^\nu \beta^{\nu} + x_H \alpha_H)}.$$ (63)
Using the first order conditions (24) and (25) shows that:

\[
\Theta^A(\beta_A) = \frac{\alpha_H x_H}{\nu} \frac{1}{x_L \alpha_L \delta^\nu \beta_T^{\nu - 1} + x_H \alpha_H}. \quad (64)
\]

\[
\Theta^T(\beta_T) = \frac{\alpha_H x_H}{\nu} \frac{1}{x_L \alpha_L \delta^\nu \beta_T^{\nu - 1} + x_H \alpha_H}. \quad (65)
\]

\(\delta^\nu \beta_T^{\nu - 1}\) is increasing in \(\delta\) and is equal to one for \(\delta = \delta^*\). It follows that \(\Theta^T(\beta_T) > \Theta^A(\beta_A)\) if and only if \(\delta < \delta^*\).

### 8.4.2. Proof of part 3

Plugging \(\delta^* \nu (\beta_T(\delta^*))^{\nu - 1} = 1\) and \(\beta_T(\delta^*) = \beta_A\) in (21) shows that \(u_L^T(\beta_T(\delta^*), \delta^*) = u_L^A(\beta_A)\). When their preferred redistribution is implemented, the utility of workers in L-sectors is equal in a closed and in an open economy at \(\delta^*\). We now show that \(u_L^T(\beta_T(\delta), \delta)\) is minimized at \(\delta = \delta^*\).

Differentiating (21) and using the envelope theorem gives:

\[
\frac{\partial u_L^T(\beta_T(\delta), \delta)}{\partial \delta} \frac{\delta}{u_L^T(\beta_T(\delta), \delta)} = x_H \alpha_H - \nu \Theta^T(\beta_T(\delta)). \quad (66)
\]

Using (65), \(\delta^* \nu (\beta_T(\delta^*))^{\nu - 1} = 1\) and that \(\delta^\nu (\beta_T(\delta))^{\nu - 1}\) is increasing in \(\delta\) shows that \(u_L^T(\beta_T(\delta), \delta)\) is minimized at \(\delta^*\).

### 8.5. Proof of Proposition 3

- For the autarkic case, we define the function:

\[
G^A(\chi) \equiv x_L^\frac{1}{\nu} u_L^A(\beta_A) - x_H^\frac{1}{\nu} u_H^A(1) = \zeta \chi \delta_A^{\frac{1}{\nu}} + \chi^{\nu - 1} + \chi^{\nu - 1} \log(\beta_A) - \zeta (1 - \chi)^{\frac{1}{2}}. \quad (67)
\]

where \(\beta_A\) is defined by (24) and where we denote \(\alpha_L x_L\) as \(\chi\). If \(G^A(\chi) \geq 0\), policy \(\beta_A\) wins the majority of votes, while policy \(\beta = 1\) wins if \(G^A(\chi) < 0\). Since \(\beta_A < \alpha_H/\alpha_L\) by assumption, \(G^A(0) = -\zeta\) and that \(G^A(1) = \zeta\).

Differentiating \(G^A(\chi)\):

\[
\frac{\partial G^A}{\partial \chi} = x_L^\frac{1}{\nu} u_L^A(\beta_A) \left( \frac{1}{\nu} - \Theta^A(\beta_A) \right) + \chi^{\nu - 1} \frac{1}{\nu} \log(\beta_A) + \chi u_H^A(1) / \nu (1 - \chi). \quad (68)
\]
where we have made use of the envelope theorem. From (64) and $\beta_A > 1$, the square bracket is positive so that $\partial G^A(\chi)/\partial \chi > 0$ and there is a unique $\chi_A$ below which $G^A(\chi) < 0$ and above which $G^A(\chi) > 0$.

- For the small open economy, we define:

$$G^T(\chi, \delta) = \zeta \chi^{\frac{1}{\nu}} + \frac{1 - \chi(1 - \chi)^{\frac{1}{\nu}}}{\chi^{\frac{1}{\nu}} - (1 - \chi)\beta^T}$$

(69)

where $\beta_T$ is defined by (25) and is itself a function of $\chi$, and where $G^T(\chi, \delta) \equiv x_L^T u_L^T(\beta_T, \delta) - x_H^T u_H^T(1, \delta)$. If $G^T(\chi, \delta) \geq 0$, policy $\beta_T(\delta)$ wins the majority of votes, while policy $\beta = 1$ wins if $G^T(\chi, \delta) < 0$. Since $\beta_T$ is bounded above by assumption, $G^T(0, \delta) = -\zeta$ and $G^T(1, \delta) = \zeta$. Differentiating $G^T(\chi, \delta)$ with respect to $\chi$ gives:

$$\frac{\partial G^T}{\partial \chi} = \frac{x_L^T \delta \chi u_L^T(\beta_T, \delta)}{1 - \chi} \left( \frac{1}{\nu} - \Theta^T(\beta_T) \right) - \left( \ln(\delta) + \frac{\chi}{\nu(1 - \chi)G^T} \right)$$

(70)

where we use the envelope theorem. From (65), $\Theta^T(\beta_T) < 1/\nu$. This shows that $\partial G^T/\partial \chi > 0$ if $G^T(\chi, \delta) = 0$, meaning there is a unique $\chi$ for which $G^T = 0$ and there exists a unique cutoff $\chi_T(\delta)$ such that policy $\beta = 1$ wins if $\chi < \chi_T(\delta)$ while policy $\beta_T(\delta)$ wins otherwise.

8.6. Proof of Proposition 4

From the appendix 8.4.2, we know that $u_L^T(\beta_T(\delta^*), \delta^*) = u_A^L(\beta_A)$ while $u_A^H(1) < u_H^T(1, \delta^*)$ since H-sectors have a comparative advantage in the open economy ($\delta^* < 1$). This implies that $G^T(\chi_A, \delta^*) < 0$ and that $\chi_T(\delta^*) > \chi_A$.

For $\delta > \delta^*$, $u_L^T(\beta(\delta), \delta)$ is increasing in $\delta$ and $u_H^T(1, \delta)$ is decreasing in $\delta$, so that $\chi_T(\delta) < 0$. Furthermore, $u_A^H(1) = u_H^T(1, 1)$ while $u_L^T(\beta_T(1), 1) > u_L(\beta_T(\delta^*), \delta^*) = u_A^H(\beta_A)$ since the real income of L-sectors increases with $\delta$.

Taking these results together, $G^T(\chi_A, 1) > 0$ and there is a unique $\bar{\delta} \in (\delta^*, 1)$ such that $\chi_T(\delta) < \chi_A$ if $\delta > \bar{\delta}$ and $\chi_T(\delta) > \chi_A$ if $\delta^* \leq \delta < \bar{\delta}$.

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For $\delta < \delta^*$, $u_T^L(\beta(\delta), \delta)$ and $u_T^H(1, \delta)$ are both decreasing in $\delta$. Differentiating $u_T^H$ (obtained from (21) and (17)) gives:
\[
\frac{\partial u_T^H(1, \delta)}{\partial \delta} \frac{\delta}{u_T^H(1, \delta)} = -\alpha_L x_L.
\]  
(71)
Combining (66) with (65) shows that:
\[
\frac{\partial u_T^L(\beta_T(\delta), \delta)}{\partial \delta} \frac{\delta}{u_T^L(\beta_T(\delta), \delta)} > -\alpha_L x_L.
\]  
(72)
Combining this result with the fact that $x^T_H u_T^H(1, \delta^*) > x^T_L u_T^L(\beta_T(\delta^*), \delta^*)$ shows that $\chi_T'(\delta) < 0$ also for $\delta < \delta^*$ and completes the proof.

8.7. Proof of Proposition 5

- $\delta \leq 1$

As described in section 6.1, and in particular equation (30), trade wins if $\chi < \min\{\chi_T(\delta), \chi_A\}$ (for $\delta = 1$, voters are indifferent between trade and autarky). If $\chi > \chi_T(\delta)$, on the other hand (redistribution is chosen at least in an open economy), L-sectors are in favor of trade by Proposition 2 and even more so if there is no redistribution in autarky. In that case, and regardless of whether H-sectors favor trade or not, trade wins because $x^T_H u_T^H(1, \delta^*) > x^T_L u_T^L(\beta_T(\delta^*), \delta^*)$ where the first inequality holds because $\chi > \chi_T$, the second as $\delta \leq 1$ and the third since H-sectors lose from redistribution. This guarantees that trade - the preferred policy of L-sectors - wins regardless of the preferred policy of H-sectors and regardless of whether redistribution takes place in autarky. Finally, if $\chi_A < \chi < \chi_T(\delta)$ (which can only arise if $\delta < \delta^*$), redistribution takes place in a closed but not in an open economy. In this case, H-sectors prefer trade while L-sectors prefer autarky (they lose redistribution and have a comparative disadvantage). Trade wins in this case as
\( x_H^1 u_H^T(1, \delta) > x_L^1 u_L^T(\beta_T, \delta) > x_L^1 u_L^A(\beta_A) \), where the first inequality holds since \( \chi < \chi_T \) and the second holds by Proposition 2.

Without redistributive policies, trade would win if and only if \( x_H^1 u_H^T(1) > x_L^1 u_L^A(1) \), i.e. if \( \chi < 1/(1+\delta^\nu) \). With the possibility to vote on redistributive policies at \( t = 2 \), however, we just showed that trade always beats autarky, which proves the second part of Proposition 5 when \( \delta \leq 1 \).

- \( \delta > 1 \)

As described in section 6.1, autarky wins if \( \chi < \min\{\chi_T(\delta), \chi_A\} \). With no redistribution in the closed and open economy, and with \( \delta > 1 \), workers in H-sectors favor autarky while L-sectors favor trade since they have a comparative advantage. However, since the H-sectors are large enough to win the vote against redistribution in an open economy, they are even more able to win the vote on trade policy as: \( x_H^1 u_H^T(1, \delta) > x_H^1 u_H^T(1, \delta) > x_H^1 u_H^T(\beta_T(\delta), \delta) \) where the first inequality holds as \( \delta > 1 \) and the second if \( \chi < \chi_T(\delta) \). If \( \chi > \chi_A \), on the other hand, redistribution happens both in a closed and an open economy. L-sectors favor trade while H-sectors favor autarky. Since L-sectors are large enough to win the vote on redistribution in autarky, they win the vote on trade policy as: \( x_L^1 u_L^T(\beta_T(\delta), \delta) > x_L^1 u_L^A(\beta_A) > x_H^1 u_H^A(1) > x_H^1 u_H^A(\beta_A) \) where the first inequality holds by Proposition 2, the second since \( \chi > \chi_A \) and the third as H-sectors lose from redistribution. Finally, if \( \chi_T(\delta) < \chi < \chi_A \), there is redistribution under trade but none in autarky. In this case, L-sectors favor trade while H-sectors favor autarky. Trade wins if:

\[
\tilde{G}(\chi, \delta) \equiv x_L^1 u_L^T(\beta_T(\delta), \delta) - x_H^1 u_H^T(1) = \zeta \chi^{\frac{1}{\nu}} \delta^{1-\chi} \frac{\chi \delta^\nu \beta_T^\nu + (1-\chi) \beta_T}{\chi \delta^\nu \beta_T^\nu + 1 - \chi} - \zeta (1-\chi)^{\frac{1}{\nu}} > 0 \quad (73)
\]
By definition of $\chi_T(\delta)$ in (69), it is immediate that $\tilde{G}(\chi_T(\delta), \delta) < 0$ while since $u_T^L(\beta_T) > u_A^A(\beta_A)$ and by (67), $\tilde{G}(\chi_A, \delta) > 0$.

To show that there is a unique $\chi$ such that $\tilde{G}$ is zero, we first use (25) repeatedly to rewrite:

$$\tilde{G}(\chi, \delta) = \zeta \chi \frac{1}{\nu} \delta^{1-\chi} \beta_T \frac{\nu - 1}{\nu} - \zeta (1-\chi) \frac{1}{\nu}. \quad (74)$$

Setting $\tilde{G}(\chi, \delta) = 0$ and plugging back in (25) shows that $\tilde{G}(\chi, \delta) = 0$ defines a relationship between $\chi$ and $\delta$ such that:

$$\delta^{\nu \chi} \left[ 1 - \left( \frac{\chi}{1-\chi} \right)^\frac{1}{\nu} \delta^{1-\chi} \right] - (\nu - 1)^{\nu-1} \nu^{-\nu} = 0. \quad (75)$$

Differentiating the left hand side with respect to $\chi$, it can be shown that there is a unique $\chi$ such that the above holds. Combined with $\tilde{G}(\chi_A, \delta) > 0$ and $\tilde{G}(\chi_T(\delta), \delta) < 0$, this shows that, for each $\delta > 1$, there is a unique $\chi_T(\delta) < \chi < \chi_A$ above which trade wins and below which autarky wins.

With the possibility of redistribution at $t = 2$, we just proved that trade wins in any case for $\delta < 1$ and if $x_T^\frac{1}{\nu} u_T^L(\beta_T, \delta) > x_H^\frac{1}{\nu} u_H^A(1)$ for $\delta > 1$. Without such a possibility however, trade beats autarky if and only if $x_T^\frac{1}{\nu} u_T^L(1, \delta) > x_H^\frac{1}{\nu} u_H^A(1)$. Since $u_T^L(\beta_T, \delta) > u_T^A(1, \delta)$, opening to trade beats autarky for a strictly larger range of parameters if redistributive policies can arise.
Figure 1: Evolution of the market subsidies to GDP ratio for various OECD countries using the COFOG (Classification of the functions of government) special classification. These subsidies are used to finance individual sectors with aims of redistribution or to solve an externality (see Kraan, Lupi, and Job (2012) for details). Source: OECD stats.

Figure 2: $\chi$ is the share of L-goods in the utility and $\delta$ the comparative advantage of L-sectors in the open economy. The left-hand graph depicts $\chi_T(\delta), \chi_A$ and $\tilde{\chi}(\delta)$ as defined in Propositions 3 and 5 for $\nu = 2$. The right-hand graph shows the parameter constellations for which autarky wins over trade at $t = 1$ if there is a vote on redistribution at $t = 2$ (colored area) and if there is no possibility to redistribute at $t = 2$ (hatched area).
9. NOT FOR PUBLICATION: Extension to a lobbying model

This section provides the formal counterpart to our descriptive analysis in section 6.2, in which we extend our results to a setup à la Grossman and Helpman (1994).

There are $N$ goods and $M$ individuals in the economy. Each consumer maximizes his utility:

$$U = q_{NUM} + \log \left( \prod_{i=1}^{N} q_i^{\alpha_i} \right), \quad \sum_{n} \alpha_n = 1$$  \hspace{1cm} (76)

subject to:

$$q_{NUM} + \sum_{n} p_n q_n = I$$  \hspace{1cm} (77)

where $p_n$ denotes the consumption price of good $n$, $I$ is the income of the consumer and $q_{NUM}$ is the consumption of the numeraire good, with a price of one. The first order conditions for the problem show that:

$$q_n = \frac{\alpha_n}{p_n}.$$  \hspace{1cm} (78)

The total demand for good $n$ is therefore given by: $M q_n = M \alpha_n / p_n$.

Plugging $q_n$ back in the budget constraint gives:

$$q_{NUM} = I - 1$$  \hspace{1cm} (79)

where we assume that the total labor force is large enough to ensure that the numeraire good is always produced. The indirect utility of consumer $j$, with income $I_j$ is therefore:

$$V(p, I_j) = I_j - 1 + \log \left( \prod_{n=1}^{N} \left( \frac{\alpha_i}{p_n} \right)^{\alpha_n} \right).$$  \hspace{1cm} (80)

On the production side, we assume that the numeraire good is produced using exclusively labor. One unit of labor produces one unit of the numeraire
good, implying that \( w = 1 \). The production of good \( n \) on the other hand uses a sector-specific capital \( K_n \) with a Cobb Douglas production function \((\eta < 1)\):

\[
y_n = K_n^\eta L_n^{1-\eta}.
\]  

(81)

Holder’s of the sector-specific factor maximize their payment by choosing the amount of labor that they employ:

\[
\max_{L_n} \bar{p}_n K_n^\eta L_n^{1-\eta} - L_n,
\]  

(82)

where \( \bar{p}_n \) denotes the production price of good \( n \). This gives:

\[
L_n = K_n \frac{\bar{p}_n (1-\eta)}{1-\eta} \frac{1}{\eta}
\]  

(83)

\[
y_n = K_n \frac{\bar{p}_n (1-\eta)^{1-\eta}}{1-\eta} 
\]  

(84)

\[
\bar{p}_n y_n = K_n \bar{p}_n (1-\eta)^{1-\eta} 
\]  

(85)

which implies that the payment to the specific factor in \( n \) is:

\[
\bar{p}_n y_n - L_n = \eta \bar{p}_n y_n.
\]  

(86)

We will consider two situations: autarky and a small open economy. In the open economy, we allow for holders of specific factors to lobby both for tariffs (as in Grossman and Helpman (1994)) and for an ad-valorem production subsidy. This allows testing whether the result of section 6.1, which shows that tariffs are dominated by production subsidies in a small open economy, also holds in the lobbying model.

The world price of each good \( n \) is denoted by \( p_n^T \). The tariff revenues per good (or costs of export subsidy) are \((p_n - p_n^T)(Mq_n - y_n)\) while the revenue from taxing the production of sector \( n \) is \((p_n - \bar{p}_n)y_n\). The total revenue of the government is:

\[
R = \sum_n (p_n - p_n^T)q_n M + \sum_i (p_i^T - \bar{p}_n)y_n.
\]  

(87)
It should be noted that, in autarky, $q_nM = y_n$, implying that the above becomes: $\sum_n (p_n - \bar{p}_n)y_n$. Under trade, with no tariffs and only production subsidies, $p_n = p_T^n$. The revenues of the government are redistributed lump-sum.

We now denote the fraction of the (voting) population which hold specific factors to $n$ as $\gamma_n$ and $l_n$ the labor held by these individuals (no individual holds specific factors in more than one sector). The combined welfare of all individuals with specific factors in $n$ is:

$$W_n = \underbrace{l_n + \eta \bar{p}_n y_n}_{l_n} - 1 + \gamma_n M \left[ \log \left( \prod_i \left( \frac{\alpha_i}{p_i} \right)^{\alpha_i} \right) \right] + \frac{R}{M}.$$  \hspace{1cm} (88)

As in Grossman and Helpman (1994), organized lobbies offer a contribution schedule, which maps the policy vector chosen by the government to campaign contributions. Equivalently, lobbies can tie their contribution to the endogenous variables affected by policies, i.e. producer and consumer prices. We follow Grossman and Helpman (1994) in assuming that the contribution schedules are differentiable and use the result that, when choosing how their contribution schedule depends on prices, members of group $n$ set truthfully:

$$\frac{\partial W_n}{\partial p_i} = \frac{\partial C_n}{\partial p_i} \text{ and } \frac{\partial W_n}{\partial \bar{p}_i} = \frac{\partial C_n}{\partial \bar{p}_i}.$$  \hspace{1cm}

The welfare function of the whole population is given by:

$$W = L + \eta \left( \sum_i \bar{p}_i y_i \right) - M + M \left[ \log \left( \prod_i \left( \frac{\alpha_i}{p_i} \right)^{\alpha_i} \right) \right] + \frac{R}{M}$$ \hspace{1cm} (89)

and the objective of the government is to maximize:

$$\sum_{n \in O} C_n(p, \bar{p}) + aW$$ \hspace{1cm} (90)

where $O$ is the set of organized sectors (i.e. contributing to the government’s
budget). The government therefore sets:

\[
\sum_{n \in O} \frac{\partial C_n}{\partial p_m} + a \frac{\partial W}{\partial p_m} = 0 \iff \sum_{n \in O} \frac{\partial W_n}{\partial p_m} + a \frac{\partial W}{\partial p_m} = 0 \tag{91}
\]

\[
\sum_{n \in O} \frac{\partial C_n}{\partial \bar{p}_m} + a \frac{\partial W}{\partial \bar{p}_m} = 0 \iff \sum_{n \in O} \frac{\partial W_n}{\partial \bar{p}_m} + a \frac{\partial W}{\partial \bar{p}_m} = 0 \tag{92}
\]

We now consider in turn the equilibrium in a small open and in a closed economy.

- **Small open economy**

In a small open economy, consumption and production prices can be chosen independently of each other. The derivatives of the welfare of individuals with specific factors in sector \( n \) with respect to all domestic consumption and production prices are:

\[
\frac{\partial W_n}{\partial p_m} = \gamma_n M \left[ -\frac{\alpha_m}{p_m} + q_m \left( \frac{p_m - \bar{p}_m}{p_m} \frac{\partial q_m}{\partial p_m} \right) \right] \forall m \tag{93}
\]

\[
\frac{\partial W_n}{\partial \bar{p}_m} = \gamma_n y_m \left( \frac{p_m - \bar{p}_m}{\bar{p}_m} \frac{\partial y_m}{\partial \bar{p}_m} \right) \forall m \neq i \tag{94}
\]

\[
\frac{\partial W_n}{\partial \bar{p}_n} = \eta y_n \left( 1 + \frac{\partial y_n}{\partial \bar{p}_n} \right) + \gamma_n y_n \left( \frac{p_n - \bar{p}_n}{\bar{p}_n} \frac{\partial y_n}{\partial \bar{p}_n} - 1 \right) \tag{95}
\]

From equation (84) and the demand function (78), we can further impose:

\[
\frac{\partial y_n}{\partial \bar{p}_n} \frac{\bar{p}_n}{y_n} = \frac{1 - \eta}{\eta} \quad \text{and} \quad \frac{\partial q_m}{\partial p_m} = -1, \tag{96}
\]

which allows to simplify:

\[
\frac{\partial W_n}{\partial p_m} = \gamma_n M q_m \frac{p_m}{p_m} \tag{97}
\]

\[
\frac{\partial W_n}{\partial \bar{p}_m} = \gamma_n y_m \left( \frac{p_m - \bar{p}_m}{\bar{p}_m} \frac{1 - \eta}{\eta} - 1 \right) \tag{98}
\]

\[
\frac{\partial W_n}{\partial \bar{p}_n} = y_n + \gamma_n y_n \left( \frac{p_n - \bar{p}_n}{\bar{p}_n} \frac{1 - \eta}{\eta} - 1 \right). \tag{99}
\]
Similarly, the derivatives of the population’s welfare $W$ with respect to the different prices are:

$$\frac{\partial W}{\partial p_m} = Mq_m \frac{p_m^T - p_m}{p_m} \quad \forall m$$ (100)

$$\frac{\partial W}{\partial \bar{p}_m} = y_m + y_m \left( \frac{p_m^T - \bar{p}_m}{\bar{p}_m} \frac{1 - \eta}{\eta} - 1 \right) \quad \forall m$$ (101)

Equations (97) and (101) show that any distortion of consumer prices away from their free trade value is costly for each lobby as well as for the whole population. Changing consumer prices does not redistribute towards any group but is simply a source of inefficiency, which no lobby and no voter should push for. Equation (98) shows that raising the producer price of good $n$ affects all individuals without a stake in sector $n$ in the same way, namely through the budget constraint of the government. Raising the producer price of $n$ has a direct costly effect (higher subsidy or lower tax revenue), and raises the production of $n$, which can raise or lower revenue depending on whether the sector is taxed or subsidized. Individuals with specific factors to $n$ however benefit from an additional redistributive effect, which raises the income ($I_n$ in (88)).

Denoting $\gamma$ as the sum of the individual $\gamma_n$ of the organized sectors ($\gamma = \sum_{n \in O} \gamma_n$) and plugging the above equations in (91) and (92) allows to rewrite the first order conditions of the government as:

$$(\gamma + a) M q_n \frac{p_n^T - p_n}{p_n} = 0$$ (102)

$$(a + \gamma) y_n \left( \frac{p_n^T - \bar{p}_n}{\bar{p}_n} \frac{1 - \eta}{\eta} - 1 \right) + (1 + a) y_n = 0$$ (103)

where $1$ is an indicator function taking value 1 if sector $n$ is organized and 0 otherwise. The above analysis shows that (i) as in section 6.1 for the voting model, there should be no tariffs in the open economy (distorting
consumption patterns is costly), and (ii) producer prices should be distorted at the rate:

\[ \bar{p}_n - p_T^n = \frac{1 - \gamma}{a + \gamma (1 - \eta)} \frac{\eta}{\eta}, \]  

(104)

which can be rewritten as:

\[ \bar{p}_n = \frac{p_T^n}{1 - \frac{1 - \gamma}{a + \gamma (1 + \eta)}} \]  

(105)

Organized sectors should face a higher price than the world price for their good (i.e. be subsidized), while non-organized sectors should face a lower price than the world price of their good (i.e. be taxed).

Note that Grossman and Helpman (1984) do not allow for production subsidies, thereby imposing that there is no difference between the home production and consumption prices \((p_n = \bar{p}_n)\). Under these conditions, the maximization of welfare requires to add the left hand sides of (102) and (103) and set the sum equal to zero, giving:

\[ \frac{\bar{p}_n - p_T^n}{\bar{p}_n} = \frac{1 - \gamma}{a + \gamma} \frac{1}{Mq_n + \frac{1 - \eta}{\eta}} \]  

(106)

which is the equivalent to the rule outlined in Proposition 2 of their paper.

- **Autarky**

The only change with the free trade case is that, in autarky, domestic production needs to equal domestic consumption. We therefore impose that \(Mq_n = y_n\), i.e.:

\[ M^\alpha p_n = K_n \bar{p}_n^\eta (1 - \eta)^\frac{1 - \eta}{\eta}, \]  

(107)

meaning that \(p_n\) and \(\bar{p}_n\) cannot be varied independently of each other. In fact:

\[ \frac{dp_n}{p_n} = \frac{1 - \eta}{\eta} \frac{d\bar{p}_n}{\bar{p}_n}. \]  

(108)
Using this in the above analysis shows that (A stands for autarky):

\[ \frac{dW^A_n}{dp_m} = \frac{\partial W_n}{\partial p_m} - \frac{1 - \eta p_m \partial W_n}{\eta p_m \partial p_m} = \gamma_n y_n \left( \frac{1 - \eta p_m - \bar{p}_m}{\eta} - 1 \right) \]  

(109)

\[ \frac{dW^A_n}{d\bar{p}_n} = y_n + \gamma_n y_n \left( \frac{1 - \eta p_n - \bar{p}_n}{\eta} - 1 \right) \]  

(110)

where the partial derivatives \( \frac{\partial W_n}{\partial p_m} \) and \( \frac{\partial W_n}{\partial p_m} \) are as given by (98) and (97).

Plugging in the first order condition of the government gives:

\[ \frac{\bar{p}_m - p_m}{\bar{p}_m} = \frac{1 - \gamma - \eta}{a + \gamma (1 - \eta)} \]  

(111)

Using (107) shows that the government sets a producer price under autarky equal to:

\[ \bar{p}_m = \left( \frac{M \alpha_m}{K_m} (1 - \eta) \frac{\eta - 1}{\eta - 1 - \gamma} \right)^{\eta} \]  

(112)

where the numerator \( M \alpha_m (1 - \eta) \frac{\eta - 1}{\eta - 1 - \gamma} \eta \) is the price that would prevail in autarky with no policy (where \( p_n = \bar{p}_n \) in (107)).

Assume that we have two sectors: \( L \) and \( H \), where sector \( L \) is lobbying while sector \( H \) is not. The ratio of specific income between the two sectors is, from (4):

\[ \frac{\bar{p}_L y_L}{\bar{p}_H y_H} = \frac{K_L}{K_H} \left( \frac{\bar{p}_L}{\bar{p}_H} \right)^{\frac{1}{\eta}} \]  

(113)

In line with our baseline model, we define \( \delta \) as the factor by which the ratio of prices in the open economy differs from the ratio of prices in the closed economy with no policy:

\[ \frac{p_L^T}{p_H^T} = \delta \left( \frac{\alpha_L K_H}{\alpha_H K_L} \right)^{\eta} \]  

(114)

Using (112) and (105) shows the equilibrium ratio of income from specific
factors in autarky and the small open economy:

\[
\left( \frac{\bar{p}_L y_L}{\bar{p}_H y_H} \right)^A = \frac{\alpha_L}{\alpha_H} \frac{1 + \frac{\gamma}{\alpha + \gamma} \frac{\eta}{1 + \eta}}{1 - \frac{1 - \gamma}{\alpha + \gamma} \frac{\eta}{1 + \eta}} \tag{115}
\]

\[
\left( \frac{\bar{p}_L y_L}{\bar{p}_H y_H} \right)^T = \frac{\alpha_L}{\alpha_H} \delta^\frac{1}{\eta} \left( \frac{1 + \frac{\gamma}{\alpha + \gamma} \frac{\eta}{1 + \eta}}{1 - \frac{1 - \gamma}{\alpha + \gamma} \frac{\eta}{1 + \eta}} \right)^{\frac{1}{\eta}} \tag{116}
\]

Since \( \eta < 1 \), the relative income of L sectors is higher under trade than in autarky, except if they have a strong comparative disadvantage.