Supplementary Online Appendix for “A Theory of Economic Disintegration”

Eckhard Janeba* Karl Schulz†
University of Mannheim University of Mannheim

October 8, 2021

Abstract

This is the Supplementary Online Appendix for “A Theory of Economic Disintegration” by Janeba and Schulz (2021). In Section 1, we demonstrate various extensions to our model. In Section 2, we consider extensions to our analysis of endogenous trade policies. References to propositions in this document refer to the results and numbering in the Online Appendix.

*University of Mannheim, Department of Economics, L7 3-5, 68131 Mannheim, Germany, janeba@uni-mannheim.de.
†University of Mannheim, Center for Doctoral Studies in Economics, L7 3-5, 68131 Mannheim, Germany, karl.schulz@gess.uni-mannheim.de.
1 Proofs for Section 2.4

1.1 Tariff Revenues and Subsidy Expenditures

We extend the notion of trade barriers to both non-tariff barriers and tariffs. That is, trade costs from country $j$ to country $i$, $\bar{t}_{ij}$, are the sum of import taxes by the domestic government in country $i$, $int_{ij} \in \mathbb{R}$, export taxes/subsidies by the foreign government, $ext_{ij} \in \mathbb{R}$, and non-tariff barriers, $\tau_{ij} \in \mathbb{R}_+$ as defined in our baseline economy. Hence, $\bar{t}_{ij} \equiv t_{ij} + \tau_{ij} \equiv int_{ij} + ext_{ij} + \tau_{ij}$. Here, we consider in the language of the trade policy literature a full set of trade policy instruments.

Notice that, from the perspective of the government, tariffs affect three margins: domestic consumer prices, trade volumes, and firm relocation. All three affect consumer surplus, revenues generated from taxing businesses, and revenues from trade taxes. Observe that, unlike in the standard Cournot relocation models, in our economy, industry-specific prices do not exhibit the
Metzler paradox, where a rise in import tariffs leads to the entry of firms domestically such that domestic consumer prices decrease. However, it may be the case for the average price. That is, a large country raises import tariffs such that firms in small countries relocate to the former country to have cheap access to the large market. This relocation makes the larger market more competitive and reduces domestic prices there.

Let us now derive the objective function of the government. Consumer surplus and business tax revenues remain unchanged. At the same time, trade taxes generate a new source of revenue. For a given industry \( ij \), the volume of exports from country \( i \) to country \( l \) is given by

\[
X_{li}^{ij} = G\left( y^{ij} \right) k_i^{ij} x_{li}^{ij} | k_i^{ij} = 1 + (1 - G\left( y^{ij} \right)) k_i^{ij} x_{li}^{ij} | k_i^{ij} = 2,
\]

whereas the import volume reads as

\[
M_{il}^{ij} = G\left( y^{ij} \right) k_i^{ij} x_{il}^{ij} | k_i^{ij} = 1 + (1 - G\left( y^{ij} \right)) k_i^{ij} x_{il}^{ij} | k_i^{ij} = 2.
\]

Observe that, by our assumption on the industry structure \( M_{il}^{ij} = 0 \) for all \( l \neq j \). To sum up, country \( i \)'s revenues from taxing imports and exports in industry \( ij \) are given by \( R_i^{ij} = \sum_{l \in \mathcal{X} \setminus \{i\}} imt_{il} M_{il}^{ij} + \sum_{l \in \mathcal{X} \setminus \{i\}} ext_{li} X_{li}^{ij} \). Therefore, we can write the overall tariff revenues in country \( i \) as \( R_i = \sum_{j \in \mathcal{X} \setminus \{i\}} R_i^{ij} \). This yields the following objective function of the government in country \( i \): \( W_i := S_i + T_i + n_i w + R_i \).

As before, the first-order condition is sufficient and there exists a unique equilibrium of the tax competition game. Apply the same steps as in the base model to obtain the equilibrium taxes

\[
t_i = 3F + \frac{1}{2K - 1} \sum_{j \in \mathcal{X} \setminus \{i\}} \Delta_i^{ij} + \frac{1}{2K - 1} \sum_{j \in \mathcal{X} \setminus \{i\}} \left( n_i^{ij} - \pi_i^{ij} \right) + \frac{1}{(K - 1)(2K - 1)} \sum_{j \in \mathcal{X}} \sum_{m \in \mathcal{X}} \Delta_j^{jm}
+
\frac{1}{2K - 1} \sum_{j \in \mathcal{X} \setminus \{i\}} \sum_{l \in \mathcal{X} \setminus \{j\}} \left[ imt_{il} \left( 2x_{il}^{ij} | k_i^{ij} = 1 - x_{il}^{ij} | k_i^{ij} = 2 \right) + ext_{li} \left( x_{li}^{ij} | k_i^{ij} = 1 - 2x_{li}^{ij} | k_i^{ij} = 2 \right) \right]
+
\frac{1}{(K - 1)(2K - 1)} \sum_{m \in \mathcal{X}} \sum_{j \in \mathcal{X} \setminus \{m\}} \sum_{l \in \mathcal{X} \setminus \{j\}} \left[ imt_{ml} \left( 2x_{ml}^{mj} | k_m^{mj} = 1 - x_{ml}^{mj} | k_m^{mj} = 2 \right) \right]
+
\frac{1}{(K - 1)(2K - 1)} \sum_{m \in \mathcal{X}} \sum_{j \in \mathcal{X} \setminus \{m\}} \sum_{l \in \mathcal{X} \setminus \{j\}} \left[ ext_{lm} \left( x_{lm}^{mj} | k_m^{mj} = 1 - 2x_{lm}^{mj} | k_m^{mj} = 2 \right) \right].
\]

Observe that for \( imt_{ml} = ext_{lm} = 0 \ \forall \ j, m, l \) we obtain Lemma 3 of the Online Appendix. The optimal business tax is, now, modified by the marginal effects of business taxation on tariff revenues through firm relocation. Since \( x_{li}^{ij} | k_i^{ij} = 1 - 2x_{li}^{ij} | k_i^{ij} = 2 = -n_i \frac{\alpha - \omega - \tilde{t}_i}{4\beta} < 0 \) and \( 2x_{li}^{ij} | k_i^{ij} = 1 -
\[ x_{ij}^{ij} |_{k_{ij} = 2} = 1 [ j = l] n_i \frac{\alpha - \tau_{ij}}{4\beta} \geq 0 \]

taxes are revised upwards for import tariffs and export subsidies.

To gain some intuition, consider a rise in the business tax in a country. As a result, firms move away from that country. Imports increase, whereas exports decline. The revenues (expenditures) from taxing imports (subsidizing exports) rise (fall).

Not surprisingly, for a given set of trade policies the forces described in the comparative statics of business taxes with respect to \( \tau_{ij} = \tau_{ji} \in \mathbb{R}_+ \) and \( \tau_{jk} = \tau_{kj} \in \mathbb{R}_+ \) (Lemma 4 of the Online Appendix) remain valid and are augmented by the effects of (non-tariff) trade costs on the marginal firm relocation effect. That is,

\[
\frac{d t_i}{d \tau_{ij}} |_{\tilde{\tau}_{ij} = \tilde{\tau}_{ji}} = \left( n_i (K-2) - 2n_j \left[ (K-1)^2 + 0.5 \right] \right) \frac{3 (K-1)}{(K-1) (2K-1)} \frac{\alpha - w - \tilde{\tau}_{ij} +}{16\beta} - \frac{1}{(K-1) (2K-1)} K n_i imt_{ij} + n_j imt_{ji} - K (K-1) n_j ext_{ji} - (K-1) n_i ext_{ij}
\]

and

\[
\frac{d t_i}{d \tau_{jk}} |_{\tilde{\tau}_{jk} = \tilde{\tau}_{kj}} = (n_j + n_k) \left[ (2K-3) \frac{\alpha - w - \tilde{\tau}_{jk} +}{16\beta} - \frac{1}{(K-1) (2K-1)} imt_{jk} n_j + imt_{kj} n_k - (K-1) ext_{jk} n_j - (K-1) ext_{kj} n_k \right].
\]

Therefore, for positive import tariffs and export subsidies, the reaction of the optimal tax in country \( i \) to a rise in \( \tau_{ij} \) and \( \tau_{jk} \), respectively, is revised downwards. The reason is that the tax of country \( i \) is upwards adjusted by the marginal effect on tariff revenues due to firm relocation. As non-tariff trade costs rise, the trade volumes decrease such that the gains in tariff revenues decline.

Furthermore, one can study the effects of tariffs on business taxes. The comparative statics of business taxes with respect to trade taxes read as

\[
\frac{d t_i}{d imt_{ij}} = n_i \frac{(7K-6) \left( \alpha - w - \tilde{\tau}_{ij} \right) - 4Kimt_{ij} + 4ext_{ij}}{(K-1) (2K-1) 16\beta}
\]

and

\[
\frac{d t_i}{d ext_{ij}} = n_i \frac{(3K-10) \left( \alpha - w - \tilde{\tau}_{ij} \right) - 4Kimt_{ij} + 4ext_{ij}}{(K-1) (2K-1) 16\beta},
\]
Table 1: *(tariff effect)* Effects of trade taxes on business tax in country $i$.

\[
\frac{dt_i}{dimt_{ji}} = n_i \frac{-\left(6(K-1)^2 - 1\right)(\alpha - w - \bar{\tau}_{ji}) + 4Kext_{ji} - 4imt_{ji}}{(K-1)(2K-1)16\beta},
\]

and

\[
\frac{dt_i}{dext_{ji}} = n_i \frac{-\left(6(K-1)^2 + 4K + 3\right)(\alpha - w - \bar{\tau}_{ji}) + 4Kext_{ji} - 4imt_{ji}}{(K-1)(2K-1)16\beta},
\]

and

\[
\frac{dt_i}{dimt_{jm}} = n_j \frac{(6K-5)(\alpha - w - \bar{\tau}_{jm}) - 4(imt_{jm} - ext_{jm})}{(K-1)(2K-1)16\beta},
\]

and

\[
\frac{dt_i}{dext_{jm}} = n_j \frac{(6K-13)(\alpha - w - \bar{\tau}_{jm}) - 4(imt_{jm} - ext_{jm})}{(K-1)(2K-1)16\beta},
\]

for $j \neq i$ and $m \neq i, j$.

There are, now, several opposing forces on consumer surpluses, profit differentials, and revenues from trade taxes. The rows of Table 1 summarize these forces and their effects on business taxes in country $i$. To give an example, suppose the domestic government in country $i$ raises tariffs on imports from country $j$ ($imt_{ij} \uparrow$). This policy makes imports from country $j$ more costly and, as a result, lowers consumer surplus in country $i$. At the same time, country $i$ becomes
ceteris paribus more attractive as a business location vis-à-vis country $j$ due to the rise in trade frictions firms in country $j$ face. On the one hand, a higher import tariff mechanically increases the size of tariff revenues, which the government influences by business taxation (positive direct effect). On the other hand, the rise in import tariffs lowers import volumes such that the gains from tariff revenues become smaller (negative indirect effect).

Let trade taxes be small for simplicity ($imt_{ij} \approx 0$ and $ext_{ij} \approx 0$) and $K > 3$. Then the relation between business taxes in country $i$, $t_i$, and import tariffs, $imt_{ij}$, is positive. However, the sign of $\frac{dt_i}{dimt_{ij}}$ is negative for large $imt_{ij}$. Therefore, the relation between domestic taxes and import tariffs is hump-shaped. Similarly, this is the case with $imt_{jm}$. The relationship between business taxes and trade taxes on firms in country $i$ ($imt_{ji}$ and $ext_{ji}$) is U-shaped. This result is similar to Proposition 1 in Haufler and Wooton (2010), although here we deal with tariffs that have revenue effects.

### 1.2 Domestic Accrual of Firm Profits

So far, we have assumed that the profits of firms do not accrue domestically or, at least, do not enter the objective function of the government. In the following, we relax this assumption. There are two noteworthy variants of firm ownership: one, where firms are owned by entrepreneurs, who enter social welfare in a country only when they locate in that country, and another one, where citizens are shareholders of the firms worldwide. The former one fits well for small corporations, whereas the latter is suited for the case of larger firms. In the following, we consider both variants.

#### 1.2.1 Entrepreneurs

Let $m_i$ be the (endogenous) number of entrepreneurs in the population of country $i$ and $\omega$ be the social marginal welfare weight of entrepreneurs relative to workers. As before, consumer surplus and tax revenues, respectively, read as

$$S_i = \sum_{j \in \mathcal{J} \setminus \{i\}} \left[ \delta_i^{ij} + \frac{\gamma^{ij} - F_{ij}}{2F_{ij}} \Delta_i^{ij} \right] + \frac{1}{2} \sum_{j \in \mathcal{J} \setminus \{i\}} \sum_{l \in \mathcal{J} \setminus \{i,j\}} \left[ \delta_i^{ij} + \frac{\gamma^{ij} - F_{ij}}{2F_{ij}} \Delta_i^{ij} \right] \quad \text{and} \quad (6)$$

$$T_i = t_i \left[ (K - 1) + \frac{1}{2F} \sum_{j \in \mathcal{J} \setminus \{i\}} (F - \gamma^{ij}) \right]. \quad (7)$$
Moreover, profits of a firm in industry \( ij \) and country \( i \) are given by

\[
\pi_i^{ij}(\mu) = \begin{cases} 
\frac{n_i}{16\beta} \left( \frac{w - 3\mu}{16\beta} \right)^2 + \frac{n_j}{16\beta} \left( \frac{w - 2\mu}{16\beta} \right)^2 + \sum_{i \in \mathcal{X} \setminus \{i, j\}} \frac{n_i}{16\beta} \left( \frac{w - 3\mu}{16\beta} \right)^2 & \text{w/ prob } (1 - G(\gamma^{ij})) \\
\frac{n_i}{16\beta} \left( \frac{w - 2\mu}{16\beta} \right)^2 + \frac{n_j}{16\beta} \left( \frac{w - 3\mu}{16\beta} \right)^2 + \sum_{i \in \mathcal{X} \setminus \{i, j\}} \frac{n_i}{16\beta} \left( \frac{w - 3\mu}{16\beta} \right)^2 & \text{w/ prob } G(\gamma^{ij}) 
\end{cases}
\]

\[
:= \begin{cases} 
\pi_i^{ij}(2) & \text{w/ prob } (1 - G(\gamma^{ij})) \\
\pi_i^{ij}(1) & \text{w/ prob } G(\gamma^{ij}) 
\end{cases}.
\]

To calculate the expected profits, one needs to keep track of the number of firms and how it affects profits. Besides, for every second industry type the mobile firm pays the relative fixed cost, when it decides to locate in country \( i \). To give an example, in the three-country setting, this would happen in \( ki \)-industries but not in \( ij \)-industries. Therefore, the expected profits of firms in \( ij \)-industries and country \( i \) can be written as

\[
\Pi_i^{ij} := G(\gamma^{ij}) \cdot \left( \pi_i^{ij}(1) - t_i \right) + (1 - G(\gamma^{ij})) \cdot \left( \pi_i^{ij}(2) - t_i \right) - \frac{1}{2} G(\gamma^{ij}) \cdot 1 \cdot \mathbb{E}(F^{ij}|F^{ij} \leq \gamma^{ij}) \\
= G(\gamma^{ij}) \left( \pi_i^{ij}(1) - t_i \right) + (1 - G(\gamma^{ij})) \left( 2\pi_i^{ij}(2) - 2t_i \right) - \frac{1}{8F} \left( (\gamma^{ij})^2 - F^2 \right).
\]

Summing over all industries gives expected total profits in country \( i \)

\[
\Pi_i := \sum_{j \in \mathcal{X} \setminus \{i\}} \Pi_i^{ij} = \sum_{j \in \mathcal{X} \setminus \{i\}} \left[ \frac{\gamma^{ij} + F}{2F} \pi_i^{ij}(1) + \frac{F - \gamma^{ij}}{2F} \pi_i^{ij}(2) - \frac{1}{8F} \left( (\gamma^{ij})^2 - F^2 \right) \right] - T_i := \Pi_i - T_i.
\]

The benevolent social planner in country \( i \), now, solves

\[
\max_{t_i} \sum_{i} \Pi_i^{ij} \left( \frac{S_i + T_i}{n_i} + w \right) + \omega m_i \frac{\Pi_i - T_i}{m_i} = \max_{t_i} S_i + (1 - \omega) T_i + n_i w + \omega \Pi_i.
\]

The first-order condition is sufficient for \( \omega < \frac{4}{3} \). The reaction function is again linear in the business taxes of the other countries. Taxes are strategic complements, and the slope is less than 1 for \( \omega < \frac{4K - 6}{3(K - 1)} \), which is always (any \( K \geq 2 \)) fulfilled for \( \omega < \frac{2}{3} \) but, for instance, also holds when \( \omega < 1 \) for \( K = 3 \).

Notice that for \( \omega = 1 \) the equilibrium of the tax competition game is indeterminate. The reason is that the reaction functions intercept for each possible combination of solutions \( \{t_i\}_{i \in \mathcal{X}} \). Hence, in the following, we consider the cases where \( \omega \neq 1 \). By the same techniques as above, we solve for \( \sum_{j \in \mathcal{X} \setminus \{i\}} (t_j - t_i) \) and plug it into the reaction function of country \( i \). This yields a
new equilibrium to the tax competition game

\[ t_i = 3F + \sum_{j \in \mathcal{X} \setminus \{i\}} \left( \frac{(1 - \omega)(K - 1) + \left( 1 - \frac{1}{2} \omega \right)}{(1 - \omega)(K - 1) \left( 1 - \frac{1}{2} \omega \right) K + (1 - \omega)(K - 1)} \right) \sum_{j \in \mathcal{X} \setminus \{i\}} \left( \Delta_j^{ij} - \omega \left( 2\pi_j^{ij}(2) - \pi_i^{ij}(1) \right) \right) + \frac{1 - \frac{1}{2} \omega}{(1 - \omega)(K - 1) \left( 1 - \frac{1}{2} \omega \right) K + (1 - \omega)(K - 1)} \sum_{j \in \mathcal{X} \setminus \{i\}} \sum_{m \in \mathcal{X} \setminus \{j\}} \left( \Delta_j^{jm} - \omega \left( 2\pi_j^{jm}(2) - \pi_j^{jm}(1) \right) \right) \]

for every \( i \in \mathcal{X} \). Observe that for \( \omega = 0 \) one obtains Lemma 3. For \( \omega > 0 \), Equation (11) is just an adjusted version of the solution in Lemma 3. Aside from modified factors, the only difference to before is that the optimal tax also accounts for the accrual of profits at home \( (2\pi_i^{ij}(2) - \pi_i^{ij}(1)) \) and, in the Nash equilibrium, profits accrued abroad \( (2\pi_j^{jm}(2) - \pi_j^{jm}(1)) \). Now, governments have an additional incentive to attract firms because their presence raises national income. As a result, the accrual of profits tends to reduce business taxes. Due to this close similarity of Equation (11) to Lemma 3 our main results carry over. This finding holds, in particular, for low \( \omega \). There may be rare exemptions when the accrual of domestic profits becomes very important. However, from an economic perspective, this case is not particularly relevant as almost all governments in the world pursue a more or less pronounced redistributive goal in their setting of business taxes.

Moreover, one can show that the extra terms in Equation (11) have intuitive comparative
statics:

\[
\frac{d \left( 2\pi_i^{ij}(2) - \pi_i^{ij}(1) \right)}{d\tau_{ij}} = -n_i \frac{4\tau_{ij}}{16\beta} - n_j \frac{2\tau_{ij} + 2(\alpha - w)}{16\beta}, \quad \frac{d \left( 2\pi_i^{ij}(2) - \pi_i^{ij}(1) \right)}{d\tau_{il}} = n_i \frac{4\tau_{il} + 2(\alpha - w - \tau_{il})}{16\beta},
\]

\[
\frac{d \left( 2\pi_i^{ij}(2) - \pi_i^{ij}(1) \right)}{d\tau_{jl}} = -n_j \frac{4\tau_{jl} + 2(\alpha - w - 2\tau_{il})}{16\beta}, \quad \text{and} \quad \frac{d \left( 2\pi_i^{ij}(2) - \pi_i^{ij}(1) \right)}{d\tau_{lm}} = 0
\]

for \( i \neq j \neq l \neq m \). A worsening of the conditions under which mobile firms in \( ij \)-industries can trade in country \( i \) with country \( j \) (\( \tau_{ij} \uparrow \)) lowers the gains from the domestic accrual of profits. As a consequence, the social planner in country \( i \) lowers the business tax by less. The same happens when trade with third countries becomes less costly (\( \tau_{il} \downarrow \)) or when trade costs between country \( j \) and third countries rise (\( \tau_{jl} \uparrow \)). The reason is that domestic competition becomes harsher as
country \( i \) becomes more attractive vis-à-vis country \( j \). The negative effect of a more competitive pricing and lower profit margins overcompensates the positive direct effect of improved trading conditions. Therefore, the accrual of extra profits from having two firms instead of one in the country in a given industry is less important. Trade costs between third countries \( \tau_{lm} \) do not matter.

### 1.2.2 Citizens as Shareholders

Now suppose that, in each country, citizens own a share \( \omega \) of firms worldwide. Then, the social planner solves \( \max_{n_i} \left( \frac{S_i + T_i}{n_i} + w \right) + n_i \frac{\omega \sum_{x \in X} (\Pi_x - T_l)}{n_i} \). The first-order condition is sufficient for \( \omega < 2 \). Then, the equilibrium of the tax competition game exists and is unique. Its solution is given by

\[
t_i = 3 (1 - \omega) \bar{F} + \frac{1}{(2 - \omega) K - 1} \sum_{j \in X \setminus \{i\}} \left( \Delta_{ij}^l - \omega \left[ 2\pi_i^j (2) - \pi_i^j (1) - \left( 2\pi_i^j (2) - \pi_i^j (1) \right) \right] \right) + \frac{1}{(2 - \omega) K - 1} \sum_{j \in X \setminus \{i\}} \left( \pi_i^j - \pi_j^i \right) + \frac{1 - \omega}{(K - 1) (2 - \omega) K - 1} \sum_{l \in X} \sum_{m \in X \setminus \{l\}} \left( \Delta_{lm}^l - \omega \left[ 2\pi_i^l (2) - \pi_i^l (1) - \left( 2\pi_m^l (2) - \pi_m^l (1) \right) \right] \right). \tag{13}
\]

Again, for \( \omega = 0 \) one gets Lemma 3. Aside from modified factors, additional terms enter the optimal tax function for \( \omega > 0 \). Our main results remain valid. In contrast to above, where the extra terms measure the accrual of profits by domestic and foreign entrepreneurs, now the extra terms downward adjust the optimal tax by the accrual of profit differentials at home \( (2\pi_i^j (2) - \pi_i^j (1) - \left( 2\pi_i^j (2) - \pi_i^j (1) \right)) \) and abroad \( (2\pi_i^l (2) - \pi_i^l (1) - \left( 2\pi_m^l (2) - \pi_m^l (1) \right)) \). The reason is that profits enter social welfare no matter where they realize as profits accrue to the citizens who are the shareholders of the firms worldwide. A shareholder in a given country, therefore, only cares about how much firms earn in one country versus another.

Furthermore, comparative statics of these extra terms are very similar to above

\[
\frac{d \left( 2\pi_i^j (2) - \pi_i^j (1) - \left( 2\pi_i^j (2) - \pi_i^j (1) \right) \right)}{d \tau_{ij}} = (n_i - n_j) \frac{2 (\alpha - w - \tau_{ij})}{16 \beta},
\]

\[
\frac{d \left( 2\pi_i^l (2) - \pi_i^l (1) - \left( 2\pi_i^l (2) - \pi_i^l (1) \right) \right)}{d \tau_{il}} = n_i \frac{2\tau_{il} + 4 (\alpha - w)}{16 \beta},
\]
\[
\frac{d \left( 2\pi_i^{ij} (2) - \pi_i^{ij} (1) - \left( 2\pi_j^{ij} (2) - \pi_j^{ij} (1) \right) \right)}{d\tau_{ij}} = -n_i \frac{2\tau_{ij} + 4(\alpha - w)}{16\beta}.
\]

and
\[
\frac{d \left( 2\pi_i^{ij} (2) - \pi_i^{ij} (1) - \left( 2\pi_j^{ij} (2) - \pi_j^{ij} (1) \right) \right)}{d\tau_{jm}} = 0
\]

with the only exception that the one with respect to \( \tau_{ij} \) now depends on the relative size of countries. The above-described intuitions carry over.

### 1.3 Arbitrary Number of Firms

We now relax the assumption that, in each industry, there are only three producing firms of which two are immobile. To be precise, in an \( ij \)-industry let \( k_i^{ij} \in \mathbb{R}_+ \) be the number of firms in country \( i \). Hence, \( k_i^{ij} + k_j^{ij} + 1 \) is the total number of firms producing in a given industry, of which only one continues to be mobile. Assume, for simplicity, that \( k_i^{ij} \) is the same for all industry types. Furthermore, one has to modify the upper bound of trade costs \( \tau_{ij} \leq \frac{\alpha - w}{k_i^{ij} + 1} \). Note that the new number of firms country \( i \) is given by

\[
i = \sum_{j \in \mathcal{X} \setminus \{i\}} k_i^{ij} + \frac{1}{2 \mathcal{F}} \sum_{j \in \mathcal{X} \setminus \{i\}} (\pi_i^{ij} - \pi_j^{ij}) + \sum_{j \in \mathcal{X} \setminus \{i\}} \tau_j. \tag{15}
\]

By the same techniques as above, one can derive the equilibrium of the tax competition game

\[
t_i = 3\mathcal{F} + 2\mathcal{F} \frac{K \sum_{j \in \mathcal{X} \setminus \{i\}} k_i^{ij} + \sum_{j \in \mathcal{X} \setminus \{i\}} \sum_{m \in \mathcal{X} \setminus \{j\}} k_j^{jm} - (K - 1)(2K - 1)}{(K - 1)(2K - 1)}
+ \frac{1}{2K - 1} \sum_{j \in \mathcal{X} \setminus \{i\}} \Delta_i^{ij} \left( \pi_i^{ij} - \pi_j^{ij} \right) + \frac{1}{(K - 1)(2K - 1)} \sum_{j \in \mathcal{X}} \sum_{m \in \mathcal{X} \setminus \{j\}} \Delta_j^{jm}. \tag{16}
\]

Relative to Lemma 3, the new optimal business tax is modified by the second term on the right-hand side. Notice, moreover, that the other terms implicitly depend on \( k_i^{ij} \) and \( k_j^{ij} \) since

\[
\Delta_i^{ij} = n_i \left( -\frac{\left( \alpha \left( k_i^{ij} + 1 \right) - w \left( k_i^{ij} + 1 \right) - \left( k_i^{ij} + 1 \right) \tau_{ij} \right)^2 - \left( \alpha \left( k_i^{ij} + 1 \right) - w \left( k_i^{ij} + 1 \right) - k_j^{ij} \tau_{ij} \right)^2}{2\beta \left( k_i^{ij} + 2 \right)^2} \right).
\]
\[
\pi_i^j - \pi_i^j = (n_i - n_j) \frac{2(\alpha - w) - \tau_{ij}}{\beta (k^{ij} + 2)^2} (k^{ij} + 1) \tau_{ij} + \left( k_i^{ij} - k_i^{ij} \right) (n_i + n_j) \frac{\tau_{ij}^2}{\beta (k^{ij} + 2)^2} (k^{ij} + 1)
+ \sum_{l \in \mathcal{X} \setminus \{i, j\}} n_l \left( \tau_{jl} - \tau_{il} \right) \frac{2(\alpha - w) - (\tau_{ij} + \tau_{il}) - \left(k_i^{ij} - k_i^{ij}\right) (\tau_{ij} - \tau_{il})}{\beta (k^{ij} + 2)^2} (k^{ij} + 1),
\]

(17)

and

\[
\Delta_j^j = n_j \frac{(\alpha (k^{ij} + 1) - w (k^{ij} + 1) - \left(k_i^{ij} + 1\right) \tau_{ij})^2 - (\alpha (k^{ij} + 1) - w (k^{ij} + 1) - k_i^{ij} \tau_{ij})^2}{2\beta (k^{ij} + 2)^2}.
\]

Therefore, the comparative statics of Lemma 4 are slightly modified

\[
\frac{dt_l}{d\tau_{ij}} = \frac{n_i (K - 2) - n_j \left[ 2(K - 1)^2 + 1 \right]}{(K - 1)(2K - 1)} \frac{(\alpha - w - \tau_{ij})(k^{ij} + 1)}{\beta (k^{ij} + 2)^2}
+ \frac{\sum_{m \in \mathcal{X} \setminus \{i, j\}} \frac{\tau_{ij} - \tau_{mj}}{\tau_{ij}} - 1}{(K - 1)(2K - 1)} n_j \frac{\tau_{ij} \left(k_i^{ij} - k_i^{ij}\right)}{\beta (k^{ij} + 2)^2},
\]

(18)

and

\[
\frac{dt_l}{d\tau_{jk}} = \frac{(2K - 3)(n_j + n_k)}{(K - 1)(2K - 1)} \frac{(\alpha - w - \tau_{jk})(k^{ij} + 1)}{\beta (k^{ij} + 2)^2} + \frac{\tau_{jk} \left(k_i^{jk} - k_i^{jk}\right)(n_j - n_k)}{(K - 1)(2K - 1)} \frac{\beta (k^{ij} + 2)^2}{\beta (k^{ij} + 2)^2}
+ \frac{2n_k (K - 1) \left(k_j^{ij} - k_j^{ij}\right) (\tau_{jk} - \tau_{kj}) + 2n_j (K - 1) \left(k_i^{jk} - k_i^{jk}\right) (\tau_{jk} - \tau_{kj})}{(K - 1)(2K - 1)} (k^{ij} + 1).
\]

(19)

Observe that for \(k_i^{ij} = k_i^{ij} = k_i^{jk}\) and \(k^{ij} = 2\) one obtains the expressions in Lemma 4. Moreover, for a similar number of immobile firms across countries, the main results hold.

One should, however, note that there is an interaction between the number of immobile firms and the above mentioned comparative statics. For instance, \(\frac{dt_l}{d\tau_{ij}}\) tends to decrease (increase) in \(k_i^{ij}\) (\(k^{ij}\)). The more immobile firms produce in country \(i\) and the higher the costs of trade, the less can the mobile firms gain from moving there. In other words, the mobile firms are more and more willing to move somewhere else as both \(\tau_{ij}\) and \(k^{ij}\) increase. Therefore, a rise in \(k_i^{ij}\) puts additional pressure on the government of country \(i\) to lower the business tax when it loses attractiveness as a business location due to a rise in \(\tau_{ij}\). A reverse argument holds for \(k_i^{ij}\).
Furthermore, notice that

\[
\frac{dt_i}{dk_{ij}^i} = \frac{2F}{(K-1)(2K-1)} \left\{ \frac{[(K-1)(k_{ij}^i+1)n_i + ((K-1)(k_{ij}^i+1)-2)n_j] \tau_{ij}^2}{(K-1)(2K-1)\beta(k_{ij}^i+2)^2} \right. \\
+ \frac{(K-1)(k_{ij}^i+1)\sum_{l \in \mathcal{X} \setminus \{i,j\}} n_l (\tau_{jl} - \tau_{il})^2}{(K-1)(2K-1)\beta(k_{ij}^i+2)^2} \right\} < 0,
\]

(20)

\[
\frac{dt_i}{dk_{jk}^i} = \frac{2F}{(K-1)(2K-1)} \left\{ \frac{\left[(K-1)(k_{ij}^i+1)+K\right)n_i + (K-1)(k_{ij}^i+1)n_j \right\} \tau_{ij}^2}{(K-1)(2K-1)\beta(k_{ij}^i+2)^2} \\
+ \frac{(K-1)(k_{ij}^i+1)\sum_{l \in \mathcal{X} \setminus \{i,j\}} n_l (\tau_{jl} - \tau_{il})^2}{(K-1)(2K-1)\beta(k_{ij}^i+2)^2} > 0,
\]

and

\[
\frac{dt_i}{dk_{jk}^i} = \frac{2F}{(K-1)(2K-1)} \left\{ \frac{1}{(K-1)(2K-1)} + \frac{n_j \tau_{jk}^2}{(K-1)(2K-1)\beta(k_{ij}^i+2)^2} \right\} > 0
\]

(21)

for \(i \neq j \neq k\). On the one hand, similar to above, a rise in \(k_{ij}^i\) tends to make the domestic market in country \(i\) more competitive. As a consequence, country \(i\)’s government competes harsher for mobile firms (lower tax). On the other hand, more immobile firms in country \(i\) mechanically raise the government’s ability to tax. Altogether, the effect of \(k_{ij}^i\) on the domestic business tax, \(t_i\), is ambiguous. Vice versa, as the degree of local competition increases abroad \((k_{ij}^j \uparrow \text{and} k_{jk}^i \uparrow\text{)}, market \(i\) becomes relatively more attractive, which improves country \(i\)’s ability to tax. Also, more immobile firms abroad mechanically raise taxes there, which positively feeds back into country \(i\)’s tax.

Let us now study the effects of firm exit and entry as a reaction to the disintegration of a country from an economic union formed by a set of countries \(\mathcal{X}_{EU}\). Suppose that, as a reaction to this economic disintegration, firms exit from the leaving market and enter the economic union holding fixed the number of firms per industry. The effect on the business tax of the leaving country and on the member countries, which experience firm entry, is ambiguous by the opposing forces described above. That is, the entry (exit) of firms in a country raises (reduces) the degree of local competition and makes that country less (more) attractive for mobile firms, while it mechanically increases (decreases) the government’s ability to tax corporations. Nonetheless, one should bear in mind that this reasoning is in the absence of employment and growth effects attached to firm relocation.
What is the effect on business taxes of third countries outside the union, \( k \in \mathcal{K} \setminus (\mathcal{K}_{EU} \cup l) \)? As we can see, the answer depends on the size of the leaving country relative to the average country inside the union:

\[
\sum_{m \in \mathcal{K}_{EU}} \left( \frac{dt_k}{dk_{lm}} - \frac{dt_k}{dk_{l}} \right) = \frac{K_{EU} (n_l - \overline{n}_{EU}) \tau^2}{(K - 1) (2K - 1) \beta (k^{ij} + 2)^2} \begin{cases} 
> 0 & \text{for } \overline{n}_{EU} < n_l \\
< 0 & \text{for } \overline{n}_{EU} > n_l.
\end{cases}
\]

The exit of firms in the leaving country and the entry into member countries, respectively, have no direct effect on the business taxes of third countries outside the union. Also, the mechanical effects of the exit and entry of firms cancel out. However, in the Nash equilibrium, the business taxes of third countries depend on the consumer surplus in the leaving country and the remaining union members. The exit of firms in the leaving country makes domestic prices in the member countries more elastic to firm relocation towards member countries. In other words, the gains in consumer surplus, which member countries realize from attracting firms by lowering taxes, rise. The size of this effect is proportional to \( \overline{n}_{EU} \). Vice versa, more firms inside the union make prices in the leaving country less elastic to firm relocation towards that country. Altogether, when a relatively large country leaves an economic union and firms exit (enter) the leaving country (member countries), third countries tend to tax more.

### 1.4 Cross-Price Effects

In the following, we deal with cross-price effects. That is, we specify preferences of the representative household in country \( i \) as in Melitz and Ottaviano (2008)

\[
u_i := z_i + \alpha \int_{\mu \in \Omega} x_i (\mu) d\mu - \frac{\beta}{2} \int_{\mu \in \Omega} x_i (\mu)^2 d\mu - \frac{\eta}{2} \left( \int_{\mu \in \Omega} x_i (\mu) d\mu \right)^2
\]

for \( \eta > 0 \). The parameters \( \alpha \) and \( \eta \) measure the substitutability between the numéraire and the differentiated varieties, whereas the parameter \( \beta \) determines the degree of product differentiation of varieties. A rise in \( \eta \) shifts down the demand for the differentiated varieties compared to the numéraire. Since we are interested in the effects of firm selection in the differentiated industries, let \( \beta > \eta \) such that consumers are sufficiently interested in consuming sufficiently differentiated varieties.

The aggregate demand functions are still linear in the industry price, but the vertical intercepts
are endogenously shifted

\[ X_i(\mu) = \frac{n_i(\alpha_i - p_i(\mu))}{\beta} \]  

(24)

where \( \alpha_i := \frac{\alpha \beta + \eta^*_i}{\beta + \eta} \) and \( \bar{p}_i := \int_{\mu \in \Omega} p_i(\mu) d\mu \). As before, optimal production quantities by firms lead to country- and industry-specific prices

\[
p^{ij}_i(\mu) = \frac{\alpha_i + 3w + k^*_j \tau_{ij}}{4} = \begin{cases} 
\frac{\alpha_i + 3w + 4\tau_{ij}}{4} & \text{if } F^{ij} \geq \gamma^{ij} \\
\frac{\alpha_i + 3w + 2\tau_{ij}}{4} & \text{if } F^{ij} < \gamma^{ij},
\end{cases}
\]

(25)

and

\[
p^{ik}_i(\mu) = \frac{\alpha_i + 3w + k^*_k \tau_{ik}}{4} = \begin{cases} 
\frac{\alpha_i + 3w + 2\tau_{ik} + \tau_{jk}}{4} & \text{if } F^{ik} \geq \gamma^{ik} \\
\frac{\alpha_i + 3w + \tau_{ik} + 2\tau_{jk}}{4} & \text{if } F^{ik} < \gamma^{ik}
\end{cases}
\]

for any \( j, k \in \mathcal{K} \setminus \{i\} \). Again, prices depend on firms’ relocation choices. Pre-tax variable profits of a firm in country \( i \) are given by

\[
\pi^{ij}_i(\mu) = \begin{cases} 
\frac{n_i(\alpha_i - w + \tau_{ij})^2}{16\beta} + \frac{n_j(\alpha_j - w + 2\tau_{ij})^2}{16\beta} + \frac{n_k(\alpha_k - w - 2\tau_{ij} + \tau_{jk})^2}{16\beta} & \text{if mobile firm locates in } i \\
\frac{n_i(\alpha_i - w + \tau_{ij})^2}{16\beta} + \frac{n_j(\alpha_j - w - 3\tau_{ij})^2}{16\beta} + \frac{n_k(\alpha_k - w - 3\tau_{ij} + 2\tau_{jk})^2}{16\beta} & \text{if mobile firm locates in } j.
\end{cases}
\]

(26)

Interestingly, the cutoff industries \( \gamma^{ij}, \gamma^{jk}, \) and \( \gamma^{ki} \) remain unchanged. Thus, cross-price effects do not directly affect firm mobility in our model.

Accordingly, tax revenues in country \( i \) still read as \( T_i := t_i \left( 3 - G \left( \gamma^{ij} \right) + G \left( \gamma^{ki} \right) \right) \). However, consumer surplus accounts for cross-price effects \( S_i = \delta^{ij}_i + G \left( \gamma^{ij} \right) \Delta^{ij}_i + \delta^{jk}_i + G \left( \gamma^{jk} \right) \Delta^{jk}_i + \delta^{ki}_i G \left( \gamma^{ki} \right) \Delta^{ki}_i \), where \( \delta^{ij}_i := n_i \left( \frac{(3\alpha_i - 3w - \tau_{ij})^2}{32\beta} \right), \delta^{jk}_i := n_i \left( \frac{(3\alpha_i - 3w - 2\tau_{ij} - \tau_{jk})^2}{32\beta} \right), \delta^{ki}_i := n_i \left( \frac{(3\alpha_i - 3w - 2\tau_{ij} + \tau_{jk})^2}{32\beta} \right) \),

\[
\Delta^{ij}_i := n_i \left( \frac{(3\alpha_i - 3w - \tau_{ij})^2}{32\beta} \right) - \frac{(3\alpha_i - 3w - \tau_{ij})^2}{32\beta}, \quad \Delta^{jk}_i := n_i \left( \frac{(3\alpha_i - 3w - 2\tau_{ij} - \tau_{jk})^2}{32\beta} \right) - \frac{(3\alpha_i - 3w - 2\tau_{ij} - \tau_{jk})^2}{32\beta},
\]

and \( \Delta^{ki}_i := n_i \left( \frac{(3\alpha_i - 3w - \tau_{ij})^2}{32\beta} \right) - \frac{(3\alpha_i - 3w - 2\tau_{ij} + \tau_{jk})^2}{32\beta} \) through the dependence of \( \alpha_i := \frac{\alpha \beta + \eta^*_i}{\beta + \eta} \) on the country’s average price level

\[ \alpha_i = \frac{4\alpha \beta}{4\beta + \eta} + \frac{\eta}{4\beta + \eta} \left( 9w + 3\tau_{ij} + 3\tau_{ik} + G \left( \gamma^{ij} \right) \tau_{ij} + G \left( \gamma^{jk} \right) \left( \tau_{ik} - \tau_{ij} \right) - G \left( \gamma^{ki} \right) \tau_{ik} \right). \]

(27)
The first-order condition with respect to the business tax

\[
\frac{d (S_i + T_i)}{dt_i} = \frac{1}{F - E} \left( \Delta^{ij}_{ij} - \Delta^{ki}_{ki} \right) + 3 \left( G(\gamma^{ij}) + G(\gamma^k) \right) - 2t_i \frac{1}{F - E} + \frac{3n_i}{16} \frac{\eta}{4 \beta + \eta} \frac{1}{F - E} \cdot (\tau_{ij} + \tau_{ki}) \left( 3 (\alpha_j - w) - \tau_{ij} - \tau_{ik} \right) - G(\gamma^{ij}) \tau_{ij} + G(\gamma^k) \tau_{ik} = 0 \quad (28)
\]

is sufficient by the second-order condition \( \frac{d^2 (S_i + T_i)}{dt_i^2} = -\frac{4}{F - E} - \frac{3n_i}{16} \beta \left( \frac{\tau_{ij} + \tau_{ki}}{F - E} \right)^2 \frac{\eta (8 \beta - 7 \eta)}{(4 \beta + \eta)^2} < 0 \). The reaction function \( t_i \) is linear in \( t_j \) and \( t_k \). One can easily find conditions under which business taxes are strategic complements, and the slope of the reaction functions is less than 1, such that the Nash equilibrium is unique. With cross-price effects, the optimal taxes are revised upward relative to \( \eta = 0 \). The higher \( \eta \), the smaller the demand for differentiated varieties compared to the numéraire, and the smaller the welfare loss from firm emigration: \( \frac{dS_i}{d\tau_i} \frac{d\tau_i}{dt_i} > 0 \). Thus, there is a welfare gain from taxing firms so that optimal taxes increase in \( \eta \).

Compared to Lemma 1, one needs to add to the comparative statics (e.g., \( \frac{d\tau_i}{d\tau_{ij}} = -\left( \frac{d^2 (S_i + T_i)}{dt_i d\tau_{ij}} \right) \left( \frac{d^2 (S_i + T_i)}{dt_i^2} \right) \)) two marginal effects, which account for the endogeneity of \( \alpha_j \) in the average price level. Observe that the average price level and, hence, \( \alpha_j \) tends to rise in trade costs. Under considerable asymmetries in the market sizes, the Metzler paradox may occur (\( \frac{d\eta}{d\tau_{ij}} < 0 \)). However, for sufficiently similar market sizes, this will not be the case. In the following, suppose that market sizes and trade costs are similar. The first adjustment regards the mentioned welfare gain (reduction in the welfare loss from firm emigration). This welfare gain rises in trade costs. The second one accounts for the endogeneity of the consumer surplus loss from taxing businesses \( \frac{d(\Delta^{ij}_{ij} - \Delta^{ki}_{ki})}{d\tau_i} \neq 0 \). A rise in trade costs increases this welfare loss. Hence, both adjustments work in opposite directions. Nonetheless, the key trade-offs and the insights from the model without cross-price effects remain unchanged.

### 1.5 Competition on Regulations

Now we endogenize the country-specific level of regulations, \( \nu^i \). In the first stage of our economy \( E \), a country \( i \) chooses not only the optimal business tax policy but also the optimal level of regulations taking all other countries’ business taxes and regulations as given. Observe that this features a situation where countries compete non-cooperatively over the setting of business
regulations. For positive taxes, country $i$’s welfare declines in $v^j$

$$
\frac{d (S_i + T_i)}{d v^i} = \frac{1}{F - E} \left( \Delta_{ij} - \Delta_{i}^{ki} - 2t_i \right) < 0,
$$

(29)

because $\Delta_{ij} < 0$.

Two negative effects on welfare add up. Firstly, a rise in $v^j$ lowers consumer surplus because it triggers firm emigration out of country $i$. This leads to a rise in the country’s price level and reduces aggregate welfare. Secondly, as firms move away from country $i$, tax revenues in that country decline. To obtain interior solutions, let $V_i$ measure the regulation surplus generated from $v^j$ in country $i \in \mathcal{K}$ in reduced form, where $\frac{dV_i}{d v^j} > 0$ and $\frac{d^2 V_i}{(d v^j)^2} < -\frac{6}{5(F - E)}$ for all $v^j$. In the context of environmental protection, a rise in environmental standards may lower air pollution in cities or reduce the risk of natural disasters. $V_i$ captures the resulting aggregate regulation surplus in country $i$. In principle, this surplus may be a function of the other countries’ regulations as well. That is, $V_i \left( v^i, \{v^j\}_{j \in \mathcal{K}} \right)$ captures cross-country complementarities in regulations. For simplicity, let us abstract from such complementarities. Even in the absence of cross-country complementarities, a country’s optimal level of regulations will be inefficiently low. The reason is that, similar to the tax competition game, the government in country $i$ does not take into account the positive externality of firm emigration on other countries’ welfare $\frac{d (S_i + T_i)}{d v^i} > 0$. This leads to an underprovision of regulations (e.g., environmental protection), and countries would gain from coordinating business regulations.

The first-order condition with respect to $v^j$

$$
\frac{d (S_i + T_i + V_i)}{d v^i} = \frac{1}{F - E} \left( \Delta_{ij} - \Delta_{i}^{ki} - 2t_i \right) + \frac{dV_i}{d v^j} = 0
$$

(30)

determines the optimal level of regulations in country $i$. By Proposition 3, $\frac{dt_i}{d v^j} = 1$ such that regulations are strategic substitutes:

$$
\frac{\partial v^j}{\partial v^j} = -\frac{2}{F - E} \frac{dt_i}{d v^j} = \frac{2}{4 + 5 \left( \frac{d^2 V_i}{(d v^j)^2} \right)} < 0.
$$

(31)

Since $\frac{d^2 V_i}{(d v^j)^2} < -\frac{6}{5(F - E)}$, the slope of the reaction functions is greater than $-1$, such that the Nash equilibrium is unique.

The first-order condition reveals that other domestic policies (here: business regulations)
interact with the optimal business tax policy. Perhaps surprisingly, for positive business taxes, the comparative statics of regulations and business taxes (Lemma 1) may point in opposite directions. For instance,

\[
\text{sign} \left( \frac{\partial \nu^l}{\partial \tau_{jk}} \right) = -\text{sign} \left( \frac{\partial t_i}{\partial \tau_{jk}} \right).
\] (32)

The intuition is that a rise in a country’s business tax (e.g., by an increase in \( \tau_{jk} \)) magnifies the size of lost tax revenues and, thus, the welfare costs of \( \nu^l \). In the optimum, this reduces a country’s level of business regulations.

### 1.6 Harmonization of Business Taxes

In this section, we deal with the effects of economic disintegration on harmonized taxes. We look at the scenario of partial harmonization (e.g., Conconi, Perroni, and Riezman (2008)) in the \( K \)-country economy described above. That is, a non-empty subset of countries, \( \mathcal{H} \), (e.g., the EU) coordinates their level of business taxation to maximize joint welfare \( \max_{\{t_m\}_{m \in \mathcal{H}}} \sum_{m \in \mathcal{H}} (S_m + T_m + n_m w) \) subject to \( t_H := t_m = t_n \ \forall m, n \in \mathcal{H} \). Under this set of constraints, the consumer surplus in the harmonized area reads as

\[
\sum_{m \in \mathcal{H}} S_m = \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \setminus \{m\}} \left[ \delta_{m}^{mj} + \frac{\gamma_{mj}^{m} - F}{2F} \Delta_{m}^{mj} \right] + \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \cup \{m\}} \left[ \delta_{m}^{mj} + \frac{\gamma_{mj}^{m} - F}{2F} \Delta_{m}^{mj} \right] + \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \setminus \{m\}} \left[ \delta_{m}^{mj} + \frac{\gamma_{mj}^{m} - F}{2F} \Delta_{m}^{mj} \right]
\]

\[
\quad + \frac{1}{2} \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \setminus \{m\}} \sum_{l \in \mathcal{H} \setminus \{m, j\}} \left[ \delta_{m}^{jl} + \frac{\gamma_{jl}^{m} - F}{2F} \Delta_{m}^{jl} \right]
\]

\[
\quad + \frac{1}{2} \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \setminus \{m\}} \sum_{l \in \mathcal{H} \setminus \{m, j\}} \sum_{\mathcal{H} \setminus \{m\}} \left[ \delta_{m}^{jl} + \frac{\gamma_{jl}^{m} - F}{2F} \Delta_{m}^{jl} \right]
\]

\[
\quad + \frac{1}{2} \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \setminus \{m\}} \sum_{l \in \mathcal{H} \setminus \{m, j\}} \sum_{\mathcal{H} \setminus \{m\}} \left[ \delta_{m}^{jl} + \frac{\gamma_{jl}^{m} - F}{2F} \Delta_{m}^{jl} \right]
\]

\[
\quad + \frac{1}{2} \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \setminus \{m\}} \sum_{l \in \mathcal{H} \setminus \{m, j\}} \sum_{\mathcal{H} \setminus \{m\}} \left[ \delta_{m}^{jl} + \frac{\gamma_{jl}^{m} - F}{2F} \Delta_{m}^{jl} \right] \quad (33)
\]
where \( \tilde{\gamma}^{mj} := \gamma^{mj} - t_m + t_j = \pi_j^{mj} - \pi_m^{mj} \) is independent from business taxes. Similarly, one can decompose tax revenues as follows

\[
\sum_{m \in \mathcal{H}} T_m = t_H \sum_{m \in \mathcal{H}} [(K - 1) + \frac{1}{2F} \sum_{j \in \mathcal{H} \setminus \{m\}} (F - \tilde{\gamma}^{mj})] + t_H \sum_{m \in \mathcal{H}} \left[(K - 1) + \frac{1}{2F} \sum_{j \in \mathcal{H} \cup \{m\}} (F - \gamma^{mj})\right].
\]

The first-order condition is given by

\[
\frac{d}{dt_H} \sum_{m \in \mathcal{H}} (S_m + T_m + n_m w) = \frac{1}{2F} \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \setminus \{m\}} \Delta^{mj} + \frac{1}{2F} \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \setminus \{m\}} \sum_{l \in \mathcal{H} \setminus \{m\}} \Delta^{lj} m
\]

\[
+ \sum_{m \in \mathcal{H}} (K - 1) + \frac{1}{2F} \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \setminus \{m\}} (F - \gamma^{mj}) - t_H \frac{1}{2F} K_H (K - K_H) = 0
\]

which is sufficient by the second-order condition

\[
\frac{d^2}{dt_H^2} \sum_{m \in \mathcal{H}} (S_m + T_m + n_m w) = \frac{1}{2F} \sum_{m \in \mathcal{H}} \sum_{j \in \mathcal{H} \setminus \{m\}} \left( -\frac{d\gamma^{mj}}{dt_m} \right) - \frac{1}{2F} K_H (K - K_H) = -\frac{K_H (K - K_H)}{F} < 0.
\]

The reaction function in the harmonized area can be written as

\[
t_H = \frac{1}{2(K - K_H)} \left( \sum_{j \in \mathcal{H} \setminus \mathcal{H} \setminus \{m\}} \tilde{\Delta}^{Hj} + \sum_{j \in \mathcal{H} \setminus \mathcal{H} \setminus \{m\}} \tilde{\Delta}^{H'j} + 3F (K - 1) - \sum_{j \in \mathcal{H} \setminus \{m\}} \tilde{\gamma}^{Hj} + \sum_{j \in \mathcal{H} \setminus \mathcal{H} \setminus \{m\}} t_j \right)
\]

where we define \( \tilde{\Delta}^{Hj} := \frac{1}{K_H} \sum_{m \in \mathcal{H}} \Delta^{mj} \), \( \tilde{\Delta}^{H'j} := \frac{1}{K_H} \sum_{m \in \mathcal{H} \setminus \{m\}} \sum_{l \in \mathcal{H} \setminus \{m\}} \Delta^{lj} \), and \( \tilde{\gamma}^{Hj} := \frac{1}{K_H} \sum_{m \in \mathcal{H}} \tilde{\gamma}^{mj} \).

In the other regions, governments choose their business taxes non-cooperatively as before

\[
\max_{t_i} S_i + T_i + n_i w \quad \text{yielding the reaction function}
\]

\[
t_i = \frac{1}{2(K - 1)} \left( \sum_{j \in \mathcal{H} \setminus \{i\}} \Delta^{ij} + 3F (K - 1) - \sum_{j \in \mathcal{H} \setminus \{i\}} \tilde{\gamma}^{ij} + \sum_{j \in \mathcal{H} \setminus \mathcal{H} \cup \{i\}} t_j + K_H t_H \right)
\]

for any \( i \notin \mathcal{H} \).

Business taxes are, as before, strategic complements, the relation is linear, and the slope is less than 1. Thus, there exists a unique interior intersection of reaction functions forming the
Nash equilibrium in this tax competition game.

The formula for the non-cooperative tax \( t_i \) in country \( i \notin \mathcal{H} \) is unaltered relative to the case without tax harmonization. The only difference is that \( K_H t_H \) replaces \( \sum_{j \in \mathcal{H}} t_j \). The reaction function in the harmonized area, \( t_H \), accounts for average effects on consumer surplus (\( \tilde{\Delta}^H \) and \( \tilde{\Delta}'^H \)) and tax revenues (\( \tilde{\gamma}^H \)) vis-à-vis other countries \( j \). Another remarkable feature is the prefactor \( \frac{1}{K-K_H} \) that is increasing in the number of countries in the harmonized area, \( K_H \). It accounts for the gain in tax revenues a country realizes from participating in the coordination of business taxes.

In the following, we derive the symmetric Nash equilibrium. Suppose that \( \tau_{ij} = \tau_{kl} \) for all \( i \neq j \) and \( k \neq l \) and let \( n_i = n_j \) for all \( i, j \). Then, \( \tilde{\gamma}^{ij} = 0 \forall i, j \), \( \tilde{\gamma}^{Hj} = 0 \forall j \), and \( \tilde{\Delta}^{ij}_l := \Delta < 0 \forall i, j \). The Nash equilibrium business taxes are given by

\[
t_H = \frac{(K-1) (2K-1)}{(K-K_H) (2K-2+K_H)} + \Delta \quad \text{and} \quad t_i = t_H - 3\tilde{\gamma} \frac{(K-1) (K_H-1)}{(K-K_H) (2K-2+K_H)}
\]

for \( i \notin \mathcal{H} \). Taxes inside the harmonized area are higher than outside (\( t_H > t_i \)). Similar to Proposition 6, one can derive the comparative statics of business taxes with respect to \( K_H \). Both \( t_H \) and \( t_i \) increase in the number of members in the harmonized area (\( \frac{dt_H}{dK} > 0 \) and \( \frac{dt_i}{dK} > 0 \). In other words, when a country disintegrates from the harmonized area, business taxes decline everywhere. The reason is that tax harmonization leads to a reduction in the degree of tax competition worldwide. As a country leaves the harmonized area, there is effectively one more player in the tax competition game leading to harsher competition and lower taxes.

### 1.7 Industry-Specific Trade Costs

In this section, we allow for heterogeneity in firms’ trade costs. To be precise, we let trade costs vary by industry types. Trade between the countries \( m \) and \( n \) costs a firm in an \( ij \)-industry \( \tau_{mn}^{ij} = \tau_{mn} + \tilde{\tau}_{mn}^{ij} \), where \( \tau_{mn} \) measures the country-pair specific level of trade costs, and \( \tilde{\tau}_{mn}^{ij} \) is an idiosyncratic component that may vary across industry types. Then, a firm’s profits in country \( i \) and industry \( ij \) read as

\[
\pi_i^{ij} (\mu) = \begin{cases} 
\frac{n_i (a-w+3\tilde{\gamma}^{ij})^2}{16\beta} + \frac{n_j (a-w+2\tilde{\gamma}^{ij})^2}{16\beta} + \frac{n_k (a-w-2\tilde{\gamma}^{ij}+\tilde{\tau}^{ij})^2}{16\beta} & \text{if mobile firm locates in } i \\
\frac{n_i (a-w+3\tilde{\gamma}^{ij})^2}{16\beta} + \frac{n_j (a-w+2\tilde{\gamma}^{ij})^2}{16\beta} + \frac{n_k (a-w-2\tilde{\gamma}^{ij}+\tilde{\tau}^{ij})^2}{16\beta} & \text{if mobile firm locates in } j.
\end{cases}
\]
Accordingly, industry thresholds are adjusted as follows

$$\gamma^{ij} = (n_j - n_i) \frac{6\bar{\pi}^{ij} (\alpha - w) - 3 \left(\bar{z}^{ij}_l \right)^2}{16\beta} + \sum_{i \in \mathcal{I} \setminus \{i, j\}} n_i \left(\bar{z}^{ij}_l - \bar{z}^{ij}_l \right) \frac{6 (\alpha - w) - 3 \left(\bar{z}^{ij}_l + \bar{z}^{ij}_l \right)}{16\beta} + t_i - t_j, \quad (40)$$

leading to the same tax revenue function $T_i = t_i \left(\frac{1}{K} + \frac{1}{2F} \sum_{j \in \mathcal{I} \setminus \{i\}} (F - \gamma^{ij})\right)$. Observe that introducing firm-specific trade costs would make firm heterogeneity in a given industry type two-dimensional. Then, $\gamma^{ij}$ may not be uniquely defined. Therefore, we focus on the setting described here.

Consumer surplus also remains qualitatively unchanged

$$S_i = \sum_{j \in \mathcal{I} \setminus \{i\}} \left[\delta^{ij}_l + \frac{\gamma^{ij} - F}{2F} \Delta^{ij}_l\right] + \frac{1}{2} \sum_{j \in \mathcal{I} \setminus \{i\}} \sum_{l \in \mathcal{I} \setminus \{i, j\}} \left[\delta^{il}_l + \frac{\gamma^{il} - F}{2F} \Delta^{il}_l\right] \quad (41)$$

with adjusted terms $\delta^{ij}_l := n_i \frac{(3a - 3w - \bar{r}_l^{ij})^2}{32\beta}, \delta^{il}_l := n_i \frac{(3a - 3w - 2\bar{r}_l^{ij} - \bar{r}_l^{il})^2}{32\beta}, \Delta^{ij}_l := n_i \frac{3a - 3w - 2\bar{r}_l^{ij} - (3a - 3w - \bar{r}_l^{ij})^2}{32\beta},$

and $\Delta^{il}_l := n_i \frac{(3a - 3w - \bar{r}_l^{il} - 2\bar{r}_l^{il})^2}{32\beta} - \Delta^{il}_l^\prime.$

The remainder of the analysis is identical to Section 2.3 of the main paper. Relative to the comparative statics of Nash equilibrium business taxes with respect to country-pair specific trade costs $\tau_{mn}$ (Lemma 4 and Proposition 5 of the Online Appendix), one has to keep track of industry-type trade cost differentials. Nonetheless, our main insights remain unchanged. Since we are interested in the effects of economic disintegration that is supposed to affect all firms in a country, we abstain from carrying out comparative statics with respect to industry-type specific trade costs $\bar{\pi}^{ij}_{mn}$. As we have shown in this section, however, our model may also speak to the effects of trade shocks that hit a country’s firms to a varying extent.

## 2 Proofs for Section 2.5

### 2.1 Welfare and Trade Costs

In this section, we prove Lemma 5 of the Online Appendix. Let business taxes be positive, suppose that trade taxes are small and let trade costs be similar $\bar{\tau}_{im} \approx \bar{\tau}_{jk}$. Then, applying the envelope theorem, welfare in country $i$ positively depends on non-tariff trade costs between two
other countries $m$ and $k$

\[
\frac{dW_i}{dt_{mk}} = \frac{1}{2F} \left( t_i - \sum_{l \in \mathcal{X} \setminus \{i\}} 1 \left[ l = m \right] \text{int}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} + \sum_{l \in \mathcal{X} \setminus \{i\}} \text{ext}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} - \Delta_i^m \right) \left( \frac{dt_m}{dt_{mk}} - \frac{\partial y^m}{\partial t_{mk}} \right) + \frac{1}{2F} \left( t_i - \sum_{l \in \mathcal{X} \setminus \{i\}} 1 \left[ l = k \right] \text{int}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} + \sum_{l \in \mathcal{X} \setminus \{i\}} \text{ext}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} - \Delta_i^k \right) \left( \frac{dt_k}{dt_{mk}} - \frac{\partial y^k}{\partial t_{mk}} \right) + \frac{1}{2F} \sum_{j \in \mathcal{X} \setminus \{i, m, k\}} \left( \frac{dt_j}{dt_{mk}} - \frac{\partial y^j}{\partial t_{mk}} \right) \left( t_i - \sum_{l \in \mathcal{X} \setminus \{i\}} 1 \left[ l = j \right] \text{int}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} + \sum_{l \in \mathcal{X} \setminus \{i\}} \text{ext}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} \right) > 0
\]

(42)

since

\[
\frac{dt_m}{dt_{mk}} - \frac{\partial y^m}{\partial t_{mk}} = 3n_m \frac{K - 2}{(K - 1)(2K - 1)} \frac{\alpha - w - \tau_{mk}}{16\beta} + 3n_k \frac{2(K - 1)K - 1}{(K - 1)(2K - 1)} \frac{\alpha - w - \tau_{mk}}{16\beta} - \frac{1}{(K - 1)(2K - 1)} \frac{K n_i m \text{int}_{km} + n_k \text{int}_{km} - K(K - 1)n_k \text{ext}_{km} - (K - 1)n_m \text{ext}_{mk}}{4\beta} > 0
\]

for small trade taxes and \( \frac{dt_j}{dt_{mk}} > 0, \frac{\partial y^j}{\partial t_{mk}} = 0, \Delta_i^m < 0, \) and \( \Delta_i^k < 0. \)

Similarly, for tariffs \( t_{mk} = \text{int}_{mk} + \text{ext}_{mk} \) that may include import and export taxes

\[
\frac{dW_i}{dim_{mk}} = \frac{1}{2F} \left( t_i - \sum_{l \in \mathcal{X} \setminus \{i\}} 1 \left[ l = m \right] \text{int}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} + \sum_{l \in \mathcal{X} \setminus \{i\}} \text{ext}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} - \Delta_i^m \right) \left( \frac{dt_m}{dim_{mk}} + \frac{dt_k}{dim_{mk}} - \frac{\partial y^m}{\partial dim_{mk}} \right) + \frac{1}{2F} \sum_{j \in \mathcal{X} \setminus \{i, m, k\}} \left( t_i - \sum_{l \in \mathcal{X} \setminus \{i\}} 1 \left[ l = m \right] \text{int}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} + \sum_{l \in \mathcal{X} \setminus \{i\}} \text{ext}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} \right) \left( \frac{dt_j}{dim_{mk}} \right) > 0 \quad \text{and}
\]

(43)

\[
\frac{dW_i}{dext_{mk}} = \frac{1}{2F} \left( t_i - \sum_{l \in \mathcal{X} \setminus \{i\}} 1 \left[ l = m \right] \text{int}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} + \sum_{l \in \mathcal{X} \setminus \{i\}} \text{ext}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} - \Delta_i^m \right) \left( \frac{dt_m}{dext_{mk}} + \frac{dt_k}{dext_{mk}} - \frac{\partial y^m}{\partial dext_{mk}} \right) + \frac{1}{2F} \sum_{j \in \mathcal{X} \setminus \{i, m, k\}} \left( t_i - \sum_{l \in \mathcal{X} \setminus \{i\}} 1 \left[ l = j \right] \text{int}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} + \sum_{l \in \mathcal{X} \setminus \{i\}} \text{ext}_{il} n_i \frac{\alpha - w - \tau_i}{4\beta} \right) \left( \frac{dt_j}{dext_{mk}} \right) > 0
\]

(44)

as

\[
\frac{dt_m}{dim_{mk}} + \frac{dt_k}{dim_{mk}} - \frac{\partial y^m}{dim_{mk}} - \frac{\partial y^k}{dim_{mk}} = n_m \frac{(12K - 11)K(\alpha - w - \tau_{mk}) - 4K \text{int}_{mk} + 4 \text{ext}_{mk}}{(K - 1)(2K - 1)16\beta} + n_k \frac{[6(K - 1)K + 1](\alpha - w - \tau_{mk}) - 4 \text{int}_{mk} + 4 \text{ext}_{mk}}{(K - 1)(2K - 1)16\beta} > 0,
\]

21
\[
\frac{dt_m}{dext_{mk}} + \frac{dt_k}{dext_{mk}} - \frac{\partial \gamma^m}{dext_{mk}} - \frac{\partial \gamma^k}{dext_{mk}} = n_m \left[ \frac{6(K-1)(2K-1) + (3K-10)}{(K-1)(2K-1)}(\alpha - w - \tilde{\tau}_{mk}) - 4Kmt_{mk} + 4ext_{mk} \right] + n_k \left[ \frac{6(K-1)K - 4K - 3}{(K-1)(2K-1)}(\alpha - w - \tilde{\tau}_{mk}) - 4imt_{mk} + 4ext_{mk} \right] > 0,
\]

\[
\Delta^{ij} < 0, \quad \frac{dt_j}{dmt_{mk}} > 0, \quad \text{and} \quad \frac{dt_j}{dext_{mk}} > 0,
\]

for small trade taxes and similar trade costs (see Section 1.1 in this Supplementary Online Appendix).

### 2.2 Repercussions on Tax Policies

In this section, we derive the augmented trade-cost effect. That is, we do not only consider the exogenously driven rise in trade costs between a leaving country \( l \) and the remaining member countries \( m \in \mathcal{K}_{EU} \), but also the endogenous downward adjustment in trade costs inside the remaining economic union.

The overall effect on the leaving country’s business tax reads as

\[
\sum_{m \in \mathcal{K}_{EU}} \frac{dt_l}{d\tau_{ml}} - \frac{1}{2} \sum_{m \in \mathcal{K}_{EU}} \sum_{j \in \mathcal{K}_{EU} \setminus \{m\}} \frac{dt_l}{d\tau_{jm}} = 3n_l (K-2) - \tilde{n}_{EU} \left[ \frac{2(K-1)^2 + (2K-3)(K_{EU} - 1) + 1}{(K-1)(2K-1)} \right] K_{EU} \frac{\alpha - w - \tau}{16\beta},
\]

which is negative for \( n_l < \tilde{n}_{EU} \frac{2(K-1)^2 + (2K-3)(K_{EU} - 1)}{K-2} \). Without the readjustment of trade policies inside the union, the according condition was \( n_l < \tilde{n}_{EU} \frac{2(K-1)^2 + 1}{K-2} \).

The augmented trade-cost effect on the remaining member countries taxes is given by

\[
\frac{dt_m}{d\tau_{ml}} + \sum_{j \in \mathcal{K}_{EU} \setminus \{m\}} \frac{dt_m}{d\tau_{jl}} - \sum_{j \in \mathcal{K}_{EU} \setminus \{m\}} \frac{dt_m}{d\tau_{mj}} - \frac{1}{2} \sum_{l \in \mathcal{K}_{EU} \setminus \{m\}} \sum_{j \in \mathcal{K}_{EU} \setminus \{m,l\}} \frac{dt_m}{d\tau_{jl}}
\]

\[
= \left( \frac{2(K-1)^2 + 2(K_{EU} - 1)(K-1) + 1}{K_{EU} \tilde{n}_{EU} - 2(K-1)n_l (K - K_{EU})} \right) \left( 2(K-1)^2 + 2K - K_{EU} \right) n_m + K_{EU} (n_l - \tilde{n}_{EU}) \frac{3}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta}.
\]

Under symmetric market sizes the equation simplifies to

\[
... = \left( \frac{2(K-1)^2 + 2K_{EU} (K-1) + 4(K - K_{EU})}{K_{EU} - 1 - 2K (K - K_{EU})} \right) \frac{3n}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta}.
\]

which is increasing in \( K_{EU} \) and positive for \( K_{EU} = 2 \). Therefore, for all \( K_{EU} \geq 2 \), the equation is
positive.

The augmented effect on third countries writes as

\[
\sum_{j \in \mathcal{X}_{EU}} \frac{dt_k}{d\tau_{ji}} - \frac{1}{2} \sum_{m \in \mathcal{X}_{EU}} \sum_{j \in \mathcal{X}_{EU} \setminus \{m\}} \frac{dt_k}{d\tau_{jm}} = -\frac{3K_{EU} (2K - 3)}{(K - 1)(2K - 1)} \frac{a - w - \tau}{16\beta} ((K_{EU} - 2) \bar{n}_{EU} - n_l), \tag{47}
\]

which is negative if \( n_l < (K_{EU} - 2) \bar{n}_{EU} \).

References


