

Epstein Zin Utility, Asset Prices, and the Business Cycle Revisited

Christopher Heiberger, Halvor Ruf

December 18, 2014

We gratefully acknowledge financial support from the German Research Foundation (Deutsche Forschungsgemeinschaft). This work is part of a research project within the Priority Programme 1578 “Financial Market Imperfections and Macroeconomic Performance”.

Abstract

This paper analyzes to what extent a number of currently prevalent building blocks of DSGE models are helpful in trying to replicate characteristic empirical figures of the German real economy within an Epstein and Zin (1989) (EZ) framework. We thereby target both, classical RBC statistics as well as asset pricing figures so that our results may be regarded as initial guidance for researchers who consider the adoption of EZ utility.

We start our model analysis with an EZ version of Jermann (1998)’s model and then add some popular feature at a time. In particular, the models considered encompass economies closely related to Uhlig (2007) and Boldrin, Christiano, and Fisher (2001) so that modeling devices such as capital adjustment costs, exogenous consumption habits, and some real labor market frictions are discussed. The numerical results are found by perturbation.

Most importantly, we find an EZ version of Jermann’s model with endogenously fluctuating labor supply to already yield simulation results in good accordance with the data.

Contents

1	Introduction	1
2	Analytical framework	3
2.1	Household	3
2.2	Firm	6
2.3	General equilibrium	11
2.4	Steady state	12
2.5	Composite good aggregation	14
2.5.1	MX.a: No habits	15
2.5.2	MX.b: Standard consumption habits	16
2.5.3	MX.c: Slowly adapting consumption habits	17
3	Numerical framework	17
3.1	Solution	17
3.2	Computation of the return series	19
3.3	Empirical targets	22
3.4	Calibration and simulation	23
4	Model analysis	26
4.1	M0: Baseline	26
4.2	M1: No labor market frictions	28
4.2.1	M1.a: No habits	29
4.2.2	M1.b: Standard consumption habits	29
4.2.3	M1.c: Slowly adapting consumption habits	30
4.3	M2: Sticky real wages	30
4.3.1	M2.a: No habits	32
4.3.2	M2.b: Standard consumption habits	32
4.3.3	M2.c: Slowly adapting consumption habits	33
4.4	M3: Predetermined labor supply	34
4.4.1	M3.a: No habits	35
4.4.2	M3.b: Standard consumption habits	35
4.4.3	M3.c: Slowly adapting consumption habits	35
4.5	M4: Sectoral frictions in the allocation of labor	36
4.5.1	M4.a: No habits	44
4.5.2	M4.b: Standard consumption habits	45
4.5.3	M4.c: Slowly adapting consumption habits	45

5	Results and discussion	46
6	Conclusion	51
A	Appendix	52
A.1	Framework	52
A.2	M0	54
A.3	M1	54
A.4	M2	55
A.5	M3	56
A.6	M4	57
A.7	Documentation of computation routines	58
	References	iii

1 Introduction

In order to be an adequate tool for economic (policy) analysis, macroeconomic models have to be able to numerically replicate empirical regularities that characterize both the asset market and the real business cycle. The necessity for such simultaneous consistency has recently been highlighted by the real economic crisis that followed on the crash of the financial market in 2008. However, since the publication of Mehra and Prescott (1985), many papers have further confirmed the diagnosis that, for a reasonable degree of risk aversion, the paradigm of modern macroeconomics—(general) equilibrium in a model economy that features an infinitely-lived representative household who maximizes his lifetime utility as an expected discounted sum of within period CRRA¹ utilities—fails to reproduce the empirical equity premium. This holds for endowment economies and even more for models with an endogenous productive sector.²

The situation can be summarized as follows. A key to the magnitude of the equity premium is found in the household's aversion to an uneven consumption trajectory, which is reflected in the level of his elasticity of intertemporal (consumption) substitution (EIS). The lower the latter is, the higher the compensation the household demands for the hazard of (further) deviation from a smooth consumption path associated with potentially procyclical asset payments. Now, if the within period utility function is chosen to display constant RRA, the above mentioned notion of lifetime utility results in a specification that rigidly obeys the restriction $EIS = RRA^{-1}$. Consequently, assuming a low enough EIS in order to produce large risk premia comes with imposing a high level of RRA.

To work around this problem, Jermann (1998) modifies the standard RBC model in two respects. First, he modifies the household's within period utility aggregation. Specifically, he assumes the household to not value leisure and to have consumption habits as in Abel (1990). Second, he complements the habit formation in that he additionally assumes capital adjustment—and therefore consumption smoothing—to be costly. This way Jermann generates a sizable equity premium, yet at the cost of a constant labor supply.

Another approach is to add real frictions in the allocation of labor to

¹We use CRRA to abbreviate constant Arrow-Pratt relative risk aversion (RRA).

²For a sketch of the corresponding literature and some intuition on why nontrivial production economies are generally performing even worse than endowment economies, see the introductory remarks in Jermann (1998).

the standard RBC framework. Particularly, Boldrin, Christiano, and Fisher (2001) consider an economy in which the productive part is decomposed into two sectors and the household faces two real frictions regarding his labor allocation. First, he has to commit himself to some labor supply before the respective period's technology level is revealed. Second, in addition his labor supply is contracted sector-specific. On the other hand, Uhlig (2007) combines the capital adjustment friction studied by Jermann (1998) with some (non-modeled) stickiness in the real wage process. Alongside, he also allows for stickiness in the consumption habit process, rendering the model relatively parameter intensive.

In summary, the above can be understood as a list of building blocks that is read from the strand of the DSGE literature, which is concerned with the joint replication of the empirical evidence on both the financial and the real economy. This is exactly the perspective we adopt in this paper. Accordingly, our approach is to assess to what extent these building blocks are helpful in trying to replicate such empirical evidence. Most importantly, we thereby allow for the more general Epstein and Zin (1989) (EZ) utility representation of the household's preferences over stochastic consumption paths. We employ this representation because it loosens the rigid entanglement of EIS and RRA common to the models above, which considerably helps us in targeting asset pricing figures. In that sense, the present paper complements the work of Heer and Maußner (2013), who report on the empirical performance of the models mentioned so far (amongst others). As in their study, our empirical targets are statistical regularities of the German real economy.

Our main findings are the following. First, by means of the simulated "data" generated by our models, the empirical fit we were able to achieve using the EZ representation displays considerable improvement to the results presented in Heer and Maußner (2013). I.e. our results suggest deviation from the standard expected utility representation. Second and most importantly, a parsimoniously specified model, namely an EZ version of the Jermann (1998) model enhanced with a composite good aggregator that considers leisure next to standard exogenous consumption habits, already generates simulation results well in line with the empirical evidence. Third, adding those real labor market frictions mentioned above does not necessarily yield further improvement in empirical performance.

The remainder of this paper is organized into four sections. Sections 2 and 3 present our framework. I.e. we lay out the analytical basis of all considered models and document our empirical targets, our calibration and

the solution technique. Section 4 presents the considered model variants and their respective results, while section 5 discusses these findings in summary. Section 6 concludes the paper.³

2 Analytical framework

This section is primarily concerned with the analytical fundament of the ensuing analysis, which is the description of the behavior of our model economies' respective agents.

2.1 Household

Throughout this paper, we will assume the existence of an infinitely-lived representative household with preferences represented by a recursive utility function of EZ's Kreps/Porteus class.⁴ We employ this representation because it allows us to address the household's attitude towards intertemporal consumption substitution and towards the uncertainty associated with future lifetime utility somewhat separately. This considerably helps us in replicating our return targets.⁵ Yet, the additional degree of freedom also comes at a cost. It namely most prominently implies a preference for either earlier or later resolution of the uncertainty regarding consumption (or, more generally, the composite good) that may be tricky to justify.⁶ We will get back to this issue at the end of the paper. The household's infinite planning horizon can be motivated by intergenerational altruism.

Essentially, the representative household maximizes his lifetime utility as of period τ , denoted by U_τ , stemming from consumption, $c_t, t \geq \tau$, and leisure, $1 - n_t, t \geq \tau$, yet to come, where n denotes labor normalized to a maximum level of 1. Thereby, U_τ is stated as a recursive two-period utility that aggregates today's within period utility from c_τ and $1 - n_\tau$, denoted by $u(c_\tau, n_\tau)$, with a certainty equivalent of random future lifetime utility

³The accompanying appendix collects the more tedious derivations and also a complete list of equilibrium conditions for every considered model (class).

⁴Cf. Epstein and Zin (1989), p. 947 et. seq..

⁵In fact, the development of more flexible utility representations was, to a certain degree, driven by the standard framework's bad empirical performance mentioned above, cf. Epstein and Zin (1989), p. 938.

⁶Cf. Epstein, Farhi, and Strzalecki (2014).

depending on tomorrow's state.⁷

Next, we assume the certainty equivalent, μ , of a risk averse expected utility maximizer with a constant rate of relative risk aversion to serve as our household's uncertainty aggregation rule for lotteries over random future lifetime utility, i.e.

$$\mu_\tau := (\mathbb{E}_\tau[U_{\tau+1}^{1-\gamma}])^{\frac{1}{1-\gamma}}, \gamma \in \mathbb{R}_{>0} \setminus \{1\}.$$

The time aggregation of these two components, resulting in the lifetime utility as of period τ , is of the CES form

$$W(u, \mu) = [(1 - \beta)u^{1-\frac{1}{\psi}} + \beta\mu^{1-\frac{1}{\psi}}]^{\frac{1}{1-\frac{1}{\psi}}}, \psi \in \mathbb{R}_{>0} \setminus \{1\}, \beta \in (0, 1).$$

Summing up, we are led to the following recursive formulation of the representative household's preferences over intertemporal consumption lotteries

$$\begin{aligned} U_\tau &= W(u(c_\tau, n_\tau), \mu_\tau) \\ &= [(1 - \beta)u(c_\tau, n_\tau)^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_\tau[U_{\tau+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}}]^{\frac{1}{1-\frac{1}{\psi}}}. \end{aligned}$$

The rationale behind this representation is much more elaborately summarized in Heiberger and Ruf (2014). Therein it is also explained in what sense it holds that γ controls the agent's attitude towards risk as a coefficient of relative risk aversion while his EIS with respect to the composite good u is determined by ψ . Following Caldara et al. (2012), we will additionally introduce

$$\theta := \frac{1 - \gamma}{1 - \frac{1}{\psi}}$$

as a parameter measuring the relative deviation from the "classic" case, where the coefficient of relative risk aversion coincides with the reciprocal of the intertemporal elasticity of substitution. We thus write

$$W(u, \mu) = [(1 - \beta)u^{\frac{1-\gamma}{\theta}} + \beta\mu^{\frac{1-\gamma}{\theta}}]^{\frac{\theta}{1-\gamma}}$$

and

$$U_\tau = [(1 - \beta)u(c_\tau, n_\tau)^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_\tau[U_{\tau+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}. \quad (1)$$

⁷More precisely, any period's composite good is an aggregation of its consumption and leisure and the within period utility mapping of this composite good is the identity mapping. We may thus, for the sake of intuition, switch between the two notions of within period utility and the composite good.

In the ensuing analysis, we consider different specifications of the composite good u . The core difference between the corresponding models will be their implied stochastic discount factor. For the sake of exposition, we thus begin with the household's necessary optimality conditions for a general composite good. In the models to come, the respective decision problem basically⁸ is

$$\begin{aligned} \max U_\tau &= W(u(c_\tau, n_\tau), (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma}}) \\ \text{s.t. } c_t &\leq w_t n_t + d_t s_t - v_t (s_{t+1} - s_t), \\ c_t &\geq 0, 0 \leq n_t \leq 1, \text{ for all } t \geq \tau, \\ &\text{given } s_\tau, \end{aligned} \tag{2}$$

where c_t , w_t , n_t , d_t , s_t , and v_t denote period t 's consumption, wage, working hours, dividend payments, the number of shares held by the household, and the share price, respectively.

We can summarize the necessary conditions for an interior optimum to the representative household's problem as follows.⁹ For all $t \geq \tau$ it has to hold that

$$V_t = [(1 - \beta)u(c_t, n_t)^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}, \tag{3}$$

$$\mathbb{E}_t \left[m_{t+1,t} \frac{d_{t+1} + v_{t+1}}{v_t} - 1 \right] = 0, \tag{4}$$

$$\frac{\partial u}{\partial c}(c_t, n_t) w_t = -\frac{\partial u}{\partial n}(c_t, n_t), \tag{5}$$

$$c_t = w_t n_t + d_t s_t - v_t (s_{t+1} - s_t), \tag{6}$$

where V_t denotes the problem's period t value function and

$$m_{t+1,t} := \beta \left(\frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t V_{t+1}^{1-\gamma}} \right)^{1-\frac{1}{\theta}} \left(\frac{u(c_{t+1}, n_{t+1})}{u(c_t, n_t)} \right)^{\frac{1-\gamma}{\theta}-1} \frac{\frac{\partial u}{\partial c}(c_{t+1}, n_{t+1})}{\frac{\partial u}{\partial c}(c_t, n_t)} \tag{7}$$

is the household's stochastic discount factor. It reflects his marginal lifetime utility evaluation of the implications of setting aside v_t units of the consumption good in period t in order to receive the uncertain reward of $d_{t+1} + v_{t+1}$

⁸Although the decision problems of the more complicated models differ from this basic framework, their treatment closely parallels the one to be outlined. See the respective sections on the necessary changes.

⁹A detailed derivation of these conditions can be found in appendix A.1.

consumption units next period. Note how

$$\left(\frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t V_{t+1}^{1-\gamma}}\right)^{1-\frac{1}{\theta}} = \left(\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma} = \left(\frac{V_{t+1}}{\mu_\tau}\right)^{\frac{1}{\psi}-\gamma}$$

makes explicit the effect of timing preferences on the household's asset valuation, especially how the standard discount factor emerges from the classic consequentialist indifference assumption $\frac{1}{\psi} = \gamma$.¹⁰

2.2 Firm

Next, we accordingly assume the existence of a representative firm. In period t it produces the amount y_t of the final good employing the households' labor force and capital k_t via a constant returns to scale Cobb-Douglas technology

$$y_t = e^{z_t} n_t^{1-\alpha} k_t^\alpha, \alpha \in (0, 1),$$

with α determining the factors' output elasticities. The firm's period t total factor productivity evolves randomly with z_t . The latter is modeled as a stationary first order autoregressive process, i.e.

$$z_{t+1} = \rho z_t + \sigma \epsilon_{t+1}, \epsilon_t \sim \text{iidN}(0, 1), |\rho| < 1. \quad (8)$$

Hence, ϵ_t can be interpreted as a technology shock.

Capital is owned and produced by the firm whose capital stock evolves as

$$k_{t+1} - (1 - \delta)k_t = \Phi\left(\frac{i_t}{k_t}\right) k_t, \quad (9)$$

where δ measures depreciation. Φ is a concave function introducing capital adjustment costs in the form employed by Jermann (1998).¹¹ We define

$$\Phi(x) := \frac{b_1}{1 - \kappa} x^{1-\kappa} + b_2, \kappa > 1, b_1 > 0. \quad (10)$$

¹⁰See Heiberger and Ruf (2014) for a discussion of the discounting implications of non-indifference towards the timing of uncertainty resolution.

¹¹Following Hayashi (1982), we interpret Φ as an "installation function" for it describes adjustment costs by means of capital accumulation rather than by means of a negative summand in the firm's definition of profit.

In order for equation (9) to be well-defined, we therefore additionally demand $i_t > 0$ and $k_t > 0$ for all $t \geq \tau$. While κ controls the speed of investment, the parameters b_1 and b_2 are chosen in order to render the steady state unaffected by adjustment costs.¹² The concavity of $\Phi(\cdot)$ implies capital adjustment costs in that it both limits the growth rate of capital and makes abrupt changes in the capital stock more investment intensive. More specifically, by rearranging (9),

$$k_{t+1} = \left(1 - \delta + \Phi\left(\frac{i_t}{k_t}\right)\right) k_t,$$

we find the function $\Phi(\cdot)$ to model the part of the capital stock's growth rate controlled through investment effort. With larger values of κ , positive deviations from i_t to δk_t have a decreasing, less than proportional effect on the capital stock while the effect of negative deviations is increasing and more than proportional. Hence, the firm's management has an incentive to avoid large deviations from i_t to δk_t . To put it another way, investment variability decreases in κ . Eventually note that $\kappa > 1$ implies that $\Phi(\frac{i_t}{k_t})$ is bound above by b_2 but falls without any bound if investment approaches 0 and that the case of no adjustment costs, i.e. $\kappa = 0$, makes the "standard" specification of $\Phi = \text{id}$ emerge.

Next, since capital is owned by the firm, period t 's profit amounts to revenue less labor costs, $y_t - w_t n_t$.¹³ The firm's owners—i.e. households—participate in these profits via dividend payments, d_t per share. Investment in the capital stock is financed through profits beyond dividend payments plus the issuance of new shares

$$i_t = y_t - w_t n_t - d_t s_t + v_t (s_{t+1} - s_t). \quad (11)$$

Eventually, period t 's cash flow cf_t is defined as profits less investment expenditures, both in t ,

$$cf_t := y_t - w_t n_t - i_t.$$

Using this definition, the financing equation (11) can be equivalently stated as

$$d_t s_t - v_t (s_{t+1} - s_t) = cf_t. \quad (12)$$

¹²Note that Φ could also be defined for $\kappa \in \mathbb{R}_{\geq 0} \setminus \{1\}$. See later on empirical evidence against $\kappa \leq 1$ and on more details about b_1, b_2 .

¹³Revenue equals output because we assume that firms never store any of their output and take y_t as numéraire.

Now, next to its dividend and stock policy, the firm's management decides over capital investment, its next period capital stock and its demand for labor. It has to balance the tradeoff between current profits and future capital resources knowing that its investment funding depends on the share price process. The management is thus not statically maximizing profits or cashflows period by period but rather maximizing its firm value as of t , denoted by fv_t . The latter is classically defined as the firm's current period cash flow plus its ex dividend market capitalization, i.e.

$$fv_t := cf_t + v_t s_{t+1}.$$

Next, the shareholders' infinite scope requires us to impose an additional constraint. It demands that, from period τ on, their appreciation of any market capitalization in infinite future vanishes. I.e. the growth rate of the firm's market capitalization has to be capped by the household's discounting behavior,

$$\lim_{t \rightarrow \infty} \mathbb{E}_\tau [m_{t,\tau} v_t s_{t+1}] = 0, \quad (13)$$

where $m_{t,\tau} := m_{\tau+1,\tau} \cdot \dots \cdot m_{t,t-1}$ for $t \geq \tau + 1$ with $m_{\tau,\tau} \equiv 1$ is the stochastic discount factor from period t to τ . Hence, following Altug and Labadie (2008), p. 265, repeatedly using (4), (12) and (13) we find

$$\begin{aligned} v_\tau s_{\tau+1} &= \mathbb{E}_\tau [m_{\tau+1,\tau} (d_{\tau+1} + v_{\tau+1}) s_{\tau+1}] = \\ &= \mathbb{E}_\tau [m_{\tau+1,\tau} (d_{\tau+1} s_{\tau+1} - v_{\tau+1} (s_{\tau+2} - s_{\tau+1}) + v_{\tau+1} s_{\tau+2})] = \\ &= \mathbb{E}_\tau [m_{\tau+1,\tau} cf_{\tau+1} + m_{\tau+1,\tau} v_{\tau+1} s_{\tau+2}] = \dots = \\ &= \mathbb{E}_\tau \left[\sum_{t=\tau+1}^{\infty} m_{t,\tau} cf_t \right], \end{aligned} \quad (14)$$

so that the firm value as of period τ is the expected present value of its cash flows to come,

$$fv_\tau = \mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} m_{t,\tau} cf_t \right].$$

In period τ , the firm's management has to choose the amount of working hours employed, the investment expenditures and next period's capital stock, while k_τ is given. In other words, the maximization problem of the

representative firm is

$$\begin{aligned}
& \max \mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} m_{t,\tau} (e^{z_t} n_t^{1-\alpha} k_t^\alpha - w_t n_t - i_t) \right] \\
& \text{s.t. } k_{t+1} = (1 - \delta)k_t + \Phi \left(\frac{i_t}{k_t} \right) k_t, \\
& \quad k_{t+1} > 0, i_t > 0, 0 \leq n_t \leq 1, \text{ for all } t \geq \tau, \\
& \text{given } k_\tau > 0.
\end{aligned} \tag{15}$$

Note that we do not explicitly consider the firm's financing in the statement of its maximization problem. To see why, note that after having determined the optimal level of investment, next period's capital stock and labor demand, for the firm it is always possible to find a respective financing that satisfies (11). To see this, note that given optimal i_t , k_{t+1} and n_t the firm's cash flow $cf_t = e^{z_t} n_t^{1-\alpha} k_t^\alpha - w_t n_t - i_t$ is determined. The constraint (11) then is satisfied for any d_t and s_{t+1} , which meet

$$d_t s_t - v_t (s_{t+1} - s_t) = cf_t, t \geq \tau,$$

where by iterating equation (4)

$$\begin{aligned}
v_t &= \mathbb{E}_t [m_{t+1,t} (d_{t+1} + v_{t+1})] = \mathbb{E}_t [m_{t+1,t} d_{t+1} + m_{t+1,t} v_{t+1}] = \\
&= \mathbb{E}_t [m_{t+1,t} d_{t+1} + m_{t+1,t} \mathbb{E}_{t+1} [m_{t+2,t+1} (d_{t+2} + v_{t+2})]] = \dots = \\
&= \mathbb{E}_t \left[\sum_{s=1}^{\infty} m_{t+s,t} d_{t+s} \right], t \geq \tau,
\end{aligned}$$

if we additionally assume $\lim_{s \rightarrow \infty} \mathbb{E}_t [m_{t+s,t} v_{t+s}] = 0$. In general, however, the resulting s_{t+1} , d_t and v_t are not uniquely determined without imposing a particular dividend policy for the firm.

Differentiating with respect to n_t , i_t and k_{t+1} and assuming an interior solution yields the first order conditions for the firm's maximization problem. First,

$$w_t = (1 - \alpha) e^{z_t} n_t^{-\alpha} k_t^\alpha, \text{ for all } t \geq \tau,$$

i.e. wages have to equal the marginal product of labor.¹⁴ Second,

$$q_t = \frac{1}{\Phi' \left(\frac{i_t}{k_t} \right)}, \text{ for all } t \geq \tau,$$

¹⁴Note that this will imply profits to be positive in equilibrium due to the assumed linear homogeneity of production by Euler's Theorem, because both capital and its marginal

where q_t is period t 's Lagrange multiplier for the capital accumulation constraint (9) divided by $m_{t,\tau}$. Third, the Euler equation

$$q_t = \mathbb{E}_t \left[m_{t+1,t} \left(\alpha e^{z_{t+1}} n_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \Phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) \right) \right]$$

must hold for all $t \geq \tau$.

Summing up, the list of optimality conditions for an interior solution to the firm's problem are for all $t \geq \tau$,

$$w_t = (1 - \alpha) e^{z_t} n_t^{-\alpha} k_t^\alpha, \quad (16)$$

$$q_t = \frac{1}{\Phi' \left(\frac{i_t}{k_t} \right)}, \quad (17)$$

$$q_t = \mathbb{E}_t \left[m_{t+1,t} \left(\alpha e^{z_{t+1}} n_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \Phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) \right) \right] \quad (18)$$

$$k_{t+1} = (1 - \delta) k_t + \Phi \left(\frac{i_t}{k_t} \right) k_t. \quad (19)$$

$$i_t = y_t - w_t n_t - d_t s_t + v_t (s_{t+1} - s_t), \quad (20)$$

$$y_t = e^{z_t} k_t^\alpha n_t^{1-\alpha}. \quad (21)$$

An (informal) remark on q . Since the price of the investment good is 1 and additional investment in period τ increases $k_{\tau+1}$ by $\Phi' \left(\frac{i_\tau}{k_\tau} \right)$, we find additional $k_{\tau+1}$ to have a price of $\left(\Phi' \left(\frac{i_\tau}{k_\tau} \right) \right)^{-1}$. Next, using the envelope theorem, in $\tau + 1$, the increase of the then given state $k_{\tau+1}$ increases the maximum firm value as of period $\tau + 1$ by

$$\alpha e^{z_{\tau+1}} n_{\tau+1}^{1-\alpha} k_{\tau+1}^{\alpha-1} - \frac{i_{\tau+1}}{k_{\tau+1}} + q_{\tau+1} \left(1 - \delta + \Phi \left(\frac{i_{\tau+1}}{k_{\tau+1}} \right) \right),$$

all variables evaluated at the optimum. By (18) we thus find q_τ to measure the expected discounted change in period τ 's maximum firm value induced

product are positive for all t given z_t . Precisely,

$$k_t \frac{\partial y_t}{\partial k_t} = \alpha y_t > 0, \text{ for all } t.$$

by an exogenous increase in the capital stock at the end of τ . Hence, equation (17) states that, in an optimum, the value of a unit of capital to the firm has to be equal to its price.¹⁵

2.3 General equilibrium

In a general equilibrium, all markets in the model economy are cleared simultaneously and the representative household as well as the representative firm mutually act optimally. In anticipation of the ensuing general equilibrium analysis, we have already denoted demand and supply variables identically. Imposing these identities on the agents' respective optimality conditions therefore already ensures a cleared labor market. Additionally, the goods market has to be cleared, i.e.

$$y_t = c_t + i_t.$$

This equation together with (3)-(6) and (16)-(21) define a general equilibrium in period t .

As mentioned above, the values for s_{t+1} , d_t and v_t are not uniquely determined in equilibrium without assuming the firm to follow a particular dividend policy. Note however, that it follows from equations (6) and (20), that the goods market clearing condition is already necessary and sufficient for the stock market to clear, too. Hence, we can ignore (4), (6) and (20) without any loss, if we are not interested in s_{t+1} , d_t and v_t .

To sum up, we list our fundamental equilibrium conditions: For all $t \geq \tau$ it must hold that

$$V_t - [(1 - \beta)u(c_t, n_t)^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}} = 0, \quad (22)$$

$$\frac{\partial u}{\partial c}(c_t, n_t)w_t = -\frac{\partial u}{\partial n}(c_t, n_t), \quad (23)$$

$$w_t - (1 - \alpha)e^{z_t}n_t^{-\alpha}k_t^\alpha = 0, \quad (24)$$

$$q_t - \frac{1}{\Phi'\left(\frac{i_t}{k_t}\right)} = 0, \quad (25)$$

¹⁵Hayashi (1982) rigorously develops this crucial role of the capital accumulation constraint's shadow price in economies with installation costs.

$$q_t - \mathbb{E}_t \left[m_{t+1,t} \left(\alpha e^{z_{t+1}} n_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \Phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) \right) \right] = 0, \quad (26)$$

$$k_{t+1} - (1 - \delta)k_t - \Phi \left(\frac{i_t}{k_t} \right) k_t = 0, \quad (27)$$

$$y_t - e^{z_t} k_t^\alpha n_t^{1-\alpha} = 0, \quad (28)$$

$$y_t - c_t - i_t = 0, \quad (29)$$

where the sequence $\{z_t\}$ follows (8) and the stochastic discount factor is determined by (7).

2.4 Steady state

We next characterize the model's deterministic steady state, i.e. the solution with

$$\sigma = 0 \text{ and } x_t = x_{t+1} =: x_{ss}, \text{ for all } t,$$

where $x_t := (k_{t+1} \ z_t \ V_t \ c_t \ n_t \ w_t \ y_t \ i_t \ q_t)^T$. First, we see that this implies $z_{ss} = 0$. Second, using (28) and (24) we find

$$y_{ss} = k_{ss}^\alpha n_{ss}^{1-\alpha}$$

and

$$w_{ss} = (1 - \alpha) \frac{y_{ss}}{n_{ss}}.$$

(27) and (25) further yield

$$\delta k_{ss} = \Phi \left(\frac{i_{ss}}{k_{ss}} \right) k_{ss}$$

and

$$q_{ss} = \frac{1}{\Phi' \left(\frac{i_{ss}}{k_{ss}} \right)}.$$

Next, as mentioned above, we do not want adjustment costs to play a role in the steady state. Thus, we have to make sure that our parametric specification of $\Phi(\cdot)$ makes

$$i_{ss} = \delta k_{ss}$$

and

$$q_{ss} = 1$$

emerge. This is achieved by demanding

$$\begin{aligned} & \Phi(\delta) = \delta \text{ and } \Phi'(\delta) = 1 \\ \Leftrightarrow & \frac{b_1}{1-\kappa} \delta^{1-\kappa} + b_2 = \delta \text{ and } b_1 \delta^{-\kappa} = 1, \end{aligned} \tag{10}$$

which is satisfied for

$$b_1 = \delta^\kappa$$

and thus

$$b_2 = \delta \left(1 - \frac{1}{1-\kappa} \right) = -\delta \frac{\kappa}{1-\kappa}.$$

Additionally, one might want the adjustment cost function $\Phi(\cdot)$ to be positive.¹⁶ This is equivalent to demanding a period's capital stock never to fall short its last period's value less depreciation. For that to be the case, the investment-to-capital ratio has to always satisfy the condition

$$\frac{i_t}{k_t} > \kappa^{\frac{1}{1-\kappa}} \delta,$$

i.e. it must always exceed $\kappa^{\frac{1}{1-\kappa}}$ times its steady state value.¹⁷

For the value function (22) we find

$$V_{ss} = u(c_{ss}, n_{ss})$$

¹⁶Cf. Jermann (1998), p. 260, who actually lists positivity as a defining property of Φ .

¹⁷Hence, if positivity is demanded, it is necessary to either impose this condition directly or to check for it in the simulation results. Note that for the factor $\kappa^{\frac{1}{1-\kappa}} = e^{\frac{\ln \kappa}{1-\kappa}}$, where $\kappa > 1$, we find

$$\frac{\ln \kappa}{1-\kappa} = -\frac{\ln \kappa - \ln 1}{\kappa - 1} \xrightarrow{\kappa \downarrow 1} -\ln'(1) = -1$$

and

$$\frac{\ln \kappa}{1-\kappa} \xrightarrow{\kappa \rightarrow \infty} 0$$

and hence the limits

$$\lim_{\kappa \downarrow 1} \kappa^{\frac{1}{1-\kappa}} = e^{-1} \text{ and } \lim_{\kappa \rightarrow \infty} \kappa^{\frac{1}{1-\kappa}} = 1,$$

demonstrating the implications of different parametrizations with respect to the hazard of generating simulation results that violate this restriction.

and, via the goods market equilibrium condition,

$$c_{ss} = y_{ss} - i_{ss}.$$

We can now express all variables' steady state values in terms of k_{ss} and n_{ss} . While the latter is generally set to some specific level, k_{ss} is eventually determined via the model's Euler equation (26)

$$k_{ss} = \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}} n_{ss}.$$

2.5 Composite good aggregation

This subsection introduces the nontrivial variations of the composite good (consumption bundle) considered in this paper in increasing order of generality. Economic intuition and a brief discussion of their respective implications for the interpretation of our key parameters, ψ and γ , are provided.

Just as the real business cycle phenomenon and also fundamental asset prices can be viewed as a consequence of very few core mechanisms of economic activity, the models considered here generate corresponding fluctuations and return series mainly through technology shocks and the induced intertemporal substitution behavior of households. The shock variability and the EIS are thus recognized as the pivotal parameters in that respect.

While our framework leaves no room to work on σ , the chosen EZ specification of lifetime utility, however, allows for a rather liberal parameterization of ψ as it loosens the strict entanglement of EIS and RRA. Hence, we are e.g. able to make the household as averse to a non-smooth composite good path as is needed to reach our targeted equity premium. Yet, we would rather want to work on the household's *consumption* behavior more directly for we also want to separately target a particular labor variability found in the data.

Primarily for this purpose, we will additionally consider habit formation solely in consumption. Precisely, after analyzing the model with a classical composite good aggregation, we will additionally allow for external habits in the sense of Campbell and Chocrane (1999). The external habit process thereby is either standard or also allows for slowly adjusting consumption habits as in Uhlig (2007).

2.5.1 MX.a: No habits

First, we will consider models using the linearly homogenous Cobb–Douglas aggregator

$$u(c_t, n_t) := c_t^\nu (1 - n_t)^{1-\nu}, \quad \nu \in (0, 1),$$

where ν controls the relative weight of consumption and leisure in the composite good, i.e. in the within period utility. In particular, we may interpret the case $\nu \geq (\leq) \frac{1}{2}$ as consumption having a larger (smaller) impact than leisure on the composite good.

It thus becomes necessary to distinguish the household’s attitude towards intertemporal substitution of consumption from his attitude towards intertemporally substituting the composite good. While the latter is determined by ψ , the former must also take c_t ’s “importance parameter” ν into account. In the present case e.g., the computation reads $\psi_c := \frac{1}{1-\nu(1-\frac{1}{\psi})}$. Hence, when we speak of the EIS, we relate to the notion of substituting the composite good. By means of ν , this, however, can be directly translated into a statement about consumption substitution. Precisely, as ψ increases c.p., so does ψ_c . Further, the chosen composite good aggregation analogously yields $\psi_{(1-n)} := \frac{1}{\frac{1}{\psi} + \nu(1-\frac{1}{\psi})}$.

Accordingly, we have to be aware of the fact that the composite good’s importance parameter ν must be considered just as much when interpreting the degree of risk aversion associated with each ingredient. E.g. in the special case of $\theta = 1$, while the composite good CRRA is γ , the consumption CRRA and leisure CRRA are $\nu\gamma$ and $(1 - \nu)\gamma$, respectively.¹⁸ In the habit variants to be outlined in the following, it is more cumbersome to derive the implications of the composite good’s specification. Nevertheless, the gist of the above reasoning carries over. We thus skip an explicit discussion of this issue in the following.

From equation (7) we can calculate the representative household’s stochastic discount factor as

$$m_{t+1,t} := \beta \left(\frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t V_{t+1}^{1-\gamma}} \right)^{1-\frac{1}{\theta}} \left(\frac{c_{t+1}^\nu (1 - n_{t+1})^{1-\nu}}{c_t^\nu (1 - n_t)^{1-\nu}} \right)^{\frac{1-\gamma}{\theta}} \frac{c_t}{c_{t+1}}. \quad (30)$$

¹⁸Cf. Swanson (2012).

2.5.2 MX.b: Standard consumption habits

In this variation, we allow for consumption habits in the aggregator u . The employed version of habits is in some sense naive, as the household now regards the current period's excess over last period's consumption level as his utility argument but does not consider this behavior in advance. From a representative agent perspective one might interpret this as the household comparing his current consumption level to the economy's previous period's overall average consumption, giving rise to the popular notion of *catching up with the Joneses*.¹⁹ Following Campbell and Chochrane (1999), we will thus refer to the studied form of habit formation as external. Interpreted either way, the fact that current consumption increases future consumption "obligations"—however recognized by the household—is meant to further smoothen the consumption behavior in equilibrium.

Formally, we now consider the aggregator

$$u(c_t, n_t) := (c_t - c_t^h)^\nu (1 - n_t)^{1-\nu}, \quad \nu \in (0, 1),$$

where c_t^h is a habit process that is strictly exogenous to the household. Note that for this composite good to be always well-defined we must impose the more restrictive constraint

$$c_t \geq c_t^h, t \geq \tau, \quad (31)$$

instead of $c_t \geq 0$. The exogenous habit process here follows

$$c_t^h := \chi c_{t-1}, \chi \in [0, 1]. \quad (32)$$

Given this functional form of u the stochastic discount factor results in

$$m_{t+1,t} = \beta \left(\frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t V_{t+1}^{1-\gamma}} \right)^{1-\frac{1}{\theta}} \left(\frac{(c_{t+1} - c_{t+1}^h)^\nu (1 - n_{t+1})^{1-\nu}}{(c_t - c_t^h)^\nu (1 - n_t)^{1-\nu}} \right)^{\frac{1-\gamma}{\theta}} \frac{c_t - c_t^h}{c_{t+1} - c_{t+1}^h}. \quad (33)$$

Steady state The steady state value for the habit variable is determined through (32),

$$c_{ss}^h = \chi c_{ss}.$$

¹⁹Cf. Abel (1990).

2.5.3 MX.c: Slowly adapting consumption habits

In this modification we generalize the process specifying the exogenous consumption habit as in Uhlig (2007). More precisely, the consumption habit is no longer some given fraction of the previous period's consumption level, but also depends on its own anteceding value and hence adjusts more slowly to variations in consumption. This behavioral generalization allows us to more finely calibrate the household's smoothing behavior.

While the composite good aggregation again takes the form as in MX.b, i.e.

$$u(c_t, n_t) := (c_t - c_t^h)^\nu (1 - n_t)^{1-\nu}, \quad \nu \in (0, 1),$$

the exogenous habit process c_t^h now follows

$$c_t^h := \lambda c_{t-1}^h + (1 - \lambda)\chi c_{t-1}, \quad \chi \in [0, 1], \lambda \in [0, 1).$$

Thus, the stochastic discount factor remains the same as in the previous variant, i.e.

$$m_{t+1,t} = \beta \left(\frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t V_{t+1}^{1-\gamma}} \right)^{1-\frac{1}{\theta}} \left(\frac{(c_{t+1} - c_{t+1}^h)^\nu (1 - n_{t+1})^{1-\nu}}{(c_t - c_t^h)^\nu (1 - n_t)^{1-\nu}} \right)^{\frac{1-\gamma}{\theta}} \frac{c_t - c_t^h}{c_{t+1} - c_{t+1}^h}.$$

Steady state Note that it follows from $\lambda \neq 1$ that still

$$c_{ss}^h = \chi c_{ss}.$$

3 Numerical framework

In this section, we report in detail on the employed solution method, the chosen empirical targets, and on how our results were computed.²⁰

3.1 Solution

Our goal is to simulate a time path of the model's equilibrium outcomes emerging from a series $\{\epsilon_t\}$ of iidN(0, 1) distributed pseudorandom shocks to the productivity level z_t . In order to do this, for each period t we have to find

²⁰In the appendix, we additionally provide a brief documentation on the employed computation routines.

the solution to the stochastic dynamic system implicitly defined by the equilibrium conditions (22)-(29), given this period's states k_t and z_t . Following Schmitt-Grohe and Uribe (2004), we denote by $h(k_t, z_t, \sigma)$ the solution for k_{t+1} and by $g_i(k_t, z_t, \sigma), i = 1, \dots, 7$, the solution for the remaining variables of this system of equations except z_{t+1} . I.e. we make explicit that the functions characterizing the solution depend on the states k_t and z_t and also on the standard deviation σ of the AR(1) noise term but are time-independent.²¹ The g_i s are called policy functions, while h is called the capital stock's dynamic. Note that the technology's dynamic is already explicitly given by (8).

Rather than solving for their exact solutions, we use the perturbation method and thus search for a local Taylor approximation of the functions h and g_i .²² The Taylor polynomials' point of expansion is the deterministic steady state $(k_{ss}, z_{ss}, 0)$. More precisely, in our equilibrium conditions, we replace k_{t+1} by the dynamic h and the remaining variables by their respective policy functions g_i . Hence, differentiating these conditions with respect to k, z and σ at the steady state yields a system of equations in which the derivatives of the dynamic and the policy functions at the steady state are the unknowns. Solving for the stable solution to this system of equations, we find the coefficients of the first order Taylor polynomials.²³ Finally, computing the equilibrium conditions' second derivatives at the deterministic steady state and inserting the already determined first derivatives of h and g_i yields a linear system of equations in the dynamic's and policies' second order derivatives. Its solution completes our necessary computations for a second order perturbation.

The appropriateness of perturbation in a DSGE model with EZ utility is documented by Caldara et al. (2012), who compare on different solution methods with regard to accuracy and computing time for several calibrations.

²¹Of course, the solution also depends on the other parameters. σ , however, plays a special role with regard to the solution as it scales the uncertainty in our model. Explicitly considering σ as an argument of h and g_i makes it possible to examine the effect of uncertainty on our solution. Note that time-independence is necessary for optimality.

²²We thereby assume sufficient smoothness of the functions h and g_i .

²³A solution is regarded "stable" if all eigenvalues of the Jacobian of the system's dynamic are less than unity in absolute value so that the states' processes are bounded.

3.2 Computation of the return series

The upcoming analysis places particular interest on the equity premium and the risk free rate. This section demonstrates their respective computation.

Return on Equity Our models' return on equity is²⁴

$$r_{t+1}^e := \frac{d_{t+1} + v_{t+1}}{v_t}.$$

As already stated, we do not compute v_t and d_t in our solution. Therefore, in order to compute the return on equity in our simulations nonetheless, we follow Heer and Maußner (2013) and make use of the equality

$$\frac{d_{t+1} + v_{t+1}}{v_t} = \frac{y_{t+1} - w_{t+1}n_{t+1} - i_{t+1} + q_{t+1}k_{t+2}}{q_t k_{t+1}}.$$

This equation holds along an equilibrium path and can be deduced as follows. First, remember that the financing constraint of the firm as well as the household's budget constraint in equilibrium determine d_t and s_{t+1} up to

$$d_t s_t - v_t (s_{t+1} - s_t) = cf_t.$$

Second, by equation (24)

$$w_t n_t = (1 - \alpha) e^{z_t} n_t^{1-\alpha} k_t^\alpha = (1 - \alpha) y_t$$

and hence

$$cf_t = y_t - w_t n_t - i_t = \alpha y_t - i_t.$$

Therefore, by the fact that k_{t+1} is known at the beginning of period t and by equations (26) and (27), we may write

$$\begin{aligned} q_t k_{t+1} &= \mathbb{E}_t \left[m_{t+1,t} \left(\alpha e^{z_{t+1}} n_{t+1}^{1-\alpha} k_{t+1}^\alpha - i_{t+1} + q_{t+1} \left(1 - \delta + \Phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) k_{t+1} \right) \right] \\ &= \mathbb{E}_t [m_{t+1,t} (\alpha y_{t+1} - i_{t+1} + q_{t+1} k_{t+2})] = \mathbb{E}_t [m_{t+1,t} (cf_{t+1} + q_{t+1} k_{t+2})] \end{aligned}$$

and thus by continuing inductively

$$q_t k_{t+1} = \mathbb{E}_t \left[\sum_{s=1}^{\infty} m_{t+s,t} cf_{t+s} \right],$$

²⁴Note that the two sector economies studied in subsection 4.5 demand a slight modification with respect to the computation of the risky return, but see later.

if we additionally assume $\lim_{s \rightarrow \infty} \mathbb{E}_t [m_{t+s,t} q_{t+s} k_{t+1+s}] = 0$.²⁵ Thus, according to (14), the term on the right hand side equals $v_t s_{t+1}$, so that

$$q_t k_{t+1} = v_t s_{t+1}. \quad (34)$$

The claim now follows from

$$\begin{aligned} \frac{d_{t+1} + v_{t+1}}{v_t} &= \frac{d_{t+1} s_{t+1} + v_{t+1} s_{t+1}}{v_t s_{t+1}} = \\ &= \frac{d_{t+1} s_{t+1} - v_{t+1} (s_{t+2} - s_{t+1}) + v_{t+1} s_{t+2}}{v_t s_{t+1}} = \\ &= \frac{d_{t+1} s_{t+1} - v_{t+1} (s_{t+2} - s_{t+1}) + q_{t+1} k_{t+2}}{q_t k_{t+1}} = \\ &= \frac{c f_{t+1} + q_{t+1} k_{t+2}}{q_t k_{t+1}} = \frac{y_{t+1} - w_{t+1} n_{t+1} - i_{t+1} + q_{t+1} k_{t+2}}{q_t k_{t+1}}. \end{aligned}$$

This allows us to compute the return on equity without having to determine d_t or v_t . To put it another way, under the assumed transversality conditions it plausibly holds that the return on the firm's capital investment equals the return on the households' (i.e. firm owners') share investment.²⁶ Besides, in the steady state the return on equity is $\frac{1}{\beta}$.

A second remark on q . By (34), the price of one share divided by the book value of the firm's capital stock per share, turns out as

$$q_t = \frac{v_t}{\frac{k_{t+1} \cdot 1}{s_{t+1}}}.$$

Hence, following the first remark on q above, in our model q in fact (also) measures the figure Tobin (1969) already found to be central in any agent's investment decision. Also does the chosen capital adjustment friction meet his suggestion, p. 21, that "the speed at which investors wish to increase the

²⁵This is another transversality condition, akin to (13). It imposes a growth cap on the value of the firm's capital stock.

²⁶Cf. Kaltenbrunner and Lochstoer (2010), who directly define the risky return via the capital investment Euler equation.

capital stock should be related [...] to q ,” for it turns out that

$$\begin{aligned}
\Phi'(x) &= b_1 x^{-\kappa} \\
\Leftrightarrow x &= \left(b_1 \frac{1}{\Phi'} \right)^{\frac{1}{\kappa}} = (b_1 q)^{\frac{1}{\kappa}} \\
\Rightarrow \frac{\partial x}{\partial q} &= \frac{1}{\kappa} b_1 (b_1 q)^{\frac{1}{\kappa}-1} \quad \text{and} \quad \frac{q}{x} = \frac{1}{b_1} x^{\kappa-1} = \frac{1}{b_1} (b_1 q)^{\frac{\kappa-1}{\kappa}} \\
\Rightarrow \frac{\partial x}{\partial q} \frac{q}{x} &= \kappa^{-1}.
\end{aligned}$$

I.e. the parameter controlling the severity of the adjustment friction, and thus the “speed of investment”, κ , is reciprocal to the elasticity of the investment-to-capital ratio with respect to q , “Tobin’s q ”.

Risk Free Return In order to be able to also approximate the risk free rate, we add it to our list of variables and find its respective necessary equilibrium condition. The latter is achieved by again applying the Lucas equation (4) to evaluate a claim on one unit of the final good with certainty at the end of *next* period. Following the reasoning before, such an asset’s price v^f would have to satisfy

$$\begin{aligned}
v_t^f &= \mathbb{E}_t [m_{t+1,t} \cdot 1], \text{ for all } t \\
&\Leftrightarrow \mathbb{E}_t [m_{t+1,t}] \frac{1}{v_t^f} - 1 = 0, \text{ for all } t.
\end{aligned}$$

Defining $r_t^f := \frac{1}{v_t^f}$ yields the sought for conditions

$$\mathbb{E}_t [m_{t+1,t}] r_t^f = 1, \text{ for all } t. \tag{35}$$

Note that this also yields a steady state value of $r_{ss}^f = \frac{1}{\beta}$.

Equity Premium The equity premium is finally computed as the expected excess return on equity beyond the risk free rate,

$$ep_{t+1} := \mathbb{E}_t [r_{t+1}^e - r_t^f], \tag{36}$$

which implies a zero steady state premium.

3.3 Empirical targets

We examine all models along their ability to replicate factual German quarterly business cycle statistics. Additionally, we try to match two asset pricing figures, the annual equity premium EP and the annual risk free rate r^f .

Since related versions of the models considered in the present paper are also examined in Heer and Maußner (2013), we decided to stick with their empirical targets to be able to compare our results. Specifically, the chosen RBC statistics are output volatility s_y , relative volatility of investment to output s_i/s_y , working hours to output s_n/s_y , wages to output s_w/s_y , and the contemporary correlation of output to working hours r_{yn} and wages to working hours r_{wn} .²⁷ The respective numerical target values are thus taken from Heer and Maußner (2009), while the empirical equity premium is from Kyriacou, Madsen, and Mase (2004). Consequently, we are left with the task of finding a reasonable target for the real risk free rate of the German economy.

Yet, the way in which Kyriacou, Madsen, and Mase compute their figure of 5.18 for the German equity premium is hard to trace. As a consequence, we determine the German risk free rate target indirectly as follows. First, we take the German prime standard share index, DAX, as our approximation of the German market portfolio and calculate its mean real return over an extended historical performance, including dividend payments.²⁸ In particular, we find a real annual return on equity of 7.67. Now, this figure must, by definition, exceed the sought for risk free rate by 5.18 in order to be consistent with the chosen equity premium target.

We want to remark that our results, i.e. the “goodness of fit” found possible for the considered models, are not particularly sensitive to the chosen

²⁷Note that all macro variables are understood as the respective real aggregate’s cyclical component, i.e. HP-filtered.

²⁸Although the DAX only covers 30 firm shares, it already “represent[s] around 80 percent of the market capitalization listed in Germany.” Cf. the official information from Deutsche Börse AG, as in December 2013, http://dax-indices.com/EN/MediaLibrary/Document/120611_DeutscheBoerse_E_WEB.pdf. The DAX history was officially prolonged backwards until December 1959. However, issues with the chaining of the preceding indices have been discussed, cf. Strehle, Huber, and Maier (1996). Hence, we do not want to pull the available data too far into the past. On the other hand, more good data would improve on the mean as our return on equity estimator. We exogenously balance this tradeoff by taking the 1973/74 oil embargo as our cutting date and thus only consider data as of 1974:Q2.

risk free target rate. Our empirical targets are summarized in table 1.

Table 1: Empirical targets

EP	r^f	s_y	s_i/s_y	s_n/s_y	s_w/s_y	r_{yn}	r_{wn}
5.18	2.49	1.14	2.28	0.69	1.03	0.4	0.27

3.4 Calibration and simulation

Calibration In our models, a unit of time equals three months. The numerical results are heavily driven by the values chosen for the parameters $\alpha, \delta, \rho, \sigma$, the RRA parameter γ and β, ψ, κ . There is direct empirical evidence for the first set of parameters so these are usually chosen very similarly by researchers.²⁹ Therefore, we consider them as fixed throughout the whole paper at the values displayed in table 2. For γ , the possible bandwidth seems less restraint. An authoritative range, $[0, 10]$, is e.g. provided by Mehra and Prescott (1985). However, in order to emphasize on the impact of the household's attitude towards intertemporal consumption substitution, we nevertheless regard the RRA parameter also as fixed. In particular, we choose $\gamma = 2$.³⁰

Table 2: Fixed Parameters

α	δ	ρ	σ	γ
0.27	0.011	0.9	0.0072	2

The remaining parameters, as well as the parameters to be introduced alongside the upcoming extensions to this framework, are understood as free within particular intervals. These intervals are fixed descriptively in the first place. The chosen calibration's (quantitative) implications will be discussed only after the parameters' final determination.

²⁹Cf. Heer and Maußner (2013).

³⁰E.g. Caldara et al. (2012) also consider this RRA calibration amongst others.

With regard to the calibration of β , the DSGE literature displays disagreement, at least within the boundaries of 0.95 (e.g. Schmitt-Grohe and Uribe (2004)) and 0.99999 (Boldrin, Christiano, and Fisher (2001)). These values thus span our considered interval.

In the case of ψ , a comparably tight interval of possible values is much less evident in the literature. Moreover, as ψ is our key parameter, we decided to leave it less restrained a priori. Thus, ψ is centered around the standard case of reciprocal EIS and RRA with 0 and 1 as excluded boundaries. However, note already that the results in Heiberger and Ruf (2014) suggest smaller values of ψ to be more likely to give rise to better empirical performance.

Further, the range of values for κ^{-1} estimated by Abel (1980) in a somewhat different setting provides us with initial empirical evidence regarding the magnitude of κ , suggested to be around 3.³¹ In accordance to that, we choose our interval for κ as the union of the respective intervals in Jermann (1998) and Heer and Maußner (2013), also considering (10), i.e. from above 1 to 9.

Eventually, the weighting parameter ν is chosen such that steady state working hours meet

$$n_{ss} = 0.13,^{32}$$

while the habit parameters χ and λ are considered free within their respective domains.

Simulation Our approach is to set the free parameters' values within the intervals above in order to match the respective models' simulation results to the German real economy at the best. We discretely optimize this match over a grid $\mathcal{A} \subset \mathbb{R}^k$, where k is the number of free parameters in the considered model, calculating the second order approximations of the policy functions g_i and the dynamic h of the model as described above for each parametrization $a \in \mathcal{A}$.

With these approximated solutions, we follow Heer and Maußner (2013) and simulate 300 time series, each of length 80, of the models' variables and

³¹Cf. Abel (1980), p. 75. Note that in order to find the estimates that correspond best to our framework, we must choose the time preference parameter that lies within our allowed interval, 0.95, and an elasticity of substitution between capital and labor of 1, due to our Cobb-Douglas technology. Abel's corresponding interval for κ^{-1} is [0.272, 0.516], i.e. [1.938, 3.677] for κ .

³²Cf. Heer and Maußner (2013).

compute the average outcomes of their moments $\sigma_y, \frac{\sigma_i}{\sigma_y}, \frac{\sigma_n}{\sigma_y}, \frac{\sigma_w}{\sigma_y}, \rho_{yn}$ and ρ_{wn} as the models' counterparts to our empirical targets in obvious notation.³³

The model's risk free rate and the equity premium are computed as the annualized time series averages of a simulation of 500,000 periods along the formulae derived in subsection 3.2. Hence, we actually have to compute *ex post* risk premia, since the computation of any period's return on equity requires knowledge over later periods' quantities.³⁴ Note that this is just in line with the typical computation of empirical return targets.

Altogether, this yields a vector $S_m(a) \in \mathbb{R}^8$ of values implied by the simulation of the model that corresponds to our chosen targets. We accordingly evaluate the models' fit to the empirical data as displayed in table 1, denoted by $S_d \in \mathbb{R}^8$, via some distance measure of the form

$$\text{dist}_A(S_m(a), S_d) := \langle A(S_m(a) - S_d), S_m(a) - S_d \rangle,$$

where A is a positive definite matrix. Within the grid, we search for the parameter values $\hat{a} \in \mathcal{A}$ minimizing this distance. The resulting minimum value is called the model's *score* and is reported alongside the respective models' artificial moments and return figures. In many cases, cf. e.g. in Boldrin, Christiano, and Fisher (2001), the weighting matrix is chosen as diagonal with the reciprocal of the estimates' respective error variances on the diagonal. We, however, follow Heer and Maußner (2013) and Uhlig (2007), in that we weight all statistics equally but quote the asset pricing quantities in percentage notation. Hence, our matching criterion is a slightly modified sum of squared differences between the model's simulated results and the respective empirical targets, where the modification is executed via

$$A = \left[\begin{array}{c|c} 100^2 \cdot I_2 & 0_{2 \times 6} \\ \hline 0_{6 \times 2} & I_6 \end{array} \right].$$

³³Note that also the model's moments are calculated from HP-filtered (artificial) time series.

³⁴For the computation of the risk free asset's return, no such complication arises. This is the case because its *ex post* return coincides with the *ex ante* return due to its risk free nature. To see this, note that the return computation can be decomposed into two steps. First, we derive the price of the risk free asset via its Lucas equation. Next period, we calculate the return by relating the payoff to this price. Yet the payoff is risk free and already known to be 1 with certainty by the time of the purchase such that the *ex post* return is identical to the reciprocal of the price.

4 Model analysis

This section provides the description of the model economies considered. For the sake of exposition, tedious derivations and the final list of respective equilibrium conditions are collected in the accompanying appendix.

4.1 M0: Baseline

We start our model analysis with an EZ variation of Jermann (1998). We also use this less complicated baseline model to demonstrate how we will constantly refer to the results obtained in the previous sections in order to keep the presentation of all models to come well-arranged.

The representative household faces the decision problem (2). For the baseline case we choose the functional form of the within-period utility function as

$$u(c_t, n_t) := c_t,$$

i.e. the household does not value leisure.

From equation (7) we can thus calculate this model's stochastic discount factor as

$$m_{t+1,t} = \beta \left(\frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t V_{t+1}^{1-\gamma}} \right)^{1-\frac{1}{\theta}} \left(\frac{c_{t+1}}{c_t} \right)^{\frac{1-\gamma}{\theta}-1}. \quad (37)$$

Since the household does not care for leisure, it is obvious that the optimal solution here has to satisfy $n_\tau = 1$ instead of equation (5) for an interior solution. The remaining optimality conditions are unchanged. Hence, in the equilibrium conditions, (23) is replaced by

$$n_t = 1, t \geq \tau.$$

The representative firm faces the basic decision problem described in (15).

The equilibrium conditions for the baseline model, (23) and (24) in subsection 2.3 have to be replaced by

$$n_t - 1 = 0,$$

and

$$w_t \leq (1 - \alpha)e^{z_t}n_t^{-\alpha}k_t^\alpha, \quad (38)$$

respectively. The complete list can be found in the appendix. Note that this model's general equilibrium cannot yield a unique wage because every wage that satisfies (38) solves the problem.

The steady state can be computed by the equations given in subsection 2.4, now complemented with

$$n_{ss} = 1.$$

Results Our baseline model's free parameters are β , ψ and κ . First, all considered parameterizations within our grid led to nearly the same output volatility. Second, correlation is not defined for n and third, because of (38), the equilibrium wage is not determined. We are thus left with three targets, namely EP , r^f , and s_i/s_y and choose the free parameters in order to exactly match the data with respect to these. The simulation results and the corresponding parameter values are displayed in tables 3 and 4.

Table 3: Results M0

EP	r^f	s_y	s_i/s_y	s_n/s_y	s_w/s_y	r_{yn}	r_{wn}	Score
Data								
5.18	2.49	1.14	2.28	0.69	1.03	0.4	0.27	
M0								
5.18	2.49	0.89	2.28	0				

Table 4: Free Parameters M0

β	ψ	κ
0.9874	0.04265	7.05

Additionally, we exemplify our previous discussion on the positivity of $\Phi(\cdot)$ for M0. The found parametrization would demand the investment never

to go below eight thousandth of the current capital stock, i.e. the investment-to-capital ratio never to go below 72% its steady state value.³⁵

As a consequence of the reduced list of targets and their matching mentioned above, we do not compute a score value for M0 and neither consider consumption habits in order to improve on the model's empirical performance.

4.2 M1: No labor market frictions

In this section's class of models, we focus on the effect of making the household appreciate leisure. This will result in a fluctuating labor supply below 1 and a unique equilibrium wage allowing us to also target our selected labor market statistics. As announced before, we will analyze this and the upcoming model classes within the three variations presented in subsection 2.5. The corresponding variants, M1.a - M1.c, will thereafter serve as our benchmark models.

We start with a brief summary of this class' structure. The following paragraphs' general statements hold up to potential habit formation.

The representative household faces exactly the decision problem (2) within all of the upcoming three settings and the specification of u will guarantee an interior solution.

The representative firm decides upon the decision problem (15) as already presented in the framework section.

The equilibrium conditions for all three variants therefore are fully characterized by the system of equations in subsection 2.3.

The steady state is computed as described in subsection 2.4.

³⁵ $\kappa^{\frac{1}{1-\kappa}} \delta = 0.7241 \cdot 0.011 = 0.008.$

4.2.1 M1.a: No habits

The stochastic discount factor $m_{t+1,t}$ is determined by (30). Further, by (23), the equilibrium wages necessarily satisfy

$$w_t = \frac{1 - \nu}{\nu} \frac{c_t}{1 - n_t}.$$

Hence, we compute ν as

$$\nu = \frac{c_{ss}}{w_{ss}(1 - n_{ss}) + c_{ss}}.$$

Results The best fit to the data that was achievable for the respective variants within M1 is collectively summarized in table 5 while the score minimizing parameter values can be read from table 6. Both tables can be found at the end of this subsection.

With regard to M1.a, we emphasize on the fact that—in line with the intuition sketched in the introductory remarks (section 1)—making the household appreciate leisure requires a notable change in the EIS parametrization in order to still be able to generate a sizeable equity premium. Specifically, in comparison to M0 the resulting ψ drops by 85 percent. This way, we are able to reproduce the empirical returns. With regard to the RBC targets, apart from the relative volatility of working hours that is too low and the almost perfectly positive labor market correlations, this model’s results are already roughly in line with the empirical data.

4.2.2 M1.b: Standard consumption habits

In this case the stochastic discount factor $m_{t+1,t}$ is given by (33) and condition (23) here reads

$$w_t = \frac{1 - \nu}{\nu} \frac{c_t - c_t^h}{1 - n_t}.$$

Hence,

$$\nu = \frac{(1 - \chi)c_{ss}}{w_{ss}(1 - n_{ss}) + (1 - \chi)c_{ss}}.$$

Results Introducing standard consumption habits, the additional free parameter χ primarily allows us to improve the fit of the labor market correlations. The score drops by nearly two thirds. With consumption habits in the model, we do not have to choose the EIS as low as in M1.a in order to replicate the return figures.

4.2.3 M1.c: Slowly adapting consumption habits

Neither the stochastic discount factor nor the equilibrium conditions are changed in comparison to M1.b.

Results The additional free parameter λ , does not help in further lowering the score. The best fit is found for a standard habit process with $\lambda = 0$ and the remaining optimal parameter values found for M1.b, so that the simulation results are identical to those of M1.b.

Table 5: Summary of Results M1

	EP	r^f	s_y	s_i/s_y	s_n/s_y	s_w/s_y	r_{yn}	r_{wn}	Score
	Data								
	5.18	2.49	1.14	2.28	0.69	1.03	0.40	0.27	
	M1: No labor market frictions								
a	5.19	2.55	1.01	2.27	0.16	0.84	1.00	1.00	1.23
b	5.27	2.50	0.92	2.16	0.12	0.96	0.37	0.26	0.40
c	5.27	2.50	0.92	2.16	0.12	0.96	0.37	0.26	0.40

4.3 M2: Sticky real wages

In this section we add a friction to the labor market by introducing a type of stickiness to the real wages as in Uhlig (2007). Primarily, this stickiness is expected to decrease the volatility of wages and thus increase the volatility of working hours, as the wages’ “buffering” of changes in the productivity of labor is limited. We first analyze the changes in the general framework, again up to potential habit formation, before reporting on the individual variants’ results.

Table 6: Free Parameters M1

	β	ψ	κ	χ	λ
M1: No labor market frictions					
a	0.9873	0.00635	6.3	-	-
b	0.988	0.00715	6.25	0.3	-
c	0.988	0.00715	6.25	0.3	0

The representative household Due to some nonmodeled friction, the household's optimality condition (5) is not necessarily fulfilled in equilibrium anymore. Instead, we implicitly define

$$\frac{\partial u}{\partial c}(c_t, n_t)w_t^f = -\frac{\partial u}{\partial n}(c_t, n_t), \quad (39)$$

introducing a separate symbol for the marginal rate of substitution, w_t^f , denoting the equilibrium wage in an economy that is free of such frictions.

The representative firm again faces the basic decision problem (15).

The equilibrium conditions As mentioned above, the wage w_t in the economy's equilibrium is no longer necessarily equal to the marginal rate of substitution w_t^f of the household. Instead it evolves as a geometric mean of the previous period's wage w_{t-1} and the marginal rate of substitution, i.e.

$$w_t = w_{t-1}^\mu (w_t^f)^{1-\mu}, \mu \in [0, 1]. \quad (40)$$

This way, high changes of the wage between two subsequent periods become less likely.

Summing up, with regard to the equilibrium conditions in subsection 2.3, condition (23) is jointly replaced by both wage equations above. The full list of equations characterizing the equilibrium is also laid out in the appendix

The Steady state Since $\mu \neq 1$, it follows from (40) that in the steady state

$$w_{ss}^f = w_{ss}.$$

We then parameterize ν again using (39) in order to ensure a steady state value of $n_{ss} = 0.13$. Hence, for all considered variants of our sticky wages economy, all the remaining steady state values are identical to their frictionless counterparts of the previous section.

Calibration The additional parameter controlling the degree of wage stickiness is considered free within its domain, i.e. $\mu \in [0, 1)$.

4.3.1 M2.a: No habits

The first variant's stochastic discount factor is given by (30) and the equilibrium condition (39) reads

$$w_t^f = \frac{1 - \nu}{\nu} \frac{c_t}{1 - n_t}. \quad (41)$$

Results The optimal parameter values and the corresponding fit for M2 are again collectively summarized in tables 7 and 8. As in model M1.a, a low value for the EIS is necessary in order to replicate the empirical equity premium. The additional stickiness parameter μ helps in dissolving the strict correlation structure between hours, output and wages found in M1.a. Plus, as anticipated, the relative volatility of hours can also be increased, while on the other hand the relative volatility of wages falls—in fact to about two thirds of its empirical value.

Altogether, the additional degree of freedom allows us to reduce the score of M1.a by more than 50 percent.

4.3.2 M2.b: Standard consumption habits

For the second setting, the stochastic discount factor is given by (33). Further, equation (39) now takes the form

$$w_t^f = \frac{1 - \nu}{\nu} \frac{c_t - c_t^h}{1 - n_t} \quad (42)$$

Results Despite the fact that the possibility of real wage stickiness results in a considerable score reduction from M1.a to M2.a, allowing for standard consumption habits leads to a model with $\mu = 0$. The optimal values of the remaining parameters are thus identical to those found in M1.b, just as the simulation results can be read from table 6.

4.3.3 M2.c: Slowly adapting consumption habits

As in the previous subsection, only alternating the external habit process compared to M2.b does neither change the stochastic discount factor nor the exact form of (39).

Results The consideration of real wage stickiness together with slowly adjusting consumption habits now again leads to $\mu \neq 0$. In comparison to M1.c

Table 7: Summary of Results M2

	EP	r^f	s_y	s_i/s_y	s_n/s_y	s_w/s_y	r_{yn}	r_{wn}	Score
Data									
	5.18	2.49	1.14	2.28	0.69	1.03	0.40	0.27	
M2: Sticky real wages									
a	5.13	2.50	1.23	2.18	0.47	0.64	0.87	0.63	0.58
b	5.27	2.50	0.92	2.16	0.12	0.96	0.37	0.26	0.40
c	5.17	2.50	1.03	2.35	0.43	0.84	0.55	0.15	0.16

Table 8: Free Parameters M2

	β	ψ	κ	χ	λ	μ
M2: Sticky wages						
a	0.9875	0.00588	5.25	-	-	0.5
b	0.988	0.00715	6.25	0.3	-	0
c	0.988	0.007775	5.4	0.49	0.8	0.65

and M2.b, simultaneously allowing for $\mu \neq 0$ and $\lambda \neq 0$ increases the relative volatility of hours by such an amount that the model score can be reduced by nearly 60 percent even though the remaining labor market statistics are hit less exactly. As displayed in table 8, the corresponding parametrization features a rather high persistence in the habit formation and a larger degree of wage stickiness as in M2.a.

4.4 M3: Predetermined labor supply

We now introduce a different friction with respect to labor supply flexibility as we follow Boldrin, Christiano, and Fisher (2001) and require the representative household to commit himself to a certain labor supply one period in advance. Hence, he cannot respond to changes in productivity directly but with a time lag of one period. Again, we first discuss the implications of this change with respect to the general framework up to potential habit formation, before stating the results for the three individual variations considered.

The representative household has to fix his labor supply before the technology shock is revealed, i.e. we consider period τ 's working hours n_τ as a given state variable and the household's decision on $n_{\tau+1}$ may not depend on $\epsilon_{\tau+1}$. Summing up, the representative household's problem reads

$$\begin{aligned} \max U_\tau &= W(u(c_\tau, n_\tau), (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma}}) \\ \text{s.t. } c_t &\leq w_t n_t + d_t s_t - v_t (s_{t+1} - s_t), \\ c_t &\geq 0, 0 \leq n_t \leq 1, \text{ for all } t \geq \tau, \\ &\text{given } s_\tau, n_\tau. \end{aligned} \tag{43}$$

While the necessary optimality conditions (3), (4), (6) remain unchanged, the condition for next period's labor supply is now given by³⁶

$$\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1} + \frac{\partial u}{\partial n}(c_{t+1}, n_{t+1}) \right) \right] = 0, t \geq \tau. \tag{44}$$

The representative firm again faces the basic decision problem (15).

The equilibrium conditions in this model are obtained by replacing equation (23) of subsection 2.3 with (44). The full list is again presented in the appendix.

The steady state With respect to the steady state values, there are no changes to the general framework described in subsection 2.4.

³⁶The detailed derivation can again be found in the appendix.

4.4.1 M3.a: No habits

The stochastic discount factor is given by (30). Further, equation (44) becomes

$$\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1} - \frac{1-\nu}{\nu} \frac{c_{t+1}}{1-n_{t+1}} \right) \right] = 0. \quad (45)$$

Results Again, this class' fit and the respective parametrizations are collectively summarized in table 9 and in table 10.

In comparison to the frictionless counterpart M1.a, the fit of both correlation targets can be improved in M3.a. Yet, in contrast to the sticky wages variant M2.a, we cannot achieve a better fit for the relative volatility of hours while the relative volatility of wages is matched a little more exactly. Altogether, this model's score is virtually the same as in M2.a, even with one free parameter less.

4.4.2 M3.b: Standard consumption habits

We now combine the assumptions of predetermined labor supply and consumption habits to see whether we can further improve our model score particularly with respect to the labor market targets. Now, the stochastic discount factor is given by (33). Moreover with $u(c_t, n_t) = (c_t - c_t^h)^\nu (1 - n_t)^{1-\nu}$ equation (44) turns out as

$$\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1} - \frac{1-\nu}{\nu} \frac{(c_{t+1} - c_{t+1}^h)}{1-n_{t+1}} \right) \right] = 0. \quad (46)$$

Results In comparison to model M1.b the relative volatility of hours decreases even more. Also the correlation between wages and working hours are not matched as exactly. Concluding, in our grid, the score for the variant with standard consumption habits cannot be made smaller than in its frictionless counterpart.

4.4.3 M3.c: Slowly adapting consumption habits

In this variant, again, the stochastic discount factor and the exact form of equation (44) are the same as for M3.b.

Results Adding the possibility of slowly adapting consumption habits to the model with predetermined hours does not further lower the achievable score. The best fit is again found by setting the additional parameter $\lambda = 0$ and the simulation results are therefore identical to those of M3.b.

Table 9: Summary of Results M3

	EP	r^f	s_y	s_i/s_y	s_n/s_y	s_w/s_y	r_{yn}	r_{wn}	Score
Data									
	5.18	2.49	1.14	2.28	0.69	1.03	0.40	0.27	
M3: Predetermined labor supply									
a	5.15	2.49	0.96	2.30	0.14	0.90	0.71	0.62	0.57
b	5.19	2.52	0.92	2.35	0.09	0.97	0.41	0.34	0.43
c	5.19	2.52	0.92	2.35	0.09	0.97	0.41	0.34	0.43

Table 10: Free Parameters M3

	β	ψ	κ	χ	λ
M3: Predetermined labor supply					
a	0.987485	0.00643	6.5	-	-
b	0.98872	0.0073	5.3	0.53	-
c	0.98872	0.0073	5.3	0.53	0

4.5 M4: Sectoral frictions in the allocation of labor

In this section, we follow the approach of Boldrin, Christiano, and Fisher (2001) and further extend our framework in that we decompose the economy's productive part into two sectors, both of which are assumed to be representable by one stand-in firm. The consumption good is produced in one sector, the investment good in the other.³⁷

³⁷Nevertheless, note that M4 is not an EZ variation of the original Boldrin, Christiano, and Fisher (2001) model for we also stick with our adjustment

As in subsection 4.4, there is a representative household who is assumed to be unable to adapt his labor supply to technology shocks in the respective periods but is committed to the hours of labor contracted prior to that period. We now, additionally, require labor to be contracted sector-specific, i.e. the household can neither switch intersectorally within a given period. To sum up, while we already analyzed the effect of predetermination of labor supply in M3, the sector mobility constraint introduces an additional friction into the framework discussed so far.

This model class' structure is considerably different to our basic framework and will thus be introduced in detail in the next paragraphs, again up to potential habit formation.

The representative household For the representative household there are two changes. Since there are two representative firms, one for the consumption sector and one for the investment good sector, he may now allocate working hours, hold shares and receive dividends from either of these. Hence, with the obvious notation, his budget constraint becomes

$$c_t \leq w_t^I n_t^I + w_t^C n_t^C + d_t^I s_t^I + d_t^C s_t^C - v_t^I (s_{t+1}^I - s_t^I) - v_t^C (s_{t+1}^C - s_t^C), t \geq \tau.$$

Further, just like in M3, the household also has to decide on his labor supply one period ahead so that his decision problem reads

$$\begin{aligned} \max U_\tau &= W(u(c_\tau, n_\tau), (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma}}) \\ \text{s.t. } c_t &\leq w_t^I n_t^I + w_t^C n_t^C + d_t^I s_t^I + d_t^C s_t^C - v_t^I (s_{t+1}^I - s_t^I) - v_t^C (s_{t+1}^C - s_t^C), \\ n_t &= n_t^I + n_t^C, \\ c_t &\geq 0, n_t^I \geq 0, n_t^C \geq 0, 0 \leq n_t \leq 1, \text{ for all } t \geq \tau, \\ \text{given } &s_\tau^I, s_\tau^C, n_\tau^I, n_\tau^C. \end{aligned} \tag{47}$$

With these changes, the optimality conditions (3)-(6) for a solution where last row's constraints do not bind are

$$\begin{aligned} V_t &= [(1 - \beta)u(c_t, n_t)^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}, \\ \mathbb{E}_t \left[m_{t+1,t} \frac{d_{t+1}^I + v_{t+1}^I}{v_t^I} - 1 \right] &= 0, \end{aligned}$$

cost assumption.

$$\mathbb{E}_t \left[m_{t+1,t} \frac{d_{t+1}^C + v_{t+1}^C}{v_t^C} - 1 \right] = 0,$$

$$\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1}^I + \frac{\frac{\partial u}{\partial n}(c_{t+1}, n_{t+1})}{\frac{\partial u}{\partial c}(c_{t+1}, n_{t+1})} \right) \right] = 0, \quad (48)$$

$$\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1}^C + \frac{\frac{\partial u}{\partial n}(c_{t+1}, n_{t+1})}{\frac{\partial u}{\partial c}(c_{t+1}, n_{t+1})} \right) \right] = 0, \quad (49)$$

$$n_t = n_t^I + n_t^C,$$

$$c_t = w_t^I n_t^I + w_t^C n_t^C + d_t^I s_t^I + d_t^C s_t^C - v_t^I (s_{t+1}^I - s_t^I) - v_t^C (s_{t+1}^C - s_t^C),$$

with the stochastic discount factor given by (7).

Note that we are only interested in interior solutions with respect to the last row of constraints because sticking to a Cobb-Douglas production technology implies labor demand to always be strictly positive in both sectors, so that in general equilibrium wages have to be set in such a way that also labor supply is strictly positive in both sectors. Particulary, as stated above, the household has to be indifferent in expectation between the wages and returns in both sectors. I.e. it must hold for all $t \geq \tau$ that

$$\mathbb{E}_t [m_{t+1,t}(w_{t+1}^I - w_{t+1}^C)] = 0$$

and further, for the problem to not be unbounded,

$$\mathbb{E}_t \left[m_{t+1,t} \left(\frac{d_{t+1}^I + v_{t+1}^I}{v_t^I} - \frac{d_{t+1}^C + v_{t+1}^C}{v_t^C} \right) \right] = 0.$$

The latter is a no arbitrage condition on the sector-specific stocks.

The representative firm in the consumption good sector produces the consumption good via the technology

$$c_t = e^{z_t} (n_t^C)^{1-\alpha} (k_t^C)^\alpha, \quad \alpha \in (0, 1),$$

where the sequence $\{z_t\}$ follows (8), and accumulates capital according to

$$k_{t+1}^C - (1 - \delta)k_t^C = \Phi \left(\frac{i_t^C}{k_t^C} \right) k_t^C,$$

with $\Phi(\cdot)$ as defined in (10). Investment goods now have to be purchased from the representative firm in the investment sector. Let p_t denote the price of investment relative to consumption. These investment expenditures are again assumed to be financed through profits beyond dividend payments plus the issuance of new shares. Hence, the equivalent to (11) here is

$$p_t i_t^C = c_t - w_t^C n_t^C - d_t^C s_t^C + v_t^C (s_{t+1}^C - s_t^C)$$

and period t 's cash flow is given by

$$cf_t^C := c_t - w_t^C n_t^C - p_t i_t^C.$$

The firm's management again maximizes its firm value, which is defined as above and can, under the respective transversality condition

$$\lim_{t \rightarrow \infty} \mathbb{E}_\tau [m_{t,\tau} v_t^C s_{t+1}^C] = 0,$$

thus be written as

$$fv_\tau^C := cf_\tau^C + v_\tau^C s_{\tau+1}^C = \mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} m_{t,\tau} cf_t^C \right].$$

In other words, the maximization problem of this sector's representative firm is

$$\begin{aligned} \max \mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} m_{t,\tau} (e^{zt} (n_t^C)^{1-\alpha} (k_t^C)^\alpha - w_t^C n_t^C - p_t i_t^C) \right] \\ \text{s.t. } k_{t+1}^C = (1 - \delta) k_t^C + \Phi \left(\frac{i_t^C}{k_t^C} \right) k_t^C, \text{ for all } t \geq \tau, \\ \text{given } k_\tau^C. \end{aligned} \quad (50)$$

The equivalent optimality conditions to (16)-(21) hence are

$$w_t^C = (1 - \alpha) e^{zt} (n_t^C)^{-\alpha} (k_t^C)^\alpha, \quad (51)$$

$$q_t^C = \frac{p_t}{\Phi' \left(\frac{i_t^C}{k_t^C} \right)},$$

$$q_t^C = \mathbb{E}_t \left[m_{t+1,t} \left(\alpha e^{z(t+1)} (n_{t+1}^C)^{1-\alpha} (k_{t+1}^C)^{\alpha-1} - p_{t+1} \frac{i_{t+1}^C}{k_{t+1}^C} + q_{t+1}^C \left(1 - \delta + \Phi \left(\frac{i_{t+1}^C}{k_{t+1}^C} \right) \right) \right) \right], \quad (52)$$

$$\begin{aligned}
k_{t+1}^C &= (1 - \delta)k_t^C + \Phi\left(\frac{i_t^C}{k_t^C}\right)k_t^C, \\
p_t i_t^C &= c_t - w_t^C n_t^C - d_t^C s_t^C + v_t^C (s_{t+1}^C - s_t^C), \\
c_t &= e^{z_t} (k_t^C)^\alpha (n_t^C)^{1-\alpha},
\end{aligned} \tag{53}$$

with $\{z_t\}$ following (8).

The representative firm in the investment good sector produces the investment good via the production function

$$i_t = e^{z_t} (n_t^I)^{1-\alpha} (k_t^I)^\alpha, \quad \alpha \in (0, 1),$$

where the sequence $\{z_t\}$ follows (8), and also accumulates capital according to

$$k_{t+1}^I - (1 - \delta)k_t^I = \Phi\left(\frac{i_t^I}{k_t^I}\right)k_t^I.$$

This firm sells an amount of i_t^C of the investment good to the firm in the consumption good sector. The remaining i_t^I is used for own investments. Its respective equivalent to (11) hence is

$$p_t i_t^I = p_t i_t - w_t^I n_t^I - d_t^I s_t^I + v_t^I (s_{t+1}^I - s_t^I),$$

or equivalently

$$p_t i_t^C - w_t^I n_t^I - d_t^I s_t^I + v_t^I (s_{t+1}^I - s_t^I) = 0$$

and period t 's cash flow is

$$cf_t^I := p_t i_t - w_t^I n_t^I - p_t i_t^C = p_t i_t^C - w_t^I n_t^I.$$

This firm's management maximizes its firm value, again defined as above. Under the respective transversality condition

$$\lim_{t \rightarrow \infty} \mathbb{E}_\tau [m_{t,\tau} v_t^I s_{t+1}^I] = 0,$$

this can be written as

$$fv_\tau^I := cf_\tau^I + v_\tau^I s_{\tau+1}^I = \mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} m_{t,\tau} cf_t^I \right].$$

Consequently, the maximization problem of the representative firm in the investment good sector is

$$\begin{aligned} \max \mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} m_{t,\tau} \left(p_t e^{z_t} (n_t^I)^{1-\alpha} (k_t^I)^\alpha - w_t^I n_t^I - p_t i_t^I \right) \right] \\ \text{s.t. } k_{t+1}^I = (1 - \delta) k_t^I + \Phi \left(\frac{i_t^I}{k_t^I} \right) k_t^I, \text{ for all } t \geq \tau, \\ \text{given } k_\tau^I. \end{aligned} \quad (54)$$

Hence, this sector's equivalent optimality conditions to (16)-(21) are

$$w_t^I = (1 - \alpha) p_t e^{z_t} (n_t^I)^{-\alpha} (k_t^I)^\alpha, \quad (55)$$

$$q_t^I = \frac{p_t}{\Phi' \left(\frac{i_t^I}{k_t^I} \right)},$$

$$q_t^I = \mathbb{E}_t \left[m_{t+1,t} \left(\alpha p_{t+1} e^{z_{t+1}} (n_{t+1}^I)^{1-\alpha} (k_{t+1}^I)^\alpha - p_{t+1} \frac{i_{t+1}^I}{k_{t+1}^I} + q_{t+1}^I \left(1 - \delta + \Phi \left(\frac{i_{t+1}^I}{k_{t+1}^I} \right) \right) \right) \right] \quad (56)$$

$$k_{t+1}^I = (1 - \delta) k_t^I + \Phi \left(\frac{i_t^I}{k_t^I} \right) k_t^I,$$

$$p_t i_t^I = p_t i_t - w_t^I n_t^I - d_t^I s_t^I + v_t^I (s_{t+1}^I - s_t^I),$$

$$i_t = e^{z_t} (k_t^I)^\alpha (n_t^I)^{1-\alpha}, \quad (57)$$

with $\{z_t\}$ following (8).

The equilibrium conditions The general equilibrium for this two sector model is characterized by the optimality conditions listed in the paragraphs above plus the condition

$$i_t = i_t^I + i_t^C.$$

Again, this condition already guarantees a cleared stock market, even if we do not solve for a solution for s_{t+1}^i, d_t^i and $v_t^i, i \in \{C, I\}$. The full list of equilibrium conditions can be found in appendix A.6.

The steady state According to our basic framework, for the steady state we demand that

$$i_{ss}^C = \delta k_{ss}^C \text{ and } i_{ss}^I = \delta k_{ss}^I$$

as well as

$$q_{ss}^C = 1 \text{ and } q_{ss}^I = 1 \text{ and } p_{ss} = 1$$

and thus parameterize Φ as before. Next, equations (52) and (56) first yield

$$k_{ss}^C = \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}} n_{ss}^C \text{ and } k_{ss}^I = \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}} n_{ss}^I. \quad (58)$$

Taking the sum,

$$k_{ss} := k_{ss}^C + k_{ss}^I = \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}} n_{ss}$$

and hence

$$i_{ss} = i_{ss}^C + i_{ss}^I = \delta k_{ss} = \delta \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}} n_{ss}.$$

Using (57) and (58) we calculate

$$\begin{aligned} i_{ss} &= (k_{ss}^I)^\alpha (n_{ss}^I)^{1-\alpha} = \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{\alpha}{\alpha-1}} (n_{ss}^I)^\alpha (n_{ss}^I)^{1-\alpha} \\ \Leftrightarrow n_{ss}^I &= \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{-\frac{\alpha}{\alpha-1}} i_{ss} = \delta \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{-1} n_{ss} \end{aligned} \quad (59)$$

and

$$n_{ss}^C = n_{ss} - n_{ss}^I.$$

With (58) and (59), we find

$$\begin{aligned} k_{ss}^I &= \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}} \delta \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{-1} n_{ss} \\ &= \delta \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{2-\alpha}{\alpha-1}} n_{ss} \end{aligned}$$

and

$$k_{ss}^C = k_{ss} - k_{ss}^I.$$

From (53) we can determine

$$c_{ss} = (k_{ss}^C)^\alpha (1 - n_{ss}^C)^{1-\alpha}.$$

(51) and (55) further yield

$$\begin{aligned} w_{ss}^C &= (1 - \alpha) \left(\frac{k_{ss}^C}{n_{ss}^C} \right)^\alpha = (1 - \alpha) \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{\alpha}{\alpha-1}}, \\ w_{ss}^I &= (1 - \alpha) \left(\frac{k_{ss}^I}{n_{ss}^I} \right)^\alpha = (1 - \alpha) \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{\alpha}{\alpha-1}} = w_{ss}^C. \end{aligned}$$

Now, all steady state variables are expressed in terms of n_{ss} . We use (48) or (49) to again set ν for all three variants of u in such way that a steady state value of

$$n_{ss} = 0.13$$

arises.

Computation of the return series The risk free return is, of course, unaffected by the extension of our basic framework to two productive sectors. What does change, though, is the computation of the return on equity in this economy.

As in the one sector case, in order to be able to derive the formulae needed to compute both sectors' return on equity, we have to impose additional conditions on our two sector economy. Precisely, for both sectors, i.e. for $i \in \{I, C\}$, we assume

$$\lim_{s \rightarrow \infty} \mathbb{E}_t [m_{t+s,t} q_{t+s}^i k_{t+1+s}^i] = 0.$$

By the same reasoning as above, we thus find

$$v_t^i s_{t+1}^i = q_t^i k_{t+1}^i \tag{60}$$

so that the two sectors' period $t + 1$ return on equity both satisfy

$$\begin{aligned} r_{t+1}^{e,C} &:= \frac{d_{t+1}^C + v_{t+1}^C}{v_t^C} = \frac{c_{t+1} - w_{t+1}^C n_{t+1}^C - p_{t+1} i_{t+1}^C + q_{t+1}^C k_{t+2}^C}{q_t^C k_{t+1}^C}, \\ r_{t+1}^{e,I} &:= \frac{d_{t+1}^I + v_{t+1}^I}{v_t^I} = \frac{p_{t+1} i_{t+1}^C - w_{t+1}^I n_{t+1}^I + q_{t+1}^I k_{t+2}^I}{q_t^I k_{t+1}^I}. \end{aligned}$$

Thus, as the overall gross return on firm shares over both sectors is naturally computed as

$$r_{t+1}^{e,C} v_t^C s_{t+1}^C + r_{t+1}^{e,I} v_t^I s_{t+1}^I,$$

we finally reach

$$\begin{aligned} r_{t+1}^e &:= \frac{r_{t+1}^{e,C} v_t^C s_{t+1}^C + r_{t+1}^{e,I} v_t^I s_{t+1}^I}{v_t^C s_{t+1}^C + v_t^I s_{t+1}^I} = \\ &\stackrel{(60)}{=} r_{t+1}^{e,C} \frac{q_t^C k_{t+1}^C}{q_t^C k_{t+1}^C + q_t^I k_{t+1}^I} + r_{t+1}^{e,I} \frac{q_t^I k_{t+1}^I}{q_t^C k_{t+1}^C + q_t^I k_{t+1}^I} \end{aligned}$$

as this economy's return on equity.³⁸

4.5.1 M4.a: No habits

The stochastic discount factor remains the same as in (30). Further, equations (48) and (49) can be written as

$$\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1}^I - \frac{1-\nu}{\nu} \frac{c_{t+1}}{1-n_{t+1}} \right) \right] = 0,$$

and

$$\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1}^C - \frac{1-\nu}{\nu} \frac{c_{t+1}}{1-n_{t+1}} \right) \right] = 0.$$

From these equations it (again) follows that

$$\nu = \frac{c_{ss}}{w_{ss}^C(1-n_{ss}) + c_{ss}} = \frac{c_{ss}}{w_{ss}^I(1-n_{ss}) + c_{ss}}.$$

Results Again, this class' variants' fit and the corresponding parametrization are collectively summarized in table 11 and in table 12.

With respect to M1.a, the lower score is again primarily ascribable to the improvement on the labor market correlation, despite the notable “overshooting” in the relative volatility of wages.

³⁸Note that this departs from Heer and Maußner (2013), who weigh each sector's return on equity by the respective sectors' capital shares only.

4.5.2 M4.b: Standard consumption habits

This variant's stochastic discount factor is given by (33). Equations (48) and (49) become

$$\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1}^I - \frac{1 - \nu}{\nu} \frac{c_{t+1} - c_{t+1}^h}{1 - n_{t+1}} \right) \right] = 0,$$

and

$$\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1}^C - \frac{1 - \nu}{\nu} \frac{c_{t+1} - c_{t+1}^h}{1 - n_{t+1}} \right) \right] = 0.$$

Therefore, we set

$$\nu = \frac{(1 - \chi)c_{ss}}{w_{ss}^C(1 - n_{ss}) + (1 - \chi)c_{ss}} = \frac{(1 - \chi)c_{ss}}{w_{ss}^I(1 - n_{ss}) + (1 - \chi)c_{ss}}.$$

Results Standard consumption habits cannot improve on the achieved fit. Thus, the best fit is found at $\chi = 0$, with the remaining parameter values chosen identically to M4.a. We observe that the more complicated two sector framework is not able to empirically perform as well as the comparable frictionless economy modeled in M1.b.

4.5.3 M4.c: Slowly adapting consumption habits

Generalizing the habit defining process with respect to M4.b, does neither change the discount factor nor the form of equations (48) and (49).

Results The consideration of slowly adjusting consumption habits neither helps to improve the data fit. Hence, the optimal fit is achieved at $\lambda = 0$, with the other parameters chosen as in M4.b.

Table 11: Summary of Results M4

	EP	r^f	s_y	s_i/s_y	s_n/s_y	s_w/s_y	r_{yn}	r_{wn}	Score
	Data								
	5.18	2.49	1.14	2.28	0.69	1.03	0.40	0.27	
	M4: Sectoral frictions in the allocation of labor								
a	5.24	2.49	1.03	2.33	0.26	1.73	0.75	0.13	0.84
b	5.24	2.49	1.03	2.33	0.26	1.73	0.75	0.13	0.84
c	5.24	2.49	1.03	2.33	0.26	1.73	0.75	0.13	0.84

Table 12: Free Parameters M4

	β	ψ	κ	χ	λ
	M4: Sectoral frictions in the allocation of labor				
a	0.9884	0.0086	3.0625	-	-
b	0.9884	0.0086	3.0625	0	-
c	0.9884	0.0086	3.0625	0	0

5 Results and discussion

This section is devoted to the collective presentation of the respective models' results and to their comparative discussion. First, the best fits achievable and the corresponding parametrizations are summarized in tables 13 and 14.

Table 13: Summary of Results

	EP	r^f	s_y	s_i/s_y	s_n/s_y	s_w/s_y	r_{yn}	r_{wn}	Score
	Data								
	5.18	2.49	1.14	2.28	0.69	1.03	0.40	0.27	
M0: Baseline	5.18	2.49	0.89	2.28	0				
M1: No labor market frictions									
a	5.19	2.55	1.01	2.27	0.16	0.84	1.00	1.00	1.23
b	5.27	2.50	0.92	2.16	0.12	0.96	0.37	0.26	0.40
c	5.27	2.50	0.92	2.16	0.12	0.96	0.37	0.26	0.40
M2: Sticky real wages									
a	5.13	2.50	1.23	2.18	0.47	0.64	0.87	0.63	0.58
b	5.27	2.50	0.92	2.16	0.12	0.96	0.37	0.26	0.40
c	5.17	2.50	1.03	2.35	0.43	0.84	0.55	0.15	0.16
M3: Predetermined labor supply									
a	5.15	2.49	0.96	2.30	0.14	0.90	0.71	0.62	0.57
b	5.19	2.52	0.92	2.35	0.09	0.97	0.41	0.34	0.43
c	5.19	2.52	0.92	2.35	0.09	0.97	0.41	0.34	0.43
M4: Sectoral frictions in the allocation of labor									
a	5.24	2.49	1.03	2.33	0.26	1.73	0.75	0.13	0.84
b	5.24	2.49	1.03	2.33	0.26	1.73	0.75	0.13	0.84
c	5.24	2.49	1.03	2.33	0.26	1.73	0.75	0.13	0.84

Since all models in the present paper are EZ variations with Cobb-Douglas composite good aggregation of the corresponding models analyzed in Heer and Maußner (2013), we first want to point out that, due to the more general utility representation, we were able to considerably improve on their reported data fit. Note that next to the extra degree of freedom associated with the EZ representation with regard to ψ , we thereby also considered β as free in order to additionally target the German real risk-free rate.

Model evaluation As foreshadowed in the analysis of M0 above, we must flexibilize the household's labor decision in order to arrive at models that

Table 14: Free Parameters

	β	ψ	κ	χ	λ	μ
M0: Benchmark	0.9874	0.04265	7.05	-	-	-
M1: No labor market frictions						
a	0.9873	0.00635	6.3		-	-
b	0.988	0.00715	6.25	0.3	-	-
c	0.988	0.00715	6.25	0.3	0	-
M2: Sticky wages						
a	0.9875	0.00588	5.25	-	-	0.5
b	0.988	0.00715	6.25	0.3	-	0
c	0.988	0.007775	5.4	0.49	0.8	0.65
M3: Predetermined labor supply						
a	0.987485	0.00643	6.5	-	-	-
b	0.98872	0.0073	5.3	0.53	-	-
c	0.98872	0.0073	5.3	0.53	0	-
M4: Sectoral frictions in the allocation of labor						
a	0.9884	0.0086	3.0625	-	-	-
b	0.9884	0.0086	3.0625	0	-	-
c	0.9884	0.0086	3.0625	0	0	-

allow for the targeting of labor market statistics. Now, M1.a shows that it is still possible to match our return targets but the corresponding calibration yields simulation results with nearly perfectly positive correlations between output and working hours and between wages and working hours as well as a rather low relative volatility of working hours. Allowing for standard consumption habits clears away these perfect correlations. As a matter of fact, the corresponding simulation results already might very well be regarded in line with the empirical evidence. Generalizing the assumed habit formation towards slowly adjustment, however, does not yield any improvement regarding the data fit. We again stress the fact that the range we allowed for ψ was broad enough in order to prevent the equity premium from dropping, as it dramatically happens in Heer and Maußner (2013).

In M2, M3 and M4, we study to what extent different real labor market frictions are—within this framework—able to help in improving on the data fit already achieved by M1.

We start with the analysis of real wage stickiness. The comparison of M1.a and M2.a shows that allowing for wage stickiness also dissolves the rigid correlation structure between output, hours and wages—albeit not as much as the introduction of habits within M1—and improves on the volatility of working hours. Now, while the consideration of real wage stickiness does not improve on the data fit under standard consumption habits, also allowing for these habits’ slow adjustment most notably further improves on the considered correlations, leading to our overall minimum score.

An alternative friction, predetermined labor supply (M3), also initially moves ρ_{yn} and ρ_{wn} towards the data. As within M1, the assumption of standard habits again further improves on these correlations while allowing for $\lambda \neq 0$ does not help in lowering the model score. It is worth pointing at the fact that M3.b and M3.c do not empirically outperform their frictionless counterparts.

Eventually, in the two sector framework, M4.a also yields better simulated labor market correlations than M1.a. Yet, the relative volatility of wages rises to nearly 170 percent of its empirical value rendering the overall fit inferior to the pure predetermined hours model. Moreover, the considered forms of habit formation cannot improve on the score. We want to stress on the fact that M4.b and M4.c are not able to yield simulation results that are as well in accordance with the data as M1.b and M1.c.

The pairwise comparison of the three considered labor market frictions clearly attributes the largest score improvement to the modeling device of sticky wages, which introduces an additional free parameter μ . Checked against the pure predetermined labor class, M4 cannot justify its more complicated structure through empirical performance.

Implications So far, our analysis was kept descriptive in that we objectively fixed intervals for the free parameters via their respective domains and partly via observable consensus in the literature. In particular, we hitherto did not bother about the found parametrizations’ behavioral implications on our representative household. We thus want to complement our analysis with a few—partly summarizing, partly normative—remarks on the resulting values of our free parameters.

First, the range of score minimizing values for β , $[0.9873, 0.98872]$, is much smaller than initially anticipated. To put it another way, the targeting of r^f does not require remarkably different levels of impatience along the models considered.

For reasons laid out in subsection 2.5, the parametrization of ψ is crucial for our models' data fit. Specifically, EIS controls the household's sensitivity to deviations from a smooth composite good path. The smaller ψ , the higher his sensitivity. Now, the only way the household can transfer consumption intertemporally is provided by our models' asset market, namely via the purchase of stocks or the riskfree security. Thus, decreasing ψ makes the household demand a higher compensation for him taking the risk of a stock investment, which leads to a larger return on equity. The range of values for ψ we actually found to optimize the data fit, $[0.00588, 0.04265]$, was already broad enough to match the empirical equity premium in all our models by an accuracy of less than a decimal. Caution must nevertheless be paid to this resulting magnitude of ψ , which is rather close to the lower boundary of the interval initially allowed. Although Hall reports on confirmatory estimates leading him to the conclusion that “the elasticity is unlikely to be much above 0.1 [...]”,³⁹ we have to be aware that the disentanglement of EIS and RRA within the EZ framework can only be partly in nature. This is because any deviation from the standard case of $\psi^{-1} = \gamma$, i.e. in our case any deviation from $\psi = 0.5$, gives rise to nonindifference towards the temporal resolution of uncertainty regarding the composite good.⁴⁰ More precisely, as in our models all score minimizing values of ψ clearly satisfy $\psi^{-1} > \gamma$, we are actually simulating economies where the stand-in agent is assumed to have a preference for *later* resolution of uncertainty. Importantly, the above interval of optimizing values for ψ noticeably indicates a deviation from the typically assumed expected utility framework.

As pointed out above, the reciprocal of κ is the elasticity of the investment-to-capital ratio with respect to Tobin's q . Thus, M0 and the classes M1-M3 roughly span its interval as $[0.14, 0.19]$, close to the value found by Jermann (1998). The two sector class M4 yields a notably higher elasticity of about 0.33.⁴¹

³⁹Cf. Hall (1988), p. 340.

⁴⁰Moreover, RRA and EIS are both identified in the absence of what respectively characterizes the other, namely intertemporality or uncertainty.

⁴¹Note that the two sector framework forces Heer and Maußner (2013), p. 19, to assume “negligible adjustment costs”, i.e. an enormous elasticity of 200.

Regarding standard consumption habits, by (31) and (32), the chosen value for χ seems to critically affect the plausibility of the assumed habit formation. The resulting range of values, $[0.3, 0.53]$, indicate a moderate and thus maybe a more easily agreeable degree of habit formation as e.g. found by Jermann (1998), Uhlig (2007) or Heer and Maußner (2013).

Among the four models M1.c, M2.c, M3.c and M4.c, only the sticky wages framework actually indicates slow adjustment of consumption habits. The score minimizing value of $\lambda = 0.8$ is of notable magnitude and close to Uhlig (2007)'s calibration with 0.9. The resulting values for μ , 0.5 and 0.65, display a medium degree of real wage stickiness, again well below the score minimizers found in Uhlig (2007) or Heer and Maußner (2013).

6 Conclusion

Within the EZ utility representation, frictionless models already yield simulation results in good accordance with the German empirical data. Amongst the considered labor market frictions, allowing for real wage stickiness leads to the most remarkable improvement in fit, while, under habit formation, predetermined labor supply, with or without additionally decomposing the production sector into two parts, could not further improve on the frictionless models' empirical performance.

In a sense, a researcher considering policy evaluation on the basis of an EZ framework might look at the information collected in tables 13 and 14 as initial guidance with respect to the specification of his DSGE economy.

With respect to the standard additive power utility model, the additional flexibility of the EZ framework seems to help in avoiding such extreme parametrizations as found necessary in Heer and Maußner (2013). The degree of additional flexibility, however, primarily hinges on the allowed magnitude of deviations from the standard case of $\theta = 1$. Yet, since there is no obvious reason for such a nonindifference towards the timing of uncertainty resolution, any large deviation from $\theta = 1$ calls for justification. It would therefore be interesting to quantitatively assess the plausibility of the implied preference for later resolution that results in our analysis. This could e.g. be done along the lines of Epstein, Farhi, and Strzalecki (2014) and Kaltenbrunner and Lochstoer (2010). While the former authors present such a quantitative measure within a long run risk (LRR) framework, the latter study how endogenous long run consumption risk arises in M0.

A Appendix

A.1 Framework

We derive the optimality conditions from (2) for the representative household's maximization problem. Since an optimal solution has to fulfill the first constraint with equality, we can plug it into the objective function. Also, in almost all of the considered cases it will be obvious that the solution has to be interior with respect to the remaining two constraints, i.e. it satisfies $c_t > 0$ and $n_t \in (0, 1)$. Hence, we state the corresponding necessary optimality conditions, i.e. we set the derivatives of the objective function U_τ , with the first constraint plugged in, equal to zero.

With respect to $s_{\tau+1}$, we find the first condition for an interior optimum

$$\begin{aligned} 0 &= \frac{\partial W}{\partial u} \frac{\partial u}{\partial c} (-v_\tau) + \frac{\partial W}{\partial \mu} \frac{1}{1-\gamma} (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma}-1} (1-\gamma) \mathbb{E}_\tau \left[U_{\tau+1}^{-\gamma} \frac{\partial U_{\tau+1}}{\partial s_{\tau+1}} \right] \\ &= -\frac{\partial W}{\partial u} \frac{\partial u}{\partial c} v_\tau + \frac{\partial W}{\partial \mu} (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma}-1} \mathbb{E}_\tau \left[U_{\tau+1}^{-\gamma} \frac{\partial U_{\tau+1}}{\partial s_{\tau+1}} \right], \end{aligned}$$

where $\frac{\partial W}{\partial u}$ is short for $\frac{\partial W}{\partial u}(u(c_\tau, n_\tau), (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma}})$, analogously for $\frac{\partial W}{\partial \mu}$. $\frac{\partial u}{\partial c}$ abbreviates $\frac{\partial u}{\partial c}(c_\tau, n_\tau)$. We thus calculate

$$\begin{aligned} \frac{\partial W}{\partial u} &= \frac{\theta}{1-\gamma} [\dots]^{\frac{\theta}{1-\gamma}-1} (1-\beta) \frac{1-\gamma}{\theta} u(c_\tau, n_\tau)^{\frac{1-\gamma}{\theta}-1} \\ &= (1-\beta) [\dots]^{\frac{\theta}{1-\gamma}(1-\frac{1-\gamma}{\theta})} u(c_\tau, n_\tau)^{\frac{1-\gamma}{\theta}-1} \\ &= (1-\beta) U_\tau^{1-\frac{1-\gamma}{\theta}} u(c_\tau, n_\tau)^{\frac{1-\gamma}{\theta}-1}, \\ \frac{\partial W}{\partial \mu} &= \frac{\theta}{1-\gamma} [\dots]^{\frac{\theta}{1-\gamma}-1} \beta \frac{1-\gamma}{\theta} (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma}(\frac{1-\gamma}{\theta}-1)} \\ &= \beta [\dots]^{\frac{\theta}{1-\gamma}(1-\frac{1-\gamma}{\theta})} (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{\theta}-\frac{1}{1-\gamma}} \\ &= \beta U_\tau^{1-\frac{1-\gamma}{\theta}} (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{\theta}-\frac{1}{1-\gamma}} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial U_{\tau+1}}{\partial s_{\tau+1}} &= \frac{\partial W}{\partial u}(u(c_{\tau+1}, n_{\tau+1}), (\mathbb{E}_{\tau+1} U_{\tau+2}^{1-\gamma})^{\frac{1}{1-\gamma}}) \frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1})(d_{\tau+1} + v_{\tau+1}) \\ &= (1-\beta) U_{\tau+1}^{1-\frac{1-\gamma}{\theta}} u(c_{\tau+1}, n_{\tau+1})^{\frac{1-\gamma}{\theta}-1} \frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1})(d_{\tau+1} + v_{\tau+1}). \end{aligned}$$

Combining these equations, we finally reach at

$$\begin{aligned}
0 &= - (1 - \beta) U_\tau^{1 - \frac{1-\gamma}{\theta}} u(c_\tau, n_\tau)^{\frac{1-\gamma}{\theta} - 1} \frac{\partial u}{\partial c}(c_\tau, n_\tau) v_\tau + \\
&\quad + \beta U_\tau^{1 - \frac{1-\gamma}{\theta}} (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{\theta} - \frac{1}{1-\gamma}} (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma} - 1} \\
&\quad \mathbb{E}_\tau \left[U_{\tau+1}^{-\gamma} (1 - \beta) U_{\tau+1}^{1 - \frac{1-\gamma}{\theta}} u(c_{\tau+1}, n_{\tau+1})^{\frac{1-\gamma}{\theta} - 1} \frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1}) (d_{\tau+1} + v_{\tau+1}) \right] \\
&= (1 - \beta) U_\tau^{1 - \frac{1-\gamma}{\theta}} u(c_\tau, n_\tau)^{\frac{1-\gamma}{\theta} - 1} \frac{\partial u}{\partial c}(c_\tau, n_\tau) v_\tau \cdot \\
&\quad \left(\beta (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{\theta} - 1} \mathbb{E}_\tau \left[U_{\tau+1}^{(1-\gamma)(1-\frac{1}{\theta})} \left(\frac{u(c_{\tau+1}, n_{\tau+1})}{u(c_\tau, n_\tau)} \right)^{\frac{1-\gamma}{\theta} - 1} \right. \right. \\
&\quad \left. \left. \frac{\frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1}) (d_{\tau+1} + v_{\tau+1})}{\frac{\partial u}{\partial c}(c_\tau, n_\tau) v_\tau} \right] - 1 \right) \\
&= (1 - \beta) U_\tau^{1 - \frac{1-\gamma}{\theta}} u(c_\tau, n_\tau)^{\frac{1-\gamma}{\theta} - 1} \frac{\partial u}{\partial c}(c_\tau, n_\tau) v_\tau \cdot \\
&\quad \mathbb{E}_\tau \left[\beta \left(\frac{U_{\tau+1}^{1-\gamma}}{\mathbb{E}_\tau U_{\tau+1}^{1-\gamma}} \right)^{1 - \frac{1}{\theta}} \left(\frac{u(c_{\tau+1}, n_{\tau+1})}{u(c_\tau, n_\tau)} \right)^{\frac{1-\gamma}{\theta} - 1} \frac{\frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1}) (d_{\tau+1} + v_{\tau+1})}{\frac{\partial u}{\partial c}(c_\tau, n_\tau) v_\tau} - 1 \right].
\end{aligned}$$

Writing V_t for the value function as of period t to the dynamic optimization problem above and using $V_\tau = U_\tau$, if we evaluate U_τ at the optimal processes it follows that⁴²

$$\mathbb{E}_\tau \left[m_{\tau+1, \tau} \frac{d_{\tau+1} + v_{\tau+1}}{v_\tau} - 1 \right] = 0,$$

where

$$m_{\tau+1, \tau} := \beta \left(\frac{V_{\tau+1}^{1-\gamma}}{\mathbb{E}_\tau V_{\tau+1}^{1-\gamma}} \right)^{1 - \frac{1}{\theta}} \left(\frac{u(c_{\tau+1}, n_{\tau+1})}{u(c_\tau, n_\tau)} \right)^{\frac{1-\gamma}{\theta} - 1} \frac{\frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1})}{\frac{\partial u}{\partial c}(c_\tau, n_\tau)} \quad (61)$$

is the household's stochastic discount factor.

Second, differentiating with respect to n_τ reveals the second optimality

⁴²Note also the positivity of lifetime utility, within period utility, marginal utility and the stock price.

condition

$$\begin{aligned}
0 &= \frac{\partial W}{\partial u}(u(c_\tau, n_\tau), (\mathbb{E}_\tau U_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma}}) \left(\frac{\partial u}{\partial c}(c_\tau, n_\tau) w_\tau + \frac{\partial u}{\partial n}(c_\tau, n_\tau) \right) \\
\Leftrightarrow \quad \frac{\partial u}{\partial c}(c_\tau, n_\tau) w_\tau &= -\frac{\partial u}{\partial n}(c_\tau, n_\tau).
\end{aligned} \tag{62}$$

A.2 M0

For a general equilibrium in M0 it has to hold that for all $t \geq \tau$

$$\begin{aligned}
V_t - [(1 - \beta)c_t^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}} &= 0, \\
n_t - 1 &= 0, \\
w_t &\leq (1 - \alpha)e^{z_t} n_t^{-\alpha} k_t^\alpha, \\
q_t - \frac{1}{\Phi' \left(\frac{i_t}{k_t} \right)} &= 0, \\
q_t - \mathbb{E}_t \left[m_{t+1,t} \left(\alpha e^{z_{t+1}} n_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \Phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) \right) \right] &= 0, \\
k_{t+1} - (1 - \delta)k_t - \Phi \left(\frac{i_t}{k_t} \right) k_t &= 0, \\
y_t - e^{z_t} k_t^\alpha n_t^{1-\alpha} &= 0, \\
y_t - c_t - i_t &= 0,
\end{aligned} \tag{63}$$

where the sequence $\{z_t\}$ follows (8) and $m_{t+1,t}$ is given by (37).

A.3 M1

For a general equilibrium in M1 it has to hold that for all $t \geq \tau$

$$\begin{aligned}
V_t - [(1 - \beta)u(c_t, n_t)^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}} &= 0, \\
\frac{\partial u}{\partial c}(c_t, n_t) w_t &= -\frac{\partial u}{\partial n}(c_t, n_t), \\
w_t - (1 - \alpha)e^{z_t} n_t^{-\alpha} k_t^\alpha &= 0,
\end{aligned}$$

$$\begin{aligned}
q_t - \frac{1}{\Phi' \left(\frac{i_t}{k_t} \right)} &= 0, \\
q_t - \mathbb{E}_t \left[m_{t+1,t} \left(\alpha e^{z_{t+1}} n_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \Phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) \right) \right] &= 0, \\
k_{t+1} - (1 - \delta)k_t - \Phi \left(\frac{i_t}{k_t} \right) k_t &= 0, \\
y_t - e^{z_t} k_t^\alpha n_t^{1-\alpha} &= 0, \\
y_t - c_t - i_t &= 0,
\end{aligned}$$

where the sequence $\{z_t\}$ follows (8) and the stochastic discount factor is determined by (7).

A.4 M2

For a general equilibrium in M2 it has to hold that for all $t \geq \tau$

$$\begin{aligned}
V_t - [(1 - \beta)u(c_t, n_t)^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}} &= 0, \\
\frac{\partial u}{\partial c}(c_t, n_t)w_t^f + \frac{\partial u}{\partial n}(c_t, n_t) &= 0, \\
w_t - w_{t-1}^\mu (w_t^f)^{1-\mu} &= 0, \\
w_t - (1 - \alpha)e^{z_t} n_t^{-\alpha} k_t^\alpha &= 0, \\
q_t - \frac{1}{\Phi' \left(\frac{i_t}{k_t} \right)} &= 0, \\
q_t - \mathbb{E}_t \left[m_{t+1,t} \left(\alpha e^{z_{t+1}} n_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \Phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) \right) \right] &= 0, \\
k_{t+1} - (1 - \delta)k_t - \Phi \left(\frac{i_t}{k_t} \right) k_t &= 0, \\
y_t - e^{z_t} k_t^\alpha n_t^{1-\alpha} &= 0, \\
y_t - c_t - i_t &= 0,
\end{aligned}$$

where the sequence $\{z_t\}$ follows (8) and $m_{t+1,t}$ is given by (7).

A.5 M3

Household Equation (44) can be derived as follows. First, note that the first restriction again has to be fulfilled with equality in an optimum. Hence substituting for $c_t, t \geq \tau$, in the objective function and differentiating with respect to $n_{\tau+1}$ yields

$$\begin{aligned}
0 &= \frac{\partial W}{\partial \mu} \frac{1}{1-\gamma} (\mathbb{E}_\tau V_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma}-1} \mathbb{E}_\tau \left[(1-\gamma) V_{\tau+1}^{-\gamma} \frac{\partial V_{\tau+1}}{\partial n_{\tau+1}} \right] \\
&= \beta V_\tau^{1-\frac{1-\gamma}{\theta}} (\mathbb{E}_\tau V_{\tau+1}^{1-\gamma})^{\frac{1}{\theta}-\frac{1}{1-\gamma}} (\mathbb{E}_\tau V_{\tau+1}^{1-\gamma})^{\frac{1}{1-\gamma}-1} \mathbb{E}_\tau \left[V_{\tau+1}^{-\gamma} \frac{\partial V_{\tau+1}}{\partial n_{\tau+1}} \right] \\
&= V_\tau^{1-\frac{1-\gamma}{\theta}} \mathbb{E}_\tau \left[\beta (\mathbb{E}_\tau V_{\tau+1}^{1-\gamma})^{\frac{1}{\theta}-1} V_{\tau+1}^{-\gamma} \frac{\partial W}{\partial u} \left(\frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1}) w_{\tau+1} + \frac{\partial u}{\partial n}(c_{\tau+1}, c_{\tau+1}) \right) \right] \\
&= V_\tau^{1-\frac{1-\gamma}{\theta}} \mathbb{E}_\tau \left[\beta (\mathbb{E}_\tau V_{\tau+1}^{1-\gamma})^{\frac{1}{\theta}-1} V_{\tau+1}^{-\gamma} (1-\beta) V_{\tau+1}^{1-\frac{1-\gamma}{\theta}} u(c_{\tau+1}, n_{\tau+1})^{\frac{1-\gamma}{\theta}-1} \cdot \left(\frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1}) w_{\tau+1} + \frac{\partial u}{\partial n}(c_{\tau+1}, c_{\tau+1}) \right) \right] \\
&= (1-\beta) V_\tau^{1-\frac{1-\gamma}{\theta}} \mathbb{E}_\tau \left[\beta (\mathbb{E}_\tau V_{\tau+1}^{1-\gamma})^{\frac{1}{\theta}-1} V_{\tau+1}^{(1-\gamma)(1-\frac{1}{\theta})} u(c_{\tau+1}, n_{\tau+1})^{\frac{1-\gamma}{\theta}-1} \cdot \left(\frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1}) w_{\tau+1} + \frac{\partial u}{\partial n}(c_{\tau+1}, c_{\tau+1}) \right) \right] \\
&= (1-\beta) V_\tau^{1-\frac{1-\gamma}{\theta}} \mathbb{E}_\tau \left[\beta \left(\frac{V_{\tau+1}^{(1-\gamma)}}{\mathbb{E}_\tau V_{\tau+1}^{1-\gamma}} \right)^{1-\frac{1}{\theta}} u(c_{\tau+1}, n_{\tau+1})^{\frac{1-\gamma}{\theta}-1} \cdot \left(\frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1}) w_{\tau+1} + \frac{\partial u}{\partial n}(c_{\tau+1}, c_{\tau+1}) \right) \right] \\
&= (1-\beta) V_\tau^{1-\frac{1-\gamma}{\theta}} \mathbb{E}_\tau \left[m_{\tau+1, \tau} u(c_\tau, n_\tau)^{\frac{1-\gamma}{\theta}-1} \frac{\frac{\partial u}{\partial c}(c_\tau, n_\tau)}{\frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1})} \cdot \left(\frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1}) w_{\tau+1} + \frac{\partial u}{\partial n}(c_{\tau+1}, c_{\tau+1}) \right) \right] \\
&= (1-\beta) V_\tau^{1-\frac{1-\gamma}{\theta}} u(c_\tau, n_\tau)^{\frac{1-\gamma}{\theta}-1} \frac{\partial u}{\partial c}(c_\tau, n_\tau) \mathbb{E}_\tau \left[m_{\tau+1, \tau} \left(w_{\tau+1} + \frac{\frac{\partial u}{\partial n}(c_{\tau+1}, c_{\tau+1})}{\frac{\partial u}{\partial c}(c_{\tau+1}, n_{\tau+1})} \right) \right],
\end{aligned}$$

where in the next to last step we used equation (7) for the stochastic discount factor.

Equilibrium conditions For a general equilibrium in M3 it has to hold that for all $t \geq \tau$

$$\begin{aligned}
V_t - [(1 - \beta)u(c_t, n_t)^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}} &= 0, \\
\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1} + \frac{\frac{\partial u}{\partial n}(c_{t+1}, n_{t+1})}{\frac{\partial u}{\partial c}(c_{t+1}, n_{t+1})} \right) \right] &= 0, \\
w_t - (1 - \alpha)e^{z_t} n_t^{-\alpha} k_t^\alpha &= 0, \\
q_t - \frac{1}{\Phi' \left(\frac{i_t}{k_t} \right)} &= 0, \\
q_t - \mathbb{E}_t \left[m_{t+1,t} \left(\alpha e^{z_{t+1}} n_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \Phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) \right) \right] &= 0, \\
k_{t+1} - (1 - \delta)k_t - \Phi \left(\frac{i_t}{k_t} \right) k_t &= 0, \\
y_t - e^{z_t} k_t^\alpha n_t^{1-\alpha} &= 0, \\
y_t - c_t - i_t &= 0,
\end{aligned}$$

where the sequence $\{z_t\}$ follows (8) and the stochastic discount factor is given by (7).

A.6 M4

For a general equilibrium in M4 it has to hold that for all $t \geq \tau$

$$\begin{aligned}
V_t &= [(1 - \beta)((c_t - c_t^h)^\nu (1 - n_t)^{1-\nu})^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}, \\
\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1}^I + \frac{\frac{\partial u}{\partial n}(c_{t+1}, n_{t+1})}{\frac{\partial u}{\partial c}(c_{t+1}, n_{t+1})} \right) \right] &= 0, \\
\mathbb{E}_t \left[m_{t+1,t} \left(w_{t+1}^C + \frac{\frac{\partial u}{\partial n}(c_{t+1}, n_{t+1})}{\frac{\partial u}{\partial c}(c_{t+1}, n_{t+1})} \right) \right] &= 0, \\
n_t &= n_t^I + n_t^C, \\
w_t^C &= (1 - \alpha)e^{z_t} (n_t^C)^{-\alpha} (k_t^C)^\alpha,
\end{aligned}$$

$$q_t^C = \frac{p_t}{\Phi' \left(\frac{i_t^C}{k_t^C} \right)},$$

$$q_t^C = \mathbb{E}_t \left[m_{t+1,t} \left(\alpha e^{z_{t+1}} (n_{t+1}^C)^{1-\alpha} (k_{t+1}^C)^{\alpha-1} - p_{t+1} \frac{i_{t+1}^C}{k_{t+1}^C} + q_{t+1}^C \left(1 - \delta + \Phi \left(\frac{i_{t+1}^C}{k_{t+1}^C} \right) \right) \right) \right]$$

$$k_{t+1}^C = (1 - \delta) k_t^C + \Phi \left(\frac{i_t^C}{k_t^C} \right) k_t^C,$$

$$c_t = e^{z_t} (k_t^C)^\alpha (n_t^C)^{1-\alpha},$$

$$w_t^I = (1 - \alpha) p_t e^{z_t} (n_t^I)^{-\alpha} (k_t^I)^\alpha,$$

$$q_t^I = \frac{p_t}{\Phi' \left(\frac{i_t^I}{k_t^I} \right)},$$

$$q_t^I = \mathbb{E}_t \left[m_{t+1,t} \left(\alpha p_{t+1} e^{z_{t+1}} (n_{t+1}^I)^{1-\alpha} (k_{t+1}^I)^{\alpha-1} - p_{t+1} \frac{i_{t+1}^I}{k_{t+1}^I} + q_{t+1}^I \left(1 - \delta + \Phi \left(\frac{i_{t+1}^I}{k_{t+1}^I} \right) \right) \right) \right]$$

$$k_{t+1}^I = (1 - \delta) k_t^I + \Phi \left(\frac{i_t^I}{k_t^I} \right) k_t^I,$$

$$i_t = e^{z_t} (k_t^I)^\alpha (n_t^I)^{1-\alpha},$$

$$i_t = i_t^C + i_t^I,$$

where the sequence $\{z_t\}$ follows (8) and the stochastic discount factor is given by (7).

A.7 Documentation of computation routines

In order to find the models' respective perturbations, we employed the `Maple-Matlab` toolbox introduced in Heiberger and Ruf (2014). For the simulation and evaluation, we essentially added two procedures. On the one hand, `mom2` computes the second moments of our models' variables. In particular, after either loading or generating 300 pseudorandom iid $N(0,1)$ shock series of length 80, it simulates the induced time paths of the state and control variables from their respective (second order) approximations. Second, depending on the user's choice, the procedure computes the second moments from the plain time paths (`mode = 0`) or particular manipulations thereof such as e.g. their natural log (`mode = 1`), their growth rates (`mode = 3`), or

log differences (`mode = 5`). Thereby, if `hp = 1`, the HP-filter is applied by calling the respective `Matlab` routine.

On the other hand, `prem_mxx_lang` computes model `Mx.x`'s simulated ex post return figures. Therefore, it first loads a pseudorandom iid $N(0,1)$ shock series of length 500,000 and then simulates the induced time paths of all variables along their (second order) approximations.⁴³ Second, it uses the models' return formulae and accordingly computes ex post averages of the risk free rate, the return on equity and the equity premium.

The programs were run on `Maple 17` and `Matlab 2013a`.⁴⁴

⁴³Note that in both `mom2` and `prem_mxx_lang`, the path to the shock series has to be specified correctly.

⁴⁴The employed version of the `Maple/Matlab` toolbox can be downloaded from http://www.wiwi.uni-augsburg.de/vwl/maussner/lehrstuhl/heiberger_en.html and http://www.wiwi.uni-augsburg.de/vwl/maussner/lehrstuhl/ruf_en.html.

References

- Abel, Andrew B. 1980. "Empirical Investment Equations: An Integrative Framework." *Carnegie-Rochester Conference Series on Public Policy* 12 (January): 39–91.
- . 1990. "Asset Prices under Habit Formation and Catching up with the Joneses." *American Economic Review* 80 (2): 38–42 (May).
- Altug, Sumru, and Pamela Labadie. 2008. *Asset Pricing for Dynamic Economies*. Cambridge University Press.
- Boldrin, Michele, Lawrence J. Christiano, and Jonas D. M. Fisher. 2001. "Habit Persistence, Asset Returns, and the Business Cycle." *American Economic Review* 91 (1): 149–166 (March).
- Caldara, Dario, Jesús Fernández-Villaverde, Juan F. Rubio-Ramírez, and Wen Yao. 2012. "Computing DSGE Models with Recursive Preferences and Stochastic Volatility." *Review of Economic Dynamics* 15 (April): 188–206.
- Campbell, John Y., and John H. Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107 (2): 205–251 (April).
- Epstein, Larry, Emmanuel Farhi, and Tomasz Strzalecki. 2014. "How Much Would You Pay to Resolve Long-Run Risk?" *American Economic Review* 104 (9): 2680–2697 (September).
- Epstein, Larry G., and Stanley E. Zin. 1989. "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica* 57 (July): 937–969.
- Hall, Robert E. 1988. "Intertemporal Substitution in Consumption." *Journal of Political Economy* 96 (April): 339–357.
- Hayashi, Fumio. 1982. "Tobin's Marginal q and Average q : A Neoclassical Interpretation." *Econometrica* 50 (1): 213–224 (January).
- Heer, Burkhard, and Alfred Maußner. 2009. *Dynamic General Equilibrium Modeling, Computational Methods and Applications*. 2nd. Springer.
- . 2013. "Asset Returns, the Business Cycle and the Labor Market." *German Economic Review* 14 (3): 372–397 (August).

- Heiberger, Christopher, and Halvor Ruf. 2014, October. “Applied Macroeconomic Analysis with Epstein Zin utility.” Mimeo. University of Augsburg.
- Jermann, Urban J. 1998. “Asset pricing in production economies.” *Journal of Monetary Economics* 41 (April): 257–275.
- Kaltenbrunner, Georg, and Lars A. Lochstoer. 2010. “Long-Run Risk Through Consumption Smoothing.” *Review of Financial Studies* 23 (8): 3190–3224 (August).
- Kyriacou, Kyriacos, Jakob Madsen, and Bryan Mase. 2004. “The Equity Premium.” *Public Policy Discussion Paper 04-10, Economics and Finance Section, School of Social Sciences, Brunel University.*
- Mehra, Rajnish, and Edward C. Prescott. 1985. “The Equity Premium: A Puzzle.” *Journal of Monetary Economics* 15 (March): 145–161.
- Schmitt-Grohe, Stephanie, and Martin Uribe. 2004. “Solving Dynamic General Equilibrium Models Using a Second Order Approximation to the Policy Function.” *Journal of Economic Dynamics and Control* 28 (January): 755–775.
- Strehle, Richard, Rainer Huber, and Jürgen Maier. 1996. “Rückberechnung des DAX für die Jahre 1955 bis 1987.” *Kredit und Kapital* 29:277–304.
- Swanson, Eric T. 2012. “Risk Aversion and the Labor Margin in Dynamic Equilibrium Models.” *American Economic Review* 102 (4): 1663–91 (June).
- Tobin, James. 1969. “A General Equilibrium Approach To Monetary Theory.” *Journal of Money, Credit and Banking* 1 (February): 15–29.
- Uhlig, Harald. 2007. “Explaining Asset Prices with External Habits and Wage Rigidities in a DSGE Model.” *American Economic Review* 97 (May): 239–243.