Locally optimal transfer free mechanisms for border dispute settlement

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10th February 2023

Abstract

Individuals living in a contested region are privately informed about their preference for citizenship in two rivalling countries. Not all frontiers are technically feasible which is why not everybody can live in his preferred country. Monetary transfers are not feasible. When citizens only care about their own citizenship and types are drawn independently, a simple mechanism with simultaneous binary messages implements a social choice function that maximizes the expected sum of local residents' payoffs. This mechanism selects a feasible allocation that maximizes the number of individuals who live in what they say is their preferred country. An approval voting mechanism implements the same social choice function but does not require any knowledge about voters' location. Sequential voting and electoral competition may instead lead to suboptimal outcomes.

Keywords: mechanism design without transfers, border dispute settlement, voting, approval voting.

JEL Codes: D71, D72, D74, F51.

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1 Introduction

Decisions about how to draw the border between two neighboring countries and, correspondingly, which citizenship the residents of the contested region should have are often at the origin of international conflict and war. While the conflicting parties often claim the entire contested region for themselves as an integral part of their national territory, splitting up a such a region peacefully is a theoretical option[1][2].

From a welfare perspective, a method for the settlement of a border dispute should (among possibly other considerations) take into account the preferences of the local residents concerned. This is not a trivial task because individual preferences about citizenship are not directly observable to others. Similarly, the intensity of citizenship preferences is only known privately. No national or international institution can claim to know for sure how important it is for a specific person to be citizen of one country rather than another.

What additionally complicates the process of determining a frontier is that not all frontier lines make equal sense from an economic or purely logistic perspective. A country should ideally be geographically connected to facilitate production, the provision of public goods, travel and transportation. Other geographical factors such as the location of rivers or mountains may add further restrictions. Actually, these constraints are a main reason why it makes little sense to let everybody simply be a citizen of his preferred country at the place where he lives.

This paper asks whether there exists a way to efficiently settle border disputes when there are obvious geographical constraints regarding the way in which borders can be drawn. The paper uses mechanism design theory to address the problem of selecting one border from a given feasible set. Since I rule out mobility, the location of the border determines the allocation

[1]This is an exercise in mathematical allocation theory that does not shed any light on the legality of rivalling claims to any specific territory. The paper studies solutions to a given allocation problem but it cannot help to decide whether the underlying claims are legally right or wrong.

[2]A historic example of a peaceful determination of a border line is the definition of the Danish-German border in 1920 which followed a referendum based procedure that was laid out in the treaty of Versailles (for details see Schlürmann, 2019). The new border split up the region of Schleswig which previously was part of Germany but inhabited by many citizens who preferred Danish citizenship. After more than 100 years, the new border is still intact and it is not an issue of political debate.

of citizenship.[3] Thus, the problem is one of assigning each citizen in the contested region either to one country A or another country B. For the sake of realism, I only consider mechanisms which do not make use of contingent financial transfers. Transfers that are conditioned on individual messages are a powerful tool in theory, but so far they play no practical role in real world border dispute settlements[4].

Before summarizing the main results, I would like to clearly spell out some limits of the present analysis. Several aspects that may play an important role when countries determine their borders are not considered here. The list includes in particular (i) externalities that the choice of a border may have on those who live in- and outside the disputed territory, e.g. because of tax base effects or security concerns, (ii) costs that may be associated with uncertainty generated by some collective choice mechanisms including in particular potential adverse effects on private and public investment and (iii) severe adverse incentive effects that may arise when an international order puts existing frontiers into question. Related to the last point, important normative aspects of existing international law are also not addressed here[5]. The present paper is an exercise in mechanism design addressing only one aspect, asymmetric information about preferences and preference intensity of local residents, that is part of a larger problem. Thus, it would be too early to directly draw practical normative conclusions from this analysis.

The first main finding of this paper is that a simple decision mechanism implements a constrained optimal social choice in dominant strategies. This mechanism asks all individuals in the contested region to report their preferred citizenship and then selects one border from the feasible set that maximizes the number of individuals who live in their preferred country.

[3] Actually, moving people is considered illegal in international law.

[4] When financial transfers are possible, a Vickrey Clarke Groves (VCG) mechanism can implement an ex post efficient decision in dominant strategies (provided that the revenue is allocated to an unconcerned third party). Similarly, with transfers, an ex post efficienct decision can be reached in a Bayesian Nash equilibrium with a d'Aspremont Gerard-Varet mechanism. An interesting question is what can be implemented with a strategically simple mechanism that uses transfers (Börgers and Li, 2019).

[5] While international law protects the integrity of existing states against attempts of a secession of part of the country, secessions have sometimes been recognized by other states after they occurred. This looks somewhat inconsistent. With a view to the mentioned incentive effects, it may make sense to make the hurdle for such an ex-post recognition particularly or prohibitively high if the seceded part is integrated into another country. These aspects are not addressed in the present analysis.

When type distributions differ across individuals, the mechanism has to be adjusted accordingly, giving more weight to citizens with stronger conditional expected valuations. While this class of mechanisms requires detailed knowledge about the locations of voters in combination with their individual reports, a simple approval voting mechanism implements the same social choice function as the unweighted mechanism without requiring this detailed information. That mechanism selects a frontier that is approved by a maximum number of local citizens.

The second main result is that not all indirect simple majority voting mechanisms are suited to replace the optimal direct mechanism. Although the welfare maximizing solution is a Condorcet winner, a sequence of votes can lead to suboptimal outcomes when voters act strategically. Accordingly, the practice of voting on frontiers or secessions must be put into question.

I also address the case where local residents do not only care about where they live themselves, but where they have preferences about the location of the border as such. In this case, voting outcomes in indirect democracies turn out to be potentially inefficient while the simple direct mechanism studied in this paper still performs optimally.

The paper is related to the seminal work of Alesina and Spolaore (1997) who analyze the optimal partitioning of a set of locations in a local public good setup and show that politically stable borders can be inefficient (see Spolaore, 2022, for a recent survey of the ensuing literature). This research focuses on an elementary trade off: having more countries increases the fixed cost of government, but it also tailors public goods to the preferences of local citizens. What distinguishes the present work is (i) that it focuses on an information aggregation problem and (ii) that it considers all possible one-stage decision mechanisms.

More generally, the paper contributes to the literature on optimal mechanisms without transfers (examples include Börgers 2004, Schmitz and Tröger, 2012, Gershkov, Moldovanu, and Shi 2017, Grüner and Tröger, 2019). From a theoretical perspective, the paper fills a gap in the mechanisms design literature, addressing a general class of allocation problems: collective decisions regarding a vector of individual specific binary outcomes with a given set of feasible outcome vectors. Binary voting is one special case of this setup which obtains when the feasibility restriction is that the individual outcome has to be the same for everyone. The mechanism put forward here then turns into the simple majority rule. Thus, the well known simple and qualified majority rules are special cases of the mechanisms that are put forward here.

2 The model

Consider two countries A and B and a contested region R that is populated by I citizens. A collective choice has to be made about how to allocate the citizens of R to the countries A and B. Call an individual outcome x_i, where the outcome $x_i = 0$ means that i becomes citizen of country A, and $x_i = 1$ means that i becomes a citizen of country B. An overall outcome is a vector $x \in \{0,1\}^I =: \mathbf{X}$. There are feasibility constraints, captured by the feasible set of allocations $\mathbf{F} \subseteq \mathbf{X}$. An example for a simple meaningful feasibility set is $\mathbf{F} = \{ x \in \mathbf{X} | \, x_1 \leq x_2 \leq ... \leq x_n \}$. This restriction obtains when the agents are ordered on a line from 1 to n, and the two countries must be connected. More complex meaningful feasibility sets can arise in the two-dimensional case.

Agents' preferences for citizenship are fully represented by a privately known type $\theta_i \in \Theta_i$ that represents the value of living in country B instead of living in country A. Thus, a negative valuation indicates a preference for country A, and a positive valuation a preference for country B. Valuations are drawn independently from a joint distribution $\hat{\phi}(\theta_1, ..., \theta_n) = \phi_1(\theta_1) \times ... \times \phi_n(\theta_n)$.

A social choice function maps all relevant information into an outcome: $x = f(\theta)$. I restrict the analysis of direct mechanisms to deterministic, transfer-free mechanisms. I show that there is an welfare maximizing mechanism that implements a corresponding social choice function in ex-post equilibrium. I define *realized social welfare* as

$$W(x, \theta) = \sum_{i=1}^{I} \theta_i x_i, \tag{1}$$

and *expected social welfare* as $E_\theta W(f(\theta), \theta)$.

We require that a social choice function is implemented in dominant strategies. Thus, an ex-ante welfare maximizing planner solves:

$$\max_{f(\theta)} E_\theta W(f(\theta), \theta) \tag{2}$$

$$\begin{aligned} s.t. \quad & f(\Theta_1 \times ... \times \Theta_I) \subseteq \mathbf{F}, \\ & f_i(\theta_i, \theta_{-i}) \geq f_i(\theta_i', \theta_{-i}) \text{ if } \theta_i > 1 \text{ for all } \theta_i, \theta_i', \theta_{-i}, \\ & f_i(\theta_i, \theta_{-i}) \leq f_i(\theta_i', \theta_{-i}) \text{ if } \theta_i < 1 \text{ for all } \theta_i, \theta_i', \theta_{-i}. \end{aligned}$$

3 The generalized majority mechanism

Consider first the case where individual types are drawn from identical symmetric and independent distributions. A straightforward way to address the design problem in this case is to maximize the number of individuals who live in what they claim to be their strictly preferred country, hereafter called the *number of fits*. For a given vector of announced types $\hat{\theta}$ and a given allocation x the *number of fits* is

$$S\left(x,\hat{\theta}\right) := \frac{1}{2}\cdot\left(\sum_{j=1}^{n} sgn\left(\hat{\theta}_j\left(x_j-\frac{1}{2}\right)\right)+n\right).$$

There may be more than one outcome that maximizes the number of fits. To select one of them, I define the *B-minimal outcome* in some (finite) set $\tilde{\mathbf{F}}\subseteq\mathbf{F}$ as the (unique) first ranked outcome in $\tilde{\mathbf{F}}$ according to the lexicographic order based on its components. According to this criterion, one discards outcomes that put the first individual country in country B if there are other outcomes in $\tilde{\mathbf{F}}$ that put the first individual in country A, and so on.

Definition 1 *The S-mechanism asks all individuals to report their types. It selects the B-minimal outcome in the set $\tilde{\mathbf{F}}\subseteq\mathbf{F}$ of alloations that maximize the number of fits $S\left(x,\hat{\theta}\right)$.*

In the general case where individual valuations are drawn from different distributions, the S-mechanisms can be weighted. I define individual probabilities of positive and negative valuations as follows:

$$\beta_i^+ := \int_{\check{\theta}>0}\phi_i\left(\check{\theta}\right)\cdot d\check{\theta},$$

$$\beta_i^- := \int_{\check{\theta}<0}\phi_i\left(\check{\theta}\right)\cdot d\check{\theta},$$

Whenever β_i^+ or respectively β_i^- are both nonzero for all individuals, I speak of a nontrivial setup. In a nontrivial setup, the conditional valuations are

$$G_i^+ := \frac{\int_{\check{\theta}>0}\phi_i\left(\check{\theta}\right)\check{\theta}\cdot d\check{\theta}}{\beta_i^+\left(\theta_i\right)},$$

$$G_i^- := \frac{\int_{\check{\theta}>0} \phi_i\left(\check{\theta}\right) \check{\theta} \cdot d\check{\theta}}{\beta_i^-\left(\theta_i\right)},$$

and the absolute valuation is

$$G_i\left(\theta_i\right) := \begin{cases} G_i^+ & \theta_i > 0 \\ 0 & \theta_i = 0 \\ G_i^- & \theta_i < 0 \end{cases}.$$

This permits to define the *weighted number of fits* as:

$$\tilde{S}\left(x, \hat{\theta}\right) := \frac{1}{2} \cdot G_i\left(\theta_i\right) \left(\sum_{j=1}^{n} sgn\left(\hat{\theta}_j \left(x_j - \frac{1}{2}\right)\right) + n \right).$$

Definition 2 *The weighted S-mechanism asks all individuals to report their types. It selects the B-minimal outcome in the set* $\tilde{\mathbf{F}} \subseteq \mathbf{F}$ *of allocations that maximize the weighted number of fits.*

To study optimality, it is important to note that incentive compatibility of a social choice function $f\left(\theta\right)$ requires that for all players the interim probability to live in country B is a step function of θ_i. This is why this mechanism can be replaced by a mechanism that just asks for the sign of the valuation.

Lemma 1 *Consider a revelation mechanism* Γ *implementing the social choice function* $f\left(\theta\right)$*. Let* $\pi_i\left(\theta_i, f\left(\cdot\right)\right) = E_{\theta_{-i}}\left[f_i\left(\theta_i, \theta_{-i}\right)\right]$*. The social choice function* $f\left(\theta\right)$ *is Bayesian incentive compatible if and only if* $\pi_i\left(\theta_i\right)$ *satisfies*

$$\begin{aligned} \pi_i\left(\theta_i\right) &= \begin{cases} \pi_i^- & \text{if } \theta_i < 0 \\ \pi_i^+ & \text{if } \theta_i > 0 \end{cases}, \\ \pi_i^- &\leq \pi_i^+, \\ \pi_i^0 &: = \pi_i\left(0\right) \in \left[\pi_i^-, \pi_i^+\right]. \end{aligned} \tag{3}$$

The social choice function is dominant strategy implementable only if this condition holds.

Proof: The "if" part is obvious. "Only if" part: Otherwise at least one citizen with nonzero valuation can increase his expected payoff by misreporting his type. *Q.E.D.*

Note that the interim probabilities π_i^-, π_i^0 and π_i^+ need not be the same for different individuals.

I introduce some more notation:

- Call $\sigma(\theta) := (sgn(\theta_1), ..., sgn(\theta_I))$ the *realized sign profile*. The *set of possible sign profiles* is $\Sigma := \{1, 0, -1\}^I$ with elements s.
- Call $\Delta(\mathbf{F})$ the *set of probability distributions over* $\mathbf{F}$.
- Call $\rho : \Sigma \rightarrow \Delta(\mathbf{F})$ a *profile-assignment rule*. Note that a profile assignment rule can be interpreted as an indirect mechanism that only permits binary signals.
- Call a profile-assignement rule *deterministic* if the outcomes are deterministic for all $s \in \Sigma$.
- Call the *set of deterministic profile assignment* rules Φ.
- Call $\sigma^{-1}(s)$ the *set of type profiles with sign profile* s.
- For any given sign profile s and any $\theta \in \sigma^{-1}(s)$ I define the following distribution function
$$\varsigma(\theta, s) = \frac{\phi(\theta)}{\int_{\sigma^{-1}(s)} \phi(x)\, dx}.$$

From any mechanism implementing some social choice function $f(\theta)$, one can construct an associated profile assignment rule in the following way.

Definition 3 *Call $\rho_{f(\cdot)}(s) \in \Delta(\mathbf{F})$ the distribution that assigns the density $\varsigma\left(\tilde{\theta}, s\right)$ to the outcome $f\left(\tilde{\theta}\right)$. We say that the step assignment rule that maps profile s into $\rho_{f(\cdot)}(s)$ corresponds to $f(\theta)$.*

Now the following holds:

Lemma 2 *If some direct revelation mechanism implements $f(\theta)$ in Bayesian Nash equilibrium (dominant strategy equilibrium), then the corresponding profile assignment rule $\rho_{f(\cdot)}(s)$ implements a social choice function $g(\theta)$ in Bayesian Nash equilibrium (dominant strategy equilibrium) with identical interim welfare for all types.*

Proof: As in Grüner and Tröger (2019), Lemma 1: Consider some direct mechanism $\Gamma = (\Theta_1, ..., \Theta_I, f(\theta))$ with a Bayesian truthtelling equilibrium. Consider the corresponding profile assignment rule $\rho_{f(\cdot)}(s)$. A citizen who chooses to report a positive (negative) valuation and who expects the other citizens to report the sign of the valuation truthfully, realizes the same interim probability of living in country B as any other citizen i type with a positive (negative) valuation does in the original equilibrium. Deviations to another announcement about the sign of the valuation yield the same interim probabilities as a deviation to a valuation with another sign under the direct mechanism Γ. Thus, a true (or false) report yield the same payoffs. Thus, there is a truthtelling Bayesian equilibrium with identical interim payoffs and welfare.

The same type of argument applies to dominant strategy equilibria. *Q.E.D.*

Based on this Lemma, one can restrict the further welfare analysis to the comparison of profile assignment rules.

Proposition 3 *(i) All weighted S-mechanisms have an ex-post equilibrium in which agents report their types truthfully.*

(ii) All weighted S-mechanisms implement an ex-post efficient social choice.

(iii) The weighted S-mechanism is a solution to (2).

Note that (iii) implies that the detail-free S mechanism is a solution to the planers problem (2) when absolute conditional valuations on both sides are the same.

Proof: (i) Equilibrium: Consider w.l.o.g. individual $i = 1$ with $\theta_1 > 0$, i.e. an individual preferring to live in country B. Consider some vector of reports $\hat{\theta} = \left(\theta_i, \hat{\theta}_{-i}\right)$ and a result that maximizes $S\left(x, \theta_i, \hat{\theta}_{-i}\right)$. An alternative individual report $\hat{\theta}_i \neq \theta_i$ with $\hat{\theta}_i > 0$ does not change $S\left(x, \hat{\theta}_i, \hat{\theta}_{-i}\right)$ for any $\hat{\theta}_{-i}$. Thus, the individual does not gain from misreporting.

Consider a false report $\theta_i \leq 0$. By reporting $\theta_i < 0$ instead of $\theta_i > 0$ the values $S\left((1, x_{-i}), \left(\theta_i, \hat{\theta}_{-i}\right)\right)$ weakly decrease for all $\hat{\theta}_{-i}$ and those of all $S\left((0, x_{-i}), \left(\theta_i, \hat{\theta}_{-i}\right)\right)$ weakly increase. Thus, the probability that individual

i is allocated to country A weakly increases and misreporting in this direction is suboptimal.

(ii) Ex-post efficiency: Consider any realization of θ and the corresponding outcome of the mechanism $f(\theta)$. A Pareto-improvement implies that all individuals that live in their preferred country continue to live there and that others that did not live in their preferred country now do. This is not possible because the social choice already maximizes $S(x,\theta)$ on $\mathbf{F}$. Thus, there is no Pareto superior outcome relative to the outcome of the S-mechanism.

(iii) Welfare: I have to show that no other incentive compatible mechanism than the weighted S-mechanism yields a higher expected payoff. I can focus on profile assignment rules with truthful reports of signs. No other profile-assignment rule can yield higher expected welfare than the S-mechanism. The reason is that in equilibrium the S mechanism transmits the entire sign profile to the planner. Not knowing the size of valuations but only their sign, the planner cannot do better than by maximizing $\tilde{S}(x,\theta)$ for all θ. *Q.E.D.*

Conditional on the true sign profile $\sigma(\theta)$ the optimal choice in $\mathbf{F}$ is the one that maximizes the weighted number of players who live in their preferred country. In the unweighted case, the best use the planner can make of the *realized sign profile* $\sigma(\theta)$ is to maximize the number of players who live in their preferred country. Moreover, any incentive compatible mechanism does not transmit more information to the planner than the sign profile. So if the planner uses a different revelation mechanism to elicit the sign profile then this other mechanism cannot deal in a better way with that profile.

4 Approval voting

Consider the following indirect mechanism. All voters can approve some subset $F_i \subseteq \mathbf{F}$. The mechanism implements the B-minimal outcome in the set of allocations that are approved by a maximum number of voters. As an example consider the case where no border is approved by any voter. In this case all citizens are allocated to country A if that is feasible. The same holds if all borders are approved by all voters.

The approval voting mechanism has a dominant strategy equilibrium in which all players approve an allocation if and only if it puts them in their preferred country. Disapproving one of these borders is suboptimal in situations where the own statement about this border is pivotal. For the same reason, approving another border would be suboptimal.

The approval voting mechanisms replicates the social choice function of the S-mechanism. In contrast to the S-mechanism, approval voting does not require that the designer possesses any knowledge about citizens' location. All that is necessary is to collect the sets of approved allocations from all citizens living in R. This may be easier to implement when the set of feasible allocations has not too many elements.

A disadvantage of the approval voting mechanism is that it permits the use of some signals F_i that are inconsistent. These are the ones that involve a contradiction regarding the preferences of individual i. In the present setup, these signals are superfluous since they are not used in equilibrium. Still, they add complexity to the mechanisms.

An important advantage of an approval voting mechanism is that it can deal elegantly with the case where the set $\mathbf{F}$ is not known to the designer. To see why, consider the following slight modification of the present model, where all elements in some subset $\check{\mathbf{F}} \subset \mathbf{X}$ provide all individuals in region R with a utility $\underline{u}$ that lies below $min_{\theta_i}(\Theta_i)$. This low utility is realized independently of the realization of θ_i. Thus, one can interpret the set $\mathbf{X} \setminus \check{\mathbf{F}}$ as the set of feasible allocations (in the sense that only these allocations do not yield a very low payoff for everybody). A Nash equilibrium exists, in which voters approve only those elements in $\mathbf{X} \setminus \check{\mathbf{F}}$ which put them in their preferred country. In other words, they disapprove those allocations that are not feasible and also those that are feasible and do not put them in their preferred country. To see why this is an equilibrium, consider that an individual faces three possible payoffs: The lowest possible payoff $\underline{u}$, the payoff for being put into country A, 0, and θ_i. Approving the allocation that yields the maximum in $\{0, \theta_i\}$ and not approving any element in $\check{\mathbf{F}}$ is part of any undominated strategy. Thus, in an equilibrium in undominated strategies, any individual can be sure that outcomes in $\check{\mathbf{F}}$ do not realize. Approving am element in $\mathbf{X} \setminus \check{\mathbf{F}}$ that does not maximize the payoff may be pivotal relative to another element in $\mathbf{X} \setminus \check{\mathbf{F}}$ that does. This explains why the above undominated strategy extends to an equilibrium profile. The result is again a welfare maximum.

The S mechanism may instead lead to other, suboptimal equilibrium allocations. To see why, consider the case with $n = 2$ and the full set of feasible allocations $X = \{0, 1\}^2$. Assume that the allocation $(1, 0)$ yields utility $-2 < min_{\theta_i}(\Theta_i)$ for both players. All other allocations yield utility $\theta_i x_i$.

Consider the case where β_1^+ is close to one and assume that player 1 announces his type sincerely. Player 2 with type $\theta_2 = -1$ announcing his type sincerely ends up with the allocation $(1, 0)$ with probability β_1^+, yielding the payoff -2. Announcing type $\theta_2 = 1$, he instead realizes a payoff of -1. Thus, his best reply is to always signal a preference for country B. A best reply of player 1 to this is to always announce the true type. To summarize:

Proposition 4 *Consider an environment in which all allocations in some set* $\check{\mathbf{F}} \subset \mathbf{X}$ *are strictly Pareto dominated by all other allocations. An approval voting mechanism has a welfare maximizing equilibrium in undominated strategies. The S-mechnism may instead have suboptimal equilibria in undominated strategies.*

5 Voting mechanisms

Those who favor a restructuring of frontiers often argue in favor of some sort of plebiscite. This raises the question whether the optimal S-mechanism can be replaced by some more conventional voting scheme. There are many ways in which one can vote on the choice from the set $\mathbf{F}$, in particular when it has more than two elements. I distinguish three setups:

(i) a direct democracy with sequential binary votes on the entire set $\mathbf{F}$,
(ii) political competition with perfectly informed policymakers, and
(iii) political competition with imperfect information.

In all four setups, I stick to simple majority rule as the rule for each vote that takes place. Since this rule cannot account for preference intensities and in order to give voting a fair chance, I assume that $G_i^+(\theta_i) = G_i^-(\theta_i)$ in what follows. Still, I assume that $\beta_i^+(\theta_i)$ and $\beta_i^-(\theta_i)$ need not be the same, i.e. I permit asymmetric probability distributions.

It is useful to first consider the case where voters vote naively in the sense that they act as if all their votes were decisive. Consider the case of a binary vote in a direct democracy in which voters may abstain. It is easy to see that any unique welfare maximizing solution wins against all other alternatives if all voters vote for their preferred alternative if they have one and abstain otherwise. Under the same assumption on voting behavior, any sequence of binary votes which covers the entire set of feasible allocations **F** leads to unique welfare maximizing solution.

An important insight is that it is necessary to vote or on the entire set of options in **F**. Considering only a subclass of feasible partitions can exclude the optimum. In particular, there are examples where one single vote leads "away" from the optimal frontier location. To see why, consider a linear world with one border, seven citizens and valuations $(1, 1, 1, -1, -1, -1, -1)$. Consider a local vote amongst the first five individuals about moving the frontier form the right of position 5 (status quo) the to left of position 1. This referendum moves the first five individuals to country B although all seven individuals should be in country A. While the move improves welfare, moving the frontier in the opposite direction would increase welfare even further. This also implies that referenda on secessions should not be held locally in a subset of the contested region. The S-mechanism must be played on the entire feasible set to make sure that welfare is maximized.

6 Strategic sequential voting

This section deals with the more interesting case where voters act strategically when they vote sequentially. Consider a sequential voting game with a known finite sequence of binary votes. The winning alternative in each voting round enters the next round. The alternative selected in the last round is implemented and counts for payoffs.

With more than two voters, any full-information or Bayesian voting game under simple majority rule has trivial equilibria where all voters vote for the same alternative. The same holds for any sequential voting game. Identical voting behavior on all stages constitutes a Nash equilibrium. In order to rule out these implausible equilibria, I restrict the following analysis to trembling hand perfect equilibria. In this section, I show that a sequential voting game may have trembling hand perfect equilibria in which the implemented social choice function does not yield a constrained welfare maximum. Thus, the S-mechanism has the advantage that its unique trembling hand perfect equilibrium always selects a welfare maximum.

A sequential Bayesian voting game is a signaling game with potentially many equilibria. In a first step, I simplify the analysis and consider the limit case where the uncertainty disappears. In that case, voter preferences are common knowledge and an example of a suboptimal trembling hand perfect equilibrium can be constructed. In a second step, I will show that the analysis extends to a nontrivial setup which is not a limit case.

The proof of the first claim (existence of an environment with an inefficient trembling hand perfect equilibrium) is by example. I consider a full information benchmark case with four homogenous groups of citizens $g = 1, ..., 4$. Group 1 has $2\hat{n}$ members, where $\hat{n} \geq 1$, group 2 has $2\hat{n} + z$ members, where $z \in \{1, ..., \hat{n}\}$, Group 3 has $\hat{n}$ members, group 4 has z members.[6] Members of groups 1 and 4 prefer living in country A, i.e. for them $\beta_i^+(\theta_i) = 0$. All others prefer living in country B, i.e. for them $\beta_i^-(\theta_i) = 0$. Expected conditional valuations are normalized to $G_i^+(\theta_i) = -G_i^-(\theta_i) = 1$. The set of feasible allocations is $\{x_1, x_2, x_3\}$:

1. x_1: Everybody lives in country B.

2. x_2: Only members of groups 1 and 3 live in country A.

3. x_3: Everybody lives in country A.

The unique expected welfare maximizing alternative is x_2. It allocates $4\hat{n} + z$ citizens in line with their preferences, while alternative x_1 allocates $3\hat{n} + z$ citizens in line with what they prefer, and x_3 only $2\hat{n} + z$ citizens. Figures 1 and 2 visualize the example in a two dimensional plane for $\hat{n} = 2$ and $z = 1$.

Consider a sequential voting game in which first there is a vote on the two alternatives x_1 and x_2, and second, the winning alternative is entering a vote against x_3.

The following is an equilibrium in undominated strategies: In the second round, everybody votes for his preferred alternative if there is one. In the second round vote amongst the alternatives $\{x_2, x_3\}$ indifferent voters (the ones in groups 1 and 3) vote for x_3. Note that nobody is indifferent in the second round vote amongst the alternatives $\{x_1, x_3\}$. In the first round, all members of groups 1 and 4 vote for alternative x_2. All others vote for alternative x_1.

[6] This is a simple special case of what will be needed below. Let group i have n_i members. The equilibrium below requires $n_2 + n_3 > n_1 + n_4$ (first and second stage requirement) and $n_1 + n_3 + n_4 > n_2$ (the other second stage vote) and $n_1 + n_2 > n_2 + n_3 \Leftrightarrow n_1 > n_3$ and $n_1 + n_2 > n_1 + n_4 \Leftrightarrow n_2 > n_4$ (optimality of allocation x_2).

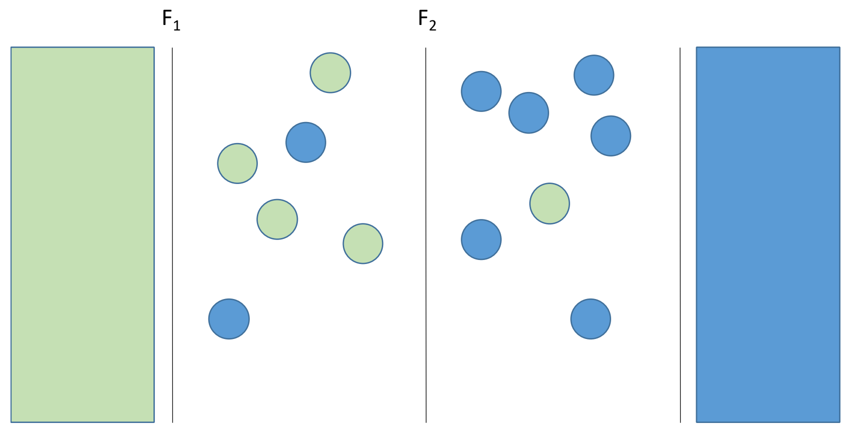

Figure 1: Green dots indicate citizens who prefer to be citizens of country A, blue dots those who prefer to be citizens of country B. F1, F2, and F3 are the three feasible borders.

Consider the votes that can possibly take place in the second round. In the second round vote on $\{x_1, x_3\}$, $3\hat{n}+z$ voters vote in favor of x_1 and $2\hat{n}+z$ vote in favor of x_3. Alternative x_1 wins that vote. In the second round vote on $\{x_2, x_3\}$, $2\hat{n}+z$ voters vote in favor of x_2 and $3\hat{n}+z$ vote in favor of x_3. Note that no one is ever pivotal along the equilibrium path.

We now show that this strategy profile is also a trembling hand perfect equilibrium. To see why, consider a sequence of totally mixed strategies where each player must play his seven non-equilibrium strategies with probability $\varepsilon > 0$ and his equilibrium strategy with probability $1-7\varepsilon$. In the two second round votes this yields a small probability for pivotality for all players, thus requiring that the second round vote is in line with voter preferences. This is the case.

The first period vote may also be pivotal. Not that, for any given probability $\varepsilon > 0$, and from the perspective of any player, there is a positive but small probability that the outcome of the second round vote on $\{x_1, x_3\}$ is x_3 and that the outcome of the second round vote on $\{x_2, x_3\}$ is x_2. Taking the almost certain second round outcomes x_1 and x_3 into account, all voters who prefer x_1 to x_3 must vote for x_1 in the first voting round and all voters who prefer x_3 to x_1 must vote for x_2. Again, this is the case.

This completes the full information case. Considering "close" asymmetric

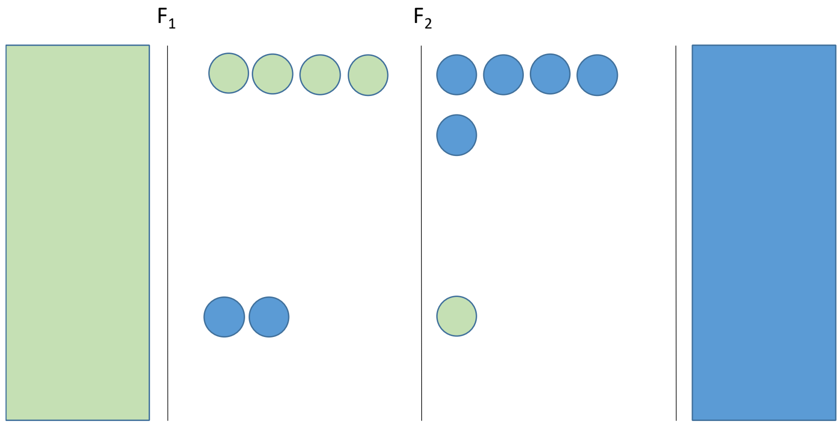

Figure 2: The four groups of citizens.

information environments leads to the following result:

Proposition 5 *Consider a sequence of binary votes that includes all feasible alternatives. There exists a full information environment in which the voting game has a trembling hand perfect equilbirum that does not maximize social welfare.*

Proof of the second part: Consider some specific asymmetric information environment with the following properties: (i) For voters in groups 1 and 4, the type is $\theta_i = 1$ with probability $1 - \delta$ (hereafter called their likely type) and $\theta_i = -1$ with probability δ. For members of groups 2 and 3 the type is $\theta_i = -1$ with probability $1-\delta$ (again called their likely type) and $\theta_i = 1$ with probability δ. Consider a strategy profile where the types $\theta_i = 1$ in groups 1 and 4 and the types $\theta_i = -1$ in groups 2 and 3 play their equilibrium strategy from the above full information example. Moreover, consider any collection of strategies for the unlikely types of all players. For small enough δ, the strategies of the more likely types satisfy the criterion of a best reply under the conditions of a trembling hand equilibrium. This is so, because they strictly prefer to vote for their preferred alternative in the second stage and in the first stage in the full information limit case. For any given strategy collection of the unlikely types, payoffs of the likely types are continuous in

δ. In the second stage, indifferent likely type voters remain indifferent for positive δ. In the first stage, no one is indifferent at $\delta = 0$ which is why the continuity argument also applies.

To extend this into a trembling hand perfect equilibrium, it remains to specify the equilibrium behavior of the unlikely types. Consider the voting behavior that is in favor of the preferred alternative in stage 2 if there is one. In case of indifference, pick any behavior. In stage 1, assume that those who favor country B vote for x_1. The same arguments as above make sure that this behavior is in line with trembling hand perfection if δ is small enough. The limit of the so described equilibrium for $\delta \to 0$ is the one that is described above. *Q.E.D.*

The proposition implies that the S-mechanism is more reliable as a tool to implement the welfare maximum than sequential voting procedures.

7 Two party competition

Proposals of referenda on secessions are often made by competing political groups or parties. Therefore, in the present context, it is useful to look into the motives of these political actors to propose specific allocations. In this section, I extend the previous analysis by a process of political competition. In this context, I show that it makes a major difference whether parties have access to citizens's information.

In a first step, consider the simple case of the competition of two parties with perfect information about voter types. Assume that one party wants to maximize the number of people living in country A, whereas the other one wants to maximize the number of people living in country B. As a tie-breaking rule, assume that indifferent voters vote for the party that shares their own country preference (alternatively, one can put an ε weight on the party winning in the utility function). Both parties offering the same solution that maximizes the number of fits is a Nash equilibrium. In equilibrium both parties receive one half of the expected votes. Any alternative platform only gains more votes if it puts more voters who previously did not fit into their preferred country than it puts voters who previously were allocated to their preferred country into the other country. Therefore the alternative platform would increase the number of fits which yields a contradiction. Moreover, there are no other symmetric equilibria and there are no asymmetric equilibria.

Next, consider the case where parties are imperfectly informed (just like anybody else). Again assume that the two parties A and B try to maximize the size of countries A and B respectively. The parties simultaneously commit to their platform in $\mathbf{F}$. The party that wins the majority of votes implements its platform. Again, I postulate the same tie-breaking rule as above. Also assume that the two options (i) everybody lives in country A and (ii) everybody in country B are in the feasible set. Then it is an equilibrium that party A proposes to put everyone in country A and party B proposes to put everyone in country B. The reason is that all A voters vote for party A no matter what party B proposes. Thus for party B, the best chance to win is to put everybody in country B to maximize the number of votes it receives from B voters. This platform both maximizes the chance of wining and the number of citizens living in country B, conditional on winning. Thus, it is a unique best reply.

Proposition 6 *Two party competition with partisan voters has a unique Nash equilibrium in which party A proposes to put every individual in country A and party B proposes to put every individual in country B. The equilibrium is not ex-post welfare maximizing.*

Proof It remains to prove uniqueness and suboptimality. Regarding a possible equilibrium where both parties offer non-extreme platforms the same argument as above can be made. To prove that welfare is not maximized, it suffices to consider the case where the realized sign profile corresponds exactly to a feasible allocation in the sense that putting everybody in his preferred country is feasible. Instead, the majority decides where individuals have to live. *Q.E.D.*

8 Conclusion

The present paper points out that a border choice mechanism that is based on binary voting decisions may result in (local) welfare losses relative to a system that is based on individual reports or approval voting. Similarly, representative democracy can lead to considerable welfare losses when voters have partisan preferences. The simple direct mechanism put forward here yields a superior result. In 1920, the Danish-German border was determined with a similar mechanism. The allocation of the border was based on the outcome of local referenda. Municipalities with a higher share of pro Danish (pro German) votes were more likely to be allocated to Denmark (Germany). Still, the decision was not to have any regional enclaves in either country. The 1920 border is still intact today, indicating that the type of mechanism may be practically robust.

Several relevant issues have not been addressed in the present paper, including (i) possible adverse incentive effects that arise when citizens or countries may expect that violence makes the use of an allocation mechanism such as the one put forward in this paper more likely, (ii) the possibility of locally correlated types, (iii) the role of voluntary ex-post mobility and (iv) the existence of sequential voting procedures with an optimal trembling hand perfect equilibrium. These issues deserve to be addressed in future research.

The present analysis relies on the assumption that the set of feasible allocations is publicly known or that some allocations yield extremely low utilities to all individuals. In the latter case, an approval voting mechanism has an equilibrium that always selects an optimal allocation. A challenging task is to consider cases in which socially unattractive solutions are associated with more unequal payoffs.

References

[1] Alesina, A. and E. Spolaore (1997) "On the Number and Size of Nations", *Quarterly Journal of Economics*, 112, 1027-1056.

[2] Börgers, T. (2004), "Costly Voting", *American Economic Review*, 94, 57-66.

[3] Börgers, T. and J. Li (2019) "Strategically Simple Mechanisms" *Econometrica*, 87, 2003-2035.

[4] Grüner, H. P. and T. Tröger (2019) "Linear Voting Rules", *Econometrica*, 87, 2037-2077.

[5] Gershkov, A., B. Moldovanu, and X. Shi (2017), "Optimal voting rules", *The Review of Economic Studies*, 84, 688-717.

[6] Schlürmann, J. (2019) *Eine Grenze für den Frieden*, Wachholtz Verlag, Kiel, Hamburg.

[7] Schmitz, P. W., and T. Tröger (2012), "The (sub-) optimality of the majority rule", *Games and Economic Behavior*, 74, 651-665.

[8] Spolaore, E. (2022) "The Economic Approach to Political Borders" in *Border Studies: A Multidisciplinary Approach* edited by Thomas M. Wilson, Chapter 7, Cheltenham: Edward Elgar.