Freedom - Why? A Mechanism Design Approach

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Abstract

According to standard mechanism design theory, the individual (freedom-) right not to accept a mechanism limits a planner's ability to achieve socially desirable (ex-post efficient or welfare maximizing) outcomes. This raises the question why and when individuals should be granted freedom rights at all. This paper studies under which conditions incentive problems on the design level justify the use of participation constraints. Depending on the environment, granting personal freedom rights may yield higher expected welfare than leaving the choice of a direct revelation mechanism to a random planner or to the electorate. The paper formalizes Hayek's conceptions of personal freedom and private sphere, and it permits the analysis of case sensitive optimal allocations of freedom rights. Two applications are studied in detail: mandatory vaccination and freedom of speech.

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1 Introduction

This paper develops a formal theory of the optimal allocation of freedom rights. A personal freedom right will be defined as an individual's right to take a specific action in full autonomy. It includes the right to voluntarily enter binding contractual arrangements regarding that action and the right to refuse any private (contractual) or public (institutional) arrangement that may limit the individual's ability to act.

An economic analysis of the allocation of personal freedom rights has to deal with two prominent and opposing theoretical results. The first one, the Coase theorem, states that *any* allocation of freedom rights is compatible with an efficient outcome. The result can be obtained if one assumes that all actors hold complete information about realized preferences and available technologies. According to the Coase theorem, granting individuals personal freedom leads to efficient bargaining outcomes. The exact allocation of freedom rights may be relevant for how well single individuals are made off, but it is not relevant for efficiency.

A very different result obtains if one instead assumes that relevant information is distributed asymmetrically. A series of papers including Hurvicz (1972), Laffont and Maskin (1979) and Myerson and Satterthwaite (1983) show that, under asymmetric information, the freedom not to participate in a conflict resolution mechanism (such as a market or a bargaining protocol) may be in conflict with (ex-post) efficiency.[1] By contrast, a benevolent designer who does not need to respect individual participation constraints can always implement an ex-post efficient social choice (D'Aspremont and Gérard-Varet, 1979)[2]. The Myerson and Satterthwaite impossibility result

[1] Güth and Hellwig (1986) prove a similar impossibility theorem in a public goods setup. Schweizer (2006) provides a very useful general classification of environments in which similar universal impossibility theorems hold.

[2] The result holds when transfers are feasible and with an appropriate equilibrium concept (Bayesian Nash equilibrium).

and the D'Aspremont and Gérard-Varet possibility result raise the fundamental question why the (freedom-) right to refuse participation in a mechanism should be granted to anybody at all.[3] This paper addresses this question.

The setup put forward in this paper is one in which relevant information is dispersed (i.e. a standard mechanism design setup). This is why it is not trivially efficient to grant freedom rights to individuals. However, I also assume that society can not simply rely on a benevolent "social planner" to select collective decision rules optimally.[4] In an attempt to build a more realistic theory of collective rule choices, I consider a set of prominent and more realistic alternative arrangements. The benchmark is a liberal and somewhat anarchic system in which all decision rights are held by individuals and all individual actions are taken non-cooperatively. The second class of systems is based on a structured political process that respects individual freedom rights but permits the emergence of institutions. Under such an arrangement, policymakers may only propose institutions (i.e. mechanisms) that guarantee the voluntary participation of all individuals concerned. This class includes in particular all market mechanism. The third type of arrangement consists of the political choice of institutions that do not need to respect any personal freedom rights.

Another design dimension that this paper considers is whether equal individuals should be required to be treated equally by a mechanism. This "formal freedom" (Giersch, 1961, p. 73) protects citizens from arbitrary and discriminatory proposals or decisions of policymakers. In the present mechanism design context, it is natural to assume that it requires that all individuals of the same type must be treated equally.

The problem studied in this paper is one of a constitutional choice at a stage where private information has not yet realized. This choice consists

[3] See also Hellwig (2006) on the role of freedom rights as constraints in welfare theory.

[4] A point that has been convincingly made by Hurvicz (2007) in his Nobel prize lecture.

of (i) allocating personal freedom rights and (ii) fixing according to which rules the decision mechanism will be determined at a later stage when the actual decision problem arises. Implicit in this formulation of the design problem is the assumption that the concrete problem is not yet known at the constitutional stage. The constitutional rules that are considered in this paper have in common that they do not require any knowledge about the set of feasible decisions, the type spaces, type distributions or preferences. Otherwise, an optimal revelation mechanism could already be chosen at the constitutional stage.

Modelling asymmetric information *and* the political process makes the optimal form of freedom rights depend on the specific environment. While personal freedom rights may be associated with inefficiencies, they can also help to avoid even more inefficient or excessively unequal outcomes. This is why freedom rights are not generally a useless constraint. This insight is derived from the analysis of two policy examples that have recently attracted considerable attention: mandatory vaccination and restrictions of freedom of speech. While the first example of a uniform externality makes the case in favor of freedom rights in form of the rule of law, the second one shows that in antagonistic environments, this may not be enough. Here, only the freedom right to reject a mechanism avoids the emergence of inefficient and highly unequal outcomes.

The concept of personal freedom put forward in this paper is inspired by the one in Hayek's (1960) book "The constitution of liberty". Hayek defines individual freedom as the absence of oppression, and oppression as an act by which one person effectively restricts the set of reasonable options that are available to another one. As a countermeasure, Hayek proposes to legally guarantee all individuals a "private sphere", defined as a set of decisions that no other person is allowed to interfere with.[5]

[5] In Hayek's view, this concept of personal freedom is distinct from the concept of freedom as an attractive set of options.

As has been noted by others previously (Stigler, 1978) Hayek's concept of personal freedom raises important questions. The world is full of externalities, and granting one individual a "private sphere" may generate diverse externalities for others. Hayek's definition of personal freedom permits very different assignments of domains of personal freedom to the individuals that compose a society. This raises the question how exactly freedom rights should ideally be allocated. Is there some "natural" or optimal way to legally define an individual's private sphere? When and if so how should somebody's private sphere be restricted? When should private action be replaced fully by some sort of collective action? The framework put forward in this paper permits to derive context sensitive answers to these questions.

The present paper also sheds some light on a debate surrounding Sen's (1970) seminal work on the impossibility of a Paretian liberal. In a social choice setup, Sen argues that any Hayekian assignment of individual decision rights is incompatible with a Pareto optimal social choice. However, his setup does not permit that free individuals may engage in efficient contracting - an option explicitly included in the present framework.

Hayek's discussion of the role and benefits of personal freedom is a verbal one. This paper embeds his concept of personal freedom in formal mechanism design theory. This is a straightforward choice since it also addresses the Hayekian information aggregation problem. It is argued here, that it also nests a concept of freedom as the absence of oppression. Taking both aspects, asymmetric information and the allocation of freedom rights, into account leads to normative results. The structure of preferences, technology and the stochastic environment determine the optimal form and allocation of freedom rights.[6][7]

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[6]The problem of self-interested political choices that is analyzed here also plays a key role in Hayek's (1960) book. The contribution of this paper is to embed it into a Bayesian mechanism design setup which makes it possible to study the welfare effects of various assignments of freedom rights.

[7]An insightful treatise of classical normative justifications for personal freedom is

This paper contributes to a broad literature in applied mechanism design. Papers in this field often take the existence and type of participation constraints for granted. A notable exception is Bierbrauer (2011a) who, in a public good setup, studies whether individuals prefer the stricter interim to an ex-ante participation constraint. By assumption, the designer of the mechanism can pocket an exogenous fraction of the project's net revenue.[8] Both constraints to some extend limit his ability to do so, and a trade-off between efficiency and surplus extraction arises. Individuals are only willing to give up the interim participation constraint if the planner is sufficiently benevolent.[9] The present paper performs related exercises, considering a broader range of design problems and institutions, and including an additional optional restriction, the rule of law. Explicitly modelling the assignment of actions to individuals, this paper also permits the analysis of the allocation of freedom rights.

The paper is also related to recent advances in the study of political competition with Bayesian mechanisms by Bierbrauer and Boyer (2016). They show that, while political competitors may want to offer a surplus maximizing social choice function in order to increase the size of the pie, they may also, in an attempt to attract voters, generate random and unequal outcomes - an insight that also plays a role in the present paper.

Vaubel (2019).

[8]Bierbrauer (2011b) uses a closely related formal framework to address the question whether a monopolist who provides an excludable public good should receive public subsidies.

[9]In the context of a selling mechanism Börgers (2015, p. 8f) argues that, in absence of a participation constraint, the seller could just keep the good and still demand infinite payments from the buyer.

2 The general setup

Consider the standard mechanism design setup with I individuals, individual specific type spaces $\Theta_1, ..., \Theta_I$ and Bernoulli utilities $u_i(x, \theta_i)$ in outcomes $x \in X$ and types $\theta_i \in \Theta_i$ with $i = 1, ..., I$. The outcome x is the result of actions. It is determined according to a function $x = g(a)$ where $a = (a_1, ..., a_n)$ is a vector of $n \geq I$ actions with $a_j \in A_j$ where $j = 1, ..., n$. All actions are elementary in the sense that they can only be taken by single individuals.[10] In some applications there is no need to distinguish between actions and outcomes. In such cases, it makes sense to fix $g(a) = a$. However, in other cases, multiple action profiles may lead to identical outcomes which is why the distinction can be useful.

In many cases, it makes sense to assume that an initial assignment of decision rights to individuals is given by nature. Think for example about a person's ability to raise the left arm which is directly assigned to the individual. However, generally, the assignment of actions to individuals may be governed by convention or law.[11]

A *noncooperative liberal system* is a system in which all individuals can take some actions, and all actions are assigned to individuals. Let $P = (P_1, ..., P_I)$ be a partitioning of the set of actions, i.e. an action a_j must be an element of some P_i with $i \in \{1, ..., I\}$. I call $P^0 = (P_1^0, ..., P_I^0)$ the initial (natural) assignment.

Based on an assignment of actions to individuals, all individuals are endowed with their respective strategy sets. Let $P_i = \left\{a_1^i, ..., a_{n_i}^i\right\}$ where a_k^i

[10] The task "carry the piano from A to B" can not be performed by a single individual. However, it can be decomposed into the actions "lift the left (right) side of the piano and help to carry it from A to B". This type of decomposition underlies the model.

[11] Note that any cost that may be associated with taking an action can be included in the utilities $u_i(x, \theta_i)$. Moreover, by assumption, the effect of a specific action on the utilities is independent of the identity of the person that has the right to determine this action. The action "i carries a suitcase from A to B" may be costly for i, but the cost is not different when i performs the task on someone else's order.

denotes the kth action available to i and n_i the number of actions that i can take. Call $\hat{S}_i(P_i) = A_1^i \times ... \times A_{n_i}^i$ the set of all combinations of actions that individual i can take and the elements of this set s_i.

In some cases it makes sense to assume that constitutional rules can assign the right to take a specific action to other individuals than those that were chosen by nature. As an example, consider again the activity "raising ones left arm". The natural assignment is that the person to which the arm is attached can directly perform it. However, the right may in principle be transferred to some other person. This requires some sort of collective enforcement.

I define

$$\Phi^P = \left(\hat{S}_1(P_1), ..., \hat{S}_I(P_I), g(\cdot)\right)$$

as the Bayesian game that results if all individuals simultaneously and independently choose the actions that have been assigned to them. The natural or anarchic mechanism is the game

$$\Phi^{P^0} = \left(\hat{S}_1\left(P_1^0\right), ..., \hat{S}_I\left(P_I^0\right), g(\cdot)\right).$$

This paper focuses on cases in which for all possible partitions P, the corresponding Bayesian game Φ^P has at least one Bayesian Nash equilibrium. In the cases that I discuss below, the equilibrium is unique with corresponding interim expected utility $\underline{u}_i(\theta_i)$. Interim participation requires that all individuals realize at least the interim utility $\underline{u}_i(\theta_i)$ for all possible types $\theta_i \in \Theta_i$.

Property rights as a specific type of freedom rights can be modeled within the present framework by assuming that specific actions are tied to the existence of an asset. The "owner" of an asset is the person that controls all the actions associated with that asset.

This paper considers the case where a political process leads to the selection of an incentive mechanism that determines the outcome x. In the standard mechanism design model, a mechanism $\Gamma = (S_1, ..., S_I, f(\cdot))$ is a collection of strategy sets and a social choice function $f(\cdot):\Theta_1\times...\times\Theta_I\rightarrow X$ that maps strategy profiles into the set of outcomes X. Implicit in this is that it can be made sure that the actions taken in society result in the outcome X. This requires that for all $\theta\in\Theta$ society can enforce that individuals play a state contingent action profile in the set $g^{-1}(f(\theta))$.

3 Institutional options

In the examples that follow, a range of alternative institutional arrangements is considered. All these arrangements have in common that they do not require any knowledge about details of the problem that they are supposed to solve. Specifically, nothing needs to be known about the properties of the set of outcomes X, the type spaces Θ_i, the joint distribution of types, or the preferences $u_i(x,\theta_i)$. What needs to be known is the set of actions $\{a_1,...,a_n\}$. In the spirit of Grossman and Hart (1990), I assume that, at a later stage, the details of the setup become available to everybody.

The benchmark arrangement is a *noncooperative liberal system* (hereafter also referred to as a system of *laissez faire*). This a setup in which (i) all actions $a_1,...,a_n$ are allocated to individuals before they receive their respective private information and (ii) all actions are exercised simultaneously and non-cooperatively after individuals receive their private information. This system represents the weakest form of institutionalization. Collective action is limited to the allocation of freedom rights $(P_1,...,P_I)$ and to their enforcement.

The role of any political system is to produce a (direct revelation) mechanism $\Gamma=(\Theta_1,...,\Theta_I,f(\cdot))$ that regulates individuals' interaction. Two prominent examples are market mechanisms and tax systems. Amongst the political systems, I distinguish those with a single self-interested ruler, hereafter called *autocracies*, from the *competitive political systems*, in which different political players propose mechanisms to the electorate. Moreover, I distinguish *systems that respect personal freedom rights* - modelled as participation constraints - from those that do not. Systems that respect personal freedom rights are defined as those requiring that state-dependent political outcomes satisfy all individuals' and all types' interim participation constraints that are associated with the existing allocation of freedom rights $(P_1, ..., P_I)$.[12,13] Obviously, any political system that operates under participation constraints either yields the same interim utility for all individuals and types as the non-cooperative liberal system or it provides a Pareto-improvement.

Finally, I distinguish systems that treat equal individuals equally - hereafter *systems that respect the rule of law* - from those that do not. In the present mechanism design environment, this requires that any two individuals who send the same signal to a mechanism must be treated equally. This implies in particular that individuals' mere identity may not play any role in the decision.

This leads to the following 2x2x2 alternatives to a laissez faire system:

1. In an *autocracy*, a randomly selected social planner chooses a social choice function $f(\theta)$ subject to the incentive compatibility constraints. The planner does not need to respect personal freedom rights.

2. In an *autocracy with participation constraints*, the ruler additionally

[12] An alternative way of modelling freedom rights would be to assume that a proposed mechanism invites individuals to participate (those who do can be considered club members) and that individuals are free to act outside the mechanism. In general, such mechanisms may implement a larger set of social choice functions $\hat{f}(\cdot)$ than the ones that respect all individuals' participation constraints. Thus, the present concept of freedom rights is particularly strict. It protects individuals against the threat of a collective punishment by those who participate in a mechanism.

[13] Note that contracting on organized markets requires that there is an entity that enforces the contract (as Hayek, 1960, remarks). Individuals can be forced to do something if an existing contract obliges them to do so, but only if they deliberately accepted the contract beforehand. Contractual arrangements may arise under some bargaining institution, e.g. in a market economy.

has to respect the participation constraints that are associated with the existing allocation of freedom rights $(P_1, ..., P_I)$. This arrangement represents what Hayek described as an autocracy that respects freedom rights. It is a questionable concept since it is not clear why the autocrat should choose to do so, but it may be practically relevant to the extent that autocratic rulers may have to secure some minimum support within society.[14]

3. An *autocracy with participation constraints under the rule of law* in addition has to treat equal individuals equally. This constraint applies to everybody except the autocrat himself.

4. An *autocracy under the rule of law* consists of the choice of a mechanism by a randomly selected social planner who has to treat equal individuals equally but does not need to respect participation constraints. Again, the equal treatment constraint applies to everybody except the autocrat himself.

5. Under *unrestricted political competition*, two vote share maximizing political entrepreneurs simultaneously propose incentive compatible direct revelation mechanisms. The competitors do not have access to voters' private information. They do not need to respect participation constraints. Voters are required to vote for one of the two mechanisms at the interim stage. This is the competitive system studied in Bierbrauer and Boyer (2016).

6. A system of *political competition with participation constraints* consists of some allocation of freedom rights $(P_1, ..., P_I)$, followed by a competitive political process. Political competitors may only propose mech-

[14] A related alternative interpretation of the arrangement is that of a profit maximizing monopolist market designer subject to a participation constraint (as in Güth and Hellwig, 1986).

anisms that satisfy all players' participation constraints for all possible realizations of types. This competitive arrangement can be thought of as one that allows policymakers to propose a market mechanism to individuals.

7. A system of *constrained political competition under the rule of law* additionally rules out discriminatory decisions. Individuals who send the same signal must be treated equally.

8. *Political competition under the rule of law* allows the political competitors to propose platforms that violate the individual rationality constraints, but it requires an equal treatment of equals.

Table 1 summarizes the arrangements considered.

4 Vaccination

4.1 Setup

Consider a large population of mass 1 indexed by $i \epsilon [0, 1]$. All individuals can either receive a vaccination ($x_i = 1$) or not ($x_i = 0$). The natural assignment is that all individuals make their own vaccination decision, in which case I write $x_i = s_i = a_i$. The collection of outcomes for all individuals is called x, and the collection of outcomes for all individuals except individual i is called x_{-i}. The share of vaccinated individuals is denoted by $s(x)$. Individual payoffs take the following linear form

$$u_i(x, \theta_i) = \begin{cases} -\alpha (1 - s(x)) & \text{if } x_i = 0 \\ -\beta (1 - s(x)) - \theta_i & \text{if } x_i = 1. \end{cases}$$

By assumption, vaccinations always reduce risks for others ($\alpha, \beta > 0$), and they may reduce the risk for the vaccinated individual itself ($\alpha \geq \beta$). The cost parameters θ_i are distributed independently and uniformly on the unit interval. I assume $1 > \alpha$, so that the highest type under no circumstances benefits from vaccination.

4.2 Non-cooperative liberal system

First note that any individual expecting a share of vaccinations s chooses to get a vaccination if

$$\theta < (\alpha - \beta)(1 - s) \in [0, \alpha - \beta].$$

The non-cooperative vaccination game has a unique Nash equilibrium x^* in which a share $s(x^*)$ of individuals remains unvaccinated. Since all subjects with lower valuations than $\theta^* := (\alpha - \beta)(1 - s(x^*))$ get vaccinated, the equilibrium vaccination share satisfies $s(x^*) = \theta^*$, and thus

$$\begin{aligned} s(x^*) &= (\alpha - \beta)(1 - s(x^*)) \\ &\Leftrightarrow \; s(x^*) = \frac{\alpha - \beta}{1 + \alpha - \beta}. \end{aligned}$$

The equilibrium is always an interior one. Equilibrium welfare is

$$\begin{aligned} W^L &= (1 - s(x))(-\alpha(1 - s(x))) + s(x)\left(-\beta(1 - s(x)) - \frac{s(x)}{2}\right) \\ &= -\frac{1}{2}\frac{\alpha^2 + 2\alpha - \beta^2}{(1 + \alpha - \beta)^2}. \end{aligned}$$

A limit case obtains when $\alpha = \beta > 0$. In this pure public good case, there is no private benefit from vaccination but a social one. Nobody gets vaccinated in equilibrium and welfare is $-\alpha$.

4.3 Welfare maximization

An uninformed mechanism designer with no access to transfers can choose the share of agents s who receive mandatory vaccination. Additionally, the designer can permit the remaining agents to opt into vaccination. Since he has one more instrument (mandatory vaccination) at his disposal, he can always at least realize the Nash equilibrium welfare.

For further use, I first derive the optimum without the possibility to opt into vaccination.

Lemma 1 *(i) When there is no possibility to opt into vaccination, a welfare maximizing planner chooses to vaccinate a share s of the population with*

$$s = \begin{cases} 0 & \text{if } \alpha < \frac{1}{4} + \frac{1}{2}\beta, \\ 1 & \text{if } \beta > \frac{1}{2}, \\ \frac{2\alpha - \beta - \frac{1}{2}}{2(\alpha - \beta)} & \text{otherwise.} \end{cases}$$

(ii) A regime with mandatory vaccination yields higher (lower) welfare than the liberal Nash equilibrium if $\alpha > \bar{\alpha} := \beta + \frac{1}{2\beta} - 1$ ($\alpha < \bar{\alpha}$).

Proof (i) Randomly vaccinating some share s of the population - without the possibility to opt in - yields welfare

$$W^R(s) = -\left(\left(1 - s\right)\alpha + s\beta\right)\left(1 - s\right) - \frac{s}{2}$$

Welfare is concave in s. Solving for the optimum yields

$$\frac{dW^R(s)}{ds} = 0 \Leftrightarrow s = \frac{2\alpha - \beta - \frac{1}{2}}{2\left(\alpha - \beta\right)}.$$

Accordingly, a boundary solution at $s = 0$ obtains iff $\alpha < \frac{1}{4} + \frac{1}{2}\beta$, and one at $s = 1$ iff $\beta > \frac{1}{2}$. A welfare maximizing planner chooses to vaccinate everybody in the latter case.

(ii) The corresponding condition is

$$\begin{aligned}-\frac{1}{2} &> -\frac{1}{2}\frac{\alpha^2+2\alpha-\beta^2}{(\alpha-\beta+1)^2} \\ &\Leftrightarrow \alpha > \bar{\alpha} := \beta + \frac{1}{2\beta} - 1.\end{aligned}$$

□

Consider next the possibility to opt into vaccination. The planner randomly vaccinates a share $\check{s}$ of the population and allows all those who do not receive a mandatory vaccination to opt in. Call the equilibrium cutoff type for a voluntary vaccination $\check{\theta}$. Individuals with a lower type than $\check{\theta}$ chose to get a vaccination if they are not already obliged to do so. Interior solutions and boundary solutions to the planner's problem may obtain.

Lemma 2 *A welfare maximizing planner chooses to randomly vaccinate a share $\check{s} < 1$ if $\beta < \frac{1}{2}$. All other individuals are allowed to opt into vaccination. There are values $\beta < \alpha < 1$ so that the optimal value for $\check{s}$ lies at the upper bound, the lower bound, or in the interior of the unit interval.*

Proof The share of individuals who receive a vaccination in equilibrium is

$$s = \check{s} + (1-\check{s})\,\check{\theta}.$$

The cutoff type $\check{\theta}$ satisfies

$$\begin{aligned}-\alpha(1-s) &= -\beta(1-s) - \check{\theta} \\ &\Leftrightarrow \check{\theta} = \frac{(\alpha-\beta)(1-\check{s})}{1+(\alpha-\beta)(1-\check{s})}.\end{aligned}$$

Thus, the share of those who receive a vaccination in equilibrium is

$$s = \check{s} + (1-\check{s})\frac{(\alpha-\beta)(1-\check{s})}{1+(\alpha-\beta)(1-\check{s})}.$$

Note that

$$\frac{ds}{d\check{s}} = \frac{1}{\left(\left(1-\check{s}\right)\left(\alpha-\beta\right)+1\right)^{2}} > 0.$$

Welfare is

$$W^{V}\left(\check{s}\right) = \left(s\left(\alpha-\beta\right)-\alpha\right)\left(1-s\right) - \frac{\check{s}}{2} - \frac{\check{\theta}}{2}.$$

The possibility to opt into vaccination guarantees that $W^{V}\left(\check{s}\right) > W^{R}\left(\check{s}\right)$ for all $\check{s} < 1$. Moreover $W^{V}\left(1\right) = W^{R}\left(1\right)$. Thus, if the optimum without opt in is not mandatory vaccination for everybody, then the optimum with opt in is not either.

The proof for the existence of an interior solution is by example. At $\alpha = 0.4$ and $\beta = 0.2$, $W^{V}\left(0.5\right) = -0.450\,41 > W^{V}\left(0\right) = -0.472\,22 > W^{V}\left(1\right) = -\frac{1}{2}$.

The proof for the existence of the two boundary solutions is also by example. Consider the case with no private vaccination benefit ($\alpha = \beta$):

$$s = \check{s}, \check{\theta} = 0$$

Welfare is linear:

$$W^{V}\left(\check{s}\right) = -\alpha + \left(\alpha - \frac{1}{2}\right)\check{s}.$$

Thus, it may be optimal to vaccinate everybody ($\alpha > \frac{1}{2}$) or nobody ($\alpha < \frac{1}{2}$). □

4.4 Autocracy

An autocrat makes vaccination mandatory for all other citizens. The autocrat does not benefit from his own vaccination and remains unvaccinated. Welfare is the same as under mandatory vaccination. This and Lemmata 1 and 2 imply:

Proposition 1 *When the welfare maximizing rate of mandatory vaccinations is an interior one, an autocrat vaccinates too many individuals, whereas, in a non-cooperative liberal system, there are fewer vaccinations than optimal. An autocratic vaccination decision yields a lower welfare than a non-cooperative liberal system if and only if the cost parameter α lies below $\bar{\alpha}$.*

4.5 System with participation constraints

In the vaccination case, participation constraints severely limit the set of implementable social choice functions.

Proposition 2 *The equilibrium outcome of any regime with participation constraints is identical to the laissez faire outcome.*

Proof One can restrict the set of available incentive compatible revelation mechanisms to a simple class. Using an argument from the voting literature (Schmitz and Tröger, 2012, Azrieli and Kim, 2014, and Grüner and Tröger, 2019), one can replace any Bayesian incentive compatible revelation mechanism by a revelation mechanism with at most two interim vaccination probabilities $\hat{\rho}_i \leq \bar{\rho}_i$ for each individual. The reason is that signals leading to intermediate vaccination probabilities would not be chosen in any equilibrium.

For any such mechanisms, the indifference condition at the cutoff value remains

$$\check{\theta} = (\alpha - \beta)(1 - s).$$

The equilibrium vaccination share for a given cutoff is

$$s = \check{\theta}\bar{\rho} + \left(1 - \check{\theta}\right)\hat{\rho}.$$

Thus, in equilibrium, the cutoff satisfies

$$\check{\theta} = \frac{(\alpha - \beta)(1 - \hat{\rho})}{1 + (\alpha - \beta)(\bar{\rho} - \hat{\rho})}, \tag{1}$$

and the equilibrium vaccination share

$$\begin{aligned} s &= \check{\theta}\bar{\rho} + \left(1 - \check{\theta}\right)\hat{\rho} & (2) \\ &= \hat{\rho} + \frac{(\bar{\rho} - \hat{\rho})(\alpha - \beta)(1 - \hat{\rho})}{1 + (\bar{\rho} - \hat{\rho})(\alpha - \beta)}. & (3) \end{aligned}$$

Consider first $\hat{\rho}_i = 0$ and $\bar{\rho}_i = 1$ for all i. The equilibrium outcome is unique. It is the laissez faire outcome x^*.

Consider next some other symmetric interim probabilities. Let $0 < \hat{\rho} < \bar{\rho} = 1$. Consider the type $\theta = 1$. The equilibrium vaccination share is

$$s = \hat{\rho} + \frac{(1 - \hat{\rho})(\alpha - \beta)}{1 + (1 - \hat{\rho})(\alpha - \beta)}(1 - \hat{\rho}).$$

For all s, type $\theta_i = 1$ picks the low vaccination probability $\hat{\rho}$. His payoff is

$$\begin{aligned} & \hat{\rho}(-\beta(1 - s) - 1) + (1 - \hat{\rho})(-\alpha(1 - s)) \\ = & -(\hat{\rho}\beta + (1 - \hat{\rho})\alpha)(1 - s) - \hat{\rho} \\ = & -(\hat{\rho}\beta + (1 - \hat{\rho})\alpha)\left(1 - \left(\hat{\rho} + \frac{(1 - \hat{\rho})(\alpha - \beta)}{1 + (1 - \hat{\rho})(\alpha - \beta)}(1 - \hat{\rho})\right)\right) - \hat{\rho} \\ = & -(\hat{\rho}\beta + (1 - \hat{\rho})\alpha)\left((1 - \hat{\rho})\left(1 - \frac{(1 - \hat{\rho})(\alpha - \beta)}{1 + (1 - \hat{\rho})(\alpha - \beta)}\right)\right) - \hat{\rho} \\ = & -(\hat{\rho}\beta + (1 - \hat{\rho})\alpha)\frac{(1 - \hat{\rho})}{1 + (1 - \hat{\rho})(\alpha - \beta)} - \hat{\rho} \end{aligned}$$

Taking the derivative yields

$$\begin{aligned} & \frac{d\left(-(\hat{\rho}\beta + (1 - \hat{\rho})\alpha)\frac{(1 - \hat{\rho})}{1 + (1 - \hat{\rho})(\alpha - \beta)} - \hat{\rho}\right)}{d\hat{\rho}} \\ = & \frac{\beta - 1}{(\alpha - \beta - \alpha\rho + \beta\rho + 1)^2} < 0. \end{aligned}$$

Thus, the interim utility of the highest type strictly decreases in $\hat{\rho}$ which is why this type refuses to participate in any mechanism satisfying $1 = \bar{\rho} > \hat{\rho} > 0$.

For any given $\hat{\rho}$, the interim payoff of type $\theta_i = 1$ from a mechanism satisfying $1 = \bar{\rho} > \hat{\rho} > 0$ is an upper bound of his payoff from a mechanism with $1 > \bar{\rho} > \hat{\rho} > 0$, because the highest type does not choose vaccination and because, according to (3), the equilibrium vaccination share increases in $\bar{\rho}$.

Finally, one has to consider mechanisms with individual specific values $\hat{\rho}_i \leq \bar{\rho}_i$. Consider any such assignment of individual specific values. Pick the individual with the highest value $\hat{\rho}_i$. If that individual has type $\theta_i = 1$, then his payoff is smaller than his payoff when everybody else has an identical low value $\hat{\rho}_j = \hat{\rho}_i$ and an identical high value $\bar{\rho}_i = \bar{\rho} = 1$. That payoff in turn is less than the outside utility. □

4.6 Unconstrained political competition

Unconstrained political competition is associated with an individual policy risk that is familiar from Colonel Blotto models of political income redistribution. Competitors have an incentive to randomly discriminate against some members of society. As part of a best reply, discrimination can guarantee majorities. In the present context, discrimination takes the form of elevated probabilities of mandatory vaccination for some but not for all individuals. An equilibrium must either be one in mixed strategies or it includes boundary solutions (everybody receives mandatory vaccination) that rule out the option of discrimination.

Proposition 3 *Any equilibrium under political competition without participation constraints must be an equilibrium in mixed strategies (with some mandatory vaccination in equilibrium) or an equilibrium in which both competitors offer general mandatory vaccination.*

Proof Consider two vote share maximizing candidates simultaneously and independently each putting up one proposal for a mechanism. The individual outcomes are two vaccination probabilities $\hat{\rho}_i \leq \bar{\rho}_i$. Any mechanism proposed in equilibrium includes that any agent who at the interim stage prefers to receive a vaccination must get a vaccination. Increasing $\bar{\rho}_i$ strictly increases the payoff of all voters $j \neq i$. It also weakly increases the payoff of voter i for all types. Thus, any mechanism that is offered in equilibrium, can be replaced by one with two signals, one triggering vaccination for sure ($\bar{\rho}_i = 1$), the other triggering vaccination with probability $\hat{\rho}_i < 1$. Note that the probabilities $\hat{\rho}_i$ can be individual specific. Moreover, the probabilities $\hat{\rho}_i$ can be used to calculate the share of mandatory vaccinations in society, $\check{s}$.

Let competition take place in such mechanisms. Consider a potential equilibrium in which both candidates do not offer full mandatory vaccination. Consider the following modification of the first candidate's platform by the second candidate. Fix $\hat{\rho}_i = 1$ for a minority of voters that previously was facing a probability $\hat{\rho}_i < 1$. The equilibrium vaccination rate s increases, since $\frac{ds}{d\check{s}}$. A majority of individuals benefits from this in equilibrium. Thus, a player playing a pure strategy can always be defeated unless his policy includes full mandatory vaccination. This directly implies the proposition. □

According to Lemma 2, laissez faire can be optimal. Proposition 3 implies that political competition never reaches this optimum.

Corollary 1 *A system that relies exclusively on voluntary vaccination cannot be the result of unconstrained political competition. This includes the cases in which it is optimal.*

4.7 Political competition under the rule of law

Under the rule of law, any two citizens who send the same message $\hat{\theta}$ under a revelation mechanism must be subject to the same probability $\rho\left(\hat{\theta}\right)$ of mandatory vaccination. For the reasons given above, two signals with two corresponding and uniform vaccination probabilities are enough to implement any incentive compatible social choice. Moreover, in equilibrium, one of the two probabilities must be one, guaranteeing that those who prefer vaccination are free to choose it. I call the second, lower vaccination probability $\hat{\rho} < 1$.

Proposition 4 *Let $\alpha = \beta > 0$. Under political competition under the rule of law, mandatory vaccination obtains for $\alpha > \frac{1}{2}$, while voluntary vaccination obtains for $\alpha < \frac{1}{2}$. The outcome always maximizes social welfare. For all $\alpha > \frac{1}{2}$, laissez faire yields a strictly lower welfare level than political competition under the rule of law. The welfare ranking of the two institutions is preserved for $\beta < \alpha > \frac{1}{2}$ when β is close enough to α.*

Proof Consider a direct mechanism with vaccination probabilities 1 and $\hat{\rho} < 1$. From (1), the following relation between the cutoff for voluntary vaccination $\check{\theta}$ and $\hat{\rho}$ holds:

$$\check{\theta} = \frac{(\alpha - \beta)(1 - \hat{\rho})}{1 + (\alpha - \beta)(1 - \hat{\rho})}.$$

Individual welfare is

$$\begin{array}{ll} -\beta(1-s) - \theta & \text{if } \theta \leq \check{\theta} \\ \hat{\rho}(-\beta(1-s) - \theta) + (1-\hat{\rho})(-\alpha(1-s)) & \text{if } \theta > \check{\theta}. \end{array}$$

This can be rewritten as

$$\begin{array}{ll} -\beta(1-s) - \theta & \text{if } \theta \leq \check{\theta} \\ \hat{\rho}((\alpha-\beta)(1-s) - \theta) - \alpha(1-s) & \text{if } \theta > \check{\theta}. \end{array}$$

From (3), the equilibrium share of unvaccinated individuals satisfies

$$\begin{aligned} (1-s) &= \left(1 - \check{\theta}\right)(1 - \hat{\rho}) \\ &= \frac{1}{1 + (\alpha - \beta)(1 - \hat{\rho})}(1 - \hat{\rho}) \\ &= \frac{1}{\frac{1}{1-\hat{\rho}} + (\alpha - \beta)}. \end{aligned}$$

Thus, individual welfare is

$$\begin{cases} -\beta \frac{1}{\frac{1}{1-\hat{\rho}}+(\alpha-\beta)} - \theta & \text{if } \theta \leq \check{\theta} \\ \hat{\rho}\left(\frac{(\alpha-\beta)}{\frac{1}{1-\hat{\rho}}+(\alpha-\beta)} - \theta\right) - \alpha \frac{1}{\frac{1}{1-\hat{\rho}}+(\alpha-\beta)} & \text{if } \theta > \check{\theta}. \end{cases} \tag{4}$$

The upper term in (4) increases in $\hat{\rho}$. Those who choose voluntary vaccination benefit from more vaccinations of others. The second derivative of the lower term is

$$-2(1-\beta)\frac{\alpha-\beta}{((\alpha-\beta)(1-\hat{\rho})+1)^3} < 0.$$

This establishes concavity of the lower term in $\hat{\rho}$. Monotonicity of the upper part and concavity of the lower part guarantee that each voter has an ideal point for $\hat{\rho}$ on the unit interval with decreasing utility in both directions. This establishes single peakedness regarding the policy variable $\hat{\rho}$ with respect to the order ">". Furthermore, note that there is a function $h(\hat{\rho})$ so that the derivative of the lower term in (4) satisfies

$$\frac{d\left(\hat{\rho}\left(\frac{(\alpha-\beta)}{\frac{1}{1-\hat{\rho}}+(\alpha-\beta)} - \theta\right) - \alpha \frac{1}{\frac{1}{1-\hat{\rho}}+(\alpha-\beta)}\right)}{d\hat{\rho}} = h(\hat{\rho}) - \theta,$$

guaranteeing that the ideal points weakly decrease as θ increases. Thus, the median voter has type $\theta = 1/2$. Median voter welfare is

$$\begin{cases} -\beta \frac{1}{\frac{1}{1-\hat{\rho}}+(\alpha-\beta)} - \frac{1}{2} & \text{if } \frac{1}{2} \leq \check{\theta} \\ \hat{\rho}\left(\frac{(\alpha-\beta)}{\frac{1}{1-\hat{\rho}}+(\alpha-\beta)} - \frac{1}{2}\right) - \alpha \frac{1}{\frac{1}{1-\hat{\rho}}+(\alpha-\beta)} & \text{if } \frac{1}{2} > \check{\theta}. \end{cases}$$

Note that

$$\begin{aligned} \frac{1}{2} &> \check{\theta} = \frac{(\alpha-\beta)(1-\hat{\rho})}{1+(\alpha-\beta)(1-\hat{\rho})} \\ &\Leftrightarrow 1+(\alpha-\beta)(1-\hat{\rho}) > (\alpha-\beta)(1-\hat{\rho}) \\ &\Leftrightarrow 1 > (\alpha-\beta)(1-\hat{\rho}), \end{aligned}$$

which by assumption ($0 \leq \beta < \alpha < 1$) always holds. Thus, the median voter does not choose voluntary vaccination for any $\hat{\rho}$. This is so because the median voter does not (even) get vaccinated in the liberal system.

Consider now the limit case $\alpha = \beta$. In this case, no one chooses voluntary vaccination, and thus $\check{s} = \hat{\rho}$. Utility of type θ_i is

$$\begin{aligned} & -\alpha\left(1-\hat{\rho}\right)-\hat{\rho}\theta_i \\ = & \left(\alpha-\theta_i\right)\hat{\rho}-\alpha. \end{aligned}$$

Thus, individuals with types $\theta_i < \alpha$ ($\theta_i > \alpha$) prefer a higher (lower) vaccination share. The equilibrium is unique. If $\alpha > \frac{1}{2}$, the unique political Nash equilibrium is both candidates offering full mandatory vaccination.

In the pure public benefits case $\alpha = \beta$, full mandatory vaccination yields higher welfare than laissez faire if

$$\begin{aligned} \alpha & > \alpha + \frac{1}{2\alpha} - 1 \\ & \Leftrightarrow \alpha > \frac{1}{2}. \end{aligned}$$

Therefore, political competition under the rule of law dominates laissez faire when the outcomes are different. By continuity of all payoff functions, equilibrium conditions and welfare in α and β, the welfare ranking extends to close enough cases for $\alpha > \frac{1}{2}$. $\square$

As a preliminary assessment of the vaccination case, one can conclude that participation constraints can make it impossible to politically reach a welfare maximizing outcome. Preserving all rights to choose the individual vaccination status is incompatible with collective welfare improvements. Instead, political competition under a rule that merely requires an equal treatment of equals may establish an outcome that is tailored to the distribution of the relevant parameters. These are the individual and social benefits and to the private costs of vaccinations.

5 Freedom of speech and religion

5.1 Setup

Consider a population with I members and individual views θ_i about some issue. All views are drawn from a binary distribution on $\{A, B\}$ with corresponding probabilities $\pi^A \geq \pi^B \in]0,1[$. Individuals can make statements $x_i = a_i \in \{A, B, N\}$ about their view on the issue, where N represents making no statement. Again, let $x_i = a_i$. All individuals derive a payoff of 1 from expressing their actual view publicly and a payoff of zero from not expressing a view. The payoff of expressing another view than the one actually held is $-\varepsilon$. Moreover, individuals are not tolerant which is why they experience a disutility $n \cdot c$ from listening to n individuals who state a view different from their own. All individuals' messages are received by all others and all messages are verifiable[15].

In what follows, individuals' public *statements* $a_i \in \{A, B, N\}$ must be distinguished from the *messages* $\hat{\theta}_i \in \{A, B\}$ that they may be required to send under a mechanism. These messages are assumed to be sent in private by the informed individual to the mechanism.[16]

5.2 Noncooperative liberal system

Consider first the case in which all individuals are free to state their views publicly. The unique Bayesian equilibrium is that everybody states his actual view. Let a individuals hold view A and $b = I - a$ individuals hold view B. When everybody expresses his view, members of the majority receive the payoff $1 - bc$, and those of the minority $1 - ac$. Welfare is $I - 2abc$. The non-

[15] Think of a complete network on Twitter. The model can also be thought of as one of exercising a religion in public.

[16] Think of a computer program representing the mechanism. As will become clear below, the optimal mechanism does not need to record the identity of the individual sending the message. This is why an anonymous survey would be sufficient to collect the relevant data.

cooperative equilibrium yields higher expected welfare than a system that always enforces complete silence if $I > 2E(ab)c$. Note that this condition does not rule out that, in a laissez faire equilibrium, some statements reduce welfare for some realizations of a. As will become clear below, welfare maximizing speech restrictions depend on the realization of a.

5.3 Welfare maximizing designer

A welfare maximizing planner can make use of a revelation mechanism to (silently) collect messages about the true realizations of a and b. The welfare maximizing social choice function can be implemented in dominant strategies.

Proposition 5 *Welfare is maximized by the following state contingent social choice:*

(i) If $c \leq \min\left\{\frac{1}{a}, \frac{1}{b}\right\}$*, grant full freedom of speech.*

(ii) If $\frac{1}{b} \geq c > \frac{1}{a}$ *restrict speech to* $\{B, N\}$*.*

(iii) $\frac{1}{a} \geq c > \frac{1}{b}$*, restrict speech to* $\{A, N\}$*.*

(iv) If $c > \max\left\{\frac{1}{a}, \frac{1}{b}\right\}$ *restrict speech to* $\{N\}$*.*

A mechanism that imposes this state contingent policy based on the messages $\left(\hat{\theta}_1, ..., \hat{\theta}_I\right)$ *has a truthtelling equilibrium in dominant strategies.*

Proof First not that no individual should be ever forced to make a statement that is not in line with its views because this adds $-\varepsilon$ to the individual's payoff without affecting anybody else.

From a utilitarian perspective, an individual with view A (B) should be allowed to state A (B) iff $1 > bc$ ($1 > ac$). This is why the policy described above maximizes social welfare for all realizations of a and b.

Given this policy and for any vector of messages $\hat{\theta}_{-i}$, stating the own view properly weakly increases individual i's probability to be allowed to make his preferred statement. Sending the correct message therefore weakly increases the probability that one is allowed to state one's own view. If all other players make truthful announcements, the probability increases strictly. □

5.4 Autocracy

Autocracy does not maximize expected social welfare because it implements a one-sided speech restriction independently of the actual realization of preferences.

Proposition 6 *(i) An autocrat restricts everybody else's speech.*

(ii) The highest welfare level that is compatible with an autocrat's view A is achieved when he always permits statement N, never permits statement B and permits statement A if and only if $c < \frac{1}{b}$.

(iii) Autocracy does not maximize expected social welfare.

Proof (i) An autocrat with view A (B) chooses one of the restrictions $\{A\}$, $\{A, N\}$, or $\{N\}$ ($\{B\}$, $\{B, N\}$, or $\{N\}$) for everybody else. The autocrat makes his preferred statement. The restrictions $\{A\}$, $\{B\}$ are always suboptimal because some individuals may be forced to misrepresent their views. The restrictions $\{A, N\}$ or, respectively, $\{B, N\}$ are superior. Still, they are not optimal unless the dictator shares the majority's view and $\frac{1}{b} > c > \frac{1}{a}$ or $\frac{1}{a} > c > \frac{1}{b}$.

(ii) The autocrat's optimum requires that statement B is not allowed whenever $b > 0$. When $b = 0$ it does not matter whether B is allowed.

For all (a, b), always permitting not to make a statement does not affect the autocrat's payoff. Permitting N benefits individuals who hold view B when stating A is allowed. The policy regarding statement A does not affect the autocrat's utility. This is why he should permit A if and only if $c < \frac{1}{b}$.

(iii) The welfare maximizing choice is given in Proposition 5. The rest follows from (ii). □

Note that, in an alternative setup where individuals benefit from other individuals' statements that are in line with their own view, an autocrat would force everybody else to state his own view, possibly resulting in additional welfare losses.

Participation constraints limit the autocrat's ability to impose speech restrictions. The autocrat benefits from forbidding any public disagreement with his own point of view. In order to make those who disagree with him participate, he must also limit the freedom of speech of some individuals who share his point of view. Such a policy only fulfills the interim participation constraints if it constitutes a Pareto improvement at the interim stage.

Proposition 7 *In an autocracy with participation constraints, the autocrat maximizes his payoff with a mechanism with the following properties:*

(i) The mechanism collects messages in $\{A, B\}$*.*

(ii) Conditional on the messages, the mechanism may impose statement N *for some individuals.*

The autocrat's optimum is suboptimal from society's perspective. However, at the interim stage, it is Pareto superior to the laissez faire outcome.

Proof Suppose the autocrat holds the view A. His objective is to silence as many B supporters as possible in expected terms. Interim participation of those who hold view B requires that he silences some A supporters in some cases. Thus, the autocrat's problem is to choose for any (a, b) a vector of speech restrictions that satisfy that, in expected terms the number of B statements is minimized subject to the interim participation constraints and the individual rationality constraints. The set of the autocrat's alternatives is the set of mappings from $\{(0, I), ..., (I, 0)\}$ to the set of lotteries over vectors of individual speech restrictions $\{\{A\}, \{B\}, \{A, N\}, \{B, N\}, \{N\}\}$. This set is convex and compact. The autocrat's payoff function is continuous on this set which is why a solution exists. □

5.5 Unconstrained political competition

Like in the vaccination example, unconstrained political competitors have an incentive to discriminate against single members of society. Discrimination takes the form of individual speech restrictions. To see why, the following definitions are useful. Consider some social choice function $x = f(\theta)$.

Definition 1 *Call* $f_A^i(A)$ *the interim probability that* $x_i = A$ *when individual* i *sends message* A. *Define* $f_A^i(B)$, $f_B^i(B)$, *and* $f_B^i(A)$ *analogously.*

Definition 2 *Call* $g_A^i(A)$ *the interim expected number of individuals who make statement* A *when individual* i *sends message* A. *Define* $g_A^i(B)$, $g_B^i(B)$, *and* $g_B^i(A)$ *analogously.*

Definition 3 *A social choice function* $f(\cdot)$ *is monotonous if*

$$f_A^i(A) \geq f_A^i(B),$$

$$f_B^i(B) \geq f_A^i(A),$$

$$g_A^i(A) \geq g_A^i(B),$$

and

$$g_B^i(B) \geq g_B^i(A).$$

The welfare maximizing state contingent policy is monotonous. Thus, monotonicity is necessary for a proper reaction of speech restrictions to the state θ. Moreover, it is sufficient to make truthtelling dominant strategy incentive compatible.

Lemma 3 *There is no Bayesian Nash equilibrium under unconstrained political competition with direct mechanisms in which (i) both players play monotonous platforms, (ii)* $f_A^i(A) > 0$ *and* $f_B^i(B) > 0$ *for all* i.

Proof Consider the competition of two candidates who maximize the expected number of votes that they receive. Both candidates do not care about the outcome itself. An incentive compatible direct mechanism collects a vector of messages θ, implements $f(\cdot)$ and yields expected interim payoffs

$U_i\left(f\left(\theta\right), \theta_i\right)$. Consider a hypothetical equilibrium in which both players offer monotonous mechanisms with the property that for some individual i $f_A^i\left(A\right) > 0$ and $f_B^i\left(B\right) > 0$. Consider the following modification of the platform that receives an expected share of votes of at least 50 percent. Silence individual i but keep all other decisions $f_{-i}\left(\cdot\right)$. The mechanism is still monotonous, incentive compatible and it makes $I-1$ individuals strictly better off. $\square$

A direct consequence is that, apart from the trivial case $c < \frac{1}{I-1}$, unconstrained competition can never be optimal.

Proposition 8 *Let $c < \frac{1}{I-1}$. There is no equilibrium under unconstrained political competition with direct mechanisms in which both players play the unique welfare maximizing platform.*

5.6 The rule of law

Under the rule of law, all individuals who send message $\hat{\theta}_i = A(B)$ have to be treated equally. Unlike in the vaccination case, this is not enough to guarantee a welfare maximizing outcome.

Proposition 9 *Consider political competition under the rule of law. Let $\pi^A > 1/2$.*

(i) There is no perfectly tolerant competitive political equilibrium.

(ii) The welfare maximizing mechanism is not an equilibrium platform.

(iii) There is an equilibrium in which both candidates restrict everybody's speech to $\{A, N\}$.

Proof (i) Consider the case where both competitors offer free speech to all individuals. Consider the alternative mechanism that lets only the expected majority speak, i.e. it restricts speech to $\{A, N\}$. This platform wins a majority of votes in expected terms when $\pi^A > 1/2$.

(ii) Consider the welfare maximizing mechanism from above. Consider the alternative mechanism that lets only the expected majority speak, i.e. it restricts speech to $\{A, N\}$. At the interim stage, all voters who hold view A strictly prefer the latter platform because it never restricts their speech and it always restricts the speech of those with view B. Therefore, the mechanism receives an expected number of votes that is larger than $I/2$. This is why, in equilibrium, any candidate offering the welfare maximizing platform get less than half of the votes in expected terms. By replicating the opponent's platform, he receives half of the votes in expected terms instead.

(iii) Consider any alternative mechanism. Such a mechanism does not increase the utility of the A voters and it only increases the utility of B voters if it restricts some A voters' statements. The expected number of votes in equilibrium is $1/2$. The expected number of votes for a deviating candidate is less than $1/2$. $\square$

Note that if $c < \frac{1}{I-1}$, it is optimal to grant full freedom of speech independently of the realization of types. This is why the following holds.

Corollary 2 *When $c < \frac{1}{I-1}$, the liberal system yields the welfare maximum, while unconstrained political competition and political competition under the rule of law do not.*

A system of political competition with participation constraints under the rule of law is biased against the expected minority view. Still, all individuals are as well of at the interim stage as in the laissez faire system.

Proposition 10 *Consider a system of political competition with participation constraints under the rule of law. There is a symmetric equilibrium that is biased against the expected minority view.*

Proof Policies need to satisfy the interim participation constraints and the rule of law. Consider the following optimization problem: Choose a mapping from (a, b) into the (identical) probabilities of speech restrictions for all those who state A or B subject to the incentive compatibility constraints and the participation constraints, that maximizes an A voter‘s interim utility. The set of options is compact and convex and the objective functions is continuous. A solution exists. If both candidates offer the same solution to this problem, then each of them receives 50% of the votes in expected terms. Any feasible deviation that makes the B voters better off at the interim stage strictly decreases utility of the A voters. This lowers the expected number of votes. □

6 Alternative assignments of decision rights

The widespread view that the individual that is affected most (or most directly) should be allowed to take a decision can be traced back to at least Adam Smith who argued that

> "(e)very man ... is much more deeply interested in what immediately concerns himself than any other man. ... And as he is fitter to take care of himself than any other person it is fit and right that it should be so."[17]

The analytical framework of this paper permits to address the question about the optimal assignment of decision rights from a welfare perspective. In the vaccination example, the vaccination of an individual changes the individual's utility by $(\alpha - \beta)(1 - s) - \theta_i$. In a large population, the effect on any other individual is minor. This is why, except for non-generic cases, individual i is affected most by decision a_i.

[17] Adam Smith's Theory of Moral Sentiments (1759, II. ii. 2) quoted according to Vaubel (2019), who also provides a detailed discussion of the intellectual origins of the argument that decisions should be allocated that way.

From a welfare perspective, mandatory vaccinations is superior to the laissez faire outcome if the social cost α that is associated with free-riding is large enough. In these cases, assigning all decision rights to the individuals that are not concerned most (i.e. for all i to individuals $j \neq i$) improves social welfare relative to non-cooperative action or equivalent systems that respect personal freedom rights. This also include autocratic regimes where all vaccination decisions are taken by a single decision maker.

At first glance, this contradicts the view that decisions should be taken by the individual that is concerned most. However, any such re-allocation of decision rights cannot tailor the outcome to the underlying cost parameters. Whenever decision rights are transferred, vaccination is enforced independently of the personal costs or benefits (measured by the difference $\alpha - \beta$).

This is different in the case where individual i holds the right to take decision a_i, since the equilibrium share of vaccinations changes with the cost parameter. Thus, on the individual level, a non-cooperative liberal system permits that the outcome is tailored to the underlying type and to the underlying public benefits. Similarly, democratic competition under the rule of law permits some tailoring on the collective level. In the pure public benefit example ($\alpha = \beta$) and for the specific distribution of types considered, it implements the welfare maximum, independently of the actual realization of the cost parameter.

Similarly, in the freedom of speech case, any assignment of decision rights that differs from the natural one (here: everybody deciding himself what to say) involves restrictions, independently of whether these restrictions are socially desirable or not. Such a reallocation makes less sense in the freedom of speech case than in the vaccination case because the outcome is completely arbitrary. The form of the speech restriction depends on the (random) preferences of the individual that holds the decision right.

7 Conclusion

It depends on the specific environment whether freedom rights increase expected social welfare. The type of externality and the underlying distribution of preferences are key. The two setups studied in this paper differ with regard to the degree of antagonism that exists in society. In the vaccination case, all individuals are aware of the fact that they derive the same benefit from additional vaccinations. The externality of not getting vaccinated works uniformly across all individuals. The only heterogeneity concerns the perceived individual cost of a vaccination. In a competitive political system under the rule of law, the policy either enforces a pro-social behavior of everybody or of nobody. Under this restriction, the equilibrium vaccination policy switches from a liberal to a restrictive policy when the social benefits from vaccinations becomes large enough. The individual freedom to reject any such collective rule is in the way of the associated welfare improvements.[18]

In the freedom of speech example, the externality instead works asymmetrically. A conflict about what may and what should not be said in public divides society into two opposing groups. In this antagonistic setup, the restriction that equals should be treated equally does not rule out the suppression of minority views. When making a specific statement is forbidden for everybody, all individuals are formally still subject to the same restriction. In other words: speech restrictions do not discriminate against specific individuals as such, they only discriminate against everyone who happens to share a particular view. In this second example, it turns out that the freedom to reject a mechanism can play a useful role. The participation constraints do not eliminate the bias against the expected minority, but they limit the degree of inequality that is associated with democratic decision making.

What both examples have in common is that the allocation of decision

[18] In the appendix I show that a similar result obtains in a public goods setup when transfers are feasible.

rights to individuals who are not concerned most is an inflexible arrangement. Those who have the authority to take a decision for someone else use this authority to switch off externalities independently of the size of social and private costs and benefits. The present analysis shows that one can do better by allocating decision rights in the "natural" way and by imposing appropriate restrictions to the set of available policy options.

Two types of extensions of the present analysis may be worth looking into. The first one concerns alternative institutions that could be established on the constitutional stage. The constitutional rules considered in this paper have in common that exactly one centralized mechanism governs the choice of the overall outcome. An alternative way to deal with the problem of regulating human behavior in the presence of externalities is to allow that society is split into two or more groups. The coexistence of multiple clubs or jurisdictions is an issue not addressed in this paper. One way to model their emergence is to consider the competition of two or more entrepreneurs who attempt to attract as many individuals as possible, offering them mechanisms that only apply to club members. Any such offer would have to be conditional on the set of participants. In the context of the vaccination model that could e.g. mean that one group lives in a regime of voluntary and another one in a regime of mandatory vaccination. Trivially, such "jurisdictional" competition may be more efficient whenever the externality is limited to the respective jurisdiction. However, when the range of the externality is fixed, such a competitive outcome is likely to just replicate the anarchic one. The second set of extensions concerns the examples that can be addressed. Auctions, redistributive taxation, public goods provision (an elaboration can be found in the appendix of this paper) and other mechanisms that make use of financial transfers are natural candidates.

8 Appendix: Monetary transfers

The purpose of the appendix is to briefly demonstrate how some of the insights of this paper extend to a case where monetary transfers are feasible. Consider the following pure public goods setup. There is a continuum of individuals indexed by $i \in [0,1]$. All individuals have access to a costly action $a_i \in \{0,1\}$. Everybody's contribution is required to produce the public good. The economy is endowed with one unit of money. I assume that individuals have property rights regarding their initial monetary holdings which is why the transfer payment from i to j is also an action. Initially, all individuals "own" one unit of money which is why, in net terms, not more than one unit can be transferred to somebody else. The monetary cost of action $a_i = 1$ is p. The individual payoffs are

$$\pi_i = \theta_i \cdot \min\{a_i \,| i \in [0,1]\} + 1 - pa_i + t_i,$$

where t_i denotes a transfer that i may receive. All incomes must be nonnegative, i.e. $1 - pa_i + t_i > 0$. Risk preferences are represented by the utilities $u(\pi_i)$ where $u(\cdot)$ is increasing and concave.

Let θ_i be distributed uniformly on $[0,1]$. Assume that p is small enough so that in the absence of transfers the provision of the public good maximizes social welfare, i.e.

$$E_{\theta_i} u(\theta_i + 1 - p) > u(1).$$

Note that this and concavity of $u(\cdot)$ imply that $p < 1/2$, which is why a majority of citizens prefers the provision of the public good.

In this setup, the role of autocratic choice and the role of participation constraints are similar to the ones from the vaccination example. Autocracy creates inequality because an autocrat appropriates all the money that is available. An autocrat with a valuation $\theta_i > p$ enforces the provision of the public good. The outcome is always suboptimal because income is distributed in the most unequal way possible. An autocrat who is subject to a participation constraint instead cannot provide the public good. Any individual with valuation $\theta_i = 0$ loses p in expected terms and refuses to participate. Laissez faire does not maximize social welfare either because no one contributes in the unique equilibrium.

The analysis of political competition can draw on the insights from Bierbrauer and Boyer (2016). The setup is a special case of their pure public goods setup with only two provision levels - no provision or a single fixed provision level. The individuals' contribution to social surplus is π_i. By assumption, the provision of the public good maximizes social welfare and therefore also the social surplus. Therefore, any equilibrium involves that both candidates offer to provide the public good (Theorem 1 of Bierbrauer and Boyer, 2016). Transfers assume the Myersonian random form (Corollary 1 of the same paper). Since everybody is risk averse, the equilibrium is not welfare maximizing.

The role of the rule of law in a competitive environment is similar to its role in the vaccination case. Since everybody must be paid the same amount, the competitive system provides the public good if and only if the median of the distribution of valuations exceeds the cost p. This outcome provides higher welfare than the outcome under unconstrained political competition.

Table 1: Institutions

Name	**Actors**	**Participation constraints, rule of law**	**Objectives**
Noncooperative liberal system	Individuals	no, no	self-interest
Autocracy with participation constraints	Random dictator	yes, no	self-interest
Autocracy with participation constraints under rule of law	Random dictator	yes, yes	self-interest
Autocracy under rule of law	Random dictator	no, yes	self-interest
Autocracy	Random dictator	no, no	self-interest
Political competition with participation constraints	Political entrepreneurs	yes, no	vote maximization
Political competition with participation constraints under the rule of law	Political entrepreneurs	yes, yes	vote maximization
Political competition under the rule of law	Political entrepreneurs	no, yes	vote maximization
Unconstrained political competition	Political entrepreneurs	no, no	vote maximization

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