Central Bank Asset Purchases as a Corrective Policy[1]

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This version: September 9, 2018

Abstract

Can unconventional monetary policies (also) play a useful role in non-crisis times? We show that central bank asset purchases in secondary markets can enhance social welfare by serving as a corrective policy under financial constraints. We develop a model with idiosyncratic risk and collateralized lending, where conventional monetary policies are neutral. Purchases of collateralized debt at above-market prices affect the allocation by driving a wedge between relevant returns for borrowers and lenders. The central bank can thereby address pecuniary externalities and can further implement welfare-dominating allocations compared to well-proven corrective debt policies. State-contingent asset purchases should be conducted in a countercyclical way to reduce financial acceleration of aggregate shocks.

JEL classification: E44, E52, D52, D62

Keywords: Unconventional monetary policy, financial constraints, pecuniary externalities, Pigouvian subsidy, financial amplification

[1] The author is grateful to Klaus Adam, George-Marios Angeletos, Felix Bierbrauer, Kai Christoffel, Ester Faia, Fiorella DeFiore, Dean Corbae, Leo Kaas, Keith Küster, Peter Karadi, Gernot Müller, Paul Pichler, Annukka Ristiniemi, Oreste Tristani, and other seminar participants at the European Central Bank, the 48. Monetary Policy Committee Meeting of the German Economic Association, the Bundesbank conference "Credit frictions and default in macroeconomics", and the CEP/SNB conference "Aggregate and distributive effects of unconventional monetary policies" for helpful comments and suggestions. Financial support from the Deutsche Forschungsgemeinschaft (SPP 1578) is gratefully acknowledged. (A previous version of this paper circulated under the title "Welfare-Enhancing Distributional Effects of Central Bank Asset Purchases".)

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1 Introduction

The European Central Bank (ECB) and US Federal Reserve (Fed), which have traditionally traded money in exchange for (short-term) treasuries, have included large-scale purchases of private debt securities in secondary markets into their set of policy instruments. According to empirical evidence, interventions in secondary markets, which have been conducted by the Fed and the ECB during and after the recent financial crisis, have altered assets prices and portfolio decisions.[3] Given that asset purchases have been shown to be beneficial under stressed financial markets,[4] the question arises whether they can play a useful role in non-crisis times, when financial market participants also face constraints, though to a lesser extent. The usefulness of asset purchases is in principle questionable, since they might reduce efficiency by distorting prices and threaten financial stability (see, e.g., Woodford, 2016). Contrary to this view, we prove – as the main novel contribution – that central bank asset purchases can serve as a corrective policy that mitigates amplification induced by financial constraints (see Davila and Korinek, 2018).[5] We further show that conventional monetary policies, like changes in nominal interest rates or in money supplied against treasury securities, cannot serve for this purpose, and that widely-applied corrective (Pigouvian) policies on borrowing can enhance social welfare only in a less successful way.

Asset purchases have distributional consequences, i.e. different effects on holders and issuers of debt, which are in principle ambiguous.[6] If, for example, a central bank offers a favorable price for specific assets in secondary markets, one might suppose that primarily agents who hold and sell these assets (i.e. savers or lenders) gain from this intervention. This argument, however, neglects that these agents, who receive liquid funds (central bank money) in exchange for less liquid assets, might further use/invest the proceeds,

[3] Krishnamurthy and Vissing-Jorgensen (2011) and Krishnamurthy et al. (2017) provide evidence on price effects via different channels of asset purchase programmes conducted by the Fed and the ECB.

[4] See, e.g., Curdia and Woodford (2011) or Del Negro et al. (2017).

[5] Amplification is induced by a financial constraint that leads to a positive feedback between asset demand, prices, and borrowing capacities, like in Lorenzoni (2008), Stein (2012), Benigno et al. (2016), or Bianchi and Mendoza (2018), who examine prudential policies. Our paper complements this literature on ex-ante policies by focussing on gains of corrective policies conducted by the central bank.

[6] Studies on distributional effects of monetary policy have so far focussed on conventional policies (see, e.g., Auclert, 2016, and references therein), which are, by construction, neutral in our model.

such that market prices and other participants in financial markets are also affected. Thus, in tranquil times, when neither asset liquidation nor liquidity hoarding is urgent, the price effects of asset purchases are central and their distributional consequences are non-trivial. Concretely, borrowers might gain when the pass-through of price effects of asset purchases reduces borrowing costs, which has in fact been aimed and observed for recent asset purchases programmes.[7]

We show that price effects of central bank purchases of private debt securities can enhance ex-ante social welfare in an environment where financial markets are not unusually stressed by crises. Instead, we acknowledge that financial markets are regularly distorted by frictions, i.e. by financial constraints faced by a subset of agents (see Kiyotaki and Moore, 1997), which induce deviations from first best and lead to inefficiencies due to pecuniary externalities, as described by Davila and Korinek (2018). We prove that secondary market interventions by the central bank, which cannot be mimicked by conventional monetary policies, can address these pecuniary externalities and can be superior to a Pigouvian tax/subsidy on debt. Specifically, lenders (i.e. the holders of eligible assets) can be incentivized to increase their supply of funds by the central bank offering an above-market price for debt purchases. This raises the lenders' effective real return on debt, while the real interest rate for borrowers falls in equilibrium, in accordance with empirical evidence on recent asset purchase programmes.[8] The wedge between the effective real returns for borrowers and lenders can be used to correct for pecuniary externalities equivalent to a Pigouvian subsidy for debt. We show that asset purchases can enhance social welfare even further by alleviating the borrowing constraint via their impact on the relative price of collateral and that they can mitigate financial acceleration of aggregate shocks. The total effect of asset purchases can principally be replicated by a combination of Pigouvian subsidies on debt and collateral, which rely on *type-specific* lump-sum taxes (see also Davila and Korinek, 2018). Asset purchases therefore serve as a particularly useful corrective policy, when the latter are not available.

[7] ECB purchases of ABS in 2014 where expected to "facilitate credit conditions" (ECB press release, 2nd Oct. 2014). Hancock and Passmore (2011) report that Fed's MBS purchases in 2008 do not only affect MBS yields, but also reduced mortgage rates.

[8] Specifically, the loan rate falls by a reduction in the (il-)liquidity premium, which accords to empirical evidence on price effects of US Federal Reserve asset purchases (see Gagnon et al., 2011).

We develop an incomplete market model with limited commitment to repay debt, which is sufficiently stylized to isolate the main mechanism, while including details of monetary policy implementation. Thus, the model is not intended to provide a description of a policy measure implemented in a specific episode, but is rather aimed to disclose potential efficiency gains from applying an established policy instrument. Too assess the corrective role of central bank asset purchases, i.e. of an exchange of assets against central bank money, we specify a monetary economy, while the main effects can also be demonstrated in a real economy by a combination of Pigouvian policies. We take a traditional view on central bank money regarding its special role to settle transactions and assume, for convenience, that money serves as the unique means of payment for non-durable consumption goods (see Lucas and Stokey, 1987). We further assume that agents differ with regard to their valuation of non-durables, giving rise to borrowing/lending in terms of money. To isolate the effects of asset purchases in secondary markets, we abstract from financial intermediation and endogenous production, such that conventional monetary policies are *neutral* with regard to the equilibrium allocation. As the main friction, we consider limited commitment by borrowers, such that borrowing is constrained by collateral (as in Kiyotaki and Moore, 1997). The central bank supplies money only against eligible assets, which solely consist of treasury securities under a conventional monetary policy regime. In addition, we account for the possibility of central bank purchases of private debt securities in secondary markets, which are non-neutral by affecting borrowers and lenders in different ways.[9] When the monetary policy rate, i.e. price of money in terms of eligible assets, is set below the marginal rate of intertemporal substitution, eligible assets are scarce and money supply is effectively rationed, such that Wallace's (1981) irrelevance result for open market operations does not apply.[10] Lenders then participate in asset purchases programs if the central bank offers an above-market price, while they supply the proceeds to borrowers leading to a lower real loan rate than in the laissez faire

[9] They can equivalently be described as a central bank collateral policy, where private debt securities are eligible. In contrast to related studies on unconventional policies (see, e.g., Curdia and Woodford, 2011, or Gertler and Karadi, 2011), the central bank does not directly trade with ultimate borrowers.

[10] Under money rationing, the central bank can simultaneously control the price and the amount of money, and can thereby implement welfare dominating allocations compared to policy regimes that satiate money demand (see also Schabert, 2015).

equilibrium, which prevails under conventional monetary policies.

To facilitate aggregation and to enable the derivation of analytical results, we apply linear-quadratic preferences and analyze an equilibrium representation with a representative lender and a representative borrower. Notably, the pecuniary externality only affects the choices of both representative agents via a modified collateral constraint, such that "distributive externalities" (see Davila and Korinek, 2018) are not relevant for the equilibrium allocation. To identify welfare gains of financial market interventions, we consider the laissez faire equilibrium and examine the problem of a social planer who maximizes welfare of ex-ante identical agents by choosing a feasible allocation, while deciding on agents' borrowing (see Bianchi and Mendoza, 2018). We ensure a non-trivial policy outcome by restricting our attention to cases where households cannot completely self-insure against liquidity risk, which requires holding a sufficiently large stock of government bonds. For this, we consider an exogenous path for short-term government debt, which we view as being well justified, given that decisions regarding the issuance of public debt instruments are typically based on public finance (rather than liquidity provision) considerations. The first best allocation is then not implementable, since borrowing can in general not be stimulated without distorting relative prices.

We show that the constrained efficient allocation can be implemented by a subsidy on debt, which is financed by a type-specific lump-sum tax (on borrowers). This Pigouvian subsidy corrects for pecuniary externalities under the collateral constraint that induces a positive feedback loop between collateral demand, prices, and the borrowing capacity, which qualitatively corresponds to the main mechanism in studies on prudential policies.[11] We establish that a central bank can exactly replicate the Pigouvian debt subsidy by a suitably sized asset purchase programme where collateralized loans are purchased at an above-market price, which drives a wedge between the effective real returns for borrowers and lenders. Moreover, we show that – compared to the constrained efficient allocation

[11] These studies, e.g., Stein (2012), Bianchi and Mendoza (2018), Jeanne and Korinek (2017), or Davila and Korinek (2018), focus on ex-ante policies under financial constraints that only bind in crises states, whereas we consider regularly binding financial constraints, as Kiyotaki and Moore (1997) or Iacoviello (2005), and examine ex-post policies that are introduced when borrowers are constrained. As shown by Bianchi (2016) and Jeanne and Korinek (2017), subsidizing borrowers ex-post (when borrowing constraints are binding) is then beneficial.

under the borrowing subsidy – asset purchases can implement welfare-dominating allocations. The reason is that asset purchases tend to increase the price of collateral relative to non-durables, which alleviates the borrowing constraint. This effect stems from the increased lenders' willingness to pay for housing relative to non-durable goods, when they receive a higher effective return on lending and thus on relinquishing non-durables. We show that this additional effect can in principle also be generated by a Pigouvian subsidy on housing. The replication of asset purchase effects via Pigouvian policies however relies on type-specific lump-sum taxes, which are typically not available.[12]

To provide numerical examples for welfare-enhancing policy interventions, we also apply a CRRA utility function. While the latter facilitates the calibration of the model, we rely on pooled end-of-period funds within households (as in Lucas and Stokey, 1987, or Woodford, 2016) when defining a competitive equilibrium with representative borrowers and lenders. For this equilibrium, which differs from the previous one solely by the agents' marginal utilities, we confirm the results derived before. Neglecting aggregate risk, we find that asset purchases can increase social welfare (measured in consumption equivalents) by up to 1% compared to the laissez faire case, while the largest contribution (about 0.9%) stems from replicating the Pigouvian subsidy on borrowing. We further introduce aggregate risk in form of a stochastic aggregate income and examine state-contingent policies under commitment. Here, we abstract from the issue of time inconsistency, which has been examined by Bianchi and Mendoza (2018) in the context of financial market interventions. We find that asset purchases should be conducted in a countercyclical way, such that borrowing is particularly stimulated in adverse states. The reason is that under adverse shocks borrowers suffer not only from a reduction in income, but also from a decline in the price of collateral (i.e. the housing price). Countercyclical asset purchases then stimulate (dampen) borrowing and thus borrowers' consumption in situations where the borrowing capacity is reduced (enhanced). Thus, asset purchases serve as an ex-post corrective policy that mitigates financial amplification and can support prudential policies that aim at reducing debt ex-ante (see, e.g., Bianchi and Mendoza, 2018); an analysis of

[12] This is shown in Appendix F, where we relate the monetary instruments under asset purchases to fiscal instruments in a corresponding non-monetary (real) economy.

this interaction being left for future research.

Our analysis of central bank purchases relates to studies on other types of unconventional monetary policies by Curdia and Woodford (2011) and Gertler and Karadi (2011), who show that direct central bank lending to ultimate borrowers can be beneficial if financial market frictions are sufficiently severe. Using an estimated preferred habitat model, Chen et al. (2012) find that changing the composition of treasury debt as under US Federal Reserve large scale asset purchase programs during the financial crisis had moderate GDP growth and inflation effects. Del Negro et al. (2017) examine government purchases of equity in response to an adverse shock to resaleability and show that the introduction of this type of policy after 2008 have prevented the US economy from a repeat of the Great Depression. Woodford (2016) extends Stein's (2012) fire sale model to assess the impact of central bank purchases of long-term treasuries on financial stability when crises are exogenously triggered. In contrast to our paper, these studies do not examine purchases of private debt in secondary markets and focus on the effects of unconventional policies on aggregate demand under stressed financial markets. Our paper further relates to Araújo et al. (2015), who show that asset purchases can exert ambiguous welfare effects under endogenous collateral constraints. In contrast to our paper, where price effects of asset purchases are based on the role of money as a means of payment, there is no special role for money in their model. The effects of debt purchases on asset prices in our model are similar to the price effects of central bank asset trades in Williamson (2016) and Rocheteau et al. (2018). The specification of central bank operations in our paper relates to Schabert (2015), who examines welfare gains from money rationing in a New Keynesian model without idiosyncratic shocks and with frictionless financial markets. Our analysis of borrowing subsidies relates to Correia et al. (2016), who apply a model with frictional intermediation and costly enforcement. They show that credit subsidies are desirable and – in contrast to our analysis – superior to monetary policy measures. Finally, our analysis of asset purchases as a corrective policy relates to the analysis of real ex-post policies, like bailouts or borrowing subsidies, which Bianchi (2016) and Jeanne and Korinek (2017) show to be beneficial in a non-monetary economy when pecuniary externalities are induced by binding financial constraints. Using a small open economy model with an occasionally binding borrowing constraint, Benigno et al. (2016) show that a "price support policy", which raises the market value of collateral, can be used to even implement the unconstrained allocation. This outcome is based on the property that this ex-post intervention does not distort further choices and cannot affect the distribution of collateral (due to a single representative agent), which crucially differs from our framework with heterogeneous agents.

In Section 2, we present the model. Section 3 provides analytical results on welfare-enhancing financial market interventions. In Section 4, we present numerical examples and analyze state-contingent asset purchases under aggregate risk. Section 5 concludes.

2 The model

In this Section, we develop an incomplete markets model with idiosyncratic preference shocks and limited contract enforcement. Major parts of the model are specified in a deliberately simple way, while it exhibits features that we view as necessary to suitably account for the way central banks have implemented asset purchase programmes. We abstract from financial intermediation, while we take a traditional view on money demand and consider that central bank money is essential because of its unique role for transactions. Concretely, money serves as the exclusive means of payment for non-durable goods and debt contracts are only available in nominal terms. To focus on the effects of asset purchases, we disregard endogenous production and price rigidities, such that conventional monetary policy measures are neutral. We explicitly model the supply of money and assume that money is supplied by the central bank only in exchange for eligible assets, which solely consist of short-term treasuries under a conventional monetary policy regime. Our particular focus is on the market for loans, where agents can – due to limited enforcement of debt contracts – only borrow against collateral and where the central bank can influence prices by purchasing collateralized loans from lenders.

2.1 Overview

The economy consists of households, a central bank, and a government. Households enter a period with money holdings and government bonds, and dispose of an exogenously given income. They can further hold a durable good (housing), which is supplied at a fixed amount. At the beginning of each period, open market operations are conducted, where the central bank sells or purchases assets outright or supplies money under repurchase agreements (repos) against treasury securities at the policy rate. Then, idiosyncratic preference shocks are realized and, subsequently, housing is traded. Households with a high realization of the preference shock tend to consume more non-durables than households with a low realization of the preference shock. Given that money serves as a means of payment for cash goods (non-durables), the former tend to borrow money from the latter. We consider a collateral constraint on private sector debt, based on limited commitment of borrowers. Importantly, we assume that these collateralized loans might be purchased by the central bank from lenders, such that the proceeds are available to extend loan supply. After cash goods are traded, labor income is paid, repos are settled, and subsequently the asset market opens. There, borrowing agents repay collateralized loans, the government issues bonds, and the central bank reinvests earnings from maturing bonds.

The central bank sets the price of money (i.e. the policy rate), decides on the amount of money that is supplied against treasuries in open market operations and via purchases of loans, and it transfers interest earnings to the government. The government issues one period bonds in an ad-hoc way and has access to lump-sum transfers. The effects of asset purchases will rely on rationed money supply, i.e. on money being supplied by the central bank only against eligible assets that are not unboundedly available. By setting the price of money below agents' marginal valuation of money, the central bank can induce scarcity of money and of eligible assets, and can influence asset prices.

2.2 Private sector

There are infinitely many and infinitely lived households i of measure one, which are characterized by identical initial stocks of wealth. Their utility increases with consumption $c_{i,t}$ of a non-durable good and holdings of a durable good, i.e. housing $h_{i,t}$; the supply of the latter being normalized to one. Households provide a fixed working time and receive labor income from non-durable goods producing firms, where each household receives income y_i, where $y_{i,t} = y_t$ and y_t denotes aggregate income that is exogenously determined with mean one. Households can differ with regard to their marginal valuation of consumption of the non-durable good due to preference shocks $\epsilon_i > 0$, which are i.i.d. across households and time. The instantaneous utility function $u_{i,t}$ of a household i is

$$u_{i,t} = u(\epsilon_i, c_{i,t}, h_{i,t}), \tag{1}$$

where $h_{i,t}$ denotes the end-of-period stock of housing. We assume that $u_{i,t}$ is strictly increasing, concave, and separable in consumption of non-durables and housing. The idiosyncratic shock ϵ_i exhibits two possible realizations, $\epsilon_i \in \{\epsilon_l, \epsilon_b\}$, with mean one, equal probabilities $\pi_\epsilon = 0.5$, and $\epsilon_l < \epsilon_b$. Households rely on money for purchases of non-durable goods, whereas we treat housing as a "credit good" (see Lucas and Stokey, 1987). They hold money $M^H_{i,t-1}$ at the beginning of each period and they can acquire additional money $I_{i,t}$ from the central bank, for which they hold eligible assets. Specifically, households can get money $I_{i,t}$ from the central bank in open market operations, where money is supplied against treasury securities $B_{i,t-1}$ discounted with the policy rate R^m_t:

$$0 \leq I_{i,t} \leq \kappa^B_t B_{i,t-1}/R^m_t. \tag{2}$$

The central bank supplies money against a fraction $\kappa^B_t \geq 0$ of randomly selected bonds under outright operations as well as repurchase agreements (see Section 2.3), implying that the non-negativity in (2) does not rule out deflationary paths. In contrast to purchases of private debt, purchases of public debt can affect the allocation only via an increase in the supply of money, while associated effects on the interest rate on treasuries will be irrelevant for the equilibrium allocation. When household i draws the realization ϵ_b (ϵ_l), which materializes after treasury open market transactions are conducted,[13] it is willing to consume more (less) than households who draw ϵ_l (ϵ_b). Hence, ϵ_b-type households tend to borrow an additional amount of money from ϵ_l-type households. We assume that borrowing and lending among households only takes place in form of short-term nominal debt at the price $1/R^L_t$. Like Jermann and Quadrini (2012) and Woodford (2016), we assume that loan contracts are signed at the beginning of the period and repaid at the end of each period. The assumption of intraperiod debt simplifies the analysis, while it

[13] The assumption that preference shocks are realized after money is supplied in open market operations against treasuries is only relevant for the case where the money supply constraint (2) is not binding.

can be shown that the main results also hold for interperiod debt (see Appendix F).

The crucial element of the model is a financial constraint, which can be microfounded as follows by limited commitment and the possibility of debt renegotiation: We assume that borrowers can threaten to repudiate the debt contract and that lenders protect themselves by collateralizing borrowers' housing. Following a repudiation, they can seize a fraction z of the borrower's housing and can sell it at the current price Q_t in the housing market. We consider the case where borrowers have all the bargaining power and are able to negotiate the loan down to the liquidation value zQ_t of their housing (see Hart and Moore, 1994). Lenders take this possibility into account, such that the debt repayment does not exceed the value of the seizable collateral. Hence, debt $-L_{i,t} > 0$ of a borrower i with housing h_i is constrained by

$$-L_{i,t} \leq zQ_t h_{i,t}, \tag{3}$$

where P_t denotes the aggregate price level, Q_t the housing price, and $z \in (0,1)$ the exogenous liquidation share of collateral. Notably, the value of collateral depends on the housing price (see 3), which will in equilibrium be affected by agents' demand for housing. This effect is not internalized by individual agents and gives rise a welfare-reducing pecuniary externality that leads to financial amplification (see Davila and Korinek, 2018). As the main object of our analysis, we consider the possibility that the central bank purchases collateralized loans in addition to treasuries: After the preference shocks are realized and loan contracts are signed, the central bank offers money in exchange for a randomly selected fraction $\kappa_t \in [0,1]$ of loans at the price $1/R_t^m$:

$$0 \leq I_{i,t}^L \leq \kappa_t L_{i,t}/R_t^m. \tag{4}$$

By purchasing loans, the central bank can thus influence lenders' valuation of collateralized loans and can induce an increase in the amount of money that is available for loan supply. For this, the price $1/R_t^m$ that the central bank pays and its relation to the market price $1/R_t^L$ are obviously decisive. Loan purchases are conducted in form of repos, where loans are repurchased by lenders before they mature (such that lenders earn the interest on loans). After loans are issued and asset purchases are conducted, the market for non-durables opens. Money is assumed to serve as the means of payment for non-durable goods, for which household i can use money holdings $M_{i,t-1}^{H}$ as well as new injections $I_{i,t}$ and $I_{i,t}^{L}$ plus/minus loans, such that the cash-in-advance constraint for household i is

$$P_t c_{i,t} \leq I_{i,t} + I_{i,t}^{L} + M_{i,t-1}^{H} - L_{i,t}/R_t^{L}. \tag{5}$$

It should be noted that the previous constraints (2)-(5) are affected by various prices, which are taken as given by private agents. Precisely, they do not take into account that their behavior affects the real price of housing $q_t = Q_t/P_t$ (see 3) and of loans $1/R_t^L$ (see 5). These pecuniary externalities, which are relevant for the equilibrium allocation of resources, can be addressed by corrective policies in a welfare-enhancing way (see below).

After consumption goods are traded, labor income is paid out in cash. Before the asset market opens, repurchase agreements are settled, i.e. agents buy back loans and treasuries under repos from the central bank, and transfers are paid. In the asset market, households repay intraperiod loans, invest in treasuries, and might trade assets among each other. Thus, the budget constraint of household i is

$$\begin{aligned} & M_{i,t-1}^{H} + B_{i,t-1} + L_{i,t}\left(1 - 1/R_t^{L}\right) + P_t y_t + P_t \tau_t \\ & \geq M_{i,t}^{H} + \left(B_{i,t}/R_t\right) + \left(I_{i,t} + I_{i,t}^{L}\right)\left(R_t^{m} - 1\right) + P_t c_{i,t} + P_t q_t \left(h_{i,t} - h_{i,t-1}\right), \end{aligned} \tag{6}$$

where $1/R_t$ denotes price of treasuries in period t and τ_t a lump-sum transfer. Maximizing $E\sum_{t=0}^{\infty}\beta^t u_{i,t}$, where the discount factor satisfies $\beta \in (0,1)$, subject to (1)-(6) and taking prices as given, leads to the following first order conditions for non-durables, holdings of treasuries and money, and additional money from treasury open market operations $\forall i \in \{b,l\}$:

$$u'(\epsilon_i, c_{i,t}) = \lambda_{i,t} + \psi_{i,t}, \tag{7}$$

$$\lambda_{i,t} = \beta R_t E_t\left[\left(\lambda_{i,t+1} + \kappa_{t+1}^{B}\eta_{i,t+1}\right)/\pi_{t+1}\right], \tag{8}$$

$$\lambda_{i,t} = \beta E_t\left[\left(\lambda_{i,t+1} + \psi_{i,t+1}\right)/\pi_{t+1}\right], \tag{9}$$

$$\overline{E}_t\psi_{i,t} = \left(R_t^{m} - 1\right)\overline{E}_t\lambda_{i,t} + \overline{E}_t R_t^{m}\eta_{i,t}, \tag{10}$$

where π_t denotes the inflation rate and $\overline{E}_t$ the expectations at the beginning of period

t before individual shocks are drawn. Further, $\lambda_{i,t} \geq 0$ is the multiplier on the asset market constraint (6), $\eta_{i,t} \geq 0$ the multiplier on the money supply constraint (2), and $\psi_{i,t} \geq 0$ the multiplier on the cash-in-advance constraint (5), where all constraints are expressed in real terms.[14] Condition (10) for money supplied against treasuries reflects that idiosyncratic shocks are not revealed before treasury open market operations are initiated. Further, the following type-specific first order conditions for loans and housing have to be satisfied, for borrowers

$$\lambda_{i,t}\left(1 - 1/R_t^L\right) - (\psi_{i,t}/R_t^L) + \zeta_{i,t} = 0, \tag{11}$$

$$u'(h_{i,t}) + \zeta_{i,t} z q_t + \beta E_t q_{t+1} \lambda_{i,t+1} - q_t \lambda_{i,t} = 0, \tag{12}$$

and for lenders, where we additionally consider the first order condition for money acquired from loan purchases $I_{l,t}^L$,

$$\lambda_{i,t}\left(1 - 1/R_t^L\right) - (\psi_{i,t}/R_t^L) + \mu_{i,t}\kappa_t = 0, \tag{13}$$

$$u'(h_{i,t}) + \beta E_t q_{t+1} \lambda_{i,t+1} - q_t \lambda_{i,t} = 0, \tag{14}$$

$$-\lambda_{i,t}\left(1 - 1/R_t^m\right) + (\psi_{i,t}/R_t^m) - \mu_{i,t} = 0, \tag{15}$$

Note that differences between the first order conditions for borrowers and lenders are due to the multiplier $\zeta_{i,t} \geq 0$ on the collateral constraint (3), which is only relevant for borrowers, and the multiplier $\mu_{i,t} \geq 0$ on the money supply constraint (4), which is only relevant for lenders. Condition (15) describes lenders' willingness to sell loans to the central bank. The conditions (11) and (13) further show that the multiplier on the cash-in-advance constraint (5) is positive if the loan rate R_t^L exceeds one, as the latter measures the price of cash goods. Further, the associated complementary slackness conditions,[15] as well as (2)-(5), (6) as an equality, and the associated transversality conditions hold.

Notably, $\lambda_{i,t} \geq 0$, $\psi_{i,t} \geq 0$, (10), and (15) imply that the policy rate is bounded from below by $R_t^m \geq 1$, if the money supply constraints (2) and (4) are not binding,

[14] Condition (8) indicates that the interest rate on government bonds is affected by a liquidity premium, stemming from the possibility to exchange a fraction κ_t^B of bonds in open market operations (see 2).

[15] The complementary slackness conditions are $\eta_{i,t}[\kappa_t^B b_{i,t-1}(\pi_t R_t^m)^{-1} - i_{i,t}] = 0$, $\zeta_{i,t}\left[zq_t h_{i,t} + l_{i,t}\right] = 0$, $\mu_{i,t}\left[\kappa_t l_{i,t}/R_t^m - i_{i,t}^L\right] = 0$, and $\psi_{i,t}\left[i_{i,t} + i_{i,t}^L + m_{i,t-1}^H - (l_{i,t}/R_t^L) - c_{i,t}\right] = 0$, where real variables are given by $b_{i,t} = B_{i,t}/P_t$, $l_{i,t} = L_{i,t}/P_t$, $m_{i,t}^H = M_{i,t}^H/P_t$, $i_{i,t} = I_{i,t}/P_t$, $i_{i,t}^L = I_{i,t}^L/P_t$.

$\eta_{i,t} = \mu_{i,t} = 0$. However, if there are binding, $\mu_{i,t} > 0$ and $\eta_{i,t} > 0$, which will be the case under an effective asset purchase policy (see Section 2.4), a policy rate below one, $R_t^m < 1$, is also feasible. Moreover, a nominal loan rate R_t^L below one is also feasible, which requires a binding collateral constraint $\zeta_{i,t} > 0$ (see 11). Hence, a zero lower bound on nominal interest rates does not generally apply in this model.

Combining (7) and (9) to $\frac{\psi_{i,t}}{u'(\epsilon_i, c_{i,t})} = 1 - \beta \frac{E_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{u'(\epsilon_i, c_{i,t})}$ shows that the liquidity constraint (5) is binding when the nominal marginal rate of intertemporal substitution $\frac{u'(\epsilon_i, c_{i,t})}{\beta E_t(u'(\epsilon_i, c_{i,t+1})/\pi_{t+1})}$ exceeds one. Then, the money supply constraint (4) is binding, $\mu_{i,t} > 0$, implying that lenders are willing to refinance loans at the central bank to the maximum amount. This is the case when the policy rate R_t^m is lower than the loan rate R_t^L, which can be seen from combining (15) with (7), (9), and (18) to

$$\frac{\mu_{i,t}}{u'(\epsilon_i, c_{i,t})} = \frac{1}{1-\kappa}\left(\frac{1}{R_t^m} - \frac{1}{R_t^L}\right). \tag{16}$$

If, however, the policy rate equals the loan rate, $R_t^m = R_t^L$, lenders have no incentive to refinance loans at the central bank and (4) becomes slack (see 16). Thus, if the central bank offers a price for loans $1/R_t^m$ that exceeds the market price $1/R_t^L$, lenders are willing to sell collateralized loans until the money supply constraint (4) is binding ($\mu_{i,t} > 0$).

The conditions for loan demand (11) and loan supply (15) reveal that the credit market allocation can be affected by the borrowing constraint (for $\zeta_{i,t} > 0$) as well as by central bank loan purchases (for $\mu_{i,t} > 0$). The borrowers' demand condition for loans (11) can – by using (7), (9), and (15) – be rewritten as

$$\frac{1}{R_t^L} = \beta \frac{E_t(u'(\epsilon_i, c_{i,t+1})/\pi_{t+1})}{u'(\epsilon_i, c_{i,t})} + \frac{\zeta_{i,t}}{u'(\epsilon_i, c_{i,t})}. \tag{17}$$

Hence, a positive multiplier $\zeta_{i,t}$ tends to raise the RHS of (17), implying a relative increase in current marginal utility of consumption, which can be mitigated by a lower loan rate. Put differently, under a binding borrowing constraint (3) the borrowers' nominal marginal rate of intertemporal substitution exceeds the loan rate. Further, the lenders' loan supply condition (13) can – by using (7) and (9) – be written as $\frac{1}{R_t^L} = \kappa_t \cdot \frac{1}{R_t^m} + (1 - \kappa_t) \cdot \beta \frac{E_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{u'(\epsilon_i, c_{i,t})}$, which implies that the loan rate depends on the lender's nominal marginal rate of intertemporal substitution as well as on the policy rate R_t^m, if the central bank purchases loans, $\kappa_t > 0$.[16] This condition can be rewritten as

$$\frac{1}{R_t^L} = \left[\frac{1-\kappa_t}{1-\kappa_t R_t^L / R_t^m}\right] \cdot \beta \frac{E_t(u'(\epsilon_i, c_{i,t+1})/\pi_{t+1})}{u'(\epsilon_i, c_{i,t})}, \tag{18}$$

showing that the term in the square brackets drives a policy induced wedge between the effective real returns for borrowers and lenders. Further note that (7), (9), and (10) imply $\frac{\overline{E}_t \eta_{i,t}}{\overline{E}_t u'(\epsilon_i, c_{i,t})} = \frac{1}{R_t^m} - \beta \frac{\overline{E}_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{\overline{E}_t u'(\epsilon_i, c_{i,t})}$, where the term $\frac{\beta \overline{E}_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{\overline{E}_t u'(\epsilon_i, c_{i,t})}$ cannot be larger than the inverse of the loan rate $1/R_t^L$ (see 17 and 18). Thus, a policy rate satisfying $1 \leq R_t^m < R_t^L$ ensures that money is scarce, such that the liquidity constraint (5) is binding, and that agents liquidate bonds as far as possible, such that the money supply constraint (2) is binding as well as (4). Money supply is then constrained by the available amount of eligible assets, i.e. bonds and loans. If however money is supplied in an unrestricted way at the policy rate R_t^m, the nominal marginal rate of intertemporal substitution will be equal to the latter and asset purchases are neutral.

2.3 Public sector

The government issues nominal bonds at the price $1/R_t$ and pays lump-sum transfers τ_t. Notably, a lump-sum tax/transfer system is not necessary to derive the main result, i.e. the welfare-enhancing role of asset purchases, and can principally be replaced by distortionary instruments. In Section 3.2, we further introduce a borrowing tax/subsidy as a means of financial market intervention, which is not specified here, for convenience. As described above, short-term government bonds serve as eligible assets for central bank operations. Hence, sufficiently large holdings of treasuries can in principle support self-insurance against illiquidity risk (see also Woodford, 1990) and thereby the implementation of the first best allocation. To ensure a non-trivial policy analysis, the supply of short-term government bonds does not support the implementation of first best (due to some unmodelled fiscal considerations) and is specified in a simple ad-hoc way. Specifically, the total amount of short-term government bonds B_t^T grows at a rate $\Gamma > 0$,

$$B_t^T = \Gamma B_{t-1}^T, \tag{19}$$

[16] Hence, a higher share of purchased loans κ_t for a given policy rate $R_t^m < R_t^L$, or a lower policy rate R_t^m for a given share of purchased loans, $\kappa_t > 0$, tend to reduce the loan rate, while the loan rate approaches the policy rate, $R_t^L \to R_t^m$, for $\kappa_t \to 1$.

given $B^T_{-1} > 0$.[17] The government further receives seigniorage revenues τ^m_t from the central bank, such that its budget constraint reads $(B^T_t/R_t) + P_t\tau^m_t = B^T_{t-1} + P_t\tau_t$.

The central bank supplies money in open market operations either outright or temporarily via repos against treasuries, M^H_t and M^R_t. It can further increase the supply of money by purchasing collateralized loans from lenders, I^L_t, i.e. it supplies money under repos against collateralized loans. At the beginning of each period, its holdings of treasuries and the stock of outstanding money are given by B^c_{t-1} and M^H_{t-1}. It then receives treasuries and loans in exchange for money. Before the asset market opens, where the central bank rolls over maturing assets, repos in terms of treasuries and collateralized loans are settled. Hence, its budget constraint reads $(B^c_t/R_t) - B^c_{t-1} + P_t\tau^m_t = R^m_t\left(M^H_t - M^H_{t-1}\right) + (R^m_t - 1)\left(I^L_t + M^R_t\right)$, showing that the central bank earns interest from bonds purchased outright and from money supply. The central bank transfers its interest earnings to the government, $P_t\tau^m_t = (1 - 1/R_t)\,B^c_t + R^m_t\left(M^H_t - M^H_{t-1}\right) + (R^m_t - 1)\left(I^L_t + M^R_t\right)$. Thus, central bank asset holdings evolve according to $B^c_t - B^c_{t-1} = M^H_t - M^H_{t-1}$. Further assuming that initial values satisfy $B^c_{-1} = M^H_{-1}$, gives the central bank balance sheet

$$B^c_t = M^H_t. \tag{20}$$

The central bank has four instruments at its disposal. It sets the policy rate R^m_t and can decide how much money to supply against a randomly selected fraction of treasuries, for which it can adjust $\kappa^B_t \in (0,1]$. The central bank can further decide whether it supplies money in exchange for treasuries either outright or temporarily via repos. Specifically, it controls the ratio of treasury repos to outright purchases $\Omega_t > 0 : M^R_t = \Omega_t M^H_t$, where a sufficiently large value for Ω_t ensures that injections are always positive, $I_{i,t} > 0$. Finally, the central bank can decide to purchase loans, i.e. to supply money temporarily against collateralized loans under repos. In each period, it therefor decides on a randomly selected share of collateralized loans $\kappa_t \in [0,1]$ that it is willing to exchange for money under repos. To give a preview, only policy regimes with loan purchases, $\kappa_t > 0$, will be non-neutral with regard to the equilibrium allocation, whereas conventional monetary

[17] Note that the growth rate Γ might affect the long-run inflation rate if the money supply constraint (2) is binding. As shown in Appendix C, the central bank can nonetheless implement a desired inflation target (different from Γ) by suited long-run adjustments of its money supply instruments.

policies, $R_t^m = R_t^L$ or $\kappa_t = 0$, will be neutral.

2.4 Equilibrium properties

In equilibrium, agents' optimal plans are satisfied and prices adjust such that all markets clear: $0 = \sum_i l_{i,t}$, $h = \sum_i h_{i,t}$, $y = \sum_i c_{i,t}$, $m_t^H = \sum_i m_{i,t}^H$, $m_t^R = \sum_i m_{i,t}^R$, $b_t = \sum_i b_{i,t}$, and $b_t^T = b_t^c + b_t$, where $l_{i,t} = L_{i,t}/P_t$, $m_{i,t}^H = M_{i,t}^H/P_t$, $m_t^R = M_t^R/P_t$, $b_{i,t} = B_{i,t}/P_t$, $b_t = B_t/P_t$, $b_t^c = B_t^c/P_t$, and $b_t^T = B_t^T/P_t$. A definition of a competitive equilibrium is given in Appendix A. Before we examine policy effects on the equilibrium allocation, we describe first best, which maximizes social welfare, i.e. welfare of ex-ante identical agents

$$E\sum_{t=0}^{\infty}\beta^t\sum_i u_{i,t}, \tag{21}$$

s.t. $h = \sum_i h_{i,t}$, and $y = \sum_i c_{i,t}$ and serves as a reference case for the subsequent analysis. Applying the law of large numbers and indexing all agents drawing ϵ_l (ϵ_b) in period t with l (b), we can summarize the first best allocation as a set of sequences $\{c_{b,t}^*, c_{l,t}^*, h_{b,t}^*, h_{l,t}^*\}_{t=0}^{\infty}$ satisfying $h_{b,t}^* + h_{l,t}^* = h$, $c_{l,t}^* + c_{b,t}^* = y$,

$$u_c(\epsilon_b, c_{b,t}^*) = u_c(\epsilon_l, c_{l,t}^*), \text{ and } h_{b,t}^* = h_{l,t}^*. \tag{22}$$

Under the first best allocation, the marginal utilities of consumption and of the end-of-period stock of housing are identical for borrowers and lenders (see 22). This will not be the case in a competitive equilibrium where the borrowing constraint (3) is binding. Only if the equilibrium lending rate R_t^L were equal to one and the supply of eligible assets sufficiently large, such that money were abundantly available, agents would be able to self-ensure against liquidity risk and the first best equilibrium would be implementable (see also Woodford, 1990). To provide a non-trivial analysis of policy interventions, this outcome is ruled out by imposing an ad-hoc specification for the supply of short-term government bonds (see 19). As an alternative way to demonstrate the welfare enhancing role of asset purchases, one could instead introduce additional frictions that render the Friedman rule undesirable/impossible (see Andolfatto, 2013, and references therein), which are neglected here for convenience.

For asset purchases to be relevant, money has to be supplied at a favorable price (see 16), which implies that money supply is rationed by the available amount of assets eligible for central bank operations. Specifically, the central bank has to set the policy rate below the lender's marginal rate of intertemporal substitution, implying $R_t^m < R_t^L$ (see 18), to ration money supply. As discussed above, a policy rate below one, $R_t^m < 1$, is then also feasible (see 15). Under a non-rationed money supply, which is equivalent to the case where the central bank supplies money in a lump-sum way (as typically assumed in the literature), the money supply constraints (2) and (4) are slack and the loan rate is identical to the policy rate $R_t^L = R_t^m$. In this case, asset purchases are irrelevant (see 16). For the subsequent analysis, we will therefore separately discuss the two cases where money supply is rationed and where money supply is not rationed, the latter being the case under a conventional monetary policy regime.

3 Analytical results

In this Section, we examine different types of policies in an analytical way. In the first part of this Section, we impose some further assumptions, which facilitate aggregation and the derivation of analytical results, and we characterize the competitive equilibrium in terms of a representative borrower and a representative lender, where conventional monetary policies are shown to be neutral. In the subsequent part of this Section, we show how asset purchases can enhance welfare by addressing pecuniary externalities and by easing borrowing conditions. We firstly analyze the constrained efficient allocation that a social planer chooses, when it decides upon agents' borrowing, and we show that it can be implemented by a Pigouvian subsidy on borrowing. Secondly, we show that asset purchases can not only implement the constrained efficient allocation under the Pigouvian subsidy, but can further increase the set of feasible allocations, including welfare-dominating allocations, by relaxing borrowing conditions. In Appendix F, we show in a real (non-monetary) economy that the welfare-enhancing impact of asset purchases can also be generated by a combination of Pigouvian subsidies on debt and collateral, which rely on type-specific lump-sum transfers.

3.1 Aggregation and conventional monetary policy

Here, we examine the benchmark case of a conventional monetary policy, where the central bank sets the policy rate equal to the loan rate, $R_t^m = R_t^L$, such that both money supply constraints (2) and (4) are not binding ($\eta_{i,t} = \mu_{i,t} = 0$), implying that asset purchases are irrelevant (see 16). Even when the central bank were willing to buy loans, lenders would then not gain from selling loans and prices would not be affected. Given that we aim at disclosing the distributional and welfare effects of financial market interventions, we apply three assumptions that allow deriving the main results in an analytical way. Firstly, we assume that preferences are given by a linear-quadratic form, which enables aggregation over individual choices, since all conditions are linear in the agents' choice variables. Once the competitive equilibrium is defined in terms of aggregate variables, we analytically derive the main results on policy interventions, which will be confirmed for alternative preferences (see Section 4).[18]

Assumption 1 *Instantaneous utility of households satisfies*

$$u(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i(\delta c_{i,t} - (1/2)c_{i,t}^2) + (\gamma h_{i,t} - (1/2)h_{i,t}^2), \tag{23}$$

where $\partial u/\partial c_{i,t} = u'(\epsilon_i, c_{i,t}) > 0$ *and* $\partial u/\partial h_{i,t} = u'(h_{i,t}) > 0$.

The cash-in-advance constraint might not be binding, which would be the case when the nominal interest rate equals one, $R_t^m = R_t^L = 1$ (see 13 for $\kappa_t = 0$). Given our specification of fiscal policy, the latter policy is not sufficient to implement first best, which would require agents to accumulate bonds and money to a sufficiently large amount to ensure the borrowing constraint never to be binding (see Section 2.2). To avoid indeterminacies due to a slack cash-in-advance constraint, we, secondly, assume that the latter is just binding even when the nominal interest rate equals one and the associated multiplier equals zero, $\psi_{l,i,t} = 0$. Alternatively, one can assume that the Friedman rule does not hold exactly, but only as a limit (analogously, for example, to Gu et al., 2016).

Assumption 2 *Agents will hold money equal to the amount of planned nominal consumption expenditures even when the multiplier on the cash-in-advance constraint equals zero.*

[18] For the case of CRRA preferences, which are introduced in Section 4, aggregation will be enabled by pooling funds within households at the end of each period.

It should be noted that Assumption 2 is made for convenience only and does not affect the main conclusions: If cash-in-advance constraints were not binding, monetary policy would apparently be irrelevant. As will be shown below, a conventional monetary policy will in fact also be irrelevant if the cash-in-advance constraint is binding (see Corollary 2). Assumption 2 will therefore not be decisive for the assessment of monetary policy. Under both Assumptions 1 and 2, we can easily aggregate by summing over all agents who draw ϵ_l in period t. Using the law of large numbers, all agents face the same probability (0.5) of drawing ϵ_l in period t, such that average holdings of money, bonds, and housing of these agents at the beginning of each period are identical. The resulting set of conditions for the representative lender are given in Appendix A.

For a non-trivial policy problem, the borrowing constraint (3) has to be binding for some borrowers, which will apparently be the case for a larger fraction of borrowers under a larger difference in the agents' valuation of consumption and for a lower liquidation value of collateral. To further facilitate aggregation over borrowers, we, thirdly, assume that the associated multiplier is strictly positive for all agents drawing ϵ_b, $\zeta_{b,i,t} > 0$, which can simply be guaranteed by a sufficiently large difference in agents' valuation of consumption relative to the liquidation value of collateral, $(\epsilon_b - \epsilon_l)/z$.

Assumption 3 *The ratio $(\epsilon_b - \epsilon_l)/z$ is sufficiently large such that the borrowing constraint (3) is binding for all agents drawing ϵ_b.*

The resulting set of conditions for the representative borrower are given in Appendix A. Now, let $x_{l,t} = 2\sum_{l,i} x_{l,i,t}$ ($x_{l,t} = 2\sum_{l,i} x_{l,i,t}$) be the value of any generic variable $x_{l,t}$ ($x_{b,t}$) of a representative agent drawing ϵ_l (ϵ_b) in period t. Applying the Assumptions 1, 2, and 3, we can write the competitive equilibrium in terms of a representative borrower and a representative lender. Notably, cash holdings and loans rather than individual net wealth positions are relevant for agents' consumption and housing choices. Thus, the combined cash-in-advance and collateral constraint will be crucial for consumption and end-of-period housing decisions of the representative borrower and the representative lender. A central element is therefore the relative price of collateral in terms of the cash good, q/R^L.

Definition 1 *With Assumptions 1-3, the competitive equilibrium under a conventional monetary policy regime can be characterized as a set of sequences* $\{c_{b,t},\, c_{l,t},\, h_{b,t},\, q_t,\, \pi_t\}_{t=0}^{\infty}$ *satisfying*

$$\epsilon_l(\delta - c_{l,t}) = \beta 0.5 E_t\left[\left(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1})\right)\left\{R_t^L/\pi_{t+1}\right\}\right], \tag{24}$$

$$\left\{R_t^L/q_t\right\}(2h_{b,t} - h)/z = \epsilon_b(\delta - c_{b,t}) - \beta 0.5 E_t[(\epsilon_b(\delta - c_{b,t+1}) + \epsilon_l(\delta - c_{l,t+1}))\left\{R_t^L/\pi_{t+1}\right\}], \tag{25}$$

$$\epsilon_l(\delta - c_{l,t})\left\{q_t/R_t^L\right\} = \gamma - (h - h_{b,t}) + \beta E_t[\epsilon_l(\delta - c_{l,t+1})\left\{q_{t+1}/R_{t+1}^L\right\}], \tag{26}$$

$$c_{b,t} - c_{l,t} = z h_{b,t} 2\left\{q_t/R_t^L\right\}, \tag{27}$$

$$y_t = c_{b,t} + c_{l,t}, \tag{28}$$

and $R_t^L = R_t^m$, *for* $\{y_t\}_{t=0}^{\infty}$ *and a sequence* $\{R_t^m \geq 1\}_{t=0}^{\infty}$ *set by the central bank.*

An agent who draws a preference shock ϵ_b in period t, borrows money from other agents to increase its consumption possibilities. Given that the loan has to be repaid at the end of the period, there are less funds available at the beginning of period $t+1$. While idiosyncratic histories of shock realizations matter for individual net wealth positions, they do not matter for the aggregate behavior of borrowers/lenders, given that all agents face the same probability of drawing ϵ_b (ϵ_l) and their behavioral relations are linear. Further note that the multiplier on the borrowing constraint satisfies

$$\zeta_{b,t} = [\epsilon_b(\delta - c_{b,t}) - \epsilon_l(\delta - c_{l,t})]/R_t^L = (2h_{b,t} - h)/(zq_t) \geq 0, \tag{29}$$

indicating that both, the housing and the consumption choice (that would ideally satisfy $h_b = h_l$ and $\epsilon_l(\delta - c_{b,t}) = \epsilon_l(\delta - c_{l,t})$, see 22), are distorted by a binding borrowing constraint ($\zeta_{b,t} > 0$). On the one hand, the marginal utility of consumption is then larger for borrowers than for lenders, $\epsilon_b(\delta - c_{b,t}) > \epsilon_l(\delta - c_{l,t})$. On the other hand, borrowers' housing exceeds lenders' housing, $h_{b,t} > h/2$, as the former is characterized by a relatively higher valuation of housing due to its ability to serve as collateral. Given that the supply of non-durables and durables is exogenous, such that $h = h_{b,t}^* + h_{l,t}^*$, and $y = c_{l,t} + c_{b,t}$, (29) implies that the equilibrium allocation equals the first best allocation (see 22) when the borrowing constraint gets irrelevant, $\zeta_{b,t} \to 0$. However, the distortion due to the cash-in-advance constraint (5) alone does not lead to an allocative inefficiency.

Corollary 1 *For the limiting case where the multiplier on the borrowing constraint approaches zero, the equilibrium allocation is identical with the first best allocation.*

Definition 1 reveals that the nominal interest rate and thus the policy rate matter jointly with either the housing price or the inflation rate. Precisely, the conditions (24)-(28) impose restrictions on the allocation, $c_{b,t}, c_{l,t}$, and $h_{b,t}$, the relative price of housing q_t/R_t^L, and the real interest rate R_t^L/π_{t+1} (see curly brackets in 24-27), but not separately on q_t, π_t, and R_t^L. Thus, conventional monetary policy measures, i.e. changes in the policy rate $R_t^m = R_t^L$, leave relative prices and the allocation unaffected, while they affect the inflation rate and the price of housing.[19]

Corollary 2 *Under a conventional monetary policy regime, changes in the monetary policy rate do neither affect relative prices nor the equilibrium allocation, while the housing price and the inflation rate increase with the nominal interest rate.*

The reason for the neutrality summarized in Corollary 2 is that conventional monetary policies can only affect equilibrium prices that are equally relevant for both agents, while the aggregate supply of durable and non-durable goods is exogenously determined. Given that changes in the monetary policy instrument R_t^m under a conventional monetary policy regime do not affect the equilibrium allocation, the latter is time-invariant if there is no aggregate risk. To facilitate comparisons between the different policy experiments, we restrict our attention to the case of time-invariant policies in the subsequent analysis. In Section 4.3, we introduce aggregate risk and analyze state-contingent policies.

3.2 Pecuniary externalities and constrained efficiency

For the remainder of this section, we abstract from aggregate risk $y_t = y$ and focus on time-invariant financial market interventions, such that neither the allocation nor prices are time-varying. Given that conventional monetary policy is neutral (see Corollary 2), we summarize this case as *laissez faire*. We aim at examining efficiency gains that can be reaped by a social planer, who takes into account how prices that are relevant for financial constraints are affected by agents' decisions. We consider a competitive equilibrium (see Definition 1) under time-invariant endogenous variables. Condition (26) then implies the price of housing relative to non-durables q/R_L to be negatively related to lenders' housing

[19]The latter effect is due to the liquidity constraint and the well-known inflation tax on cash goods, implying that higher interest rates reduce the demand for non-durables and raise housing demand.

$h - h_b$ and positively related to lenders' consumption c_l,

$$\frac{q}{R^L} = \frac{\gamma - (h - h_b)}{(1-\beta)\epsilon_l(\delta - c_l)}. \tag{30}$$

Notably, an increase in the relative price q/R^L tends to raise the difference between consumption of borrowers and lenders, as shown by the consolidated cash-in-advance and collateral constraint (see 27), since it increases borrowers' consumption possibilities by raising the price of collateral in terms of consumption. Yet, the impact of the demand for housing and consumption on the relative price q/R^L is not internalized by individuals, giving rise to inefficiencies induced by a pecuniary externality. Notably, the latter, which corresponds to Davila and Korinek's (2018) "collateral externality", is the single relevant externality in our model, since further (zero-sum) "distributional externalities" are not relevant for the consumption and housing choices of the representative borrower and the representative lender (due to the irrelevance of individual net wealth positions under Assumptions 1-3).

For example, borrowers do not internalize that an increase in their housing (thus a decrease in lenders' housing) tends to increase the relative price q/R^L as lenders' willingness to pay for housing increases. Hence, there might exists an uninternalized positive feedback loop between collateral demand, prices, and borrowing, as in Stein (2012), Jeanne and Korinek (2013), or Bianchi and Mendoza (2018). Notably, these studies, which focus on prudential financial regulation, restrict their attention on financial constraints that are only binding in crisis states where asset prices decline, such that ex-ante interventions should reduce debt to mitigate adverse deleveraging effects. Considering regularly binding constraints, like Kiyotaki and Moore (1997) or Iacoviello (2005), we instead examine ex-post interventions that are introduced while borrowers are constrained. In these states, policies should benefit borrowers, as also shown by Bianchi (2016) and Jeanne and Korinek (2016) for bailouts and borrowing subsidies.

Like Bianchi and Mendoza (2018), we consider a problem of a social planer who accounts for all competitive equilibrium conditions, including the borrowing constraint, and selects a feasible allocation, while choosing the amount that agents borrow. Given that there is no time variation, the problem of a social planer, who maximizes ex-ante social welfare (21), can then – by using the primal approach – be summarized as

$$\max_{c_l, c_b, h_b} \{u(\epsilon_b, c_b, h_b) + u(\epsilon_l, c_l, h - h_b)\}/(1-\beta), \tag{31}$$
$$\text{s.t.} \quad y = c_l + c_b,\ c_b - c_l \leq 2zh_b \cdot \left(q/R^L\right), \text{ and (30).}$$

In contrast to private agents, the social planer takes into account that changes in the allocation alter the relative price q/R^L (see 30). The solution to the planer's problem (31) leads to the constrained efficient allocation. To implement the latter, we consider a (non-monetary) financial market intervention that alters the cost of borrowing. Specifically, we suppose that the social planer can influence private borrowing by a Pigouvian tax/subsidy on debt τ^L, while it transfers/collects the funds in cash to/from borrowers in a lump-sum way. In contrast to open economy models, where all (domestic) agents are borrowers (see e.g. Benigno et al., 2016, or Bianchi and Mendoza, 2018), a welfare-enhancing corrective policy in an economy with heterogenous agents therefore relies on type-specific tax/transfers (see e.g. Davila and Korinek, 2018). Then, the borrower's effective real interest rate $r^{\tau}_{b,t}$ is

$$r^{\tau}_{b,t} = \frac{R^L_t/\pi_{t+1}}{1-\tau^L}, \tag{32}$$

and the borrower's loan price net of taxes is $(1-\tau^L)/R^L_t$, which is financed by a lump-sum transfer/tax equal to $\tau^R_t = \tau^L l_t/R^L_t$ that only applies to borrowers.[20] This intervention affects the marginal costs of borrowing and can thereby correct for inefficiencies induced by pecuniary externalities under financial constraints. Now, consider a competitive equilibrium (see Definition 1) under time-invariant endogenous variables and with the Pigouvian tax/subsidy. The borrowers' consumption Euler equation (25) then changes to $(1-\tau^L)\epsilon_b(\delta - c_b) = \beta 0.5[(\epsilon_b(\delta - c_b) + \epsilon_l(\delta - c_l)) \left\{R^L/\pi\right\} + \left\{R^L/q\right\}(2h_b - h)/z$. Using condition (30) to substitute out q/R^L in the latter and in (27), and (24) to substitute out the real interest rate, yields

$$(1-\tau^L)\epsilon_b(\delta - c_b) - \epsilon_l(\delta - c_l) = (2h_b - h)\,\frac{(1-\beta)\epsilon_l(\delta - c_l)}{z(\gamma - h + h_b)}. \tag{33}$$

[20]Thus, the tax/subsidy and the lump-sum transfers/taxes enter the budget constraint (6) and the goods market constraint (5) of borrowers.

Given the constrained efficient allocation, (33) determines the associated optimal tax/subsidy rate τ^L. The following proposition summarizes the main results.

Proposition 1 *The constrained efficient allocation of the representative agents economy without aggregate risk can be implemented by a Pigouvian subsidy on borrowing, $\tau^L < 0$, if but not only if $z/(1-\beta) \geq 1$. Compared to the laissez-faire case ($\tau^L = 0$), the Pigouvian subsidy raises borrowers' consumption and housing as well as the real interest rate R^L/π, which is associated with a decline in lenders' consumption and housing.*

Proof. See Appendix B. ■

Proposition 1 implies that a financial market intervention that stimulates borrowing can enhance social welfare if $z/(1-\beta) \geq 1$. The latter condition is sufficient to ensure that the relative price of collateral increases with borrowers' expenditures, which closely relates to condition 1 in Davila and Korinek's (2018). An increase in borrowing, which is induced by a Pigouvian subsidy $\tau^L < 0$, tends to increase borrowers' consumption and has to be supported by a larger stock of housing held by borrowers. As implied by (30) and $c_l = y - c_b$, the price of housing in terms of consumption, q/R^L, increases with the latter, $\partial(q/R^L)/\partial h_b > 0$, and decreases with the former, $\partial(q/R^L)/\partial c_b < 0$. If the impact of housing demand on the relative price q/R^L dominates, which can be shown to be the case under the following condition (see proof of Proposition 1)

$$\frac{(1-\tau^L)u_{c_b} - u_{c_l}}{u_{c_b} - u_{c_l}} = \frac{1 + \left[\partial(q/R^L)/\partial h_b\right] \cdot h_b/(q/R^L)}{1 + zh_b \cdot \left[\partial(q/R^L)/\partial c_l\right]} > 1, \tag{34}$$

where $\partial(q/R^L)/\partial c_l = u_{h_l}(-u_{c_lc_l}/u_{cl}^2)/\left(1-\beta\right) > 0$ and $\partial(q/R^L)/\partial h_b = (-u_{h_lh_l}/u_{c_l})/\left(1-\beta\right) > 0$, the constrained efficient allocation requires a borrowing subsidy $\tau^L < 0$. Given that housing is a durable good, permanent adjustments in housing demand are associated with relatively large price changes, i.e. amplified by the multiplier $1/(1-\beta)$, such that condition (34) is ensured by $z/(1-\beta) \geq 1$, where $z \in (0,1)$ accounts for the assumption that the liquidation value of housing is less than one. In this case, which is likely to be satisfied by reasonable values for the parameters β and z (see Section 4.1), borrowing is (constrained) inefficient in a competitive equilibrium, given that the private agents do not internalize the favorable effects of increased housing demand on the relative price q/R^L. The social planer can then correct for this pecuniary externality by a borrowing subsidy $\tau^L < 0$ (financed by a lump-sum tax on borrowers, $\tau_t^R = -\tau^L l_t/R_t^L > 0$), which internalizes changes in the relative price q/R^L induced by agents' demand.[21] The subsidy causes agents to borrow more, leading to an increase in borrowers' consumption and housing (see Proposition 1). Notably, the subsidy tends to reduce borrowing costs r_b^τ (see 32), while it simultaneously raises the real interest rate R^L/π, causing an increase in loan supply and a decline in lenders' consumption and housing.

3.3 Welfare-enhancing asset purchases

We now turn to the effects of central bank purchases of collateralized loans, $\kappa_t > 0$ (see 4). Firstly, we will show that an asset purchase policy can be equivalent to a Pigouvian subsidy on debt that implements the constrained efficient allocation, which is characterized in Proposition 1, by driving a wedge between the relevant real returns for borrowers and lenders (see 18). Thus, asset purchases can enhance social welfare by addressing pecuniary externalities, which is not possible under a conventional monetary policy regime (see Corollary 2). Secondly, we will show that compared to the case of a Pigouvian subsidy (as described in Section 3.2) the central bank can enlarge the set of feasible equilibria and can even implement allocations that welfare-dominate the constrained efficient allocation by increasing the relative price of collateral.[22]

Given that asset purchases are not in general effective, as discussed in Section 2.4, the central bank has to offer an above-market price for loan purchases to affect relevant prices and the equilibrium allocation. Thus, it has to set the policy rate below the market loan rate $R_t^m < R_t^L$, which implies that money supply is effectively rationed by holdings of eligible collateral, i.e. the money supply constraints in terms of treasuries and collateralized loans (2) and (4) are binding (see 16),

$$i_{i,t} = \kappa_t^B 0.5 b_{t-1}/(\pi_t R_t^m) \text{ for } i \in \{l, b\} \text{ and } i_{l,t}^L = \kappa_t l_t / R_t^m. \quad (35)$$

If, $R_t^m < R_t^L$, purchases of loans $\kappa_t > 0$ drive a wedge between the borrowers' and the lenders' effective real interest (loan) rate, such that the pecuniary externalities discussed

[21] Given that $z \geq 1 - \beta$ is just a sufficient condition, a violation of this condition does not necessarily imply the opposite result.

[22] As shown for a corresponding model of a real economy, the latter effect relates to a subsidy for housing (see Appendix F).

in Section 3.2 can be addressed like with the Pigouvian subsidy $\tau^L < 0$. Concretely, under an asset purchase regime, the effective real return for a lender $r^{ap}_{l,t}$ is distorted by the wedge $\frac{1-\kappa_t}{1-\kappa_t R^L_t/R^m_t} \geq 1$ (see 18), whereas the borrowers' real interest rate $r^{ap}_{b,t}$ is not directly affected by the policy instruments (in contrast to the Pigouvian subsidy, see 32):

$$r^{ap}_{b,t} = \frac{R^L_t}{\pi_{t+1}} \text{ and } r^{ap}_{l,t} = \frac{R^L_t}{\pi_{t+1}} \frac{1-\kappa_t}{1-\kappa_t R^L_t/R^m_t}. \tag{36}$$

Notably, the wedge $\frac{1-\kappa_t}{1-\kappa_t R^L_t/R^m_t}$ increases with the fraction of purchased loans κ and with the price discount R^L_t/R^m_t. Moreover, by additionally purchasing eligible assets the central bank increases the overall amount of funds available for lending compared to the case of a conventional monetary policy. Using (35) to rewrite the binding liquidity constraints (5) and taking differences yields

$$c_{b,t} - c_{l,t} = zh_{b,t} \left[2 - \kappa_t R^L_t/R^m_t\right] \left\{q_t/R^L_t\right\}, \tag{37}$$

where we substituted out loans with the binding borrowing constraint (63). Comparing (27) with (37), suggests that asset purchases adversely affect the difference between borrowers' and lenders' consumption relative to borrowers' housing in the first instance, since loan purchases endow lenders rather than borrowers with additional money. However, the higher effective return for lenders increases their willingness to pay for housing relative to non-durables. Hence, to identify the ultimate impact on the equilibrium allocation, changes in the relative price q_t/R^L_t also have to be taken into account (see below).

Applying the same aggregation procedure as in Section 3.1, using (19), (20), and $B^T_t = B^c_t + B_t$, and eliminating the multiplier $\lambda_{l,t}$ with $\lambda_{l,t} = \beta E_t[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}]$ in (55), we can characterize an equilibrium under money rationing, i.e. $R^m_t < R^L_t$, in terms of a representative borrower and a representative lender.

Definition 2 *With Assumptions 1-3, a competitive equilibrium under money rationing can be characterized as a set of sequences* $\{c_{b,t}, c_{l,t}, h_{b,t}, m^H_t, b_t, q_t, R^L_t, \pi_t\}^\infty_{t=0}$ *satisfying (25),* *(28), (37),*

$$\epsilon_l(\delta - c_{l,t}) = \beta 0.5 E_t[(\epsilon_b(\delta - c_{b,t+1}) + \epsilon_l(\delta - c_{l,t+1}))(R_t^L/\pi_{t+1})]\frac{1-\kappa}{1-\kappa R_t^L/R_t^m}, \quad (38)$$

$$q_t \beta E_t \left[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}\right] \quad (39)$$

$$= \gamma - (h - h_{b,t}) + \beta^2 E_t q_{t+1}\left[0.5(\epsilon_l(\delta - c_{l,t+2}) + \epsilon_b(\delta - c_{b,t+2}))/\pi_{t+2}\right],$$

$$c_{b,t} = 0.5(1+\Omega_t)m_t^H + z q_t h_{b,t}/R_t^L \quad (40)$$

$$(1+\Omega_t)m_t^H = \kappa_t^B b_{t-1}\pi_t^{-1}/R_t^m + m_{t-1}^H \pi_t^{-1}, \quad (41)$$

$$b_t + m_t^H = \Gamma\left(b_{t-1} + m_{t-1}^H\right)/\pi_t, \quad (42)$$

and the transversality conditions, for $\{y_t\}_{t=0}^{\infty}$ *and sequences* $\{0 \leq \kappa_t < R_t^m/R_t^L,\ \kappa_t^B > 0,\ \Omega_t > 0,\ R_t^m < R_t^L\}_{t=0}^{\infty}$ *set by the central bank, given* $m_{-1}^H > 0$*, and* $b_{-1} > 0$*.*

Evidently, there are more instruments available for the central bank when it supplies money in a rationed way compared to a conventional monetary policy regime or a Pigouvian tax/subsidy. In fact, the fraction of bonds eligible for open market operations κ_t^B and the repo share Ω_t can be adjusted to support a particular equilibrium allocation and associated prices, such that (40)-(42) are always satisfied. Given that only these three conditions impose restrictions on the sequences $\{b_t,\ m_t^H,\ \kappa_t^B,\ \Omega_t\}_{t=0}^{\infty}$, there is one degree of freedom left for the policy instrument κ_t^B or Ω_t. Notably, the long-run inflation rate π in principle depends on the growth rate of treasuries Γ (see 42). Yet, the central bank can implement a desired inflation rate and an inflation target by suited long-run adjustments of its instruments κ_t^B and Ω_t, as shown in Appendix C. By setting κ_t^B (or Ω_t) accordingly, the central bank can actually use inflation as a choice variable, while (40)-(42) can be ensured to be satisfied by suited choices of Ω_t (or κ_t^B). Under money rationing, the central bank therefore has three instruments at its disposal, namely, the inflation rate π_t, the policy rate R_t^m, and the share of purchased loans κ_t to affect the equilibrium allocation, i.e. $c_{b,t}$, $c_{l,t}$, and $h_{b,t}$, and the associated prices, i.e. q_t and R_t^L.

Replication of the subsidy on borrowing Hence, a competitive equilibrium under money rationing, as given in Definition 2, can be summarized by a set of sequences $\{c_{b,t}$, $c_{l,t}$, $h_{b,t}$, q_t, $R_t^L\}_{t=0}^{\infty}$ satisfying (25), (28), (37)-(39) for a monetary policy setting $\{\kappa_t$, R_t^m, $\pi_t\}_{t=0}^{\infty}$. Notably, inflation only affects (25) and (38), where it appears jointly with the loan rate to measure the real interest rate R_t^L/π_{t+1}. For convenience, we introduce the price discount s_t, i.e. the ratio between the loan rate and the policy rate

$$s_t = R_t^L / R_t^m \geq 1,$$

which serves as a policy instrument (instead of R^m). Now, suppose again that there is no aggregate risk and that monetary policy is time-invariant, $\pi_t = \pi \geq 0$, $R_t^m = R^m \in [1, R^L)$ and $\kappa_t = \kappa \geq 0$. Using that all variables are then time-invariant and substituting out q and R^L/π with (39) and (38) in (25) and (37), a competitive equilibrium under money rationing can then be reduced to a set $\{c_b, c_l, h_b\}$ satisfying $y = c_l + c_b$,

$$\left[\frac{1-\kappa}{1-\kappa s}\right] u_{c_b} = u_{c_l}\left(1 + (1-\beta)\left(u_{h_l} - u_{h_b}\right)/(z u_{h_l})\right), \tag{43}$$

$$\frac{c_b - c_l}{h_b} = \frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}} \cdot \left[\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)}\right], \tag{44}$$

(where $u_{c_b} = \epsilon_b(\delta - c_b)$, $u_{c_l} = \epsilon_l(\delta - c_l)$, $u_{h_l} = \gamma - (h - h_b)$, and $u_{h_b} = \gamma - h_b$) given the policy instruments $\kappa \in (0, 1/s)$ and $s = R^L/R^m > 1$. For a given allocation $\{c_b, c_l, h_b\}$ and policy $\{\kappa, s\}$, the borrowers' real rate r_b^{ap} is determined by (24).[23] Notably, the instruments, κ and s, affect the RHS of (44) not only by altering the supply of money (see 37), but also via the relative price q/R^L. The reason is that asset purchases raise the lenders' return on lending cash and, thus, their willingness to pay for housing relative to non-durables, which can be seen from combining (38) and (39):

$$\frac{q}{R^L} = \frac{u_{h_l}}{(1-\beta)u_{c_l}} \cdot \frac{1-\kappa}{1-\kappa s}. \tag{45}$$

We can now easily show that the constrained efficient allocation can be implemented by a suited asset purchase regime setting $\{\kappa, s\}$. For this, we compare the latter with the corresponding conditions under the Pigouvian subsidy $\widetilde{\tau}^L < 0$ (see Proposition 1) that implements the constrained efficient allocation $\{c_l, c_b, h_b\}$ satisfying $y = c_l + c_b$,

$$\left[1 - \widetilde{\tau}^L\right] u_{c_b} = u_{c_l}\left(1 + (1-\beta)\left(u_{h_l} - u_{h_b}\right)/(z u_{h_l})\right), \tag{46}$$

$$\frac{c_b - c_l}{h_b} = \frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}} \cdot [2], \tag{47}$$

[23] Inflation, which induces changes in the nominal rate R^L, can still be chosen by the central bank.

where (46) stems from (33), and (47) from combining (27) and (30). The comparison of the terms in square brackets in (43)-(44) with the corresponding terms in (46)-(47) immediately reveals that the implementation of the constrained efficient allocation under the Pigouvian subsidy on borrowing requires both monetary policy instruments, s and κ, to simultaneously satisfy $\frac{1-\kappa}{1-\kappa s} = 1 - \widetilde{\tau}^L$ and $\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)} = 2$, and therefore

$$\kappa = -\widetilde{\tau}^L > 0 \text{ and } s = 2/(1 - \widetilde{\tau}^L). \tag{48}$$

Thus, the central bank can replicate the Pigouvian subsidy by purchasing loans up to a fraction κ that equals the subsidy rate and by offering a price $1/R^m = (1/R^L) \cdot 2/\left(1 + \kappa\right)$ (see 48), which is in principle feasible for $\kappa \in (0, 1/s)$ and $s > 1$. This equivalence result is summarized in the following proposition.

Proposition 2 *Suppose that money supply is rationed and $z \geq 1 - \beta$. Then, the constrained efficient allocation under the Pigouvian subsidy on borrowing can be implemented by the central bank via asset purchases.*

As shown above, the central bank can implement the constrained efficient allocation under the Pigouvian subsidy by purchasing loans, such that its instruments κ and s satisfy (48). In fact, the Pigouvian subsidy on debt directly alters the effective borrowers' real interest rate r_b^τ (see 32), whereas asset purchases directly change the effective lenders' real interest rate r_l^{ap} (see 36). As in the case of the Pigouvian subsidy $\widetilde{\tau}^L < 0$, asset purchases tend to reduce the equilibrium real loan rate R^L/π, which is the relevant rate for borrowers (see 36), compared to the case without interventions (see also Section 4.2). At the same time the lender's effective real rate r_l^{ap} increases, such that the representative borrower (lender) consumes more (less) than without asset purchases (see Proposition 1).

Further enhancement of social welfare An asset purchase policy can, however, not only affect the real interest rates of borrowers and lenders, r_b^{ap} and r_l^{ap}, in different ways (like the Pigouvian debt subsidy), but can further relax borrowing conditions. Yet, first best cannot be implemented with an asset purchase policy. To see this, recall that under first best $u_{c_b} = u_{c_l}$ holds (see 22). Condition (43) would in this case imply $\frac{1-\kappa}{1-\kappa s} > 1$ to equal $1 + \frac{1-\beta}{z} \frac{u_{h_l} - u_{h_b}}{u_{h_l}}$ and thus $u_{h_l} > u_{h_b} \Rightarrow h_b > h_l$, which violates the second requirement for a first best allocation (see 22). Intuitively, because an increase in borrowing has to be associated by a higher value of collateral, i.e. housing, asset purchases cannot implement the first best equilibrium.

However, an asset purchase policy can implement allocations that welfare-dominate the constrained efficient allocation under the Pigouvian debt subsidy. Importantly, the two instruments $\{\kappa, s\}$ do not affect the private sector behavior in identical ways (see 43 and 44), since a higher price discount s (either induced by a lower policy rate R^m or a higher inflation rate π) increases the amount of money supplied per loan, whereas a higher κ increases the fraction of purchased loans. To enhance social welfare compared to the constrained efficient allocation, the central bank can, on the one hand, ease the constraint imposed on the consumption differential relative to borrowers' housing $(c_b - c_l)/h_b$ (see 44) compared to the Pigouvian debt subsidy (47) by setting $\{\kappa, s\}$ to satisfy

$$\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)} > 2 \tag{49}$$

$\Leftrightarrow s > 2/(1+\kappa)$. Given that the borrowing constraint is effectively relaxed under (49), the consumption differential relative to borrowers' housing, which is under a binding borrowing constraint inefficiently small, can be increased compared to the constrained efficient allocation. On the other hand, any change in the instruments also affects the wedge $\frac{1-\kappa}{1-\kappa s}$ between the effective real returns for lenders and borrowers (see 25 and 38). Hence, the central bank can use two distinct channels for its two instrument $\{\kappa, s\}$. Specifically, it can relax borrowing conditions by ensuring (49) and simultaneously steer relative prices in an efficient way (considering 45), which includes addressing the pecuniary externality associated with the borrowing constraint. Since the term $\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}$ that alters the tightness of the borrowing constraint (44) is monotonically increasing in s, but not in κ, an optimal choice of both instruments would be associated with an infinitely large value for s, while κ has to be adjusted to avoid adverse effects of price distortions on the allocation (see 43). This is confirmed below in the numerical analysis (see Section 4.2). For the subsequent analysis, we therefore examine the problem for a given value for the price discount s and assess the optimal fraction of purchased loans κ. By treating one instrument (s) as given, the optimal policy problem is then analogous to the problem in (31). Concretely, to manipulate relative prices in an efficient way, the choice for κ for a given s has to satisfy

$$\frac{\frac{1-\kappa}{1-\kappa s}\cdot u_{c_b}-u_{c_l}}{u_{c_b}-u_{c_l}}=\frac{1+h_b\left(-u_{h_lh_l}\right)/u_{h_l}}{1+\frac{1}{1-\beta}h_bu_{h_l}\left(-u_{c_lc_l}\right)u_{c_l}^{-2}\cdot\frac{z}{2}\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}},\tag{50}$$

which corresponds to the condition for the optimal borrowing subsidy (34). Hence, an asset purchase policy $\kappa\in(0,1/s)$ and $s>1$ that satisfies (49) and (50) implements an allocation that welfare-dominates the constrained efficient allocation under a Pigouvian debt subsidy. This result is summarized in the following proposition.

Proposition 3 *Suppose that money supply is rationed and there is no aggregate risk. Then, the first best equilibrium cannot be implemented, while the central bank can implement allocations via asset purchases that welfare-dominate allocations that are implementable under a Pigouvian subsidy on borrowing.*

When the central bank raises the term $\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)}$ (above 2) by a larger the price discount s than implied by (48), it relaxes the borrowing constraint (see 44). To avoid distorting relative price (see 45), the wedge $\frac{1-\kappa}{1-\kappa s}$ then has to take a lower value (see 50). Social welfare can thus be enhanced compared to the constrained efficient allocation under the Pigouvian debt subsidy, by increasing s and reducing κ compared to (48). In Appendix F, where we set up a real (non-monetary) economy, we show that the total effects of asset purchases can principally also be generated by a combination of Pigouvian subsidies on debt and housing, for which type-specific lump-sum taxes have to be available. A closely related portfolio of instruments has also been applied by Davila and Korinek (2018) for the implementation of a constrained efficient allocation under financial constraints. Proposition 3 further indicates that subsidizing savings via asset purchases can be more efficient than a direct borrowing subsidy due to the impact on the relative price of collateral (see 45).

4 Numerical results

In this section, we provide numerical examples illustrating the analytical results derived in the previous section. To facilitate the calibration of the model, we introduce a more standard (CRRA) utility function. Applying such a utility function, however, implies that we cannot easily aggregate over individual households as in Section 3. To simplify the analysis of financial market interventions, we abstract from implications of an endogenous distribution of agents' net wealth. For this, we assume that funds are pooled within households at the end of each period, such that household members are identical at the beginning of each period before they split up into borrowers and lenders. An equilibrium in terms of a representative borrower and a representative lender then only differs from the previous version by non-linear – instead of linear – marginal utilities. In the last part of this section, we further introduce aggregate risk via a random aggregate income and demonstrate that state contingent asset purchases should be conducted in a countercyclical way.

4.1 A version with CRRA preferences

We consider infinitely many households of measure one, which consist of infinitely many members i. In each period, ex-ante identical household members draw the idiosyncratic preference shock, which induces some members to borrow and others to lend. Like Lucas and Stokey (1987) or Woodford (2016), we assume that at the end of each period (after loans are repaid) household members obtain equal shares of total household wealth, such that they are again equally endowed before new preference shocks are drawn in the next period. Thus, aggregation is facilitated by a redistribution of wealth within each household (rather than by linearity of agents' behavioral relations). We assume that period utility of household member i is given by a separable CRRA utility function

$$u^{CRRA}(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i \frac{c_{i,t}^{1-\sigma} - 1}{1-\sigma} + \gamma \frac{h_{i,t}^{1-\sigma} - 1}{1-\sigma}, \text{ where } \gamma, \sigma > 0, \tag{51}$$

and $i \in \{b, l\}$, such that Assumption 1 (and thus 23) does not apply. We further allow for aggregate risk in form of a random aggregate income y_t, which will be examined in Section 4.3. Specifically, we assume that log aggregate income follows an AR1 process

$$\log y_t = \rho \log y_{t-1} + \varepsilon_{y,t}. \tag{52}$$

Otherwise, the model presented in Section 2 is unchanged, such that the competitive equilibrium in terms of a representative borrower and a representative lender is identical to those given in Definitions 1 and 2, except for marginal utilities being non-linear in this version (see Definition 4 in Appendix D). As in the case of linear-quadratic preferences, it can be shown that a Pigouvian subsidy on debt, $\tau^L < 0$ implements a constrained efficient allocation (see Appendix E), which can also be implemented via asset purchases, given that Proposition 2 apparently holds for both types of preferences (see 43-47).

Calibration To solve the model numerically, we have to assign values for the elasticity of intertemporal substitution σ, the discount factor β, the utility weight for housing γ, the liquidation value of collateral z, the degree of household heterogeneity $\Delta\epsilon = \epsilon_b - \epsilon_l$, the autocorrelation coefficient ρ of the AR1 process, and the standard deviation of the innovations σ_ε. We further have to assign values to the growth rate of treasuries Γ and for the repo share Ω. Given that both are not relevant for the equilibrium allocation (see Appendix C), we apply the values $\Gamma = \pi$ and $\Omega = 1$, for convenience. We interpret a model period as one year and calibrate the model consistent with postwar US data. We estimate the process (52) using (linearly detrended) annual US data for real gdp per capita (for 1947-2008), leading to $\rho = 0.752$ and $\sigma_\varepsilon = 0.0216$. The inverse of the elasticity of intertemporal substitution σ is set equal to 2, which is a typical value applied in business cycle studies. The constant liquidation value of collateral z is set equal to 0.55, which is similar to values applied in related studies (see Iacoviello, 2005).

For the remaining three parameters, β, γ, and $\Delta\epsilon$, we apply values that allow to match three targets for the reference case without financial market interventions. Notably, the data samples are not aligned due to limited availability. The first target is the mean share of installment loans to income of 21% (1998-2004, Survey of Consumer Finances), which correspond to the specification in our model, where loans are demanded for consumption rather than for housing. The second target is the mean yield on MBS of 6.6% for pre-2009 US data, taken from Hancock and Passmore (2011), which corresponds to the rate on collateralized loans $R^L - 1$. The third target is the cross sectional standard deviation of real log consumption of 0.64 (see De Giorgi and Gambetti, 2012). While it is not possible to exactly match all three targets, our choice $\beta = 0.8$, $\gamma = 0.002$, and $\Delta\epsilon = 0.76$ yields to a reasonable match given by an interest rate on collateralized debt 1.06, a loan to income share of 0.2, and a standard deviation of real log consumption of 0.6.

4.2 Welfare gains of asset purchases without aggregate risk

We first consider the case without aggregate risk ($\sigma_\varepsilon = 0$) and compute the equilibrium allocation and associated prices for different policy regimes. As a reference case, we consider a laissez-faire regime, i.e. where monetary policy is conducted in a conventional way (and thus neutral, see Corollary 2) and no Pigouvian subsidy is applied. Figure 1 shows how asset purchases, which require monetary policy instruments to satisfy $\kappa \in (0, 1/s)$ and $s > 1$, affect relative prices and the equilibrium allocation. The effects are computed for a range of values $\kappa \in (0, 0.75)$ and $s \in (1, 1.2)$, while inflation is fixed at a high value that guarantees a positive equilibrium loan rate ($\pi - 1 = 7\%$). All variables are expressed in terms of percentage deviations from their corresponding laissez faire values.

Higher values for the share of purchased loans κ as well as a larger price discount s increase the effects of the central bank intervention, while their impact on prices and quantities is not unambiguous. As revealed in the first row of Figure 1, higher values for κ and s reduce the real rate for borrowers r_b, while they tend to raise the real rate for lender r_l. Notably, the latter is not the case for combinations of a low s and a high κ, where lenders receive a large amount of cash at a small discount, such that they tend to consume more and their real rate r_l is below the laissez faire case (< 0). Overall, the price effects of the policy interventions can be relatively large, in particular for the relative housing price q/R^L, which can be more than twice as large as in the laissez faire case for a combination of high κ and high s. Consistently, borrowers' consumption also tend to increase in these cases, except for combinations of a low s and a high κ (see above). The effects on borrowers' housing reveal that a larger share of purchased loans κ tend to raise collateral demand, which is not generally the case for a larger discount s, where the central bank supplies more money per loan unit.

In the last two rows of Figure 1 we present the welfare effects of asset purchases. For this, we compute the utility values for a representative borrower and a representative lender, $u(\epsilon_b, c_b, h_b)$ and $u(\epsilon_l, c_l, h_l)$, and present the permanent consumption equivalents for borrowers' and lenders' welfare, $v_b^{ce} = [(1-\sigma)\, u(\epsilon_b, c_b, h_b)]^{1/(1-\sigma)}$ and $v_l^{ce} = [(1-\sigma)\, u(\epsilon_l, c_l, h_l)]^{1/(1-\sigma)}$, as well as for ex-ante social welfare, $v^{ce} = [(1-\sigma)\, 0.5\{u(\epsilon_l, c_l, h_l) + u(\epsilon_b, c_b, h_b)\}]^{1/(1-\sigma)}$. Apparently, welfare of lenders falls with larger values for κ and s,

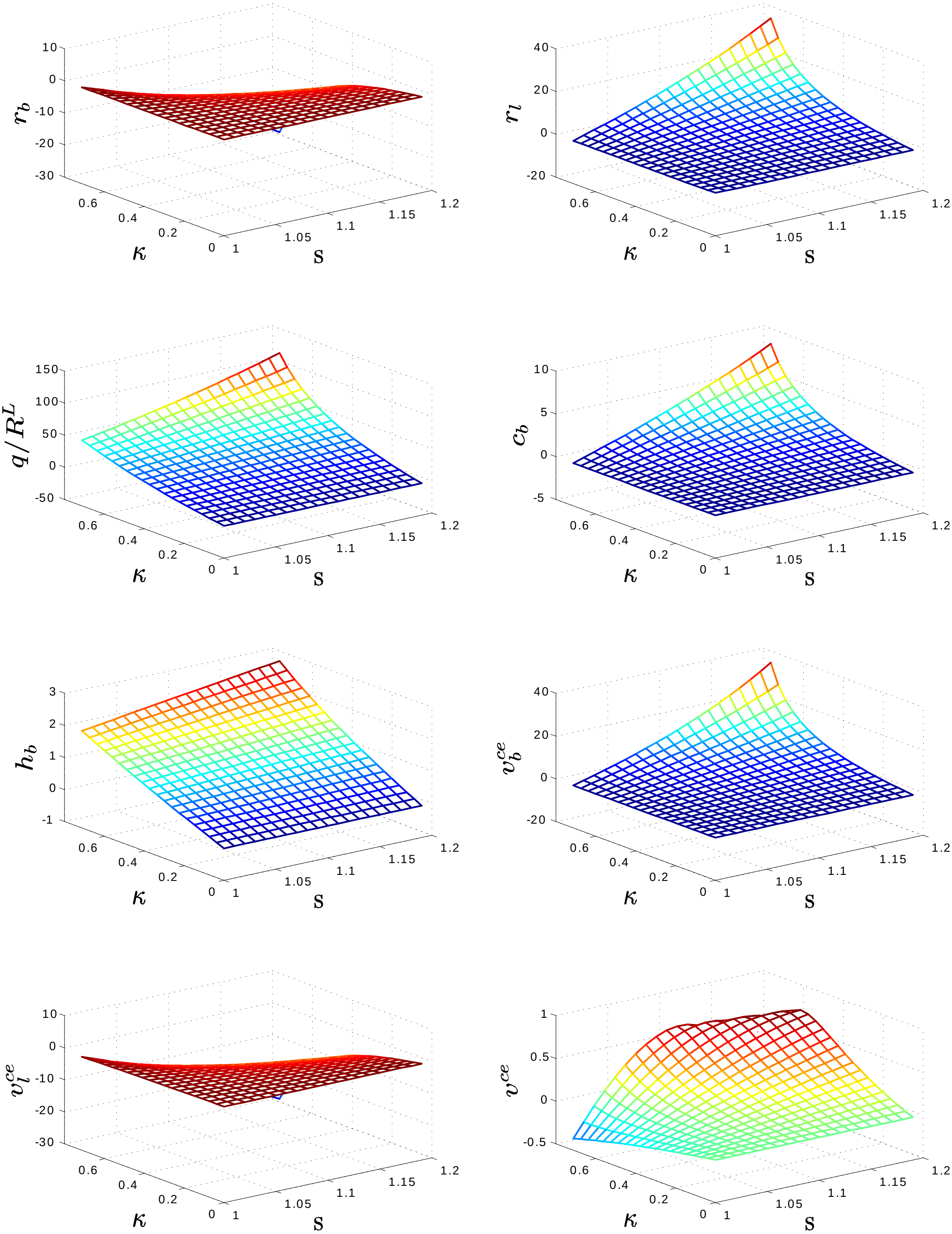


Figure 1: Effects of κ and s for $\pi = 1.07$ (in % deviations from laissez faire values)

whereas welfare of borrowers tends to increase. Notably, the welfare gain for borrowers increases by more than 30% compared to the laissez faire case. Yet, the impact on ex-ante social welfare is much smaller, since the two welfare components partly offset each other. For small values of s, the distortive effects of asset purchases dominate and social welfare decreases with κ compared to the laissez faire case. For larger values of s, welfare tends to increase in κ and s, while changes in welfare are non-monotonic as the loss of lenders strongly increases for combinations of large κ and s values. In total, the welfare gain does not exceed 1% of social welfare under laissez faire.

To assess how monetary policy instruments under asset purchases should be combined in an efficient way (see also Section 3.3), we vary the price discount $s > 1$ and compute the optimal fraction of purchased loans κ according to the efficiency condition (50) as well as to (43), (44), and $y = c_l + c$, where we now apply the utility function (51). Figure 2 shows the effects of a change in s for the benchmark value for the liquidation share of collateral, $z = 0.55$ (black solid line) and a lower value $z = 0.45$ (red dashed line), while we mark the values of the constrained efficient allocation under the Pigouvian debt subsidy with (blue) circles ($\kappa = -\widetilde{\tau}^L = 0.31$ for $z = 0.55$ and $\kappa = -\widetilde{\tau}^L = 0.38$ for $z = 0.45$). All values are now given in absolute terms, except for social welfare v^{ce}, which is again given in terms of percentage deviations from the corresponding laissez faire value.

As shown in the first line of Figure 2, a higher price discount s is accompanied by a lower share of purchased loans κ, in accordance with the efficiency condition (50), and by a higher value for the term $\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}$, implying a relaxation of the borrowing constraint, which is confirmed by an increase in the consumption differential relative to borrowers' housing . The lower value of κ in fact reduces the wedge $\frac{1-\kappa}{1-\kappa s}$ to correct for distortionary effects on market prices and to address the pecuniary externality. As a consequence, the relative price of housing q/R^L, which is larger than in the laissez faire case, decreases with s (see 45). These effects are accompanied by a decrease in borrowers' housing and thus an increase in lenders' housing induced by the enhanced lenders' willingness to save. Overall, an increase in s and the associated reduction in κ enhance ex-ante social welfare compared the constrained efficient allocation under the Pigouvian debt subsidy by relaxing the collateral requirement (see also Proposition 3). Concretely, the welfare gain

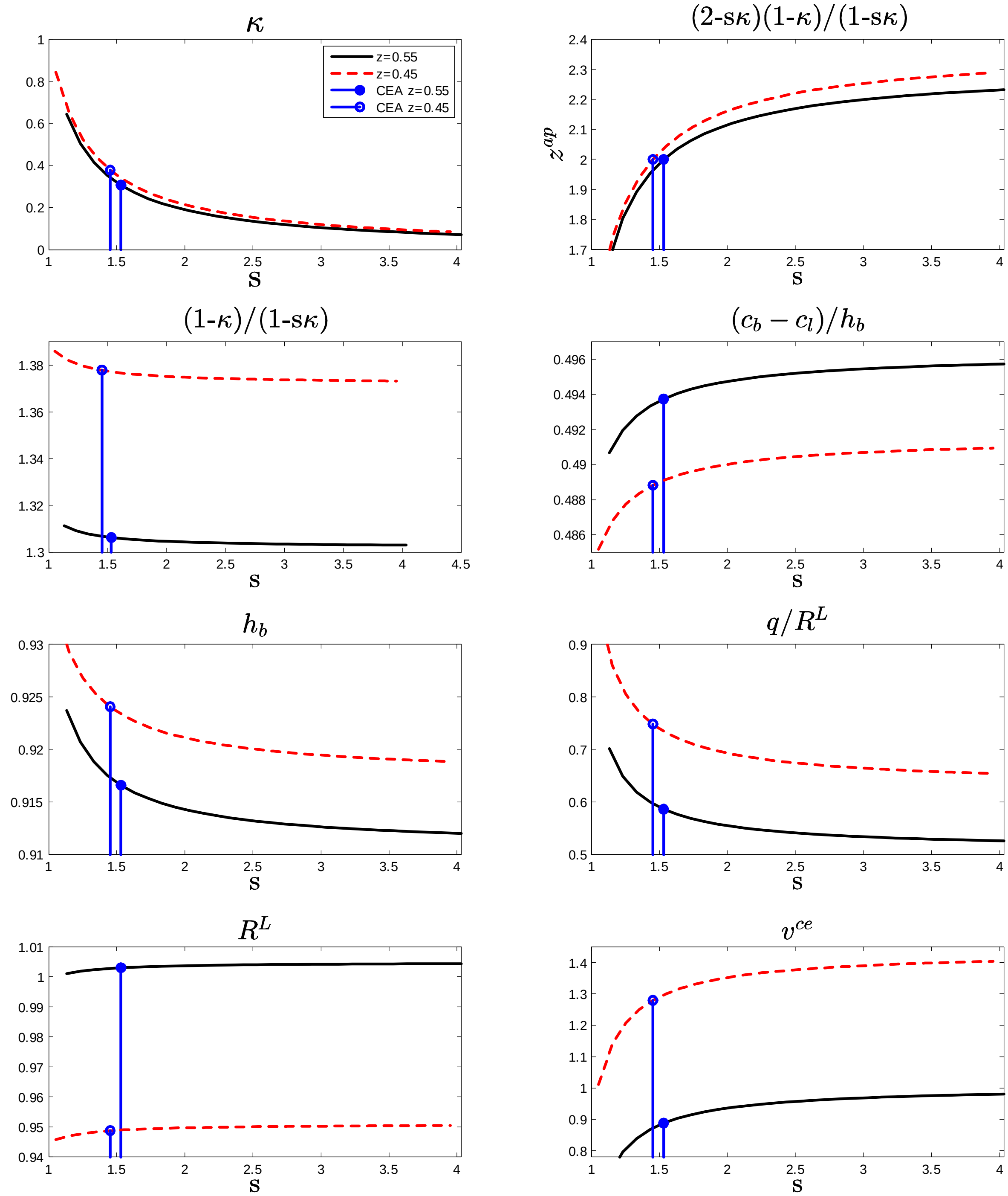


Figure 2: Equilibrium objects under asset purchases and the constrained efficient allocation for variations in s [Note: All values are given in absolute terms, except for social welfare, which is in % deviations from laissez faire values.]

under the Pigouvian subsidy of about 0.9% (compared to laissez faire) can be increased up to 1%, with diminishing gains for larger values for s. For a more severe collateral constraint, (see red dashed line for $z = 0.45$), the effects of changes in s are analogous, while, intuitively, the absolute welfare gain from policy interventions is larger.

4.3 Aggregate risk and state contingent asset purchases

We now introduce aggregate risk, by considering a positive standard deviation σ_ε for aggregate income (see 52), and examine state contingent asset purchases. Due to aggregate income shocks, welfare losses stemming from credit market imperfections can be amplified in the short-run, i.e. when the economy deviates from a stationary equilibrium due to $\varepsilon_t \neq 0$. To understand the welfare-enhancing role of state contingent interventions, consider first the laissez faire case. When the economy is hit by an adverse income shock, $\varepsilon_t < 0$, consumption of durables and non-durables of all agents decreases. As the lenders' demand for housing shifts downward (see 30), the housing price and thereby the value of collateral fall, which tends to tighten agents' borrowing capacity. In such a situation, a policy that tends to increase the collateral price and stimulates borrowing can be welfare enhancing, which can be achieved by asset purchases at above-market prices.

To identify welfare enhancing state contingent asset purchases, we set-up the problem of a social planer under commitment, where we disregard the issue of time-inconsistency and restrict our attention to time-invariant processes of the solution to the policy plan (see Appendix G).[24] As discussed in Section 4.2, an optimal choice for the fraction of purchased loans κ_t and the price discount s_t would lead to an infinitely large value for the latter, while the welfare gains beyond the constrained efficient allocation under the Pigouvian debt subsidy are limited (see Figure 2). Hence, for the subsequent analysis we focus on exactly the latter case and show how this asset purchase policy responds to aggregate shocks. Precisely, we set the means κ and s to replicate the Pigouvian subsidy without aggregate risk (as in 48), while we set s_t in a state contingent way keeping κ_t constant at κ. Following large parts of the literature on optimal policies, we disregard the issue of time-inconsistency and restrict our attention to time-invariant processes of

[24] An analysis of time consistent corrective policies in an environment with an occasionally binding financial constraint can be found in Bianchi and Mendoza (2018)

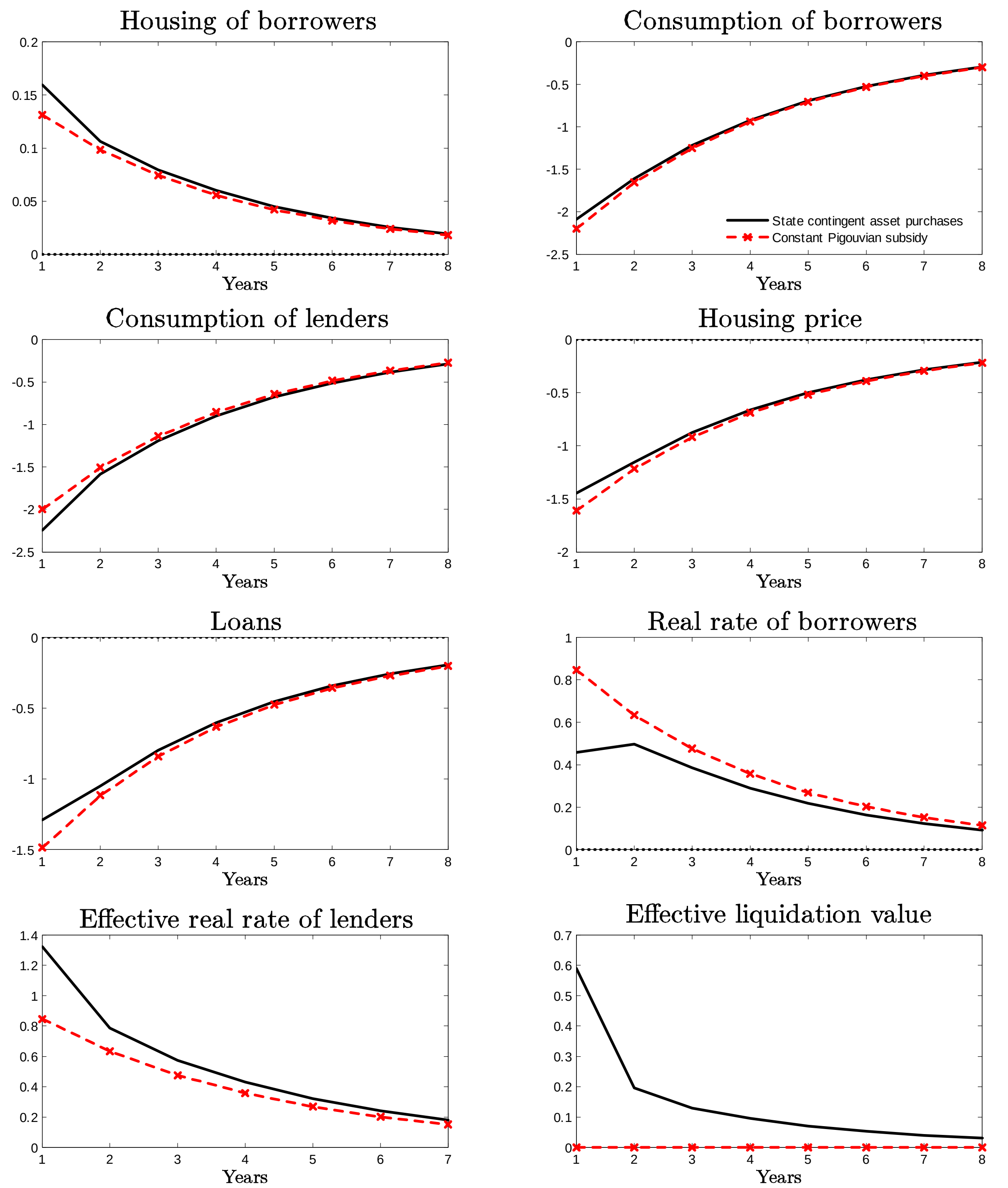


Figure 3: Responses to a minus one st.dev. aggregate income shock (in % deviations from a non-stochastic mean)

the policy plan solution. To avoid the policy plan under commitment to exhibit a unit root, we introduce a fixed depreciation of housing to the fraction δ_h and an equally sized newly constructed housing supply, to ensure a fixed supply h over time (see Appendix G).[25] This assumption has neither an impact on the mechanism nor on the main results. The model is solved applying a second order perturbation method.

Figure 3 presents impulse responses to a negative income shock by one standard deviation, which hits all agents equally. The black solid line shows the responses under the state contingent asset purchase policy, while the red dashed line with crosses shows the responses for a constant Pigouvian debt subsidy (with identical long-run equilibrium allocations). Apparently, the adverse shock reduces consumption of borrowers and lenders. Compared to the constant Pigouvian subsidy, state contingent interventions reduce the increase in the borrowers' real rate $r_{b,t}^{ap}$ and amplifies the increase in the lenders' real rate $r_{l,t}^{ap}$. While the differences in the responses of housing and consumption under both regimes are relatively small, differences in the interest rates are much more pronounced. State contingent asset purchases mitigate adverse income shock effects on borrowers' consumption and housing by stimulating borrowing via a relaxation of the borrowing constraint, i.e. asset purchases raise the effective liquidation value of collateral $\widetilde{z}_t = \frac{z}{2}\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)}$ (see last panel of Figure 3). Hence, state contingent asset purchases should be countercyclical in the sense that they stimulates (dampens) borrowing in downturns (upturns).

Finally, we assess how an asset purchase policy should respond to an exogenous worsening of financial conditions and examine an unexpected change in the liquidation value of collateral z_t.[26] A comparison of the responses under the state contingent asset purchase policy and under a constant Pigouvian debt subsidy, shows that the former mitigates the adverse effects of the liquidation value shock on loans and on borrowers' consumption (see Appendix H). By raising the price discount s_t, which tends to lower (raise) the borrowers' (lenders') real interest rate, the central bank reduces the fall in loans, which is associated with an increase in borrowers' housing and thus in the price of housing. While borrowers' consumption decreases under a constant Pigouvian policy, the state contingent asset

[25] For the numerical analysis we set δ_h to 1%, for convenience.

[26] Concretely, we assume that z_t is generated by $\log z_t = \rho \log z_{t-1} + (1-\rho)\log z_{t-1} + \varepsilon_{z,t}$, where $\varepsilon_{z,t}$ is i.i.d. with mean zero and standard deviation σ_ε.

purchase policy can stabilize borrowers' consumption subsequent to the liquidation value shock. Overall, responses to both types of shocks imply that the ex-post asset purchase policy stimulates borrowing in adverse states and, symmetrically, mitigates the build-up of debt in favorable states of the economy. Thereby, a state contingent asset purchase policy reduces the acceleration of macroeconomic shocks due to a positive feedback loop between collateral demand, prices, and borrowing.

5 Conclusion

Is there a useful role for central bank asset purchases in non-crisis times? This paper shows that central bank asset purchases in secondary markets can enhance social welfare as a corrective policy that mitigates financial amplification. The central bank can incentivize lenders to enhance the supply of funds by purchasing debt at an above-market price. This causes the borrower's real interest rate to fall relative to the effective real interest of lenders, which allows to address pecuniary externalities induced by a collateral constraint. We prove that asset purchases can be a superior corrective policy compared to a Pigouvian subsidy for borrowing, given that they can implement welfare-dominating allocations compared to the latter. While the total effect of asset purchases can in principle also be generated by a combination of Pigouvian subsidies on debt and housing that rely on type-specific lump-sum taxes/transfers, asset purchases are particularly useful when the latter are not available (or not implementable). Our analysis suggests that a saving subsidy can – due to its impact on the relative price of collateral – be a superior corrective policy compared to well-proven ex-post policies that subsidize on borrowing. We further show that state-contingent asset purchases should be conducted in a countercyclical way to reduce acceleration of aggregate shocks via financial frictions. Asset purchases can therefore contribute to financial stability, in addition to macroprudential financial regulation. In this paper, we do not analyze (ex-post) asset purchases together with ex-ante regulation to isolate novel effects of unconventional monetary policy, leaving a joint analysis for future research.

6 References

Andolfatto, D., 2013, Incentive-feasible Deflation, Journal of Monetary Economics 60, 383-390.

Araújo A., S. Schommer, and M. Woodford, 2015, Conventional and Unconventional Monetary Policy with Endogenous Collateral Constraints, American Economic Journal: Macroeconomics 7, 1-43.

Auclert, A., 2016, Monetary Policy and the Redistribution Channel, manuscript, Princeton University.

Benigno. G., H. Chen, C. Otrok, A. Rebucci, and E.R. Young, 2016, Optimal Capital Controls and Real Exchange Rate Policies: A Pecuniary Externality Perspective, Journal of Monetary Economics 84, 147-165.

Bianchi, J., 2016, Efficient Bailouts?, American Economic Review 106, 3607-59.

Bianchi, J., and E. Mendoza, 2018, Optimal Time-Consistent Macroprudential Policy, Journal of Political Economy 126, 588-634.

Chen, H., V. Curdia, and A. Ferrero, 2012, The Macroeconomic Effects of Large-Scale Asset Purchase Programs, Economic Journal 122, 289-315.

Correia I, De Fiore F., P. Teles, and O. Tristani, 2016, Credit Subsidies, European Central Bank Working Paper Series No. 1877.

Curdia, V. and M. Woodford, 2011, The Central-Bank Balance Sheet as an Instrument of Monetary Policy, Journal of Monetary Economics 58, 54-79.

Davila, E. and A. Korinek, 2018, Pecuniary Externalities in Economies with Financial Frictions, Review of Economic Studies 85, 352–395.

De Giorgi, G. and L. Gambetti, 2012, Consumption Heterogeneity over the Business Cycle, Barcelona GSE Working Paper Series Working Paper 64.

Del Negro, M., G. Eggertsson, A. Ferrero and N. Kiyotaki, 2017, The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities, American Economic Review 107, 824-57.

Gagnon, J., Raskin, M., Remache, J., and B. Sack, 2011, The Financial Market Effects of the Federal Reserve's Large-Scale Asset Purchases, International Journal of Central Banking 7, 3-43.

Gertler, M. and P. Karadi, 2011, A Model of Unconventional Monetary Policy, Journal of Monetary Economics 58, 17-34.

Gu, C., Mattesini, F., and R. Wright, 2016, Money and Credit Redux, Econometrica 84, 1-32.

Hancock D. and W. Passmore, 2014, Did the Federal Reserve's MBS Purchase Program lower Mortgage Rates?, Journal of Monetary Economics 58, 498-514.

Hart, O., and J. Moore, 1994, A Theory of Debt Based on the Inalienability of Human Capital, Quarterly Journal of Economics 109, 841–879.

Iacoviello, M., 2005, House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle, American Economic Review 95, 739-764.

Jeanne, O. and A. Korinek, 2013, Macroprudential Regulation Versus Mopping Up After the Crash, NBER Working Papers 18675.

Jermann, U., and V. Quadrini, 2012, Macroeconomic Effects of Financial Shocks, American Economic Review 102, 238-271.

Kiyotaki, N. and J.H. Moore, 1997, Credit Cycles, Journal of Political Economy 105, 211-248.

Krishnamurthy, A. and A. Vissing-Jorgensen, 2011, The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy, Brookings Papers on Economic Activity, 215–287.

Krishnamurthy, A., Nagel, S., and A. Vissing-Jorgensen, 2017, ECB Policies Involving Government Bond Purchases, Review of Finance, forthcoming.

Lucas, R.E., Jr., and N.L. Stokey, 1987, Money and Interest in a Cash-in-Advance Economy, Econometrica 55, 491-513.

Rocheteau, G., R. Wright, and S.X. Xiao, 2018, Open Market Operations, Journal of Monetary Economics, forthcoming.

Schabert, A., 2015, Optimal Central Bank Lending, Journal of Economic Theory, 157, 485-516.

Stein, J.C., 2012, Monetary Policy as Financial-Stability Regulation, Quarterly Journal of Economics 127, 57-95.

Wallace, N., 1981, A Modigliani-Miller Theorem for Open-Market Operations, American Economic Review 71, 267-274.

Williamson, S.D., 2016, Scarce Collateral, the Term Premium, and Quantitative Easing, Journal of Economic Theory 164, 136-165.

Woodford, M., 1990, "Public Debt as Private Liquidity," American Economic Review 80, 382-388.

Woodford, M., 2016, Quantitative Easing and Financial Stability, NBER Working Paper No. 22285.

Appendix

A Competitive equilibrium

Definition 3 *A competitive equilibrium is a set of sequences* $\{c_{i,t},\ l_{i,t},\ i_{i,t},\ i^L_{i,t},\ \zeta_{i,t},\ \lambda_{i,t},\ h_{i,t},\ m^H_{i,t},\ b_{i,t},\ m^H_t,\ b_t,\ b^T_t,\ \pi_t,\ R^L_t,\ q_t\ \}_{t=0}^{\infty}$ *satisfying for all* $i \in [0,1]$

$$
\begin{aligned}
\lambda_{i,t} &= \beta E_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}], \\
\frac{1}{R^L_t} &= \beta \frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})} + \frac{\zeta_{i,t}}{u'(\epsilon_i, c_{i,t})} \\
&\quad \textit{or } \frac{1}{R^L_t} = \beta \frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})} \cdot \frac{1-\kappa_t}{1-\kappa_t R^L_t/R^m_t}, \\
c_{i,t} &= i_{i,t} + i^L_{i,t} + m^H_{i,t-1}\pi_t^{-1} - l_{i,t}/R^L_t \ \ \textit{if } \psi_{i,t} > 0, \\
&\quad \textit{or } c_{i,t} \le i_{i,t} + i^L_{i,t} + m^H_{i,t-1}\pi_t^{-1} - l_{i,t}/R^L_t \ \ \textit{if } \psi_{i,t} = 0, \\
R^m_t i_{i,t} &= \kappa^B_t b_{i,t-1}\pi_t^{-1} \quad \textit{if } \eta_{i,t} > 0, \\
&\quad \textit{or } R^m_t i_{i,t} < \kappa^B_t b_{i,t-1}\pi_t^{-1} \quad \textit{if } \eta_{i,t} = 0, \\
R^m_t i^L_{i,t} &= \kappa_t l_{i,t} \quad \textit{if } \mu_{i,t} > 0 \\
&\quad \textit{or } R^m_t i^L_{i,t} \le \kappa_t l_{i,t} \quad \textit{if } \mu_{i,t} = 0, \\
-l_{i,t} &= z q_t h_{i,t} \quad \textit{if } \zeta_{i,t} > 0, \\
&\quad \textit{or } -l_{i,t} \le z_t q_t h_{i,t} \quad \textit{if } \zeta_{i,t} = 0, \\
q_t \lambda_{i,t} &= u_{h,i,t} + \zeta_{i,t} z q_t + \beta E_t q_{t+1}\lambda_{i,t+1}, \\
i_{i,t} &= (1+\Omega_t) m^H_{i,t} - m^H_{i,t-1}\pi_t^{-1}, \\
b^T_t &= b_t + m^H_t, \\
b^T_t &= \Gamma b^T_{t-1}/\pi_t,
\end{aligned}
$$

$0 = \sum_i l_{i,t}$, $h = \sum_i h_{i,t}$, $y = \sum_i c_{i,t}$, $b_t = \sum_i b_{i,t}$, *and* $m^H_t = \sum_i m^H_{i,t}$, *where the multipliers* $\psi_{i,t}$, $\mu_{i,t}$, *and* $\eta_{i,t}$ *satisfy*

$$
\begin{aligned}
\psi_{i,t} &= u'(\epsilon_i, c_{i,t}) - \beta E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right] \ge 0, \\
\mu_{i,t} &= [(1/R^m_t) - (1/R^L_t)]u'(\epsilon_i, c_{i,t})/(1-\kappa) \ge 0, \\
\sum_i \eta_{i,t} &= \sum_i [u'(\epsilon_i, c_{i,t})/R^m_t] - \beta E_t \sum_i \left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right] \ge 0,
\end{aligned}
$$

and the transversality conditions, a monetary policy setting $\{R^m_t \ge 1,\ \kappa^B_t > 0,\ \kappa_t \in [0,1],\ \Omega_t > 0\}_{t=0}^{\infty}$, *given* $\Gamma > 0$, $\{y_t\}_{t=0}^{\infty}$, *and initial values* $m^H_{i,-1} = m^H_{-1} > 0$, $b_{i,-1} = b_{-1} > 0$, $h_{i,-1} = h_{-1} = 1$ *and* $b^T_{-1} > 0$.

Let $c_{l,t} = 2\sum_{l,i} c_{l,i,t}$, $h_{l,t} = 2\sum_{l,i} h_{l,i,t}$, $l_{l,t} = 2\sum_{l,i} l_{l,i,t}$, $\lambda_{l,t} = 2\sum_{l,i} \lambda_{l,i,t}$, $i_{l,t} = 2\sum_{l,i} i_{l,i,t}$, $c_{b,t} = 2\sum_{b,i} c_{b,i,t}$, $h_{b,t} = 2\sum_{b,i} h_{b,i,t}$, $l_{b,t} = 2\sum_{b,i} l_{b,i,t}$, $\lambda_{b,t} = 2\sum_{b,i} \lambda_{b,i,t}$, and $i_{b,t} = 2\sum_{b,i} i_{b,i,t}$. Based on the Assumptions 1 and 2 and the law of large numbers, the set of conditions that describe the behavior for a representative lender, i.e. a representative agent drawing ϵ_l in period t, is given by

$$
\begin{aligned}
& 0.5m_{t-1}^{H}\pi_t^{-1} + 0.5b_{t-1}\pi_t^{-1} + l_{l,t}\left(1 - 1/R_t^L\right) + 0.5y_t + 0.5\tau_t && (53)\\
&= m_{l,t}^{H} + (b_{l,t}/R_t) + 0.5\left(i_{l,t} + i_{b,t}\right)\left(R_t^m - 1\right) + c_{l,t} + q_t\left(h_{l,t} - 0.5h\right) \\
\lambda_{l,t} &= \epsilon_l(\delta - c_{l,t})/R_t^L, && (54)\\
q_t\lambda_{l,t} &= \gamma - h_{l,t} + \beta E_t q_{t+1} 0.5\left(\lambda_{l,t+1} + \lambda_{b,t+1}\right), && (55)\\
\frac{\epsilon_l(\delta - c_{l,t})}{R_t^L} &= \beta E_t\left[\frac{0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))}{\pi_{t+1}}\right], && (56)\\
c_{l,t} &= 0.5\left(i_{l,t} + i_{b,t}\right) + 0.5m_{t-1}^{H}\pi_t^{-1} - l_{l,t}/R_t^L, && (57)
\end{aligned}
$$

where $m_{l,t-1}^H = \Sigma_{l,i} m_{l,i,t-1}^H = 0.5m_{t-1}^H$, $b_{l,t-1} = \sum_{l,i} b_{l,i,t-1} = 0.5b_{t-1}$, and $h_{l,t-1} = \sum_{l,i} h_{l,i,t-1} = 0.5h$. Note that (57) accounts for treasury open market operations being conducted before idiosyncratic shocks are drawn. Applying the Assumptions 1 and 3, the set of conditions describing the behavior of a representative borrower, i.e. a representative agent drawing ϵ_b in period t, is

$$
\begin{aligned}
& 0.5m_{t-1}^{H}\pi_t^{-1} + 0.5b_{t-1}\pi_t^{-1} + l_{b,t}\left(1 - 1/R_t^L\right) + 0.5y_t + 0.5\tau_t && (58)\\
&= m_{b,t}^{H} + (b_{b,t}/R_t) + 0.5\left(i_{l,t} + i_{b,t}\right)\left(R_t^m - 1\right) + c_{b,t} + q_t\left(h_{b,t} - 0.5h\right), \\
\lambda_{b,t} &= \beta E_t\left[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}\right], && (59)\\
q_t\lambda_{b,t} &= \gamma - h_{b,t} + \zeta_{b,t} z q_t + \beta E_t q_{t+1} 0.5\left(\lambda_{l,t+1} + \lambda_{b,t+1}\right), && (60)\\
\frac{\epsilon_b(\delta - c_{b,t})}{R_t^L} &= \beta E_t\left[\frac{0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))}{\pi_{t+1}}\right] + \zeta_{b,t}, && (61)\\
c_{b,t} &= 0.5\left(i_{l,t} + i_{b,t}\right) + 0.5m_{t-1}^{H}\pi_t^{-1} - l_{b,t}/R_t^L, && (62)\\
-l_{b,t} &= z q_t h_{b,t}, && (63)
\end{aligned}
$$

where $m_{b,t-1}^H = \Sigma_{b,i} m_{b,i,t-1}^H = 0.5m_{t-1}^H$, $b_{b,t-1} = \sum_{b,i} b_{b,i,t-1} = 0.5b_{t-1}$, and $h_{b,t-1} = \sum_{b,i} h_{b,i,t-1} = 0.5h$. Using that $h = h_{l,t} + h_{b,t}$, $l_t = l_{l,t} = -l_{b,t}$, and that (54), (56), and (59) imply $\lambda_t = \lambda_{b,t} = \lambda_{l,t}$, and substituting out $\zeta_{b,t}$, λ_t, and l_t, leads to the set of conditions (24)-(28).

B Proof of Proposition 1

The social planer problem (31), which aims at maximizing ex-ante social welfare, can be written as

$$\max_{c_b,c_l,h_b}\min_{\chi_1,\chi_2}(1-\beta)^{-1}\ \{[u(\epsilon_b,c_b,h_b)+u(\epsilon_l,c_l,h-h_b)] \\ +\chi_1\left[y-c_b-c_l\right]+\chi_2\left[2zh_b\cdot(q/R^L)-c_b+c_l\right]\}, \tag{64}$$

where $(q/R^L)=\frac{1}{1-\beta}u_{h_l}/u_{c_l}$. The first order conditions are $u_{c_b}=\chi_1+\chi_2$,

$$u_{c_l}=\chi_1-\chi_2\left(1+2zh_b\cdot\left[\partial(q/R^L)/\partial c_l\right]\right),$$
$$u_{h_l}-u_{h_b}=\chi_2\left(2z\cdot\left(q/R^L\right)+2zh_b\cdot\left[\partial(q/R^L)/\partial h_b\right]\right),$$

where $\partial(q/R^L)/\partial c_l=\frac{u_{h_l}}{1-\beta}(-u_{c_lc_l})/u_{cl}^2>0$ and $\partial(q/R^L)/\partial h_b=\frac{1}{1-\beta}(-u_{h_lh_l})/u_{c_l}>0$. Substituting out the multipliers χ_1 and χ_2 as well as (q/R^L) with $(q/R^L)=\frac{1}{1-\beta}u_{h_l}/u_{c_l}$, we get the following condition for the constrained efficient allocation

$$\frac{u_{c_b}-u_{c_l}}{u_{h_l}-u_{h_b}}\frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}}=\frac{1+zh_b\cdot\left[\partial(q/R^L)/\partial c_l\right]}{1+\left[\partial(q/R^L)/\partial h_b\right]\cdot h_b/(q/R^L)}. \tag{65}$$

To disclose the implications for the tax/subsidy rate, which is associated with this plan, we compare (65) with the competitive equilibrium condition (33), which can be rewritten as

$$\frac{(1-\tau^L)u_{c_b}-u_{c_l}}{u_{h_l}-u_{h_b}}\frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}}=1. \tag{66}$$

Apparently, the LHS of (66) differs from the LHS of (65) only by the tax rate τ^L, while the RHSs differ due to the derivatives of the relative price (q/R^L). For $z(q/R^L)\cdot\left[\partial(q/R^L)/\partial c_l\right]<\left[\partial(q/R^L)/\partial h_b\right]\Leftrightarrow$

$$z(q/R^L)\frac{-u_{c_lc_l}}{u_{c_l}}<\frac{-u_{h_lh_l}}{u_{h_l}}, \tag{67}$$

the RHS of (65) is smaller than one, implying a subsidy $\tau^L<0$. Inserting the derivatives of the utility function (23), $u_{c_l}=\epsilon_l(\delta-c_l)$, $u_{c_lc_l}=-\epsilon_l$, $u_{h_l}=\gamma-(h-h_b)$, $u_{h_lh_l}=-1$, and using the constraints $c_b-c_l=2zh_b(q/R^L)$ and $y=c_l+c_b$, the inequality (67) can be rewritten as

$$\frac{1-\beta}{z}\epsilon_l\left(\frac{c_b-c_l}{2h_b}\right)^2<1.$$

Using that (29) implies $h_b \geq 0.5h$, $\epsilon_l < 1$, and that $y = h = 1$, we can conclude that $(1-\beta)/z \leq 1$ is a sufficient condition for (67) and thus for a borrowing subsidy to be required for the implementation of the constrained efficient allocation $\tau^L < 0$.

We further seek to identify the impact of the subsidy on consumption and housing of the representative borrower. For this, we apply the competitive equilibrium conditions (28), (66), and $c_b - c_l = 2zh_b\frac{\gamma-(h-h_b)}{(1-\beta)\epsilon_l(\delta-c_l)}$, and substitute out c_l with $c_l = y - c_b$ to get $F(\boldsymbol{\tau}^L, h_b, c_b) = 0$ and $G(h_b, c_b) = 0$, where

$$\begin{aligned}
F(\boldsymbol{\tau}^L,h_b,c_b) &= \frac{(1-\boldsymbol{\tau}^L)\cdot\epsilon_b(\delta-c_b)-\epsilon_l(\delta-y+c_b)}{(2h_b-h)\,(1-\beta)\epsilon_l(\delta-y+c_b)(1/z)}-\frac{1}{\gamma-h+h_b},\\
G(h_b,c_b) &= 2zh_b\frac{\gamma-(h-h_b)}{(1-\beta)\epsilon_l(\delta-y+c_b)}-2c_b+y.
\end{aligned}$$

The partial derivatives of $G(h_b, c_b)$, where G_x abbreviates $\partial G/\partial x$, are given by

$$G_{h_b}=2z\frac{2h_b-h+\gamma}{\epsilon_l\,(\delta-y+c_b)\,(1-\beta)}>0,\quad G_{c_b}=-2\left(\frac{z}{\epsilon_l}\frac{h_b\,(\gamma-h+h_b)}{(1-\beta)\,(c_b-y+\delta)^2}+1\right)<0,$$

implying $\partial h_b/\partial c_b = -G_{c_b}/G_{h_b} > 0$. The partial derivatives of $F(\boldsymbol{\tau}^L, h_b, c_b)$ are given by

$$\begin{aligned}
F_{\boldsymbol{\tau}^L} &= -\frac{\epsilon_b\,(\delta-c_b)}{\epsilon_l\,(1-\beta)\,(2h_b-h)\,(\delta-y+c_b)\,(1/z)}<0,\; F_{h_b}=-\frac{2\gamma-h}{(2h_b-h)\,(\gamma-h+h_b)^2}<0,\\
F_{c_b} &= -\frac{2\epsilon_b\,(\delta-y/2)\left(1-\tau^L\right)}{\epsilon_l\,(\delta-y+c_b)^2\,(1-\beta)\,(2h_b-h)\,(1/z)}<0.
\end{aligned}$$

Thus, consumption of the representative borrower decreases with the tax rate, since

$$\partial c_b/\partial\tau^L=-(G_{h_b}F_{\tau^L})/(F_{c_b}G_{h_b}-F_{h_b}G_{c_b})<0.$$

Hence, introducing a subsidy $\tau^L < 0$ increases consumption and housing of the representative borrower (by $\partial h_b/\partial c_b > 0$). Given that consumption (housing) of lenders decreases for a given aggregate supply (stock of housing), the lenders' consumption Euler equation (24), which can be written as $1 = \beta 0.5[1 + \epsilon_b(\delta - c_b)/(\epsilon_l(\delta - c_l))]\left(R^L/\pi\right)$, further implies that the real interest rate R^L/π increases with the subsidy. ■

C Monetary policy instruments under asset purchases

Suppose that government bonds are supplied at a rate that is not identical to the inflation target, $\Gamma \neq \pi^*$. Then, the real value of the total stock of bonds $b_t^T = b_t + m_t^H$ might grow or shrink in a long-run equilibrium at a constant rate Γ/π (see 42). The money demand condition (40) then requires for constant steady state values c_b, R^L, h_b, q, and z, that the term $\widetilde{m}_t = (1+\Omega_t)m_t^H$ is also constant in the long-run. Rewriting (41), and (42)as $\kappa_t^B b_t = R_t^m \pi_t[\widetilde{m}_t - \widetilde{m}_{t-1}(1+\Omega_{t-1})^{-1}\pi_t^{-1}]$ and $[b_t + \widetilde{m}_t/(1+\Omega_t)] = \Gamma\left[b_{t-1} + \widetilde{m}_{t-1}/(1+\Omega_{t-1})\right]/\pi_t$, and substituting out b_t and b_{t-1} , gives

$$\begin{aligned} &\left[\frac{\pi_t R_t^m}{\kappa_t^B}\left(\widetilde{m}_t - \frac{\widetilde{m}_{t-1}\pi_t^{-1}}{1+\Omega_{t-1}}\right) + \frac{\widetilde{m}_{t-1}}{1+\Omega_{t-1}}\right] \\ &= \frac{\Gamma}{\pi_{t-1}}\left[\frac{\pi_{t-1} R_{t-1}^m}{\kappa_{t-1}^B}\left(\widetilde{m}_{t-1} - \frac{\widetilde{m}_{t-2}\pi_{t-1}^{-1}}{1+\Omega_{t-2}}\right) + \frac{\widetilde{m}_{t-2}}{1+\Omega_{t-2}}\right]. \end{aligned} \tag{68}$$

Taking the limit $t \to \infty$ of both sides of (68), we can use that for a constant long-run inflation rate π and a constant policy rate R^m a steady state is characterized by a constant value for $\widetilde{m}_t$. The term in the square brackets in (68), i.e. the total supply of short-term treasuries, grows/shrinks with the constant rate Γ/π. When the growth rate of bonds exceeds the inflation rate, $\Gamma > \pi$, this can be guaranteed by a permanently shrinking value for κ_t^B. Thus, the central bank can let κ_t^B shrink at the rate $\pi/\Gamma < 1$ and can let the share of money supplied outright, $1/\Omega_t = M_t^H/M_t^R$, go to zero in the long-run, i.e. it can set κ_t^B and $1/\Omega_t$ according to $\lim_{t\to\infty} \kappa_t^B/\kappa_{t-1}^B = \pi/\Gamma$ and $\lim_{t\to\infty} 1/\Omega_t = 0$ if $\Gamma > \pi$.

For $\Gamma < \pi$, the term in the square bracket in (68) permanently shrinks, which can not be supported by a growing value κ_t^B without violating the restriction $\kappa_t^B \leq 1$. In this case, the central bank can keep κ_t^B constant and can let the share Ω_t of money supplied under repos grow in a long-run equilibrium. For $\Gamma < \pi$, it can thus set Ω_t in the long-run according to $\lim_{t\to\infty}(1+\Omega_t)/(1+\Omega_{t-1}) = \pi/\Gamma > 1$.

D A CRRA version with representative agents

Definition 4 *A competitive equilibrium of the economy with preferences satisfying (51) and wealth redistribution within households is of a set of sequences* $\{c_{b,t},\ c_{l,t},\ \pi_t,\ R_t^L,\ h_{b,t},\ q_t,\ b_t,\ b_t^T,\ m_t^H\}_{t=0}^{\infty}$ *satisfying*

$$\begin{aligned}
&(1-\tau^L)\epsilon_b c_{b,t}^{-\sigma}/R_t^L \\
&= \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma}+\epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]+\gamma((h-h_{b,t})^{-\sigma}-h_{b,t}^{-\sigma})/[q_t z],
\end{aligned} \tag{69}$$

$$\epsilon_l c_{l,t}^{-\sigma}/R_t^L = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma}+\epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}, \tag{70}$$

$$q_t\epsilon_l c_{l,t}^{-\sigma}\frac{1/R_t^L-\kappa_t/R_t^m}{1-\kappa_t} = \gamma\left(h-h_{b,t}\right)^{-\sigma}+\beta E_t\left[q_{t+1}\epsilon_l c_{l,t+1}^{-\sigma}\frac{1/R_{t+1}^L-\kappa_{t+1}/R_{t+1}^m}{1-\kappa_{t+1}}\right], \tag{71}$$

$$c_{b,t}-c_{l,t} \leq zq_t h_{b,t}\left[(2/R_t^L)-(\kappa_t/R_t^m)\right], \tag{72}$$

$$c_{l,t}+c_{b,t} = y_t, \tag{73}$$

$$0.5(1+\Omega_t)m_t^H \geq c_{b,t}+z_t q_t h_{b,t}/R_t^L, \tag{74}$$

$$\kappa_t^B b_{t-1}\pi_t^{-1}/R_t^m \geq (1+\Omega_t)m_t^H - m_{t-1}^H\pi_t^{-1}, \tag{75}$$

$$b_t^T = \Gamma b_{t-1}^T/\pi_t, \tag{76}$$

$$b_t^T = b_t + m_t^H, \tag{77}$$

the transversality conditions, a monetary policy setting $\{R_t^m \geq 1,\ \kappa_t \in [0,1],\ \kappa_t^B > 0,\ \Omega_t > 0\}_{t=0}^{\infty}$*, a tax/subsidy* τ^L*, given* $\{y_t\}_{t=0}^{\infty}$*,* $\Gamma > 0$*,* $b_{-1}^T > 0$*,* $b_{-1} > 0$*, and* $m_{-1}^H > 0$*.*

The first best allocation apparently satisfies $\epsilon_b c_{b,t}^{-\sigma} = \epsilon_l c_{l,t}^{-\sigma}$ and $h_{b,t} = h_{l,t} = 2h$. Under binding borrowing, liquidity, and money supply constraints, a competitive equilibrium without aggregate risk consists of a set $\{c_l,\ c_b,\ R^L,\ h_b,\ q\}$ satisfying

$$1/R^L = \beta\left(c_l^{\sigma}/\epsilon_l\right)0.5(\epsilon_b c_b^{-\sigma}+\epsilon_l c_l^{-\sigma})\pi^{-1}\frac{1-\kappa}{1-\kappa R^L/R^m}, \tag{78}$$

$$(1-\tau^L)\epsilon_b c_b^{-\sigma} = R^L\beta 0.5(\epsilon_b c_b^{-\sigma}+\epsilon_l c_l^{-\sigma})\pi^{-1}+R^L\left(\gamma/qz\right)\left((h-h_b)^{-\sigma}-h_b^{-\sigma}\right), \tag{79}$$

$$\gamma(h-h_b)^{-\sigma} = q\left(1-\beta\right)\epsilon_l c_l^{-\sigma}\frac{1/R^L-\kappa/R^m}{1-\kappa}, \tag{80}$$

$$c_b - c_l = zqh_b[(2/R^L)-(\kappa/R^m)], \tag{81}$$

$$y = c_l + c_b, \tag{82}$$

for a monetary policy setting $\{1 \leq R^m < R^L,\ \kappa \in [0,1),\ \pi > \beta\}$, and a tax/subsidy τ^L. Once the set $\{c_l,\ c_b,\ R^L,\ h_b,\ q\}$ is determined, the values m^H and b are given by $m^H = \left(c_b - zqh_b/R^L\right)\frac{1}{0.5(1+\Omega)}$ and $b = \frac{R^m\pi}{\kappa^B}\left(1+\Omega-\pi^{-1}\right)m^H$ given κ^B and Ω.

E Pigouvian subsidy under CRRA preferences

In this Appendix, we consider an economy under CRRA preferences and pooling of wealth within households as summarized in Definition 4. We will show that a constrained efficient allocation is again associated with a lump-sum financed borrowing subsidy, as already shown for the case of linear-quadratic preferences (see Proposition 1). Consider the economy as given in Definition 4 for $y_t = y$, $R^m = R^L$, and $\pi_t = \pi$. Given that conventional monetary policy measures do not affect the allocation (see corollary 2) and we restrict the tax/subsidy rate also to be constant, the equilibrium allocation and prices are time-invariant. Hence, the set $\{c_l,\, c_b,\, R^L,\, h_b,\, q\}$ has to satisfy (79), (82)

$$\epsilon_l c_l^{-\sigma}/R^L = \beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})/\pi, \tag{83}$$

$$c_b - c_l \leq zqh_b 2/R^L, \tag{84}$$

$$\gamma(h - h_b)^{-\sigma} = q\beta(1-\beta)0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})/\pi, \tag{85}$$

given $\{\tau^L, \pi\}$. Substituting out the housing price q with (85) in (84), leads to

$$0 \leq zh_b 2\frac{\gamma(h - h_{b,t})^{-\sigma_h}}{(1-\beta)\epsilon_l c_l^{-\sigma}} - c_b + c_l, \tag{86}$$

where we further used (83) to substitute out the real rate R^L/π.

Proposition 4 *Consider an economy without aggregate risk, with preferences satisfying (51), and wealth redistribution within households. The constrained efficient allocation can be implemented by a Pigouvian subsidy on borrowing, if but not only if $\epsilon_b/\epsilon_l \leq 3^\sigma$.*

Proof. The problem of a social planer, who aims at maximizing social welfare (21) by setting the tax/subsidy rate τ^L, can again be summarized as (64). Likewise, a constrained efficient allocation is associated with a borrowing subsidy if (67) is satisfied. Applying the partial derivatives of the CRRA utility function (51), $u_{c_l} = \epsilon_l c_l^{-\sigma}$, $u_{c_l c_l} = -\sigma u_{cl}/c_l$, $u_{h_l} = \gamma(h - h_b)^{-\sigma}$, $u_{h_l h_l} = -\sigma u_{hl}(h-h_b)^{-1}$, we can rewrite (67) as $((c_b/c_l) - 1)(h/h_b - 1) - 2 < 0$. Since the ratio c_b/c_l is smaller under a binding borrowing constraint than under first best, $(c_b^*/c_l^*) = (\epsilon_b/\epsilon_l)^{1/\sigma}$ and $h/2 \leq h_b \Leftrightarrow (h/h_b) - 1 \leq 1$ holds, we can conclude that $((c_b/c_l) - 1)(h/h_b - 1) - 2 < ((\epsilon_b/\epsilon_l)^{1/\sigma} - 1)(h/h_b - 1) - 2 \leq (\epsilon_b/\epsilon_l)^{1/\sigma} - 3$. Hence, the constrained efficient allocation requires a subsidy, $\tau^L < 0$, if but not only if the preference shock satisfies $\epsilon_b/\epsilon_l \leq 3^\sigma$. ■

F Corrective policies in a non-monetary economy

To relate an asset purchase policy to real (fiscal) instruments, we examine a real version of the model. In contrast to the previous versions, we disregard money and bonds, and consider interperiod, i.e., one-period, real loans, instead of intraperiod nominal loans. We consider a continuum of households with a continuum of members. The budget constraint for a household member drawing ϵ_b is

$$\left(1 + r_{t-1}^L\right) l_{b,t-1} + y_t \geq (1 - \tau_t^L) l_{b,t} + c_{b,t} + q_t \left(h_{b,t} - h_{b,t-1}\right) + \tau_{b,t}^R, \tag{87}$$

and $\left(1 + r_{t-1}^L\right) l_{l,t-1} + y_t \geq l_{l,t} + c_{l,t} + q_t \left(h_{l,t} - h_{l,t-1}\right)$ for a household member drawing ϵ_l, where r_{t-1}^L is the real loan rate. As in Section 4.2, household members share financial wealth and durables at the end of each period. Notably, $\tau_t^L > 0$ (< 0) is a tax (subsidy) on borrowing ($l_{i,t} < 0$) and $\tau_{b,t}^R > 0$ is a *borrower-specific* lump-sum tax/transfer, where implementation of a Pigouvian tax/subsidy on debt requires $\tau_{b,t}^R = \tau_t^L l_{b,t}$. The collateral constraint corresponds to the nominal version (4)

$$-l_{b,t} \leq z q_t h_{b,t}. \tag{88}$$

The collateral value is measured before maturity, which can be rationalized by the borrowers' ability to renegotiate the debt contract before debt matures. Maximizing lifetime utility subject to (87) and (88) leads to

$$\lambda_{b,t} = u_{c_b,t},\ \lambda_{l,t} = u_{c_l,t}, (1 - \tau_{b,t}^L)\lambda_{b,t} = \beta 0.5 \left(1 + r_t^L\right) E_t \left(\lambda_{b,t+1} + \lambda_{l,t+1}\right) + \zeta_{b,t},$$
$$\lambda_{l,t} = \beta 0.5 \left(1 + r_t^L\right) E_t \left[0.5 \left(\lambda_{b,t+1} + \lambda_{l,t+1}\right)\right], q_t \lambda_{l,t} = u_{h_l,t} + \beta E_t q_{t+1} 0.5 \left(\lambda_{b,t+1} + \lambda_{l,t+1}\right),$$
$$q_t \lambda_{b,t} = u_{h_b,t} + \zeta_{b,t} z q_t + \beta E_t q_{t+1} 0.5 \left(\lambda_{b,t+1} + \lambda_{l,t+1}\right).$$

Combining the binding budget constraints to $-2l_{b,t} = (c_{b,t} - c_{l,t}) + q_t \left(h_{b,t} - h_{l,t}\right)$, and substituting out multipliers and loans with the binding constraint (88), we can define a competitive equilibrium as a set of sequences $\{c_{b,t},\, c_{l,t},\, r_t^L,\, h_{b,t},\, q_t\}_{t=0}^{\infty}$ satisfying

$$u_{c_l,t} = \beta E_t \left[0.5(u_{c_l,t+1} + u_{c_b,t+1}) \left(1 + r_t^L\right)\right], \tag{89}$$

$$(1 - \tau_t^L) u_{c_b,t} = u_{c_l,t} + \left[(u_{h_l,t} - u_{h_b,t}) q_t^{-1} + (u_{c_b,t} - u_{c_l,t})\right]/z, \tag{90}$$

$$u_{c_l,t}q_t = u_{h_l,t} + \beta E_t[0.5(u_{c_l,t+1} + u_{c_b,t+1})q_{t+1}], \tag{91}$$

$$c_{b,t} - c_{l,t} = q_t \left(2\widehat{z}h_{b,t} + h\right), \tag{92}$$

and $y_t = c_{b,t} + c_{l,t}$, given $\{\tau_t^L\}_{t=0}^{\infty}$, where $\widehat{z} = z - 1/2$ is positive for reasonable values for z. Notably, the constraint on the consumption differential (92) closely relates to the corresponding condition in the monetary economy (see 27). The conditions (89) and (90), which determine the real interest rate and the tax/subsidy rate for a given allocation, do not constitute binding constraints for a social planer who chooses a welfare maximizing allocation subject to (91), (92), and $y = c_b + c_l$. Considering the case without aggregate risk the problem of the social planer can be summarized as

$$\begin{aligned} \max_{c_b,c_l,h_b} \min_{\chi_1,\chi_2} (1-\beta)^{-1} \; \{&[u(\epsilon_b, c_b, h_b) + u(\epsilon_l, c_l, h - h_b)] \\ &+ \chi_1 \left[y - c_b - c_l\right] + \chi_2[(2\widehat{z}h_b + h) \cdot q - c_b + c_l]\}, \end{aligned} \tag{93}$$

where $q = u_{h_l}/\Xi$ and $\Xi = u_{c_l} - \beta 0.5(u_{c_l} + u_{c_b}) > 0$. The first order conditions are

$$\begin{aligned} u_{c_b} &= \chi_1 + \chi_2 \left(1 - (2\widehat{z}h_b + h) \cdot [\partial q/\partial c_b]\right), \\ u_{c_l} &= \chi_1 - \chi_2 \left(1 + (2\widehat{z}h_b + h) \cdot [\partial q/\partial c_l]\right), \\ u_{h_l} - u_{h_b} &= \chi_2 \left(2\widehat{z} \cdot q + (2\widehat{z}h_b + h) \cdot [\partial q/\partial h_b]\right), \end{aligned}$$

and can – by substituting out the multipliers χ_1 and χ_2 – be combined to

$$\frac{u_{c_b} - u_{c_l}}{u_{h_l} - u_{h_b}} \frac{u_{h_l}}{\Xi} \widehat{z} = \frac{2 + (2\widehat{z}h_b + h) \cdot ((\partial q/\partial c_l) - (\partial q/\partial c_b))}{2 + (2\widehat{z}h_b + h) \cdot (\partial q/\partial h_b) / (q\widehat{z})}. \tag{94}$$

Combining (89) and (90), gives the competitive equilibrium condition $(1 - \tau_t^L)u_{c_b,t} = u_{c_l,t} + [(u_{h_l,t} - u_{h_b,t})q_t^{-1} + (u_{c_b,t} - u_{c_l,t})]/z$, which simplifies without aggregate risk to

$$\left(\frac{(1 - \tau^L)u_{c_b} - u_{c_l}}{(u_{h_l} - u_{h_b})} z - \frac{u_{c_b} - u_{c_l}}{u_{h_l} - u_{h_b}} \right) \frac{u_{h_l}}{\Xi} = 1. \tag{95}$$

It can easily be seen that the RHS of (94) is smaller than one if

$$(z - 1/2)q\left[(\partial q/\partial c_l) - (\partial q/\partial c_b)\right] < \partial q/\partial h_b, \tag{96}$$

which implies for the LHSs of (94) and (95): $\frac{u_{c_b}-u_{c_l}}{u_{h_l}-u_{h_b}}\frac{u_{h_l}}{\Xi}\widehat{z} < \frac{(1-\tau^L)zu_{c_b}-zu_{c_l}-(u_{c_b}-u_{c_l})}{u_{h_l}-u_{h_b}}\frac{u_{h_l}}{\Xi} \Rightarrow$ $0 < \frac{1}{z2}(1-u_{c_l}/u_{c_b}) < -\tau^L$.

Hence, the implementation of a constrained efficient allocation requires a subsidy if (96) holds, which can by using the partial derivatives $\partial q/\partial c_b = u_{h_l}\beta 0.5 u_{c_b c_b}\Xi^{-2} < 0$, $\partial q/\partial c_l = -u_{h_l}u_{c_l c_l}(1+\beta 0.5)\,\Xi^{-2} > 0$, and $\partial q/\partial h_b = -u_{h_l h_l}/\Xi > 0$, be rewritten as

$$(z-1/2)q\frac{-u_{c_l c_l}+\beta 0.5\,(u_{c_l c_l}-u_{c_b c_b})}{u_{c_l}-\beta 0.5(u_{c_l}+u_{c_b})} < \frac{-u_{h_l h_l}}{u_{h_l}}. \tag{97}$$

Comparing (96) and (97) with the corresponding expressions for the monetary economy, i.e. $z(q/R^L)\cdot\left[\partial(q/R^L)/\partial c_l\right] < \left[\partial(q/R^L)/\partial h_b\right]$ and $z(q/R^L)\frac{-u_{c_l c_l}}{u_{c_l}} < \frac{-u_{h_l h_l}}{u_{h_l}}$ (see 67), shows that the conditions for the constrained efficient allocations are qualitatively unchanged. Specifically, a subsidy on borrowing implements the constrained efficient allocation when changes in housing demand dominates changes in consumption demand with regard to their impact on the (relative) price of housing (as also considered in Proposition 2).

To see how the effects of central bank asset purchases can in principle be replicated by fiscal instruments, we additionally introduce a Pigouvian tax/subsidy τ_t^h on end-of-period housing, financed by lump-sum taxes/transfers. Notably, to neutralize the effects of the housing tax/subsidy on households' budgets, lump-sum taxes/transfers have to be type-specific (see also Davila and Korinek, 2018). Hence, the budget constraints are then given by

$$\left(1+r_{t-1}^L\right)l_{b,t-1}+y_t \geq (1-\tau_t^L)l_{b,t}+c_{b,t}+q_t\left(\left(1+\tau_t^h\right)h_{b,t}-h_{b,t-1}\right)+\tau_{b,t}^R+\tau_{b,t}^h,$$

and $\left(1+r_{t-1}^L\right)l_{l,t-1}+y_t \geq l_{l,t}+c_{b,t}+q_t\left(\left(1+\tau_t^h\right)h_{l,t}-h_{l,t-1}\right)+\tau_{l,t}^h$, where $\tau_{b,t}^h = -q_t\tau_t^h h_{b,t}$ and $\tau_{l,t}^h = -q_t\tau_t^h h_{l,t}$. The conditions (90) and (91) then change to $(1-\tau_t^L)u_{c_b,t} = u_{c_l,t} + [(u_{h_l,t}-u_{h_b,t})q_t^{-1}+\left(1+\tau_t^h\right)(u_{c_b,t}-u_{c_l,t})]/z$ and $\left(1+\tau_t^h\right)q_t u_{c_l,t} = u_{h_l,t}+\beta E_t q_{t+1}0.5(u_{c_l,t+1}+u_{c_b,t+1})$.

Without aggregate risk, the housing price then satisfies $q = u_{h_l}/[(1+\tau^h)u_{c_l}-\beta 0.5(u_{c_l}+u_c)]$ and the consumption difference is given by

$$c_b-c_l = \frac{u_{h_l}}{(1+\tau^h)\,u_{c_l}-\beta 0.5\,(u_{c_l}+u_c)}\cdot\left(\widehat{z}h_b+h\right).$$

A comparison with (44) shows that a subsidy on housing, $\tau^h < 0$, can in principle replicate the additional effect of asset purchases (while it also alters the subsidy rate on debt). By raising the price of collateral the borrowing constraint can be relaxed, which can enhance efficiency compared to the allocation where only the Pigouvian subsidy on debt is available.

G Asset purchases under aggregate risk

Suppose that housing depreciates every period at the rate δ_h, while new housing is constructed at the same rate, such that total supply again equals h. For an individual agent, the investment decision in housing is then described by

$$q_t \epsilon_l c_{l,t}^{-\sigma} \frac{1/R_t^L - \kappa_t/R_t^m}{1-\kappa_t} = \gamma \left(h - h_{b,t}\right)^{-\sigma} + \beta E_t (1-\delta_h) q_{t+1} \epsilon_l c_{l,t+1}^{-\sigma} \frac{1/R_{t+1}^L - \kappa_{t+1}/R_{t+1}^m}{1-\kappa_{t+1}}, \quad (98)$$

instead of (71). For the analysis of an optimal asset purchase policy, we apply the conditions (69), (70), (72), (73), and (98) and define

$$\widetilde{z}_t = \frac{z}{2} \frac{(2-\kappa_t s_t)(1-\kappa_t)}{1-\kappa_t s_t}, \qquad x_t = \frac{q_t}{R_t^L} \frac{1-\kappa_t s_t}{1-\kappa_t}, \quad (99)$$

where $s_t = R_t^L / R_t^m$. Further combining (69) and (70) to

$$\epsilon_b c_{b,t}^{-\sigma} \left(q_t / R_t^L\right) = x_t \epsilon_l c_{l,t}^{-\sigma} + \gamma((h-h_{b,t})^{-\sigma} - h_{b,t}^{-\sigma})/z, \quad (100)$$

and recalling that the policy maker has two instruments at his disposal to adjust the two terms $\frac{(2-\kappa_t s_t)(1-\kappa_t)}{1-\kappa_t s_t}$ and $\frac{1-\kappa_t s_t}{1-\kappa_t}$, the constraints for an optimal choice of the set of sequences $\{c_{b,t}, c_{l,t}, h_{b,t}, x_t, \widetilde{z}_t\}_{t=0}^{\infty}$ are given by (73)

$$c_{b,t} - c_{l,t} \leq \widetilde{z}_t h_{b,t} 2 x_t, \quad (101)$$

$$\epsilon_l c_{l,t}^{-\sigma} x_t = \gamma (h-h_{b,t})^{-\sigma_h} + \beta E_t \epsilon_l c_{l,t+1}^{-\sigma} (1-\delta_h) x_{t+1}, \quad (102)$$

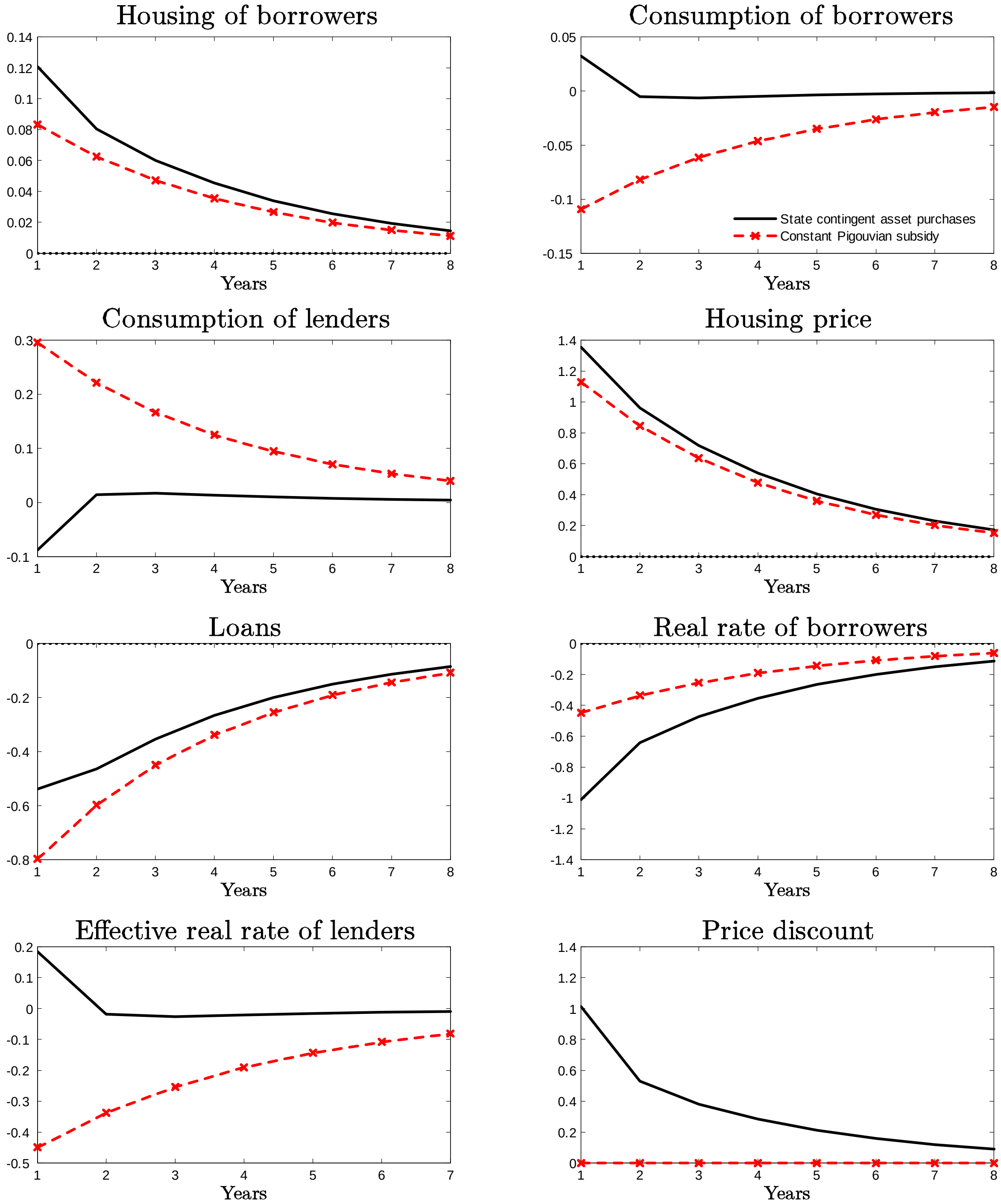
while q_t / R_t^L is determined by (100). For a given sequence $\{\widetilde{z}_t\}_{t=0}^{\infty}$, the planer problem under commitment can be summarized as $\max_{\{c_{b,t}, c_{l,t}, h_{b,t} x_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t [u(\epsilon_b, c_{b,t}, h_{b,t}) + u(\epsilon_l, c_{l,t}, h - h_{b,t})]$ subject to (73), (101), and (102). Neglecting the conditions for period $t=0$, the solution has to satisfy the following first order conditions $0 = \epsilon_b c_{b,t}^{-\sigma} - \mu_t - \lambda_t$, $0 = \epsilon_l c_{l,t}^{-\sigma} + \mu_t - \lambda_t - \sigma \psi_t \epsilon_l c_{l,t}^{-\sigma-1} x_t + \sigma \psi_{t-1} (1-\delta_h) \epsilon_l c_{l,t}^{-\sigma-1} x_t$, $0 = \gamma \left(h_{b,t}^{-\sigma_h} - ((h-h_{b,t})^{-\sigma_h}\right) + \mu_t \widetilde{z}_t 2 x_t - \psi_t \sigma_h \gamma (h-h_{b,t})^{-\sigma_h - 1}$, and $0 = \mu_t \widetilde{z}_t h_{b,t} 2 + \psi_t \epsilon_l c_{l,t}^{-\sigma} - \psi_{t-1}(1-\delta_h) \epsilon_l c_{l,t}^{-\sigma}$, where

λ_t, μ_t, and ψ_t denote the multiplier on the constraints (73), (101), and (102), respectively. Notably, the last condition would imply a unit root in the multiplier ψ_t under a binding borrowing constraint, $\mu_t > 0$, if there were no depreciation of housing ($\delta_h = 0$). Eliminating λ_t and μ_t with the first two conditions, leads to

$$\begin{aligned} 0 = &\gamma \left(h_{b,t}^{-\sigma_h} - ((h - h_{b,t})^{-\sigma_h}\right) + \left(\epsilon_b c_{b,t}^{-\sigma} - \epsilon_l c_{l,t}^{-\sigma}\right) \widetilde{z}_t x_t \qquad (103) \\ &+ \left(\psi_t - \psi_{t-1}(1 - \delta_h)\right) \sigma \epsilon_l c_{l,t}^{-\sigma-1} x_t \widetilde{z}_t x_t - \psi_t \sigma_h \gamma (h - h_{b,t})^{-\sigma_h - 1} , \\ 0 = &\left(\epsilon_b c_{b,t}^{-\sigma} - \epsilon_l c_{l,t}^{-\sigma}\right) \widetilde{z}_t h_{b,t} + \left(\psi_t - \psi_{t-1}(1 - \delta_h)\right) \left(\sigma \epsilon_l c_{l,t}^{-\sigma-1} x_t \widetilde{z}_t h_{b,t} + \epsilon_l c_{l,t}^{-\sigma}\right) . \qquad (104) \end{aligned}$$

The optimal plan given $\{\kappa_t\}_{t=0}^{\infty}$ is a set of sequences $\{h_{b,t}, c_{b,t}, c_{l,t}, x_t, s_t, \psi_t, \widetilde{z}_t, q_t/R_t^L\}_{t=0}^{\infty}$ satisfying (73), (99)-(104). Alternatively, κ_t can be endogenized given $\{s_t\}_{t=0}^{\infty}$.

H Additional figures



Responses to a minus one st.dev. liquidation value shock (in % deviations from a non-stochastic mean)