Public Policy and Sovereign Risk*

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Abstract

How does the presence of sovereign risk affect the conduct of public policy? To answer this question, this paper studies optimal monetary and fiscal policy without commitment for a model with nominal public debt and strategic sovereign default. When a government might default on debt payments, public policy changes in the short and the long run relative to a scenario without positive sovereign risk, which is induced by prohibitively high default costs. Risk of default increases the volatility of interest rates, impeding the government's ability to smooth tax distortions across states and thereby amplifying the impact of aggregate shocks on the economy. It also limits public debt accumulation which reduces the government's incentive to use surprise inflation on average. Although sovereign risk can have sizable quantitative implications for the behavior of public policy, it is found to only have negligible welfare consequences.

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1 Introduction

While sovereign default was viewed as an emerging market phenomenon for a long time, the recent European debt crisis has illustrated its ongoing relevance for developed economies (see e.g. Lane, 2012). Even the federal government of the United States, whose debt instruments have usually been treated as risk-less by market participants and economists alike, faces increased concerns about the sustainability of its debt, as highlighted, for instance, by its credit-rating downgrade in 2011. Events like the debt-ceiling crisis of 2013 or comments made by then-presidential nominee Donald J. Trump about his potential willingness to consider debt restructuring as a policy option for the federal government further fueled such concerns.[1] Thinking about how the possibility of sovereign default in the future might constrain policy makers in developed economies has thus become more than just an interesting thought experiment.

The contribution of this paper is to study how the presence of sovereign risk affects the conduct of monetary and fiscal policy. More specifically, I study optimal monetary and fiscal policy without commitment for a representative-agent cash-credit good economy with flexible prices and endogenous sovereign risk.[2] In the model, a benevolent government finances the supply of a public good by setting a labor income tax rate, choosing the money growth rate, issuing nominal non-contingent long-term bonds and deciding whether to honor its outstanding debt payments or not. The default decision is modeled as a binary choice (see Eaton and Gersovitz, 1981). Following the quantitative sovereign default literature (see Aguiar and Amador, 2014), a default is costly because it entails an exogenous direct cost and triggers a debt restructuring process that determines the debt recovery rate and the amount of time the government spends in financial autarky. In addition, the model also features a novel endogenous cost of default for monetary economies.[3]

As is common in the optimal policy literature, I consider a closed economy. This paper thus contributes to the study of domestic debt default which, despite being a historically recurring phenomenon with severe economic consequences, has not received a lot of attention in the sovereign default literature (see Reinhart and Rogoff, 2011). In a closed economy, a default does not redistribute resources from foreign lenders to domestic citizens. The government may still choose not to repay its debt to relax its budget constraint and reduce distortionary taxes. The model is calibrated to the US economy, assuming that the costs of default are sufficiently high to rule out equilibrium default. Reducing these costs then allows to study how risk of default affects public policy.

I study the Markov-perfect equilibrium of the public policy problem (see Klein, Krusell, and Rios-Rull, 2008). Since the government optimizes sequentially, it cannot commit to future policies and does not internalize that its current decisions affect household expectations in previous periods. The government is however aware that expected future policy will depend on its own borrowing

[1] See e.g. https://www.washingtonpost.com/news/wonk/wp/2013/10/04/absolutely-everything-you-need-to-know-about-the-debt-ceiling/ and https://www.nytimes.com/2016/05/07/us/politics/donald-trumps-idea-to-cut-national-debt-get-creditors-to-accept-less.html for details.

[2] As is standard in the optimal policy literature (see Chari and Kehoe, 1999), I assume that there is only one benevolent policy maker, referred to as the government, who is in charge of both, monetary and fiscal policy. See Roettger (2016) for a sovereign default model with independent monetary and fiscal authorities that allows for political economy frictions.

[3] More specifically, a default directly affects the government budget by changing inflation expectations, which matter for seigniorage revenues via the household money demand. In contrast to the exogenous default cost, the endogenous cost varies with the size of the debt position. It alone is however not sufficient to generate plausible quantitative features.

decision because it will affect the incentive to reduce the real debt burden via default or inflation in future periods. The option to default thus matters for the government's response to exogenous shocks by allowing it to adjust the real debt burden as well as by affecting the cost of borrowing and thus the attractiveness of debt as a shock absorber.

Compared to an otherwise identical economy without sovereign risk, induced by prohibitively high costs of default, economies with equilibrium default feature lower average inflation. Since the gains of inflation decline when a default takes place, inflation is lower when default is chosen instead of repayment. This direct effect of default on average inflation is however of negligible size. Instead, the key mechanism that leads inflation to be lower when the default option is available is an indirect one. The attractiveness and hence the probability of default increases with the size of the public debt position and in response to adverse shocks, i.e. when productivity is low and/or the need for public spending is high. With default risk, the bond price becomes more debt elastic in bad states and the marginal revenue from debt issuance accordingly decreases faster. Consequently, the government borrows less which reduces its incentive to adjust real debt payments via inflation on average. The increased sensitivity of the bond price to adverse shocks also impedes the government's ability to smooth tax distortions across states. Relative to an economy without default option, public policy is thus more volatile, amplifying the impact of exogenous shocks on the economy.

From a welfare perspective, it is not obvious whether it is desirable to endow the government with the option to default. As in the literature on unsecured consumer credit (see e.g. Chatterjee, Corbae, Nakajima, and Rios-Rull, 2007; Livshits, MacGee, and Tertilt, 2007), there exists a trade off when introducing the option to default in an incomplete markets setting without commitment. With only distortionary taxes available, the government would like to smooth tax distortions across states by occasionally running a budget deficit (or surplus), following the logic of Barro (1979). While the option to default allows to make debt payments (more) state contingent, the risk of default makes debt issuance more expensive, especially in adverse states, which entails welfare losses due to more volatile public policy. In the model economy studied in this paper, there is an additional indirect effect of default that can result in welfare gains. As in Martin (2009) and Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008), the government accumulates positive average debt positions because of its lack of commitment and a monetary friction. By increasing the cost of borrowing, risk of default renders public debt accumulation less attractive, reducing average debt and - as a result - inflation. A welfare exercise reveals that the option to default results in net welfare gains. Since the magnitude of the welfare gains is however of negligible size, one could argue that, for the United States, lack of commitment to debt service might not be particularly important from a welfare perspective.

Related Literature This paper is related to the literature on optimal Markov-perfect monetary and fiscal policy with nominal government debt. Martin (2009, 2011, 2013) extensively studies the short-and long-run properties of public debt and inflation when the government lacks commitment. In particular, he shows that a monetary economy with discretionary policy and nominal public debt can generate positive public debt positions of plausible size. For a similar model environment, Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008) show how public policy and welfare depend on whether debt is indexed to inflation or not. Among other things, they find that without commitment welfare can be lower when debt is indexed. In a model with nominal rigidities, Niemann, Pichler, and Sorger (2013) study how the presence of lack of commitment and nominal government debt affects the persistence of inflation. Despite highlighting the role of lack of commitment for public policy, these studies maintain the assumption that public debt always has to be repaid, thus abstracting from the commitment problem related to sovereign default.

This work is also related to a number of recent papers that study domestic debt default. For a model with incomplete markets and idiosyncratic income risk, D'Erasmo and Mendoza (2016) show that a sovereign default can occur in equilibrium as an optimal distributive policy. Pouzo and Presno (2016) investigate the robustness of Aiyagari, Marcet, Sargent, and Seppälä (2002)'s incomplete markets model of optimal taxation by considering a policy maker who cannot commit to debt payments, finding that risk of default increases the volatility of tax rates. Sosa-Padilla (forthcoming) studies Markov-perfect fiscal policy for a model where a sovereign default triggers a banking crisis. Pei (2017) considers optimal fiscal policy without commitment for an economy in which government bonds are valued by firms as collateral, generating an endogenous default cost channel.[4] All of these papers feature real economies and hence do not discuss monetary policy.

Furthermore, this paper relates to the quantitative sovereign default literature that studies how risk of default affects business cycles in small open economies.[5] Within this literature, the studies that are closest to this paper are Cuadra, Sanchez, and Sapriza (2010), Nuño and Thomas (2016) and Du and Schreger (2016). Cuadra, Sanchez, and Sapriza (2010) study a production economy with endogenous fiscal policy but abstract from monetary policy and - as is common in the sovereign default literature - look at a small open economy that trades real bonds with foreign investors. Nuño and Thomas (2016) consider a small open endowment economy with nominal defaultable debt and a benevolent government that chooses monetary policy under discretion. The authors find that the economy tends to be better off when the government issues foreign currency debt or joins a monetary union since this eliminates its inflation bias. Du and Schreger (2016) study a model of a small open economy where the government borrows in local currency from foreign investors, enabling it to reduce the real debt burden by using inflation. Since domestic entrepreneurs have liabilities denominated in foreign currency but earn revenues in local currency, inflation hurts firm balance sheets by depreciating the local currency.[6] In contrast to these papers, the closed economy model studied in this paper does not rely on the assumption that the government is impatient relative to its creditors to generate empirically plausible debt levels (see Martin, 2009).

In independent and contemporaneous work, Sunder-Plassmann (2017) also studies public policy for a monetary economy with sovereign default. There are however several differences between our studies. Allowing the government of a small open economy to issue a bond portfolio that involves fixed shares with respect to the denomination and ownership of public debt, her paper studies how the debt structure shapes macroeconomic outcomes and compares the model predictions to observations for a sample of emerging economies. By contrast, I investigate how the possibility of default affects

[4] Niemann and Pichler (2017) study the sustainability of public debt for an economy with financial frictions but without positive risk of default.

[5] See Aguiar and Amador (2014) and Aguiar, Chatterjee, Cole, and Stangebye (2016) for recent surveys of this literature.

[6] Na, Schmitt-Grohé, Uribe, and Yue (2018) also develop a quantitative sovereign default model where the government can devalue the local currency but consider external debt that is denominated in foreign currency.

the optimal conduct of monetary and fiscal policy in the absence of commitment, using a model that is calibrated to the US economy as the baseline scenario. Further differences between our studies are that I allow for long-term government bonds, endogenous debt recovery and endogenous public spending, whereas Sunder-Plassmann (2017) only considers one-period bonds, full debt default and exogenous government expenditures.

Recently, Aguiar, Amador, Farhi, and Gopinath (2013) have also developed a model to jointly study inflation and sovereign default when a government cannot commit to future policy. However, their analysis differs from mine in several ways. First, their model features a small open endowment economy that is not subject to fundamental shocks and borrows from abroad. Second, the authors assume that the government experiences an ad-hoc utility cost of inflation. Third, in the spirit of Cole and Kehoe (2000), they exclusively focus on self-fulfilling debt crises.

Layout The rest of the paper is organized as follows. Section 2 presents and discusses the model that is analyzed quantitatively in Section 3. The welfare implications of sovereign default are addressed in Section 4. Section 5 concludes.

2 Model

The model extends a monetary economy with flexible prices and distortionary labor taxes (see Lucas and Stokey, 1983; Martin, 2009) by introducing long-term government bonds (see Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012), strategic sovereign default (see Eaton and Gersovitz, 1981; Arellano, 2008) and endogenous debt recovery (see Hatchondo, Martinez, and Sosa-Padilla, 2016).

Time is discrete, starts in $t = 0$ and goes on forever. The economy is populated by a unit mass continuum of homogeneous infinitely-lived households and a benevolent government. Taking government policies and prices as given, households optimize in a competitive fashion. They supply labor n_t to produce the marketable good y_t, using a production technology to be specified below. In addition, they choose consumption of a cash good c_{1t} and a credit good c_{2t}, and decide on money ($\tilde{m}_{t+1}$) and nominal government bond ($\tilde{b}_{t+1}$) holdings. The unit price of a government bond is denoted as q_t. While all assets are nominal and thus subject to inflation risk, only government bonds are subject to default risk.

To finance the supply of a public good g_t and outstanding nominal debt payments $\delta\tilde{B}_t$, the government chooses from a set of policies that includes public spending g_t, the money growth rate μ_t, a linear labor income tax rate τ_t, the binary default decision $d_t \in \{0,1\}$, and issuance of nominal non-state contingent long-maturity bonds $\tilde{I}_t$. Following Hatchondo and Martinez (2009), government bonds are modeled as perpetuities that promise to pay an infinite stream of coupon payments that decline geometrically over time, where the coupon parameter $\delta \in (0,1]$ governs the average maturity of debt $1/\delta$ and the size of coupon payments. More specifically, a bond issued in period t promises to pay the nominal cash flow $\tilde{p}_t\delta(1-\delta)^{j-1}$ in periods $t+j$, for $j \geq 1$, with $\tilde{p}_t$ denoting the price of consumption in terms of $\tilde{m}_t$.[7] The memory-less nature of these perpetuities implies that the law of motion for the stock of nominal government debt can be written recursively as $\tilde{B}_{t+1} = (1-\delta)\tilde{B}_t + \tilde{I}_t$.[8] A default on outstanding public debt occurs when $d_t = 1$ is chosen, while the government fully repays its obligations for $d_t = 0$. In the default case, the government is excluded from financial markets until debt repayment to bond holders is settled (see Hatchondo, Martinez, and Sosa-Padilla, 2016).

The government's credit status is given by the indicator variable $h_t \in \{0,1\}$. If $h_t = 0$, the government has access to the bond market, whereas it is in financial autarky for $h_t = 1$. Given the credit status at the end of the previous period h_{t-1}, the law of motion for h_t is

$$h_t = [\zeta_t(1-e_t) + 1 - \zeta_t]\, h_{t-1} + d_t(1-h_{t-1}).$$

If the government enters period t with a good credit status ($h_{t-1} = 0$) and defaults ($d_t = 1$), its credit status switches to $h_t = 1$. Conditional on having left the previous period $t-1$ in autarky, with probability ϑ, in period t the government receives the offer to repay the fraction $\omega \in [0,1]$ of its outstanding debt and immediately leave autarky in return (see Hatchondo, Martinez, and Sosa-Padilla, 2016).[9] The acceptance decision is denoted as $e_t \in \{0,1\}$, where $e_t = 1$ means that the offer is accepted. As in Hatchondo, Martinez, and Sosa-Padilla (2016), even if the offer to repay the reduced debt burden is declined, the debt position is nevertheless reduced to ωB_t.[10] For the model formulation it will be useful to define the indicator variable $\zeta_t \in \{0,1\}$, which equals one if the government receives a repayment offer and zero if not. If the government does not accept an offer, i.e. $e_t = 0$, it remains in autarky ($h_t = 1$) and might receive a new offer in the next period, again with probability ϑ. Conditional on not being in autarky, the government will have access to the bond market until it chooses to default.

Endogenizing the debt recovery rate as well as the duration of financial autarky is relevant for a number of reasons. By allowing for a positive and endogenous haircut, the model can account for the empirical observations that default events rarely lead to haircuts of 100% and that debt recovery rates vary with the size of public debt (Cruces and Trebesch, 2013). Importantly, an endogenous haircut also matters from a policy perspective. For the government, default and inflation are imperfect substitutes since they can both reduce the real debt burden if outstanding debt is denominated in local currency. The degree of substitution between the two policy options depends on how flexibly they can be used to adjust debt payments. Allowing for partial default is crucial to capture this policy dimension. On the one hand, default events typically involve reductions of debt payments that are larger and more sudden compared to what an inflationary monetary policy could accomplish. On

[7] Du and Schreger (2016) consider similar nominal perpetuities in a model with sovereign default and risk-neutral foreign investors.

[8] The same aggregate law of motion emerges if only random maturity zero-coupon bonds (see Chatterjee and Eyigungor, 2012) are issued by the government, where $1/\delta$ is the average maturity of a bond.

[9] Examples of sovereign default models that endogenize the recovery rate by modeling debt renegotiation between a small open economy and foreign investors are Yue (2010) and Bai and Zhang (2012).

[10] This assumption reduces the notation needed for the model formulation because the acceptance decision e_t can in this case be characterized by the same policy functions as the default decision d_t. Note that it also implies that, even with a constant offer rate ω, the government can choose the size of the haircut by waiting for enough reductions of its debt before finally accepting an offer and settle.

the other hand, while a government can arguably affect the size of a haircut, a default is usually followed by a potentially lengthy debt restructuring process that cannot be entirely controlled by the government and ultimately determines the debt recovery rate. The debt restructuring process in this paper is able to capture this trade off between the potentially larger adjustment of debt payments that a default can accomplish relative to inflation and the associated uncertainty about the ultimate size and timing of debt reduction.

Given the debt restructuring process outlined above, the nominal payoff associated with holding a sovereign bond at the beginning of period t is

$$\begin{aligned} k_t \quad = \quad & \mathbf{1}_{\{h_{t-1}=0 \,\wedge\, d_t=0\}} \left[\delta + (1-\delta)\, q_t\right] + \mathbf{1}_{\{(h_{t-1}=0 \,\wedge\, d_t=1) \,\vee\, (h_{t-1}=1 \,\wedge\, \zeta_t=0)\}} q_t \\ & + \mathbf{1}_{\{h_{t-1}=1 \,\wedge\, \zeta_t=1 \,\wedge\, e_t=0\}} \omega q_t + \mathbf{1}_{\{h_{t-1}=1 \,\wedge\, \zeta_t=1 \,\wedge\, e_t=1\}} \left[\omega(\delta + (1-\delta)\, q_t)\right], \end{aligned}$$

where $\mathbf{1}_{\{\cdot\}}$ denotes the indicator function, which equals one if the statement in curly brackets is true and zero otherwise. The indicator functions allow to express the size of debt payments received by the household from the governments as well as the value of its beginning-of-period bond holdings conditional on the government's credit status in the previous period h_{t-1}, whether a repayment offer has been made (ζ_t) and accepted (e_t), and the government's repayment decision d_t.

2.1 Private Sector

Households derive utility from private consumption, leisure and a public good. Their preferences are given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ U(c_{1t}, c_{2t}, l_t, g_t, \theta_t) - h_t \psi_t \right\} \right],$$

with discount factor $\beta \in (0,1)$ and period utility function

$$U(c_{1t}, c_{2t}, l_t, g_t, \theta_t) = u(c_{1t}, c_{2t}, l_t) + \theta_t v(g_t).$$

The functions $u : \mathbb{R}^3_+ \to \mathbb{R}$ and $v : \mathbb{R}_+ \to \mathbb{R}$ are both twice continuously differentiable and satisfy $u_1, u_2, u_l, v_g > 0$ and $u_{11}, u_{22}, u_{ll}, v_{gg} < 0$. As in Martin (2013), I allow for a random variable θ_t that affects the marginal utility of the public good. In the remainder, this shock will also be referred to as a spending shock since it leads to fluctuations in the need for public spending. The spending shock θ_t is persistent and governed by a stationary first-order Markov process with discrete support $\Theta \subseteq \mathbb{R}_+$ and conditional transition probabilities $\Pr(\theta_{t+1}|\theta_t)$. Following Bianchi, Hatchondo, and Martinez (2018), households face an additive utility loss ψ_t in periods of default and financial autarky ($h_t = 1$). This loss is taken as given by the households and will be discussed in detail below.

Households have initial assets $(\tilde{b}_0, \tilde{m}_0)$ and take as given prices $\{\tilde{p}_t, q_t\}_{t=0}^{\infty}$ and government policies $\{d_t, e_t, g_t, \mu_t, \tau_t, \tilde{B}_{t+1}\}_{t=0}^{\infty}$. The aggregate money stock evolves according to $\tilde{M}_{t+1} = (1+\mu_t)\tilde{M}_t$. Households also take as given the government's credit status $\{h_t\}_{t=0}^{\infty}$, the offer process $\{\zeta_t\}_{t=0}^{\infty}$ and changes in the value of the public good $\{\theta_t\}_{t=0}^{\infty}$. Productivity $\{a_t\}_{t=0}^{\infty}$ is subject to random shocks as well and follows a stationary first-order Markov process with continuous support $\mathbb{A} \subseteq \mathbb{R}_+$ and transition function $f_a(a_{t+1}|a_t)$.

Households maximize their expected lifetime utility subject to the period budget constraint

$$(1-\tau_t)a_t n_t + \frac{\tilde{m}_t}{\tilde{p}_t} + k_t \frac{\tilde{b}_t}{\tilde{p}_t} \geq c_{1t} + c_{2t} + \frac{\tilde{m}_{t+1}}{\tilde{p}_t} + q_t \frac{\tilde{b}_{t+1}}{\tilde{p}_t},$$

the cash-in-advance constraint

$$\frac{\tilde{m}_t}{\tilde{p}_t} \geq c_{1t}.$$

and the time constraint

$$1 \geq l_t + n_t.$$

A role for money is introduced into the model economy by tying cash-good consumption to beginning-of-period household money holdings via a cash-in-advance constraint (see Clower, 1967; Svensson, 1985). The time endowment of one is divided between enjoying leisure l_t and supplying labor n_t. Labor supply is used to produce a marketable good according to the linear technology $y_t = a_t n_t$.[11]

2.2 Public Sector

The government's budget constraint is

$$g_t - \tau_t a_t n_t = \frac{\tilde{M}_{t+1} - \tilde{M}_t}{\tilde{p}_t} + (1-h_t)\frac{q_t\left(\tilde{B}_{t+1} - (1-\delta)\tilde{B}_t\right) - \delta\tilde{B}_t}{\tilde{p}_t}.$$

In the default (and autarky) case ($h_t = 1$), the government has to finance public spending g_t with income tax revenues $\tau_t a_t n_t$ and seigniorage $\tau_t^m \equiv \left(\tilde{M}_{t+1} - \tilde{M}_t\right)/\tilde{p}_t$. In the repayment case ($h_t = 0$), it additionally has to make debt payments but can access the bond market and issue new debt.

Following the quantitative sovereign default literature (see e.g. Arellano, 2008; Cuadra, Sanchez, and Sapriza, 2010), a default entails two types of exogenous costs for the economy. First, the government is excluded from the bond market in the default period and remains in autarky until it accepts an offer to repay its debt.[12] Second, the economy experiences a direct loss, governed by $\psi : \mathbb{A} \to \mathbb{R}$, which, following Bianchi, Hatchondo, and Martinez (2018), is modeled in terms of utility in this paper.[13]

While this cost specification is arguably ad hoc, it has a number of advantages. First, it allows me not to take a stand on how a sovereign default is propagated through the economy. As argued by Bianchi, Hatchondo, and Martinez (2018), it is difficult to tell empirically whether default events cause output to decline or whether they simply tend to take place during recessions. It is thus not surprising that evidence on the real costs of default is mixed and that there is no consensus on which

[11] It is straightforward to modify the model to include a representative firm that is owned by households and produces the homogeneous good y_t, using labor supplied by households at a real wage w_t. Assuming that the labor market is competitive, the behavior of the economy will not change.

[12] Note that households can still trade the distressed government bonds among each other when the government is in financial autarky.

[13] Previous versions of this paper included a resource loss in terms of productivity as in Cuadra, Sanchez, and Sapriza (2010), which did not lead to qualitatively different results and hardly changed results quantitatively. The advantages of using utility rather than resource costs of default are discussed below.

propagation mechanism is the most relevant one (see Panizza, Sturzenegger, and Zettelmeyer, 2009). Second, the default cost specification used in this paper enables me to analyze the impact that lack of commitment to debt repayment has on public policy in a transparent and flexible way as it allows me to directly control the attractiveness of default via $\psi(\cdot)$. Third, modeling the direct costs of default as additive and in terms of utility rather than output (or productivity) allows to disentangle the direct costs of default, captured by $\psi(\cdot)$, from the indirect costs of default, which arise due to more debt-elastic bond prices that impede tax smoothing.

As will be discussed in Section 2.6, the model also features an endogenous default cost channel specific to monetary economies that is novel to the literature. This channel is due to the effect that default has on inflation expectations, which in turn matter for household money demand and the revenues that the government receives from issuing money. In contrast to $\psi(\cdot)$, the size of this default cost also depends on the amount of debt that the government has defaulted on. The cost channel is however not sufficient to generate empirically plausible features, like sizable countercyclical default risk, and sustain large debt positions, such that the model still requires an additional direct default cost like $\psi(\cdot)$.[14]

2.3 Private Sector Equilibrium

In a household optimum, the first-order conditions

$$
\begin{aligned}
\frac{u_l(t)}{u_2(t)} &= (1-\tau_t)a_t, && (1)\\
1 &= \beta \mathbb{E}_t\left[\frac{u_1(t+1)}{u_2(t)}\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right], && (2)\\
q_t &= \beta \mathbb{E}_t\left[\frac{u_2(t+1)}{u_2(t)}k_{t+1}\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right], && (3)
\end{aligned}
$$

need to be satisfied and the household budget constraint as well as the time constraint hold with equality.

In addition, the following complementary slackness conditions need to be satisfied as well:

$$\lambda_t = u_1(t) - u_2(t) \geq 0, \tilde{m}_t/\tilde{p}_t - c_{1t} \geq 0, \lambda_t\left(\tilde{m}_t/\tilde{p}_t - c_{1t}\right) = 0,$$

with λ_t denoting the Kuhn-Tucker multiplier on the cash-in-advance constraint.[15]

Intuitively, the cash-in-advance constraint is binding whenever the marginal utility of cash-good consumption exceeds the marginal utility of credit-good consumption. The inequality

$$u_1(t) - u_2(t) \geq 0, \tag{4}$$

needs to hold in equilibrium to satisfy $\lambda_t \geq 0$.

[14] Similarly, D'Erasmo and Mendoza (2016) also have to supplement their endogenous redistributive cost of default with a reduced form default cost as in Arellano (2008) to generate plausible quantitative features.

[15] In a household optimum, the household budget constraint holds with equality.

Condition (1) characterizes the optimal household labor supply decision, whereas conditions (2) and (3) are the Euler equations for money and government bond holdings. Since both assets are nominal, they need to compensate households for expected (gross) inflation $\tilde{p}_{t+1}/\tilde{p}_t$. Government bonds additionally reflect default and bond price risk, as given by k_{t+1}.

As in Cooley and Hansen (1991), I normalize nominal variables by the beginning-of-period aggregate money stock $\tilde{M}_t$, $x_t \equiv \tilde{x}_t/\tilde{M}_t$ for $x \in \{B, b, m, p\}$.[16] This normalization renders the model stationary and implies that the inflation rate in period t is given as

$$\pi_t \equiv \frac{p_t\,(1+\mu_{t-1})}{p_{t-1}} - 1,$$

such that inflation equals money growth in the long run and an increase in p_t directly raises inflation π_t.

After normalizing nominal variables, the Euler equations become

$$1 = \beta \mathbb{E}_t \left[\frac{u_1(t+1)}{u_2(t)} \frac{p_t}{p_{t+1}} \frac{1}{1+\mu_t} \right], \tag{5}$$

$$q_t = \beta \mathbb{E}_t \left[\frac{u_2(t+1)}{u_2(t)} k_{t+1} \frac{p_t}{p_{t+1}} \frac{1}{1+\mu_t} \right]. \tag{6}$$

For the economy, the goods and asset market clearing conditions are as follows:

$$\begin{aligned} a_t n_t &= c_{1t} + c_{2t} + g_t, \\ b_{t+1} &= B_{t+1}, \\ m_{t+1} &= 1. \end{aligned}$$

If (aggregate) real balances $1/p_t$ are sufficiently high, households equalize marginal utility across cash- and credit-good consumption, i.e. condition (4) holds with equality. If not, they are cash-constrained and the allocation of consumption is distorted. As in Martin (2009), in a monetary equilibrium, i.e. an equilibrium in which money is valued and the price index is well-defined,

$$c_{1t} = 1/p_t,$$

needs to hold. Note that this still allows for an unconstrained consumption allocation if the cash-in-advance constraint is just binding, i.e. when $\lambda_t = 0$ and $u_1(t) = u_2(t)$ hold simultaneously.

2.4 Public Policy Problem

In this section, I formulate the public policy problem. The government is benevolent and sets its policy instruments to maximize the expected life-time utility of the households, anticipating the response of the private sector to its policies. The government cannot commit itself to a state-contingent (Ramsey) policy plan for all current and future policies but optimizes from period to period in-

[16]Note that, by construction, the normalized aggregate money stock is constant and equal to one.

stead. To analyze its decision problem, I restrict attention to stationary Markov-perfect equilibria (see Klein, Krusell, and Rios-Rull, 2008), such that the optimal decisions of the government in any period will be characterized by time-invariant functions that only depend on the minimal payoff-relevant state of the economy in that respective period. In the model, this state consists of the beginning-of-period debt-to-money ratio B_t, labor productivity a_t, spending shock θ_t, the government's credit status h_t and the repayment offer ζ_t. By requiring the government to only condition its decisions on the current payoff-relevant aggregate state, the Markov-perfect equilibrium concept rules out the possibility that the government can make promises that are not optimal ex post. The reason for this is that at the start of a period, the government does not care about the past and only considers its payoff in current and future periods.[17] By construction, the government thus is ensured to act in a time-consistent way.

The Markov-perfect policy problem will be formulated recursively. In the remainder, I will thus adopt the notation of dynamic programming, i.e. time indices are dropped and a prime is used to denote next period's variables. Define $s \equiv (a, \theta)$ and $\mathbb{S} \equiv \mathbb{A} \times \Theta$. Given the aggregate state at the start of a period, the government takes as given the policy function $\mathcal{D}(B', s')$, that determines next period's default decision, as well as the functions $\mathcal{X}^r(B', s')$ and $\mathcal{X}^d(B', s')$, with $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{G}, \mathcal{N}, \mathcal{P}, \mathcal{Q}\}$, which determine credit-good consumption, public spending, labor supply, the price index and the bond price in the next period for the case of repayment (r) and default (d).[18] Expectations of these variables enter the household optimality conditions (5) and (6) and thus matter for the allocation in the current period.[19] Despite lacking the ability to commit to future policies, the government fully recognizes today that it affects (expected) future policies via its choice of B', which in turn have an effect on the behavior of the private sector in the current period. In a stationary Markov-perfect equilibrium, the policy functions that govern future decisions then coincide with the policy functions that determine current public policy for all states.

As in Klein, Krusell, and Rios-Rull (2008), one can interpret the formulation of the public policy problem as a Markov-perfect game played between successive governments. Following this interpretation, in each period, a different government is in charge of choosing public policy. Each government then chooses its optimal strategies, taking as given the optimal response of the government in the next period.

In every period, the government anticipates how the private sector responds to its actions as given by the private sector equilibrium conditions.[20] Applying the normalization of nominal variables introduced earlier, the government budget constraint can be written as

$$g_t - \tau_t a_t n_t = \frac{\mu_t}{p_t} + (1 - h_t) \frac{q_t \left((1 + \mu_t) B_{t+1} - (1 - \delta) B_t \right) - \delta B_t}{p_t},$$

where $\tilde{M}_{t+1} = (1 + \mu_t) \tilde{M}_t$ is used as well.

[17] The focus on Markov-perfect strategies also rules out the possibility of reputational considerations based on complex trigger strategies as in Chari and Kehoe (1990, 1993).

[18] Remember that cash-good consumption c_1 is directly linked to the price index p via the cash-in-advance constraint.

[19] Households do not have a strategic impact on future government policies but form rational expectations about them based on the policy functions listed above.

[20] The government thus plays a Stackelberg game against the (passive) private sector in every period.

Using the household optimality conditions (1), (5)-(6), the definition of payoff k_t, and the aggregate resource constraint, the government budget constraint can be written as

$$\begin{aligned}
&\beta \mathbb{E}_{s'|s}\left[u_1'\left\{\frac{1-\mathcal{D}\left(B',s'\right)}{\mathcal{P}^r\left(B',s'\right)}+\frac{\mathcal{D}\left(B',s'\right)}{\mathcal{P}^d\left(B',s'\right)}\right\}\right] \\
&+\beta \mathbb{E}_{s'|s}\left[u_2'\left\{\begin{array}{c}\frac{1-\mathcal{D}\left(B',s'\right)}{\mathcal{P}^r\left(B',s'\right)}\left(\delta+(1-\delta)\,\mathcal{Q}^r\left(B',s'\right)\right)\\ +\frac{\mathcal{D}\left(B',s'\right)}{\mathcal{P}^d\left(B',s'\right)}\mathcal{Q}^d\left(B',s'\right)\end{array}\right\}\right]B' \\
&-u_2\left\{\delta+(1-\delta)\frac{\mathbb{E}_{s'|s}\left[u_2'\left(\begin{array}{c}\frac{1-\mathcal{D}\left(B',s'\right)}{\mathcal{P}^r\left(B',s'\right)}\left(\delta+(1-\delta)\,\mathcal{Q}^r\left(B',s'\right)\right)\\ +\frac{\mathcal{D}\left(B',s'\right)}{\mathcal{P}^d\left(B',s'\right)}\mathcal{Q}^d\left(B',s'\right)\end{array}\right)\right]}{\mathbb{E}_{s'|s}\left[u_1'\left(\frac{1-\mathcal{D}\left(B',s'\right)}{\mathcal{P}^r\left(B',s'\right)}+\frac{\mathcal{D}\left(B',s'\right)}{\mathcal{P}^d\left(B',s'\right)}\right)\right]}\right\}(B/p) \\
&-u_l n+u_2 c_2=0,
\end{aligned} \tag{7}$$

for the repayment case and as

$$\begin{aligned}
\beta \mathbb{E}_{s'|s}\left[\vartheta\times\left\{u_1'\left(\frac{\mathcal{D}\left(\omega B',s'\right)}{\mathcal{P}^d\left(\omega B',s'\right)}+\frac{1-\mathcal{D}\left(\omega B',s'\right)}{\mathcal{P}^r\left(\omega B',s'\right)}\right)\right\}+(1-\vartheta)\times u_1'\frac{1}{\mathcal{P}^d\left(B',s'\right)}\right] \\
-u_l n+u_2 c_2=0,
\end{aligned} \tag{8}$$

for the default (and autarky) case.

This constraint can be seen as the period implementability constraint for the government.[21] Note that $e'=1-\mathcal{D}\left(\omega B',s'\right)$ was used for the derivation of the constraint in the default case. Declining an offer to repay can hence be thought of as defaulting on it (see Hatchondo, Martinez, and Sosa-Padilla, 2016).

In addition to the implementability constraint, the government has to respect the constraints

$$0 = an-1/p-c_2-g, \tag{9}$$

$$0 \leq u_1-u_2. \tag{10}$$

The household budget constraint is satisfied by Walras' Law, given the government budget constraint and the market clearing conditions.

As in sovereign default models with risk-neutral investors (see Arellano, 2008), the model allows to express the bond price as a function of an arbitrary (and hence potentially off-equilibrium) end-of-period debt position B' and the current values of the random variables s (see Martin, 2009). For the repayment case, the bond price is given as

$$q^r\left(B',s\right)=\frac{\mathbb{E}_{s'|s}\left[u_2'\left(\frac{1-\mathcal{D}\left(B',s'\right)}{\mathcal{P}^r\left(B',s'\right)}\left(\delta+(1-\delta)\,\mathcal{Q}^r\left(B',s'\right)\right)+\frac{\mathcal{D}\left(B',s'\right)}{\mathcal{P}^d\left(B',s'\right)}\mathcal{Q}^d\left(B',s'\right)\right)\right]}{\mathbb{E}_{s'|s}\left[u_1'\left(\frac{1-\mathcal{D}\left(B',s'\right)}{\mathcal{P}^r\left(B',s'\right)}+\frac{\mathcal{D}\left(B',s'\right)}{\mathcal{P}^d\left(B',s'\right)}\right)\right]}, \tag{11}$$

[21] The derivation of the implementability constraint can be found in Appendix A.1.

whereas it is given as

$$q^d\left(B',s\right)=\frac{\mathbb{E}_{s'|s}\left[\begin{array}{c}\vartheta\times\omega\left\{u_2'\left(\begin{array}{c}\frac{1-\mathcal{D}\left(\omega B',s'\right)}{\mathcal{P}^r\left(\omega B',s'\right)}\left(\delta+(1-\delta)\,\mathcal{Q}^r\left(\omega B',s'\right)\right)\\+\frac{\mathcal{D}\left(\omega B',s'\right)}{\mathcal{P}^d\left(\omega B',s'\right)}\mathcal{Q}^d\left(\omega B',s'\right)\end{array}\right)\right\}\\+(1-\vartheta)\times u_2'\frac{1}{\mathcal{P}^d\left(B',s'\right)}\mathcal{Q}^d\left(B',s'\right)\end{array}\right]}{\mathbb{E}_{s'|s}\left[\begin{array}{c}\vartheta\times\left\{u_1'\left(\frac{\mathcal{D}\left(\omega B',s'\right)}{\mathcal{P}^d\left(\omega B',s'\right)}+\frac{1-\mathcal{D}\left(\omega B',s'\right)}{\mathcal{P}^r\left(\omega B',s'\right)}\right)\right\}\\+(1-\vartheta)\times u_1'\frac{1}{\mathcal{P}^d\left(B',s'\right)}\end{array}\right]},\tag{12}$$

for the the case of default (and autarky). These bond price functions are derived by combining the demand conditions for money and bonds, (5) and (6), and using the definition of bond payoff k.

Although the government cannot borrow in periods of autarky, it can still affect the end-of-period debt position B'. To see this, recall that B' is the end-of-period debt-to-money ratio $\tilde{B}'/\tilde{M}'$. While the numerator of this ratio (the nominal debt value) is fixed to $\tilde{B}$ due to financial autarky ($\tilde{I}=0$), the denominator (the end-of-period money stock) might change and is equal to $(1+\mu)\tilde{M}$. With definition $B=\tilde{B}/\tilde{M}$, it then follows that in periods of default (and autarky)

$$B'=\frac{B}{1+\mu},$$

holds, which can be rewritten as

$$0=B'-B\times\left(\beta\mathbb{E}_{s'|s}\left[\left\{\begin{array}{c}\vartheta\times\left\{u_1'\left(\frac{\mathcal{D}\left(\omega B',s'\right)}{\mathcal{P}^d\left(\omega B',s'\right)}+\frac{1-\mathcal{D}\left(\omega B',s'\right)}{\mathcal{P}^r\left(\omega B',s'\right)}\right)\right\}\\+(1-\vartheta)\times u_1'\frac{1}{\mathcal{P}^d\left(B',s'\right)}\end{array}\right\}\times\frac{p}{u_2}\right]\right)^{-1},\tag{13}$$

by eliminating the money growth rate via condition (5) and rearranging terms.

Let $\mathbb{B}\equiv[\underline{B},\overline{B}]$ be the set of feasible aggregate debt values, with $-\infty<\underline{B}\leq 0$ and $0<\overline{B}<\infty$. Conditional on having a good credit standing ($h=0$), the decision problem of the government solves the following functional equation:

$$\mathcal{V}(B,s)=\max_{d\in\{0,1\}}\left\{(1-d)\mathcal{V}^r(B,s)+d\mathcal{V}^d(B,s)\right\},$$

with the value of repayment given as

$$\begin{aligned}\mathcal{V}^r(B,s)\quad&=\quad\max_{c_2,g,n,p,B'\in\mathbb{B}}u\left(1/p,c_2,1-n\right)+\theta v(g)+\beta\mathbb{E}_{s'|s}\left[\mathcal{V}(B',s')\right]\\&\qquad\text{s.t. }(7),(9)-(11),\end{aligned}$$

and the value of default (and autarky) as

$$\begin{aligned}\mathcal{V}^d(B,s)\quad&=\quad\max_{c_2,g,n,p,B'\in\mathbb{B}}\left\{\begin{array}{c}u\left(1/p,c_2,1-n\right)+\theta v(g)-\psi(a)\\+\beta\mathbb{E}_{s'|s}\left[\vartheta\mathcal{V}(\omega B',s')+(1-\vartheta)\,\mathcal{V}^d(B',s')\right]\end{array}\right\}\\&\qquad\text{s.t. }(8)-(10),(12),(13).\end{aligned}$$

If the government is in financial autarky, it solves the same problem as in the default case. When in autarky, the government will have the offer to regain access to financial markets in the subsequent period with probability ϑ. With probability $1-\vartheta$, it will not receive an offer and remain in financial autarky.

2.5 Equilibrium

The Markov-perfect equilibrium is defined as follows:

Definition 1 *A stationary Markov-perfect equilibrium consists of two sets of functions* $\{\mathcal{D},\mathcal{B}^r,\mathcal{C}_2^r,\mathcal{G}^r,\mathcal{N}^r,\mathcal{P}^r,\mathcal{Q}^r,\mathcal{V},\mathcal{V}^r\}:\mathbb{B}\times\mathbb{S}\rightarrow\{0,1\}\times\mathbb{B}\times\mathbb{R}_+^5\times\mathbb{R}^2$ *and* $\{\mathcal{B}^d,\mathcal{C}_2^d,\mathcal{G}^d,\mathcal{N}^d,\mathcal{P}^d,\mathcal{Q}^d,\mathcal{V}^d\}:\mathbb{B}\times\mathbb{S}\rightarrow\mathbb{B}\times\mathbb{R}_+^5\times\mathbb{R}$*, such that for all* $(B,s)\in\mathbb{B}\times\mathbb{S}$*:*

$$\mathcal{D}(B,s)=\underset{d\in\{0,1\}}{\arg\max}\left\{(1-d)\mathcal{V}^r(B,s)+d\mathcal{V}^d(B,s)\right\},$$

$$\begin{aligned}\{\mathcal{X}^r(B,s)\}_{\mathcal{X}\in\{\mathcal{C}_2,\mathcal{G},\mathcal{N},\mathcal{P},\mathcal{B}\}} \quad &= \quad \underset{c_2,g,n,p,B'\in\mathbb{B}}{\arg\max}\; u(1/p,c_2,1-n)+\theta v(g)+\beta\mathbb{E}_{s'|s}\left[\mathcal{V}(B',s')\right] \\ & \quad\quad s.t.\;(7),(9)-(11),\end{aligned}$$

$$\begin{aligned}\left\{\mathcal{X}^d(B,s)\right\}_{\mathcal{X}\in\{\mathcal{C}_2,\mathcal{G},\mathcal{N},\mathcal{P},\mathcal{B}\}} \quad &= \quad \underset{c_2,g,n,p,B'\in\mathbb{B}}{\arg\max}\left\{\begin{array}{c} u(1/p,c_2,1-n)+\theta v(g)-\psi(a) \\ +\beta\mathbb{E}_{s'|s}\left[\vartheta\mathcal{V}(\omega B',s')+(1-\vartheta)\mathcal{V}^d(B',s')\right]\end{array}\right\} \\ & \quad\quad s.t.\;(8)-(10),(12),(13),\end{aligned}$$

as well as

$$\mathcal{V}(B,s)=(1-\mathcal{D}(B,s))\times\mathcal{V}^r(B,s)+\mathcal{D}(B,s)\times\mathcal{V}^d(B,s),$$

$$\mathcal{V}^r(B,s)=u(\mathcal{P}^r(B,s)^{-1},\mathcal{C}_2^r(B,s),1-\mathcal{N}^r(B,s))+\theta v(\mathcal{G}^r(B,s))+\beta\mathbb{E}_{s'|s}\left[\mathcal{V}(\mathcal{B}^r(B,s),s')\right],$$

$$\begin{aligned}\mathcal{V}^d(B,s) \quad = \quad & u(\mathcal{P}^d(B,s)^{-1},\mathcal{C}_2^d(B,s),1-\mathcal{N}^d(B,s))+\theta v(\mathcal{G}^d(B,s))-\psi(a) \\ & +\beta\mathbb{E}_{s'|s}\left[\vartheta\mathcal{V}(\omega\mathcal{B}^d(B,s),s')+(1-\vartheta)\mathcal{V}^d(\mathcal{B}^d(B,s),s')\right],\end{aligned}$$

and

$$\begin{aligned}\mathcal{Q}^r(B,s) &= q^r(\mathcal{B}^r(B,s),s), \\ \mathcal{Q}^d(B,s) &= q^d(\mathcal{B}^d(B,s),s).\end{aligned}$$

The equilibrium definition highlights the stationarity of the policy problem as the functions that solve the decision problem of the government in a given period coincide with the policy functions that govern the optimal decisions of the government in future periods.

2.6 Discussion

Before moving to the quantitative model analysis, it is helpful to first discuss some key model features and how they affect public policy. As discussed in detail by Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008) and Martin (2009, 2011, 2013), the government's borrowing behavior will crucially depend on how its budget is affected by marginal changes in end-of-period debt. If the government borrows more, it receives additional revenues from bond issuance, which allows it to reduce tax distortions or increase public spending (see Barro, 1979). However, since the policy maker optimizes from period to period, he lacks commitment to future policies and higher end-of-period debt will change expectations of future allocations and prices. These changes in turn feed back into the government budget of the current period via the bond price, affecting marginal bond revenues today. In a monetary economy, the government also raises revenues from money issuance, as given by the first term on the LHS of (7). These revenues reflect the households' money demand, which, like the demand for bonds, depends on expectations of future outcomes, in particular the price index. Intuitively, how households value money today depends on how much they expect it to be worth in the future, which depends on future debt as it controls the government's future policy incentives. The households' demand for money depends especially on the extend to which they expect to be cash constrained. Ceteris paribus, a higher valuation of money in the current period implies that the government has to issue more money to implement a particular price index, resulting in higher revenues from money issuance and a relaxation of the government budget.

In the long run, the sign and size of the debt position crucially depend on how household money demand and hence the government's money revenues change with B' (see Diaz-Gimenez, Giovannetti, Marimon, and Teles, 2008; Martin, 2009, 2011, 2013). More specifically, an empirically plausible positive long-run debt position requires that money revenues increase when more debt is issued. This positive marginal effect is needed to counteract the simultaneous decline in marginal revenues caused by a lower bond price, reflecting an increase in expected inflation, bond price risk and default risk.[22] Whether this property holds in the model depends on household preferences since they govern the demand for money and therefore how money revenues change with B'. For the quantitative analysis, I ensure that the model exhibits this property by using a specification of the utility function $u(\cdot)$ that generates an empirically plausible (positive) average debt position for the government (see Martin, 2009).

If the government has the option to default on debt payments, the "money-demand channel" not only matters for borrowing. It also implies an endogenous default cost channel for monetary economies that is novel to the literature. In contrast to bond revenues, money revenues also enter the government budget constraint in periods of autarky (see the first term on the LHS of (8)). When the government decides whether to repay or default, it therefore takes into account how a default affects money revenues by changing expectations of future policies. These expectations crucially depend on the evolution of public debt after default, which is governed by the debt restructuring process. In

[22] Looking at Markov-perfect public policy in a real economy setting with endogenous government spending and without default, Debortoli and Nunes (2013) show - for analytical and quantitative examples - that long-run debt only deviates from zero for a small range of empirically implausible parameter values. Similar results are found by Krusell, Martin, and Rios-Rull (2005) for a related model with exogenous government spending.

financial autarky, public debt service is suspended and there is no incentive for the government to use inflation to reduce the real debt burden. Furthermore, when the government finally exits financial autarky, it will do so with a lower debt position as long as $\omega < 1$, which also reduces its incentive to use inflation. As a result, a default leads to a drop in expected inflation, which directly affects how much money the government needs to issue to implement a particular price index. The sign and size of the associated change in money revenues again depends on how the households' valuation of money changes with B'. Importantly, a model specification that implies positive long-run debt also implies a positive "money cost of default". In contrast to the utility cost $\psi(\cdot)$, the endogenous cost varies with the amount of defaulted debt if $\omega > 0$. It is however also a function of $s = (a, \theta)$ since these values are used to form expectations. Finally, note that the exogenous default costs directly influence the endogenous cost by affecting the amount of time spent in autarky as well as the size of the final haircut.

3 Quantitative Analysis

In this section, the role of sovereign default for public policy is investigated. Because the model cannot be solved analytically due to the discrete default option, numerical methods are applied.[23] The next section presents the model specification. Simulation results are presented and discussed in Section 3.2.

3.1 Model Specification

To explore the model properties by computational means, functional forms and parameters need to be chosen.

Functional Forms Productivity follows a log-normal AR(1)-process,

$$a_t = a_{t-1}^{\rho_a} \exp\left(\sigma_a \varepsilon_{a,t}\right), \ \varepsilon_{a,t} \overset{i.i.d.}{\sim} N(0,1).$$

The support and transition probabilities of the spending shock θ_t, Θ and $\Pr(\theta_{t+1}|\theta_t)$, are obtained by discretizing the process

$$\theta_t = \theta_{t-1}^{\rho_\theta} \exp\left(\sigma_\theta \varepsilon_{\theta,t}\right), \ \varepsilon_{\theta,t} \overset{i.i.d.}{\sim} N(0,1),$$

via the Rouwenhorst method as proposed by Kopecky and Suen (2010). To reduce the computational burden, the process is discretized into a two-state process with $\Theta = \left\{\theta_{low}, \theta_{high}\right\}$. The results are however robust with respect to the number of θ-states. The shocks $\varepsilon_{a,t+i}$ and $\varepsilon_{\theta,t+j}$ are uncorrelated for all $i, j \in \mathbb{N}$.

The household utility function is specified as

$$U(c_1, c_2, l, g, \theta) = \gamma_1 \frac{c_1^{1-\sigma_1} - 1}{1-\sigma_1} + \gamma_2 \frac{c_2^{1-\sigma_2} - 1}{1-\sigma_2} + \gamma_l \frac{l^{1-\sigma_l} - 1}{1-\sigma_l} + \theta \gamma_g \frac{g^{1-\sigma_g} - 1}{1-\sigma_g},$$

[23] Appendix A.2 contains details regarding the numerical computation of the equilibrium.

with $\gamma_1, \gamma_2, \gamma_l, \sigma_i > 0$, $i \in \{1, 2, g, l\}$, and $\gamma_g = 1 - \gamma_1 - \gamma_2 - \gamma_l$.[24]

Following Bianchi, Hatchondo, and Martinez (2018), the utility loss of default is specified as

$$\psi(a) = \max\{0, \psi_0 + \psi_1 \ln a\},$$

with $\psi_0, \psi_1 \geq 0$.

This specification implies that it is overproportionally more costly to default in a boom than in a recession. In the quantitative sovereign default literature, it is well known that this feature is crucial for countercyclical sovereign risk to emerge (see e.g. Aguiar and Amador, 2014), which is a property that is consistent with empirical evidence (see Tomz and Wright, 2007). Furthermore, this property also implies that default risk, and hence the bond price, does not rapidly change in response to an increase in debt, such that the government will accumulate debt levels that give rise to non-negligible default risk (see Aguiar and Gopinath, 2006, for details.)

Parameters A model period corresponds to one year. The selected model parameters are listed in Table 1. As in Martin (2009, 2013), they are chosen to replicate certain short- or long-run properties of the US economy for the time period 1962-2006. For the baseline calibration, I abstract from fluctuations in the value of the public good and set $\theta = 1$. Following Martin (2013), the parameters for the productivity process are set to match the autocorrelation and standard deviation of US log real GDP, resulting in the values $(\rho_a, \sigma_a) = (0.72, 0.0252)$. Targeting an empirically plausible average debt maturity of four years, the model parameter δ is set to 0.25. The discount factor β is set to 0.96 to match an annual real risk-free rate of 4%. The parameters γ_1, γ_2 and γ_l are chosen to yield an average cash-credit good ratio of 0.37, an average public spending-to-GDP ratio of 18% and an average working time of 0.3, respectively (see Martin, 2009). For the leisure elasticity parameter σ_n, I choose a rather standard value of 3. The parameter σ_g is set to 2.

The chosen utility function implies that the parameter σ_1, which governs the elasticity of cash-good consumption, is crucial for the size and sign the long-run debt position. Importantly, the government only has an incentive to accumulate positive debt for $\sigma_1 > 1$ (see Diaz-Gimenez, Giovannetti, Marimon, and Teles, 2008; Martin, 2009, 2011, 2013, for details). I choose a value of $\sigma_1 = 1.822$ to match an average annual debt-to-GDP ratio of 30.80%. Targeting the US average annual inflation rate of 4.40%, the credit-good parameter σ_2 is set to 1.81. The incentive to default and hence the probability thereof critically depend on the ψ-parameters, the offer probability ϑ and the offer rate ω. The offer parameter ω is set to 0.63 as in Hatchondo, Martinez, and Sosa-Padilla (2016) to target an empirically plausible average haircut between 37% and 40% for the simulated model versions with equilibrium default (see Cruces and Trebesch, 2013). The probability of receiving an offer ϑ is set to 0.5. The results are not sensitive to the exact value used for ϑ, assuming the parameters for the utility cost of default are adjusted to keep the average default probability unchanged. Since the United States did not experience default during the considered time period, the baseline calibration assumes the utility cost of default to be sufficiently high to rule out default in equilibrium. This model version yields the same results as a model without default option and will

[24]For $\sigma_i = 1$, $i \in \{1, 2, g, l\}$, household utility is logarithmic for the respective variable.

Parameter	Description	Value
β	Discount factor	0.9600
δ	Debt maturity parameter	0.2500
γ_1	Cash-good weight	0.0256
γ_2	Credit-good weight	0.1395
γ_l	Leisure weight	0.8256
ϑ	Offer probability	0.5000
ρ_a	Persistence of productivity	0.7200
ρ_θ	Persistence of spending	0.8000
σ_1	Cash-good curvature	1.8220
σ_2	Credit-good curvature	1.8100
σ_g	Public-good curvature	2.0000
σ_l	Leisure curvature	3.0000
σ_a	Std. dev. productivity shock	0.0252
σ_θ	Std. dev. spending shock	0.0884
ω	Offer rate	0.6300

Table 1: Parameter values for baseline calibration without default

be referred to as ”baseline economy”. For the model versions with equilibrium default, I keep all model parameters from the baseline calibration except the utility cost parameters, which are set as follows. First, I set $\psi_0 = 0$ and choose ψ_1 to match an annual default probability of 1% which is line with experiences of many emerging economies ($\psi_1 = 4.84$). By raising ψ_0 and keeping all other model parameters unchanged, I can then lower the default probability and study how the incentive to default affects public policy in a transparent way. The model with equilibrium default will be referred to as ”default economy”.

For the calibration with productivity and spending shocks, I keep the parameters from the baseline calibration but set the parameters for the spending process to $(\rho_\theta, \sigma_\theta) = (0.8, 0.0884)$, targeting the autocorrelation and standard deviation of US public spending-to-GDP for the time period 1962-2006 (see Martin, 2013). As will become clear in the next section, the long-run properties of this economy with spending shocks will be quite close to that of the baseline calibration with productivity shocks only.

3.2 Results

For the model versions without spending shocks, Table 2 presents the averages of statistics calculated for 3500 simulated economies with 2500 periods each. The first 500 observations of each sample are discarded to eliminate the role of initial conditions. Output is given in logs and real terms, debt-to-GDP in terms of end-of-period debt divided by nominal GDP.

Average debt and inflation are lower for the model versions with default and both increasing with the utility cost of default. The possibility of default reduces average inflation through a direct and an indirect effect. When the government chooses to default, there is no incentive to use inflation to reduce the real debt burden anymore since there is no debt service in periods of default and financial autarky. As a result, inflation is lower in such periods on average compared to periods of repayment.

	Baseline	$\psi_0 = 0.04$	$\psi_0 = 0.02$	$\psi_0 = 0.00$
Mean				
Default probability	-	0.0034	0.0061	0.0114
Debt-to-GDP	0.3087	0.2817	0.2797	0.2626
Tax rate	0.1803	0.1822	0.1825	0.1838
Inflation rate	0.0442	0.0333	0.0315	0.0239
Haircut	-	0.3768	0.3803	0.3844
Years in autarky after a default	-	2.0525	2.0877	2.1177
Nominal yield	0.0877	0.0795	0.0811	0.0772
Standard deviation				
Output	0.0275	0.0277	0.0278	0.0278
Tax rate	0.0019	0.0031	0.0039	0.0044
Spending-to-GDP	0.0004	0.0004	0.0005	0.0005
Inflation rate	0.0240	0.0263	0.0284	0.0300
Nominal yield	0.0047	0.0137	0.0197	0.0251
Correlation with output				
Debt-to-GDP	-0.3992	0.0128	0.1645	0.2272
Tax rate	-0.9866	-0.9407	-0.9447	-0.9487
Spending-to-GDP	-0.9979	-0.9941	-0.9918	-0.9906
Inflation rate	-0.2104	-0.1175	-0.0579	-0.0113
Nominal yield	-0.6548	-0.3400	-0.3252	-0.3435

Table 2: Selected model statistics (productivity shocks only)

The role of this direct effect is however limited by the frequency of default and does not contribute much to the average inflation rate. The indirect effect of default on inflation is related to how risk of default affects the government's borrowing behavior. As can be seen in panel a) of Figure 1, the probability of default $\mathbb{E}_{s'|s}[\mathcal{D}(B',s')]$ increases with borrowing and is higher in low productivity states, reflecting the government's incentive to default in bad times.[25]

The bond price schedule $q^r(B',s)$ is depicted in panel b) of Figure 1. The presence of default risk strongly reduces the bond price in low productivity states and raises the cost of debt issuance in recessions compared to the baseline economy. This mechanism discourages the government from issuing as much debt as in an economy without default and thereby restricts the build up of public debt positions that would make higher inflation more attractive. When the cost of default is lower, its attractiveness and hence its probability increase for a given debt position. This raises the responsiveness of the bond price schedule in recessions for lower ψ_0-values, which amplifies the indirect effect outlined above and makes average debt and inflation increase with ψ_0. Less average debt also implies that the tax base of the income tax increases relative to that of inflation. Hence, the benefit of raising inflation is lower, leading to a higher average labor tax rate in the baseline economy.[26] While the accumulation of debt crucially depends on the government's ability to collect seigniorage (see Section 3.1), the average seigniorage-to-GDP ratio is rather small and of empirically plausible

[25] In economies with spending shocks, the default probability also increases in high (bad) θ-states.

[26] According to Martin (2013), the US tax revenue-to-GDP ratio was 18.2% for the time period 1962-2006, which is very close to the respective value predicted by the baseline model (18.07%).

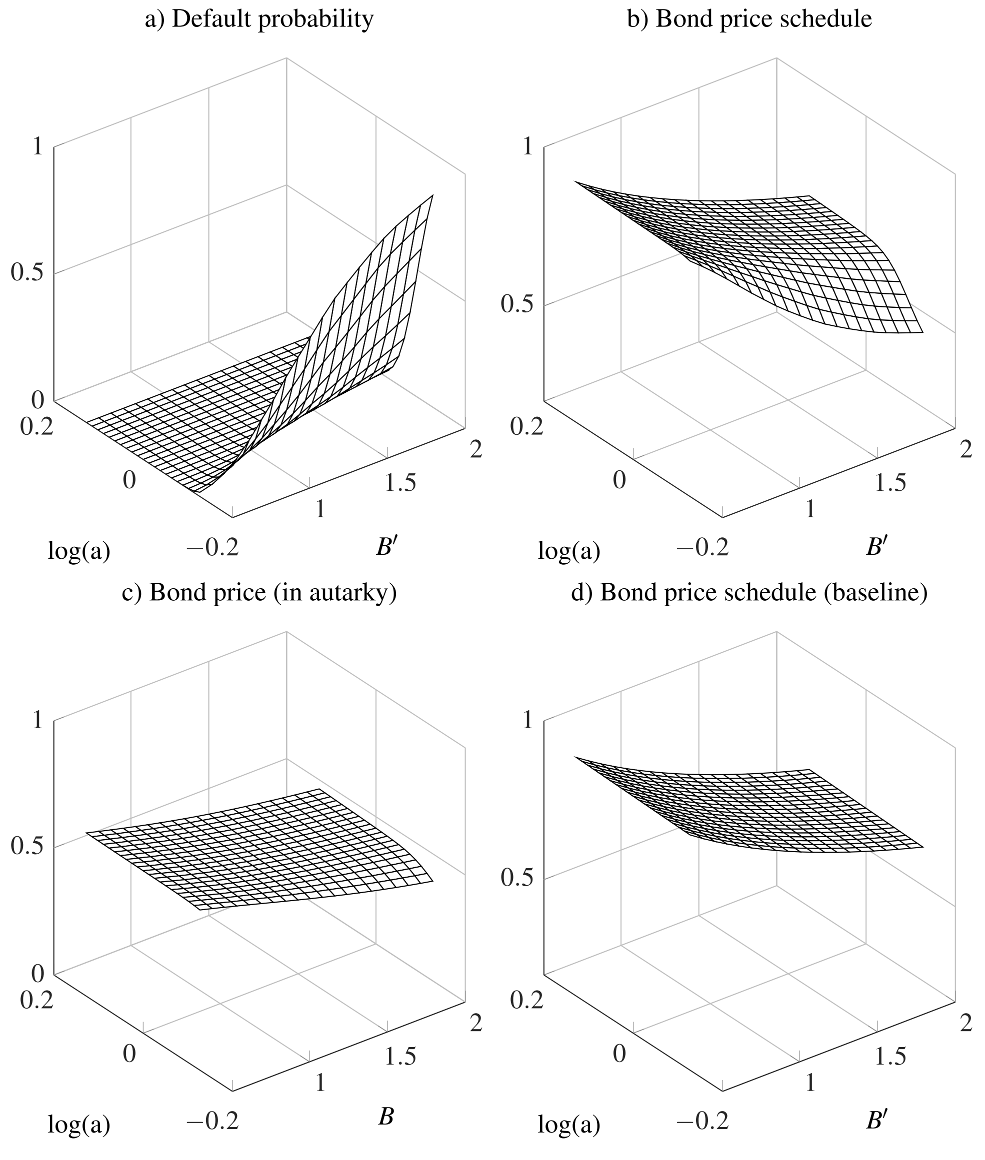

Figure 1: Default probability $\mathbb{E}_{s',|s}[\mathcal{D}(B',s')]$, bond price schedule $q^r(B',s)$ and bond price $\mathcal{Q}^d(B,s)$ for the model economy with $\psi_0 = 0.04$, as well as the bond price schedule for the baseline model (for an economy with productivity shocks only)

size (1%).[27]

While there is a clear positive relationship between ψ_0 and average debt as well as inflation, the effect that lowering the utility cost of default has on the average default frequency is not clear ex ante. Although a lower value for ψ_0 increases the incentive to default for a given debt position, it also lowers the average debt position, which might in turn reduce the incentive to default on average. However, for the simulated economies, the first effect dominates the latter, resulting in a negative relationship between the cost of default and the default frequency.

The default option also affects the cyclical behavior of the economy via its effect on borrowing conditions. Its impact on the government's borrowing behavior can be seen by looking at the cyclicality of public debt. While borrowing is countercyclical for the baseline model, it becomes (more) procyclical as ψ_0 goes down.[28] Since productivity is persistent, a negative shock to productivity raises the risk of default as the incentive to default is more likely to be strong in the subsequent period. The high debt elasticity of the bond price in low-productivity states forces the government to issue less debt in order to avoid an even larger decline of the bond price. As a result, the government has to resort to larger adjustments of inflation and taxes to finance debt payments and government spending.[29] By contrast, in the no-default economy, borrowing conditions do not deteriorate very much in response to a negative productivity shock, allowing the government to effectively smooth tax distortions across states, which translates into a lower degree of macroeconomic volatility.

The bond pricing consequences of the default option are not obvious. Following Du and Schreger (2016), the nominal yield of a bond i_t is defined as the internal rate of return that satisfies

$$q_t = \sum_{s=1}^{\infty} \frac{CF_{t+s}}{(1+i_t)^s},$$

where CF_{t+s} denotes the promised payment in period $t+s$. Due to the perpetuity structure of the bond, the nominal yield is simply given as $i_t = \delta/q_t - \delta$ (see Du and Schreger, 2016).

Despite the higher average default frequency, the average nominal yield is lower in economies with equilibrium default since these types of economies experience less inflation on average (see Table 2). The cost of default does not only affect decision making in periods of repayment but also has a direct effect on the outcome of the debt scheduling process. Since a lower value for ψ_0 implies that staying in financial autarky becomes less costly, it makes waiting for a better settlement offer more attractive for the government. As a result, the lower utility default cost parameter ψ_0 is, the higher the average haircut and the average spell in financial autarky become (see Table 2).

The Role of Spending Shocks As can be see in Table 2, the volatility of g/y is almost constant over the business cycle and therefore counterfactually low. Given that many papers in optimal policy

[27] Using the same definition of seigniorage as in the model, Aisen and Veiga (2008) calculate that average seigniorage is 0.3% of GDP for the United States.

[28] The cyclicality of debt in the absence of sovereign risk crucially depends on model parameter σ_g. As in Martin (2013), who considers $\sigma_g = 1$ in a related setting, assuming $\sigma_g \leq 1$ results in procyclical borrowing even without default risk. Furthermore, government spending becomes procyclical in this case as well.

[29] This mechanism is related to the one studied by Cuadra, Sanchez, and Sapriza (2010) in a model of a small open economy with real one-period government debt. The authors show that countercyclical default risk can rationalize the procyclical consumption taxation observed in emerging economies.

	Baseline	$\psi_0 = 0.04$	$\psi_0 = 0.02$	$\psi_0 = 0.00$
Mean				
Default probability	-	0.0041	0.0069	0.0117
Debt-to-GDP	0.3067	0.2892	0.2704	0.2614
Tax rate	0.1807	0.1820	0.1834	0.1842
Inflation rate	0.0436	0.0360	0.0282	0.0235
Haircut	-	0.3780	0.3811	0.3856
Years in autarky after a default	-	2.0727	2.0925	2.1372
Nominal yield	0.0870	0.0830	0.0775	0.0771
Standard deviation				
Output	0.0279	0.0280	0.0281	0.0282
Tax rate	0.0071	0.0080	0.0084	0.0091
Spending-to-GDP	0.0111	0.0111	0.0111	0.0111
Inflation rate	0.0292	0.0305	0.0313	0.0325
Nominal yield	0.0071	0.0224	0.0255	0.0299
Correlation with output				
Debt-to-GDP	-0.0535	0.0804	0.1307	0.2013
Tax rate	-0.1028	-0.2382	-0.2764	-0.3270
Spending-to-GDP	-0.3426	-0.3472	-0.3492	-0.3527
Inflation rate	-0.1323	-0.0543	-0.0222	-0.0221
Nominal yield	-0.0791	-0.1440	-0.1763	-0.2317

Table 3: Selected model statistics (productivity and spending shocks)

literature (see Chari and Kehoe, 1999) consider fluctuations in (the need for) public spending as a key motive for debt issuance, it is interesting to see how the economy behaves once the behavior of public spending in the model is in line with its empirical counterpart. Table 3 lists selected moments for model economies with productivity and spending shocks. While the long-run values and the volatility of the model variables are hardly affected by the addition of spending shocks, the cyclicality of the variables changes quite a lot. Similar to a low productivity shock, a high spending shock puts pressure on the government's budget, such that government has an incentive to borrow more in these states. Given that an increase in public spending crowds out private consumption and raises output through a standard wealth effect on labor supply, spending shocks make borrowing become less countercyclical relative to an economy hit by productivity shocks only. While it is more attractive to borrow in response to a high spending shock, the incentive to default is also higher in such states, such that sovereign risk increases with θ as well.

4 Welfare Implications of Sovereign Default

In this section, I discuss the welfare implications of sovereign default. With full commitment, the option to default will not decrease welfare since the government would otherwise refrain from using it.[30] Without commitment, this is not necessarily the case anymore. More specifically, as is known from the literature on unsecured consumer credit (see e.g. Chatterjee, Corbae, Nakajima,

[30] For a small open economy with non-state contingent real debt and costly sovereign default, Adam and Grill (2017) shows that welfare is increased when the policy maker can commit to a state-contingent default plan.

	$\psi_0 = 0.04$	$\psi_0 = 0.02$	$\psi_0 = 0.00$
Productivity shocks only			
Δ	0.0165	0.0141	0.0189
Δ (w/o default cost)	0.0193	0.0215	0.0349
Both types of shocks			
Δ	0.0089	0.0175	0.0187
Δ (w/o default cost)	0.0134	0.0273	0.0351

Table 4: Welfare measure Δ (in %) for different model versions

and Rios-Rull, 2007; Livshits, MacGee, and Tertilt, 2007), there exists a trade off when introducing the option to default (allowing to file for bankruptcy) in an incomplete markets environment without commitment. On the one hand, the indebted government (consumer) receives the ability to make debt payments state contingent. On the other hand, this flexibility comes at the cost of higher borrowing costs that compensate lenders for the increased risk of default and reduce the ability to smooth tax distortions (consumption) across states via debt issuance. In the model economy studied in this paper, there is an additional channel that can lead to welfare gains from having the option to default (at non-negligible costs). The model features a long-run borrowing motive that stems from the presence of two frictions, lack of commitment and a liquidity constraint (see e.g. Diaz-Gimenez, Giovannetti, Marimon, and Teles, 2008, or Martin, 2009). By limiting public debt accumulation via more sensitive bond prices, the default option reduces average inflation and the misallocation of consumption compared to the no-default setting.[31]

To evaluate whether the addition of the default option to the set of policy instruments is welfare enhancing, welfare measure Δ is calculated. It measures the increase in consumption that households in the baseline economy without default need to be given in each period to achieve the same expected lifetime utility as in an economy with default:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U(c_{1t}^{\text{base}},(1+\Delta)\,c_{2t}^{\text{base}},l_t^{\text{base}},g_t^{\text{base}},\theta_t)\right]=\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left\{U\left(c_{1t}^{\psi_0},c_{2t}^{\psi_0},l_t^{\psi_0},g_t^{\psi_0},\theta_t\right)-h_t^{\psi_0}\psi_t\right\}\right].$$

Variables in the baseline economy without equilibrium default are denoted as x_t^{base}, whereas the respective variables in a default economy with utility cost parameter ψ_0 are denoted as $x_t^{\psi_0}$, $x \in \{c_1, c_2, g, h, l\}$. Expected lifetime utility is calculated for both types of economies by averaging realized lifetime utility of 3500 samples with simulated time series of effective length $T = 2000$ each. To isolate the effect of the direct utility loss $h_t\psi_t$ on household welfare, I also compute the welfare measure Δ under the (counterfactual) assumption that households do not experience the utility cost in periods of default (and autarky).

The calculated values for the welfare measure are reported in Table 4. The main finding is that consumption c_2 needs to be increased for households in the baseline economy regardless of which default economy it is compared to and which types of shocks it is experiences, i.e. the default option

[31] Similarly, Nakajima (2012, 2017) studies the welfare effects of bankruptcy reforms when consumers exhibit an overborrowing bias due to time-inconsistent preferences, such that more restrictive borrowing conditions could enhance welfare.

is welfare enhancing. The computed Δ-values are however very small, ranging from only 0.0089% to 0.0189% of annual credit-good consumption when accounting for the utility cost of default, and from 0.0134% to 0.0351% when ignoring it. Since these welfare gains are of negligible size, one could argue that, from a welfare perspective, the model predicts that lack of commitment to repayment is not particularly important for the case of the United States. Due to the additive separable utility function $U(\cdot)$, one can directly check the sources of the welfare gains by investigating how the individual components of expected lifetime utility are affected when varying ψ_0. Doing so reveils that the welfare gains from the default option reflect the increase in cash-good consumption due to lower average inflation, which more than compensates households for the increased inflation volatility. In terms of leisure and credit-good consumption, households turn out to lose because of the default option, whereas the utillity from the public good does not change much.

5 Conclusion

To understand the implications of sovereign risk for public policy, this paper has studied optimal monetary and fiscal policy without commitment for a cash-credit economy with nominal debt and endogenous government default. When the government might default on debt payments, public policy changes in the short and the long run relative to a scenario without positive sovereign risk, induced by prohibitively high default costs. Risk of default increases the sensitivity of interest rates to borrowing, impeding the government's ability to smooth distortions across states and amplifying the impact of aggregate shocks on the economy. It also limits public debt accumulation which reduces the government's incentive to use surprise inflation on average. Although sovereign risk can have sizable quantitative implications for the behavior of public policy, it is found to only have negligible welfare consequences.

A Appendix

A.1 Derivation of the Implementability Constraint

I will only derive the implementability constraint for the repayment case. The constraint for the default case is derived similarly. First, take the household optimality conditions (1), (5)-(6) and rewrite them (in recursive notation) as

$$\begin{aligned}
\tau &= 1-\frac{u_l}{u_2}\frac{1}{a}, \\
\frac{1+\mu}{p} &= \beta \mathbb{E}_{s'|s}\left[\frac{u_1'}{u_2}\frac{1}{p'}\right], \\
\frac{(1+\mu)\,q}{p} &= \beta \mathbb{E}_{s'|s}\left[\frac{u_2'}{u_2}k'\frac{1}{p'}\right].
\end{aligned}$$

After using these expressions to eliminate the terms on the LHS of these equations in the government budget constraint

$$g-\tau an+\frac{1+(\delta+(1-\delta)\,q)B}{p}=(1+\mu)\,\frac{1+qB'}{p},$$

one obtains

$$g-an+\frac{u_l}{u_2}n+1/p+(\delta+(1-\delta)\,q)\,(B/p)=\beta \mathbb{E}_{s'|s}\left[\frac{u_1'}{u_2}\frac{1}{p'}\right]+\beta \mathbb{E}_{s'|s}\left[\frac{u_2'}{u_2}k'\frac{1}{p'}\right]B'.$$

Now, use $q=q^r\left(B',s\right)$ and eliminate an via the resource constraint $an=1/p+c_2+g$,

$$g-(1/p+c_2+g)+\frac{u_l}{u_2}n+(\delta+(1-\delta)\,q^r(B',s))\,(B/p)=\beta \mathbb{E}_{s'|s}\left[\frac{u_1'}{u_2}\frac{1}{p'}\right]+\beta \mathbb{E}_{s'|s}\left[\frac{u_2'}{u_2}k'\frac{1}{p'}\right]B'.$$

After multiplying both sides of the equation with u_2 and using the policy functions to replace next period's variables, one arrives at the implementability constraint (7).

A.2 Numerical Solution

The task of the numerical solution algorithm is to find the policy, bond price and value functions $\mathcal{X}^j(B,a,\theta)$, $\mathcal{X}\in\{\mathcal{B},\mathcal{C}_2,\mathcal{G},\mathcal{N},\mathcal{P},\mathcal{Q},\mathcal{V}\}$, $j\in\{r,d\}$.[32] Following Martin (2009) and Hatchondo, Martinez, and Sapriza (2010), I use value function iteration and approximate these functions on discrete grids for debt, productivity and the spending shock, employing Chebyshev polynomials to evaluate objects at off-grid debt values and linear interpolation for function evaluations at a-values that are not on the grid. The solution algorithm involves the following steps:

1. Construct discrete grids for debt, $\bar{\mathbb{B}}\equiv\{B_1,B_2,...,B_{N_B}\}$, productivity, $\bar{\mathbb{A}}\equiv\{a_1,a_2,...,a_{N_a}\}$ and the spending shock, $\Theta\equiv\left\{\theta_1,\theta_2,...,\theta_{N_\theta}\right\}$.

[32] The policy functions for the default and acceptance decisions can be calculated based on the value functions $\mathcal{V}^r(\cdot)$ and $\mathcal{V}^d(\cdot)$.

2. Choose initial values for the policy and value functions $\mathcal{X}^j_{start}(B,a,\theta)$, for $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{G}, \mathcal{N}, \mathcal{P}, \mathcal{Q}, \mathcal{V}\}$ and $j \in \{r,d\}$, for all $(B,a,\theta) \in \bar{\mathbb{B}} \times \bar{\mathbb{A}} \times \Theta$.

3. Set $\mathcal{X}^j_{next} = \mathcal{X}^j_{start}$, $j \in \{r,d\}$ and fix an error tolerance ε.

4. For all $(B,a,\theta) \in \bar{\mathbb{B}} \times \bar{\mathbb{A}} \times \Theta$, find the optimal policies $\mathcal{X}^j_{new}(B,a,\theta)$, $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{G}, \mathcal{N}, \mathcal{P}\}$, and the associated bond prices $\mathcal{Q}^j_{new}(B,a,\theta)$ and values $\mathcal{V}^j_{new}(B,a,\theta)$, for $j \in \{r,d\}$.

5. If $||\mathcal{X}^j_{new}(B,a,\theta) - \mathcal{X}^j_{next}(B,a,\theta)||_\infty < \varepsilon$, for $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{G}, \mathcal{N}, \mathcal{P}, \mathcal{Q}, \mathcal{V}\}$ and $j \in \{r,d\}$, go to Step 6, else set $\mathcal{X}^j_{next} = \mathcal{X}^j_{new}$, $j \in \{r,d\}$, and repeat Step 4.

6. Use $\mathcal{X}^j_{new}(\cdot)$, for $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{G}, \mathcal{N}, \mathcal{P}, \mathcal{Q}, \mathcal{V}\}$ and $j \in \{r,d\}$, as approximations of the respective equilibrium objects in the infinite-horizon economy.

The debt grid is constructed based on $N_B = 20$ Chebyshev nodes, whereas an equidistant grid with $N_A = 19$ points is used for productivity. Increasing the number of grid points for either dimension does not affect the results. As discussed in Section 3.1, the grid for the spending shock consists of two points ($N_\theta = 2$) which, together with respective transition probabilities, are obtained by discretizing a log-normal AR(1)-process via the Rouwenhorst method. For the model version without spending shocks ($N_\theta = 1$), the grid reduces to the singleton $\Theta = \{1\}$.

As is known in the literature (see e.g. Krusell, Martin, and Rios-Rull, 2005; Martin, 2009), there might be multiple Markov-perfect equilibria in models with infinitely-lived agents. In particular, there could be equilibria with discontinuous policy functions which do not arise in the infinite-horizon limit of a finite-horizon model version. To avoid such equilibria, I follow Martin (2009) and solve for the infinite-horizon limit of a finite-horizon model version.[33] In practice, this means that I compute the value and policy functions for the final period problem where no borrowing takes place and use these objects as initial values $\mathcal{X}^j_{start}$, $j \in \{r,d\}$, for Step 2. [34]

For a given state $(B,a,\theta) \in \bar{\mathbb{B}} \times \bar{\mathbb{A}} \times \Theta$, the objective of the government is the sum of two parts, flow utility $U(1/p, c_2, 1-n, g, \theta) - h \times \psi(a)$ and (in the repayment case) the continuation value $\beta \mathbb{E}_{a',\theta'|a,\theta}\left[\mathcal{V}_{next}(B',a',\theta')\right]$, with $\mathcal{V}_{next}(B,a,\theta) = \max\left\{\mathcal{V}^r_{next}(B,a,\theta), \mathcal{V}^d_{next}(B,a,\theta)\right\}$.[35] The optimal policies for Step 4 then are computed as follows. I use a sub-routine that calculates the optimal static policies c_2, g, n and p for a given state $(B,a,\theta) \in \bar{\mathbb{B}} \times \bar{\mathbb{A}} \times \Theta$ and an arbitrary, i.e. possibly off-grid, borrowing value $\hat{B}'$. More specifically, these static polices are computed by using a sequential quadratic programming algorithm (see e.g. Nocedal and Wright, 1999, for details). Using the static policy sub-routine, the policies $\{c_2, g, n, p\}$ can be expressed as functions of $\hat{S} = (B,a,\theta,\hat{B}')$: $\{\hat{\mathcal{C}}^j_{2,new}(\hat{S}), \hat{\mathcal{G}}^j_{new}(\hat{S}), \hat{\mathcal{N}}^j_{new}(\hat{S}), \hat{\mathcal{P}}^j_{new}(\hat{S})\}$, $j \in \{r,d\}$. As a result, given $(B,a,\theta) \in \bar{\mathbb{B}} \times \bar{\mathbb{A}} \times \Theta$, the government objective (for the repayment case with $h = 0$) can be expressed as a function of $\hat{B}'$ as well:

$$\hat{\mathcal{V}}^r_{new}(\hat{S}) = U(\mathcal{P}^r_{new}(\hat{S})^{-1}, \hat{\mathcal{C}}^r_{2,new}(\hat{S}), 1 - \hat{\mathcal{N}}^r_{new}(\hat{S}), \hat{\mathcal{G}}^r_{new}(\hat{S}), \theta) + \beta \mathbb{E}_{a',\theta'|a,\theta}\left[\mathcal{V}_{next}(\hat{B}',a',\theta')\right].$$

[33] As pointed out by Martin (2009), using a Svensson (1985)-type beginning-of-period cash-in-advance constraint in a finite-horizon model requires a terminal money value for a monetary equilibrium to exist. Otherwise, households will not be willing to invest in money in the final period and by backward induction not in any of the previous periods. The impact of the final-period value of money vanishes over time and does not affect the final results.

[34] The final-period values of the two bond prices are set to zero for all states.

[35] The repayment case is only used as an example. The same logic applies to the default (and autarky) case.

For each discrete grid point combination $(B,a,\theta) \in \bar{\mathbb{B}} \times \bar{\mathbb{A}} \times \Theta$, the optimal debt policy $\mathcal{B}^j_{new}(B,a,\theta)$, $j \in \{r,d\}$, then is computed via a global non-linear optimizer, calling the static policy routine to calculate the objective function $\hat{\mathcal{V}}^j_{new}(\hat{S})$ for each candidate debt value $\hat{B}'$. The optimal policies $\mathcal{X}^j_{new}(B,a,\theta)$, $\mathcal{X} \in \{\mathcal{C}_2,\mathcal{G},\mathcal{N},\mathcal{P}\}$, then are found by computing $\hat{\mathcal{X}}^j_{new}(\hat{S})$, $\mathcal{X} \in \{\mathcal{C}_2,\mathcal{G},\mathcal{N},\mathcal{P}\}$ for the optimal borrowing value $\mathcal{B}^j_{new}(B,a,\theta)$. Similarly, the equilibrium bond price $\mathcal{Q}^j_{new}(B,a,\theta)$ and the value $\mathcal{V}^j_{new}(B,a,\theta)$ are obtained by evaluating $q^j_{new}(\hat{B}',a,\theta)$ and $\hat{\mathcal{V}}^j_{new}(B,a,\theta,\hat{B}')$ for the optimal debt policy, respectively. The algorithm iterates on the policy, bond and value functions until the maximum absolute difference between the function values obtained in two subsequent iterations is below $\varepsilon = 10^{-5}$ for all $(B,a,\theta) \in \bar{\mathbb{B}} \times \bar{\mathbb{A}} \times \Theta$.

To approximate expected values in an accurate way, one needs to account for the default threshold. This can be seen by looking at the expected option value of default:

$$\begin{aligned}\mathbb{E}_{a',\theta'|a,\theta}\left[\mathcal{V}_{next}(B',a',\theta')\right] \quad = \quad & \textstyle\sum_{\theta'\in\Theta} \Pr\left(\theta'|\theta\right)\left\{\int_0^{\hat{a}(B',\theta')} \mathcal{V}^d_{next}(B',a',\theta')f_a(a'|a)da'\right. \\ & \left. + \int_{\hat{a}(B',\theta')}^{\infty} \mathcal{V}^r_{next}(B',a',\theta')f_a(a'|a)da'\right\}.\end{aligned}$$

As in Hatchondo, Martinez, and Sapriza (2010), Gauss-Legendre quadrature nodes and weights are used to approximate the integrals above. The default threshold $\hat{a}(B,\theta)$ satisfies $\mathcal{V}^r_{next}(B,\hat{a}(B,\theta),\theta) - \mathcal{V}^d_{next}(B,\hat{a}(B,\theta),\theta) = 0$ and is computed via bisection method.

In quantitative models of sovereign default with long-term debt, a positive debt recovery rate can incentivize the government to maximally increase its debt in periods prior to default (see Chatterjee and Eyigungor, 2015, or Hatchondo, Martinez, and Sosa-Padilla, 2016, for details). Following Chatterjee and Eyigungor (2015), I rule out such counterfactual borrowing behavior by imposing the restriction that the probability of default is not permitted to exceed the upper bound $\iota \in [0,1]$ whenever the government issues positive amounts of debt.[36] For all model versions, I set $\iota = 0.7$, which, as in Chatterjee and Eyigungor (2015), results in a constraint that is loose enough to not bind for the model simulations.

[36] Chatterjee and Eyigungor (2015) rationalize such a restriction as an underwriting standard.

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