Limited Market Participation, Housing and Asset Prices

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Abstract

We study the impact of limited asset market participation in a model with housing and heterogeneous beliefs. Households can invest in up to three possible assets: stocks, bonds and real estate. However, only a subset of investors can invest into the stock market. Furthermore, households have different beliefs about growth rate of the aggregate economy. Following Kurz (1994) we restrict the set of beliefs to the subset of rational beliefs. Our simulation results show that with heterogeneous beliefs restricting some households from investing into the stock market increases volatility of the price-ratio while the price-dividend ratio is nearly unaffected. Additionall, increasing participation in financial markets makes financial wealth more volatile but also decreases inequality in financial wealth.

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1 Introduction

In the late 80s, most house households didn't hold stocks as part of their portfolio. However, as can be seen from Figure 1, in the past 30 years the fraction of households that hold stocks have greatly increased. And while stock market participation had a peak in 2007 in 2014 it was close to the 2007 levels.[1] In contrast, the share of households that do own their own residence has been very stable and much higher than stock ownership.[2]

One function of financial markets is to share risks across households and if only a fraction of households do hold stocks risk sharing is limited and should in turn affect financial markets. In particular, full consumption insurance (or complete markets) implies that households equalize they marginal rate of substitution across for all states and households are only exposed to aggregate risk. Or, in other words, with complete markets the results are the same as in a representative agent economy. Thus, if a large fraction of households do not trade on financial markets consumption insurance is greatly reduced which and theoretical predictions from models with incomplete markets should be different from those with complete markets. Indeed, theoretical results predicts that limited participation increase volatility of prices (Allen and Gale (1994), Chabakauri (2015) Buss, Uppal, and Vilkov (2017)), affect the equity premium (Basak and Cuoco (1998), Guvenen (2009),Guo (2004) Favilukis (2013)). Furthermore, increased financial innvation may affect market participation (Calvet, Gonzalez-Eiras, and Sodini (2004), Iachan, Nenov, and Simsek (2015)).

However, the above mentioned models maintain a one-good assumption and we extend the literature by studying a the effects of limited participation in a multi good environment. In light of the increased participation in financial and the high ownership rate in real estate we study an exchange economy

[1] Our number include also indirect owner ship via mutual funds or IRAs but these include non-equity assets and hence our number are an upper bound. However, our numbers for 2001 are consistent with Gomes and Michaelides (2007)

[2] Mankiw and Zeldes (1991) are among the first to provide evidence of limited market participation. A closely related phenomenon in international finance is the home-bias puzzle, i.e. the fact that households do not hold enough foreign assets.

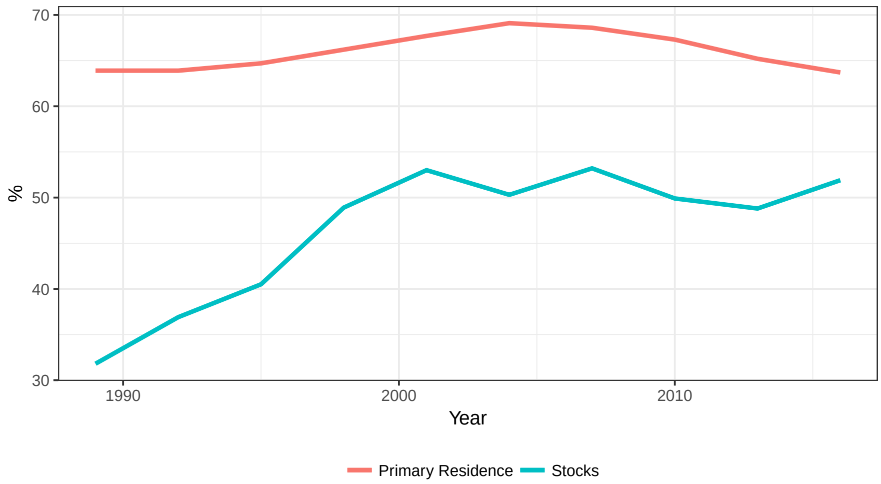

Figure 1: Fraction of households that do hold stocks (direct or indirect) and primary residence. Source: Survey of Consumer Finance

with limited participation and housing. Households can trade in up to three assets: Bonds, Stocks and Housing with trading in bonds limited by a collateral constraint on housing stock. Stocks and housing also differ in another important point. While stocks pay dividends to shareholds, housing provides housing services which affect utility of households. Thus, housing has a dual role in the economy. One as a durable consumption good, one as collateral for borrowing. To facilitate trade in the economy we assume that households have heterogeneous beliefs.[3]

There is a growing evidence that distorted beliefs are one of the drivers of housing boom-and-bust cycles (see e.g. Case, Quigley, and Shiller (2003), Piazzesi and Schneider (2009),Case, Shiller, and Thompson (2012),Kuchler and Zafar (2015)). In particular, usually households that were optimistic about future house price growths increased leverage in order to buy real estate which then itself contributed to the stepp increase in house prices.

To model beliefs we follow Kurz (1994) and Kurz and Schneider (1996) and restrict the beliefs to the set of *Rational Beliefs*. Under the principle of rational

[3]In the model with limited participation heterogeneous beliefs is not needed to facilitate trade between agents as agents have different exposure to aggregate shocks in the economy. However, in the case of full participation all agents have the same exposure to aggregate shocks and in the absence of any idiosyncratic risk there is no trade in the economy.

beliefs households agree on the empirical distribution of all macroeconomic variables. However, because of unobservable structural breaks and limited availability of data households there is no convergence of households' beliefs towards the true -not empirical- distribution of the economy. These beliefs are rational in the sense that the long-run distribution of households' beliefs has to coincide with the empirical distribution which implies that beliefs cannot be simply rejected by the data.

Because of the dual role of housing in our model, the model has two main channels which relates shocks in the housing market to other financial markets. As in Piazzesi, Schneider, and Tuzel (2007) one channel is *composition risk* arising from the non-separability of preferences over housing and non-housing consumption. The other one is a *collateral effect* as households can only borrow against collateral and in our model only housing serves as a collateral (Lustig and Van Nieuwerburgh (2005), Lustig and Van Nieuwerburgh (2006)).

Literature

Similar to stock prices, house prices also seem to be excessively volatile in the sense that volatility of house prices cannot be explained only by fundamentals (see e.g. Campbell et al. (2009) or Hott et al. (2009)). One strand of literature that tries to explain the excess volatility of house prices argues that distorted beliefs are an explanation of excess volatility (see Gelain and Lansing (2014), Piazzesi and Schneider (2009), Burnside, Eichenbaum, and Rebelo (2016),Adam, Kuang, and Marcet (2012)). The focus of these models is on house prices and they do not aim to explain other variables such as volatility of stock prices.

A different perspective is taken by Piazzesi, Schneider, and Tuzel (2007), Lustig and Van Nieuwerburgh (2005) and Lustig and Van Nieuwerburgh (2006). They try to take explain asset pricing properties by adding a durable consumption good called housing to the model. And while adding housing to the model improves the fit of the model they do not aim to explain house prices.

Favilukis, Ludvigson, and Van Nieuwerburgh (2017) study a general equilibrium model with production and heterogeneous agents and rational expectations to explain returns on housing and equity. In their model limited participation on financial markets arises from the fact that households have to pay a fixed fee in order to trade and relaxing credit constraints is the most important factor to generate large swings in house prices.

Another strand of literature studies the importance of credit constraints in determining house prices. Stein (1995) studies a static model model with downpayment requirements and concludes that distribution of debt leves may explain why boom-bust cycles are more pronounced in some cities than others. Ortalo-Magne and Rady (2006) study a life-cycle model with down payment constraints and idiosyncratic risk and argue that income volatility of 'young' households may explain some of the excess volatility of housing prices. Landvoigt (2017) studies a calibrated life-cycle model of housing demand and argues that initial equity requirements were lax prior to the boom. We extend this strand of literature by examining the joint properties of real estate and equity prices in a general equilibrium model.

The rest of the paper is structured as follows. In section 2 we lay out the model and discuss the construction of rational beliefs. In section 3 we discuss the parameterization of the economy and present the results of the simulation exercise. Section 4 concludes the paper.

2 The Model

2.1 The Setup

Consider an endowment with two consumption goods, one durable and one non-durable, in infinite horizon. Time runs from $t = 0$ to ∞. There are I types of investors:

$$i \in \mathcal{I} = \{1, 2, ..., I\}$$

in the economy. These consumers might differ in many dimensions including their preferences and their endowment of final good e^h. The consumers might also differ in their initial endowment of a real asset that pays off real dividends. However, in this paper we focus on the heterogeneity of beliefs over the evolution of the exogeneous states of the economy. There S possible exogeneous states:

$$s \in \mathcal{S} = \{1, 2, ..., S\}.$$

The state captures both aggregate uncertainty (e.g. dividends) and idiosyncratic shocks. The evolution of the economy is captured by the realizations of the shocks over time: $s^t = (s_0, s_1, ..., s_t)$. We assume that the shocks follow a markov-process with the transition probabilities $\pi(s, s')$.

Stocks: There is one real asset in the economy that pays state-dependent dividends $d(s_t)$. Agents can purchase $\theta_t^i := \theta^i(s^t)$ units of the asset. The ex-dividend price of the dividend-paying asset in history s^t is denoted by $q_t := q(s^t)$. Agents are also not allowed to short-sell the asset and total supply is normalized to 1.

We are agnostic about the reason of non-participation and instead model limited participation by exogeneously excluding agents from trading stocks.

Housing stock: In addition to investments in a dividend-paying asset, agents can purchase $h_t^i = h^i(s^t)$ units of housing stock, which can also be used as collateral for borrowing. The ex-dividend price of the asset in history s^t is denoted by $q_t^h = q^h(s^t)$. Consumers are also not allowed to short sell housing stock. Furthermore, the total supply of the housing stock is normalized to 1.

Bond: In addition to purchasing real assets, consumers can also borrow subject to collateral constraints. The agents borrow by selling $b_t^i = b^i(s^t)$ units of a one-period bond which pays one unit of the consumption good in the next period at price $p_t = p(s^t)$ and use their holdings of the real asset as collateral. In particular, we consider a collateral constraint of the following form:

$$b_t^i + (1 - m) h_t^i \min_{s^{t+1}|s^t} (q_{t+1}^h). \tag{1}$$

Here, m can be interpreted as the Loan-to-Value ratio.

Do note that we assume that households can only use housing to secure their credit and not stocks. Even though this assumption seems rather restrictive the Survey of Consumer Finances reports that in 2016 78.8% of the amount of debt were secured by real estate[4]. The large category of debt were installment loans, which is typically associated with consumer loans, with 16% of the amount of debt being installment loans. Thus, our assumption that agents can only use housing as collateral is in line with the empirical evidence that most debt is secured with real estate.

Consumers: We are now turning our attention to the consumers. First, we make the following assumptions:

Assumption 1. 1. *Each agent believes the economy is Markovian.*

2. *Each agent believes that no single agent can affect the equilibrium.*

In each state s^t, each consumer is endowed with some endowment $e_t^h = e^h(s_t)$ units of the consumption good. The aggregate endowment of the economy with the consumption is given by $\bar{e}_t = \sum_{i \in \mathcal{I}} e_t^i + d_t$ and the growth rate is denoted by $g_t = \frac{\bar{e}_t}{\bar{e}_{t-1}}$. Furthermore, we assume that they have recursive preferences as in Epstein and Zin (1989) and Weil (1990) which are the intertemporal generalization of Kreps and Porteus (1978). With recursive preferences the temporal resolution of uncertainty matters and preferences are not separable over time.

Consumers take the sequence of prices $\{p_t, q_t\}$ as given and maximize the following recursive utility function:

$$U_t^i = \left((1-\beta) \left(\tilde{c}_t^i \right)^{\frac{1-\gamma^i}{\rho^i}} + \beta E_{Q_t^i} \left[\left(U_{t+1}^i \right)^{1-\gamma^i} | \mathcal{F}_t \right]^{\frac{1}{\rho^i}} \right)^{\frac{\rho^i}{1-\gamma^i}}, \tag{2}$$

with β as the subjective discount factor, γ^i as the coefficient of relative risk aversion, and the Elasticity if Intertemporal Substitution $\psi^h \geq 0$. The parameter ρ^i is defined as $\rho^i := (1-\gamma^i)/(1-\frac{1}{\psi^i})$. And Q_t^i represents the subjective beliefs of agent h subject to the information set $\mathcal{F}_t$. Furthermore, agents get services from owning housing stock. In particular, we assume that housing

[4]In particular 69.4% were primary residence and 9.4% were other residential property

enters an agents' utility:[5]

$$\tilde{c}_t^i = \left[\omega c_t^{i\,(\epsilon-1)/\epsilon} + (1-\omega) h_t^{i\,(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)}, \tag{3}$$

with ϵ the elasiticity of intratemporal substitution between housing services and the consumption good. The specification converges to Cobb-Douglas if $\epsilon \to 1$ and the two goods are perfect complements when $\epsilon \to 0$. And α represents the relative weights of housing and consumption good in the utility function.

An investors attitude towards composition risk determined by the relation between ϵ and ρ. In the case of $\epsilon > \rho$ investors value intratemporal consumption smoothing higher than intertemporal consumption smoothing, whereas in the case of $\rho > \epsilon$ investors focus more on intertemporal consumption than intratemporal consumption smoothing.

The maximization problem of unconstrained agents is subject to the following normalized intertemporal budget constraint:

$$c_t^i + q_t \theta_t^i + q_t^h h_t^i \leq e_t^i + b_{t-1}^i + \theta_{t-1}^i (q_t + d_t) + h_{t-1}^i q_t^h \tag{4}$$

the short-sale constraints:

$$\theta_t^i \geq 0, \quad h_t^i \geq 0 \tag{5}$$

and the margin constraint

$$b_t^i + (1-m) h_t^i \min_{s^{t+1}|s^t} (q_{t+1}^h) \geq 0. \tag{6}$$

The maximization problem of constrained agents is subect to the following

[5] An alternative timing assumption would be that housing doesn't yield utility in the current period but in the next period. While it does not change the results qualitatively it has the downside that financial wealth share is not enough to describe the endogeneous state and we need to include housing stock as an additional state variable. Hence, we increase the computational burden without adding any additional insight.

normalized budget constraint:

$$c_t^i + q_t^h h_t^i \leq e_t^i + b_{t-1}^i + \theta_{t-1}^i (q_t + d_t) + h_{t-1}^i q_t^h \tag{7}$$

the short-sale constraints:

$$\theta_t^i \geq 0, \quad h_t^i \geq 0 \tag{8}$$

and the margin constraint

$$b_t^i + (1-m) h_t^i \min_{s^{t+1}|s^t} (q_{t+1}^h) \geq 0. \tag{9}$$

As we are in an economy with aggregate growth risk, it is useful to redefine the variables in terms of fraction of aggregate output. Hence, the utility function becomes

$$U_t^i = \left((1-\beta) \left(\tilde{c}_t^i \right)^{\frac{1-\gamma^i}{\rho^i}} + \beta E_{Q_t^i} \left[\left(U_{t+1}^i g_{t+1} \right)^{1-\gamma^i} | \mathcal{F}_t \right]^{\frac{1}{\rho^i}} \right)^{\frac{\rho^i}{1-\gamma^i}}. \tag{10}$$

With the corresponding budget constraints being

$$c_t^i + q_t \theta_t^i + q_t^h h_t^i \leq e_t^i + \frac{b_{t-1}^i}{g_t} + \theta_{t-1}^i (q_t + d_t) + h_{t-1}^i q_t^h \tag{11}$$

for the unconstrained agent and

$$c_t^i + q_t^h h_t^i \leq e_t^i + \frac{b_{t-1}^i}{g_t} + h_{t-1}^i q_t^h. \tag{12}$$

for the constrained agent. Additionally, it is also worthwhile to look at the first order conditions with respect to housing

$$\begin{aligned} q_t^h &= -\frac{\lambda_t^{ic}}{\lambda^{ib}} \min_{s^{t+1}|s^t} (1-m) q_{t+1}^i + \frac{1-\beta}{\lambda_t^{ib}} \left(U_t^i\right)^{\frac{\rho^i}{1-\gamma^i}-1} \left(c_t^i\right)^{\frac{1-\gamma^i}{\rho^i}-1} \frac{\partial c_t^i}{\partial h_t^i} \\ &\quad + \beta \left(U_t^i\right)^{\frac{\rho^i}{1-\gamma^i}-1} E_{Q_t^i}\left[\left(U_{t+1}^i g_{t+1}\right)^{1-\gamma^i}\right]^{\frac{1}{\rho^i}-1} \\ &\quad E_{Q_t^i}\left[\left(U_{t+1}^i g_{t+1}\right)^{-\gamma^i} \frac{\lambda_{t+1}^{ib}}{\lambda_t^{ib}} g_{t+1} q_{t+1}^h\right]. \end{aligned}$$

The first order conditions can be broken down into three parts. The first part is the collateral value of housing while the third part is the expected value for selling the house in the next period. More interesting, however, is the second part of the equation. This is the marginal value of owning an additional housing unit. Here, we interpret this term as the imputed rent for housing. Do note that the imputed rent can be different across investors.

This description of the investors optimization problem implies that demand correspondences depend on the portfolio choices made in the previous period, the state of the economy s_t, prices as well as the beliefs $\mathcal{Q}_t^i$. In particular, both types of investors have the following demand correspondence for bonds and housing:

$$h_t^i = h^i\left(h_{t-1}^i, \theta_{t-1}^i, b_{t-1}^i, p_t, q_t, q_t^h, s_t, \mathcal{Q}_t^i\right) \quad (13)$$

$$b_t^i = b^i\left(h_{t-1}^i, \theta_{t-1}^i, b_{t-1}^i, p_t, q_t, q_t^h, s_t, \mathcal{Q}_t^i\right) \quad (14)$$

For the constrained investors the demand correspondence for stocks is 0:

$$\theta_t^i = 0. \quad (15)$$

While for the unconstrained investor it is

$$\theta_t^i = \theta^i\left(h_{t-1}^i, \theta_{t-1}^i, b_{t-1}^i, p_t, q_t, q_t^h, s_t, \mathcal{Q}_t^i\right) \quad (16)$$

2.2 Equilibrium

Market clearing conditions: The market clearing conditions for our model are

1. The market for stocks clears:

$$\sum_{i \in \mathcal{I}} \theta_t^i = 1 \text{ for all } t = 1, \ldots \tag{17}$$

2. The market for housing clears:

$$\sum_{i \in \mathcal{I}} h_t^i = 1 \text{ for all } t = 1, \ldots \tag{18}$$

3. The bond market clears

$$\sum_{i \in \mathcal{I}} b_t^i = 0 \text{ for all } t = 1, \ldots \tag{19}$$

4. The market for the consumption good clears

$$\sum_{i \in \mathcal{I}} c_t^i = \sum_{i \in \mathcal{I}} e_t^i + d_t = 1 \text{ for all } t = 1, \ldots \tag{20}$$

Taken together the market clearing conditions and the demand correspondences imply that in equilibrium prices are a function of the current state of the economy s_t, the beliefs $\left(\mathcal{Q}_t^i\right)_{i \in \mathcal{I}}$ and the endogeneous state variables. In our model the endogeneous state variables are the portfolio choices made in the previous period. Thus, the equilibrium map is

$$\begin{bmatrix} q_t^h \\ q_t \\ p_t \end{bmatrix} = f\left(\left(h_{t-1}^i, \theta_{t-1}^i, b_{t-1}^i\right)_{i \in \mathcal{I}}, s_t, \left(\mathcal{Q}_t^i\right)_{i \in \mathcal{I}}\right) \tag{21}$$

2.3 The Structure of Beliefs

So far, we have taken the beliefs $\mathcal{Q}_t^h$ as given and we are now turning to the construction of rational beliefs.

Let X denote the state-space of data and observable $(s_t, p_t, q_t, (\theta_t^i, b_t^i, c_t^i)_{i\in\mathcal{I}})$ for all t and X^∞ the state space for the entire sequence. The Borel σ field generated by X^∞ will be denoted as $\mathcal{B}(X^\infty)$. The true stochastic process of the economy is described by a stochastic dynamic system $(X^\infty, \mathcal{B}(X^\infty), T, \Pi)$, where T denotes the shift-transformation[6] and Π the true probability measure.

We define now a rational belief:

Definition 1. *(Rational Beliefs) A sequence of effective beliefs $\{Q_t^i\}_{t=0}^\infty$ are a rational belief if the sequence is stable and ergodic and it induces a stationary measure that is equivalent to the one induced by the empirical measure Π.*

This definition states that rational beliefs are compatible with the empirical data[7] which makes it impossible to reject a rational belief by examining the data. However, rational beliefs still allows for mistakes as the definition does not require the belief to the true probability. It is important to note the rational beliefs principle rules out fixed (or dogmatic) beliefs, unless they believe that the empirical distribution is the true distribution. Do note that this definition of rational beliefs does not require agents to know the equilibrium map (21), instead agents deduce the relationships between variables on the observable data.

However, the definition of rational beliefs in the sense that it does tell us how we should construct rational beliefs or how agents should learn from the available data. Furthermore, a belief on X^∞ is a rather complicated object and it may prove impossible to check stability. Hence, instead modelling the learning process we pose the problem differently: Given the dynamic system $(X^\infty, \mathcal{B}(X^\infty), T, \Pi)$ we construct a sequence of effective beliefs that are rational beliefs.

To include the beliefs of the agents we follow Kurz and Schneider (1996) expand the probability space and include the sequence of generating variables $(n_t^i)_{t=1}^\infty$. Now, investor i forms a belief Q_t^h on $\left((X \times \mathcal{N}^i)^\infty, \mathcal{B}\left((X \times \mathcal{N})^\infty\right)\right)$, where $\mathcal{N}^i := \{0, 1\}$ denotes the state space of n_t^i, and $\mathcal{B}\left((X, \mathcal{N}^i)^\infty\right)$ is the

[6]The shift transformation T is defined as $x_{t+1} = Tx_t$. It is not assumed to be invertible, i.e. $T^{-1}x_{t+1} \neq x_t$, which implies that any future evolution is not associated with a unique past.

[7]A formal definition is provide in Appendix A

Borel σ-field generated by $\left(X \times \mathcal{N}^i\right)^\infty$. Now let $n^{it} := (n_1^i, n_2^i, ..., n_t^i)$, i.e. the history of generating variables n_t^i up to period t. Then, each finite history n^{ht} determines agent $i's$ effective belief in period t denoted by $Q_t^i(A) = Q^h(A|n^{it})$ for $A \in \mathcal{B}\left(X^\infty\right)$, which is a probability measure on $\left(X^\infty, \mathcal{B}\left(\Sigma^\infty\right)\right)$. The analysis is simplified by the following assumption

Assumption 2. *The marginal distribution for n_t^i with respect to Q_t^i is i.i.d. with $Q^i(n_t^h = 1) = \mu^h$.*

Assumptions 1 and 2 imply that the effective belief Q_t^i is solely determined by the generating variable n_t^i, i.e. $Q_t^i(A) = Q^i(A|n_t^i)$ for $A \in \mathcal{B}\left(\Sigma^\infty\right)$. Hence, we interpret the variable n_t^i as describing the state of belief of investor i in period t.

For example, the belief Q^i supports a regime switching model[8], then n_t^h describe the regimein which investor i believes the economy is. For instance, if $n_t^i = 1$ agent i may be optimistic about the economy while $n_t^i =$ corresponds to a pessimistic state of belief.

Furthermore, even though households switches between beliefs are i.i.d. the description above allows us to model a wide range of joint dynamics between beliefs and states. Before we state the Conditional Stability Theorem, we introduce some important notation.

Let Π_k^i denote the conditional probability of $\hat{\Pi}^i$ given a particular sequence of effective beliefs $k \in (\mathcal{N}^i)^\infty$:

$$\hat{\Pi}_k^i(\cdot) : (\mathcal{N}^i)^\infty \times \mathcal{B}(X^\infty) \mapsto [0, 1] \tag{22}$$

For each $A \in \mathfrak{B}(X^\infty)$, $\hat{\Pi}_k^h$ is a measurable function of k and for each k, $\hat{\Pi}_k^h(\cdot)$ is a probability on $(N^\infty, \mathcal{B}(X^\infty))$. For $A \in \mathcal{B}(X^\infty)$ and $B \in \mathcal{B}((\mathcal{N}^h)^\infty)$, we have

$$\hat{\Pi}^h(A \times B) = \int_{k \in B} \hat{\Pi}_k^h(A)\bar{\mu}^h(dk). \tag{23}$$

[8]Regime switching processes are popular in econometrics to model non-linearities in macroeconomic time series (e.g. Hamilton (1989))

Also, as we noted above,

$$\Pi(A) = \hat{\Pi}^h(A \times (\mathcal{N}^i)^\infty), \forall A \in \mathcal{B}(X^\infty) \tag{24}$$
$$\bar{\mu}^i(B) = \hat{\Pi}(X^\infty \times B), \forall B \in \mathcal{B}((\mathcal{N}^h)^\infty).$$

If $(\Omega^i, \mathfrak{B}^i, T, \hat{\Pi}^h)$ is a stable dynamical system with a stationary measure $m^{\hat{\Pi}^i}$, we define the two marginal measures of $m^{\hat{\Pi}^i}$ as follows:

$$m(A) := m^{\hat{\Pi}^i}(A \times (\mathcal{N}^i)^\infty), \forall A \in \mathcal{B}(X^\infty) \tag{26}$$
$$m_{Q^i}(B) := m^{\hat{\Pi}}(X^\infty \times B), \forall B \in \mathcal{B}((\mathcal{N}^i)^\infty). \tag{27}$$

Also, let $\hat{m}_k$ denote the stationary measure of $\hat{\Pi}^h_k$, which is a measure on $(X^\infty, \mathfrak{B}(X^\infty))$. Given the construction of the dynamical system, we have the following theorem:

Theorem 1. *(Conditional Stability Theorem, Kurz and Schneider (1996)).*
Let $(\Omega^h, \mathcal{B}^i, T, \hat{\Pi}^i)$ be a stable and ergodic dynamical system. Then,

1. *$(X^\infty, \mathcal{B}^\infty, T, \hat{\Pi}^i_k)$ is stable and ergodic for $\hat{\Pi}^i$ a.a. k.*
2. *$\hat{m}^i_k$ is independent of k, $m^i_k = m = \Pi$.*
3. *If $(X^\infty, \mathfrak{B}(X^\infty), T, \hat{\Pi}^i_k)$ is stationary, then the stationary measure of $\hat{\Pi}^h_k$ is Π. That is*

$$\hat{m}^i_k = m = \Pi.$$

So far, our discussion on constructing rational beliefs did assume that agents do not know the equilibrium map (21). However, to simplify the computational model we assume that agents do know the equilibrium map (21)[9]. This implies that once agents have chosen their portfolios next periods prices depend only on the state s_t and the distribution of belief $(Q^i_t)_{i\in\mathcal{I}}$. This simplifies the construction of a computational model as agents need to form beliefs only over the exogeneous variables, i.e. they form beliefs over $(s_t, (n^i_t)_{i\in\mathcal{I}})$

[9]This assumption is similar to assumptions made in the literature on bayesian learning.

Example. *To illustrate the ideas, we consider an example similar to our simulation model discussed in a later section. Consider an economy with two exogeneous states (e.g. high dividends and low dividends) and two agents. Both agents can be either optimistic in the sense that she assigns a higher probability on higher dividends then empirically observed or pessimistic in the sense that she assigns a lower probability on high dividends than empirically observed. Now, our state-space consists of* 8 *states. In particular, we have the tuple* $\{d_t, n_t^1, n_t^2\}$. *Now, in period* t *agents form beliefs not only over dividends but also over the distribution of future generating variables* $\{n_{t+1}^1, n_{t+1}^2\}$.

This implies that the sequence of effective beliefs of a household Q_t^h *must have the same stationary distribution as the tuple* $\{d_t, n_t^1, n_t^2\}_{t=0}^{\infty}$. *If the tuple has a Markov transition matrix* Γ *and the beliefs are represented by two transition matrices* F_H^h, F_L^h *the rationality condition implies that*

$$\mu^h F_H^h + (1 - \mu^h) F_L^h = \Gamma. \tag{28}$$

□

Now, using the generating variable, we can rewrite the portfolio choice (13)-(16) in terms of generating variables rather than beliefs Q_t^h:

$$h^i t = h_t^i \left(\theta_{t-1}^i, b_{t-1}^i, p_t, q_t, s_t, n_t^h \right) \tag{29}$$

$$b_t^i = \theta^h \left(\theta_{t-1}^i, b_{t-1}^i, p_t, q_t, s_t, n_t^h \right). \tag{30}$$

$$\theta_t^i = \theta^h \left(\theta_{t-1}^i, b_{t-1}^h, p_t, q_t, s_t, n_t^i \right) \text{ (unconstrained)} \tag{31}$$

$$\theta^i = 0 \text{ (constrained)} \tag{32}$$

We define the stochastic primitives y_t as follows:

$$y_t = \left(s_t, (n_t^h)_{h \in \mathcal{H}} \right) \quad \forall t. \tag{33}$$

The state space of the stochastic primitives is now $\mathcal{Y}$. We assume that the $\{y_t\}_{t=0}^{\infty}$ is a stable Markov process with a time homogeneous transition probability $P : \mathcal{Y} \to \mathcal{P}(\mathcal{Y})$, where $\mathcal{P}(\mathcal{Y})$ denotes the space of probability measures on $\mathcal{Y}$.

3 Quantitative Analysis

In this section we focus on the quantitative analysis of the model. In section 3.1 we discuss how to apply the structure for rational beliefs as outlined in the previous section into a simulation framework and the parameterization of the model. Section 3.2 discusses the results of the baseline model and in section 3.3 we discuss the role of the collateral constraint and composition risk.

3.1 The Simulation Model

We assume that there are only 2 agents in the economy, that is, $H = 2$. We also assume that there are two growth states, i.e. $g_t \in \{\bar{g}, \underline{g}\}$.

The empirical distribution $\{g_t\}$ follows a markov-process:

$$\Psi = \begin{bmatrix} \phi & 1-\phi \\ 1-\phi & \phi \end{bmatrix}. \tag{34}$$

Then, the stationary transition probability matrix has to satisfy the following conditions:

- the empirical distribution for the process g_t is specified by transition probability matrix Ψ.
- the marginal distribution for n_t^h is i.i.d with frequency of $\{n_t^h = 1\} = \alpha^h$.

Here, we use a specification similar to Kurz and Motolese (2001) as we know that the beliefs are compatible with the stationary distribution and it can generate large fluctuations. Furthermore, this specification allows for correlation between the three variables (g_t, n_t^1, n_t^2). We assume that the 8×8 matrix Γ has the following structure:

$$\Gamma = \begin{bmatrix} \phi A & (1-\phi)A \\ (1-\phi)A & \phi A \end{bmatrix} \tag{35}$$

A is a 4×4 matrice defined by 6 parameters (α^1, α^2, a) and $a = (a^1, a^2, a^3, a^4)$

as follows:

$$A = \begin{bmatrix} a^1 & \alpha^1 - a^1 & \alpha^2 - a^1 & 1 + a^1 - \alpha^1 - \alpha^2 \\ a^2 & \alpha^1 - a^2 & \alpha^2 - a^2 & 1 + a^2 - \alpha^1 - \alpha^2 \\ a^3 & \alpha^1 - a^3 & \alpha^2 - a^3 & 1 + a^3 - \alpha^1 - \alpha^2 \\ a^4 & \alpha^1 - a^4 & \alpha^2 - a^4 & 1 + a^4 - \alpha^1 - \alpha^2 \end{bmatrix} \tag{36}$$

We also have to specify the transition probability matrices that represent the beliefs of the agents. As noted above, agent $i \in \{1, 2\}$ in period t uses F_1^i when his generating variable is $n_t^1 = 1$ and F_2^i when his generating variable is $n_t^1 = 0$. The rationality of belief condition implies that

$$\alpha^h F_1^i + (1 - \alpha^h) F_2^i = \Gamma. \tag{37}$$

Thus to fully pin down a traders' belief we only have to specify F_1^i while F_2^h can be inferred from Γ and F_1^i. The matrix F_1^h is parametrized by η^i as follows:

$$F_1^i(\eta^i) = \begin{bmatrix} \phi \eta^i A & (1 - \eta^i \phi) A \\ (1 - \phi) \eta^i A & (1 - (1 - \phi) \eta^i) A \end{bmatrix} \tag{38}$$

From the above equation one can see that if $\eta^i > 1$ a trader places more weight on the growth states, i.e. he is overly optimistic that the economy grows when his beliefs are given by F_1^i. Furthermore, the larger the η^i implies a more optimistic trader. Furthermore, parameter α^i determines the frequency of optimistic beliefs, when $\alpha^i = 0.5$ then optimistic and pessimistic have the same frequency while $\alpha^i > 0.5$ implies that a trader is more often optimistic then pessimistic. This has also implications for pessimistic beliefs. In particular if $\eta^i > 1$ and $\alpha^i > 0.5$ then beliefs are more asymmetrically distributed to satisfy the rationality condition.

For the beliefs of the agents we follow Kurz and Motolese (2001) and set $(a_1, a_2, a_3, a_4) = (0.5, 0.14, 0.14, 0.14)$. Furthermore, we assume that $\alpha^1 = \alpha^2 = \alpha = 0.57$. The maximum value for η is $1/0.57 \approx 1.7$. In the baseline case we will examine two case for η. One is $\eta = 1.00$, i.e. the situation where all agents believe that the empirical distribution is the true distribution. The other case

Variable	d_t	γ	ψ	ϵ	ω	β
Value	0.15	2.0	1.5	1.25	0.90	0.96

Table 1: Parameterization of the baseline model

is $\eta = 1.60$, i.e. the case where η is close to the maximum. Do note that we assume the parameters are the same for both agents which implies that beliefs for both agents are symmetric.

Following Mehra and Prescott (1985) we consider the following transition probability matrix for Ψ:

$$\Psi = \begin{bmatrix} 0.43 & 0.57 \\ 0.57 & 0.43 \end{bmatrix}, \tag{39}$$

and set $\overline{g} = 1.054$ and $\underline{g} = 0.982$. And in line with the literature we set the dividends d_t to $d_t = 0.15$.

Our choices for preferences follow the literature. We set the time-preference parameter to $\beta = 0.96$, the coefficient of relative risk-aversion is set to $\gamma = 2.0$ which is standard in the literature. On the other hand, for the value of the EIS there is a bigger range of estiamtes. Some authors estimate a rather low value for the EIS, for example Hall (1988) estimates a value much smaller than 1, while several asset pricing models (e.g., Collin-Dufresne, Johannes, and Lochstoer (2014) or Bansal and Yaron (2004)) have used a EIS greater than 1. An EIS greater than 1 is needed to capture the negative correlation between consumption volatility and the price/dividend-ratio. For the baseline model we set the elasticity of intertemporal substitution for both agents to $\psi = 1.5$, a value which is in line with the asset pricing literature. Table 1 summarizes the parameterization of the model.

Following Kubler and Schmedders (2002) we map portfolio choices into one variable W_t, called financial wealth, i.e.

$$W_t^i = \theta_{t-1}^i(q_t + d_t) + h_{t-1}^i q_t^h + \frac{b_{t-1}^i}{g_t}. \tag{40}$$

Market clearing conditions imply that the total financial wealth in the economy

	Stock		House		Bond		Risk Premium	
η	1.00	1.60	1.00	1.60	1.00	1.60	1.00	1.60
Full Participation	0.0405	0.7989	0.0406	10.1281	0.0031	0.0136	0	0.0211
Limited Participation	0.7065	0.7958	1.6354	16.3686	0.0076	0.0114	0.015	0.0282

Table 2: Standard Deviation of stocks, house prices, bonds and risk premium under rational expectations ($\eta = 1.00$) and heterogeneous beliefs ($\eta = 1.60$)

is $q_t + d_t + q_t^h$. We define the financial wealth share of an investor i as the financial wealth of an investor divided the total financial wealth in the economy:

$$w_t^i = \frac{\theta_{t-1}^i(q_t + d_t) + h_{t-1}^i q_t^h + \frac{b_{t-1}^i}{g_t}}{q_t + d_t + q_t^h} \tag{41}$$

Because of the short-sale and the borrowing constraint, the financial wealth share is bounded between 0 and 1 and fully describes the endogeneous state.

The model is solved using a policy function iteration. The results are then obtained by a Monte Carlo simulation and the length of an individual simulation is $T = 500$ periods. The number of Simulations for each economy is $N = 5000$. Initial financial wealth is set to $\omega^1 = 0.5$ and for the initial portfolio we set $b^1 = 0$ and $h^1 = 0$. For the case of limited participation only agent 1 holds stocks while in the case of limited participation the initial stock holdings are set to $\theta^1 = 0.5$.

3.2 Results for the Baseline model

First, we consider the results for the baseline model. In particular we consider two extreme cases. For the first case we set $\eta = 1.00$ for both investors. Hence, both investors believe that the empirical distribution is the true underlying distribution and there are no differences in beliefs with both agents. We refer to this case as the case of 'rational expectations'. In the other case, we set $\eta = 1.6$ which is close to the maximum feasible value. In this case investors don't believe that the empirical distribution is the correct distribution and there is now considerable disagreement among investors.

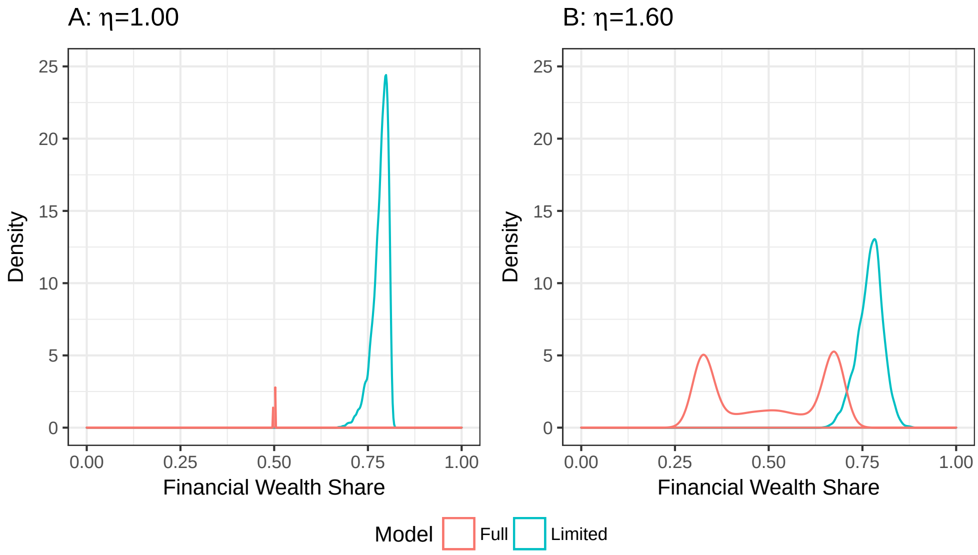

Figure 2: Distribution of financial wealth after 500 years in the simulated economy for economies with limited market participation and full market participation. Panel A displays the case of homogeneous expectations and panel B shows the case of heterogeneous expectations.

Tabel 2 displays the simulation results for these two cases. In general, volatility is higher under heterogeneous beliefs than under rational expectations because of the speculative motive to trade.

Under rational expectations moving from limited participation to full participation reduces the volatility of the Price-Dividend ratio, the Price-rent ratio and the bond price volatility. This result is in line with the literature.

However, under heterogeneous beliefs the picture is less clear. While the volatility of the Price-Dividend ratio increases only by 0.4% the volatility of the Price-rent ratio drops significantly by about 70%. The direction of the change in volatility of the Price-Dividend ratio is consistent with the one-good model by Buss, Uppal, and Vilkov (2017) the magnitude of the change is much smaller in our model.

To understand these results we will now look more closely at the behavior of the endogeneous state variables ω_t^1.

Figure 2 shows the wealth distribution across the two agents after 500 years. Under rational expectations ($\eta = 1.00$) and Full participation the wealth dis-

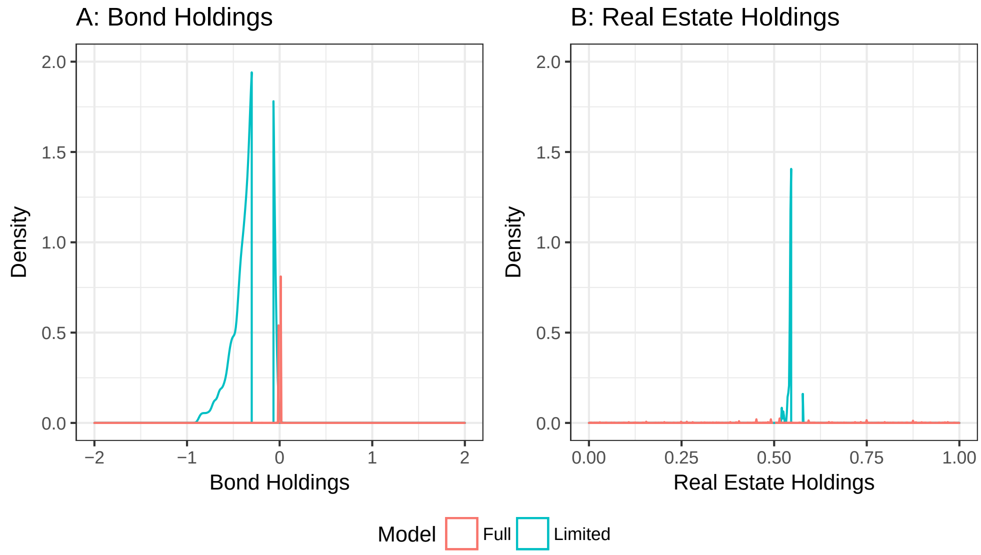

Figure 3: Distribution of bond holdings and real estate holdings after 500 years in economies with limited and full participation and homogeneous beliefs. Panel A shows bond holdings while panel B shows real estate holdings.

tribution is concentrated on 0.5 which is not really surprising as there are no other shocks in the economy than aggregate growth shocks. As both agents have the same exposure to these growth shocks there is no need for risk sharing in the economy and hence wealth does not change over time. Moving on to the case of limited participation we see that financial wealth is now unequally distributed across agents. In particular financial wealth is concentrated at $\omega^1 = 0.75$ as agent 1 holds all of the risky asset. In addition distribution of financial wealth is slightly spread out, because with limited participation both agents have now different exposures to aggregate shocks. As agent 1 holds an asset with risky dividends a need for risk sharing arises and households start to trade.

Under heterogeneous beliefs and limited participation we have a similar result as before, i.e. a single peak close to $\omega^1 = 0.75$. Yet under heterogeneous beliefs the wealth distribution is spread out a bit further, because in addition to the sharing risks of aggregate shocks there is also a speculative motive due to differences in beliefs.

With full participation and heterogeneous beliefs we have a bimodal distri-

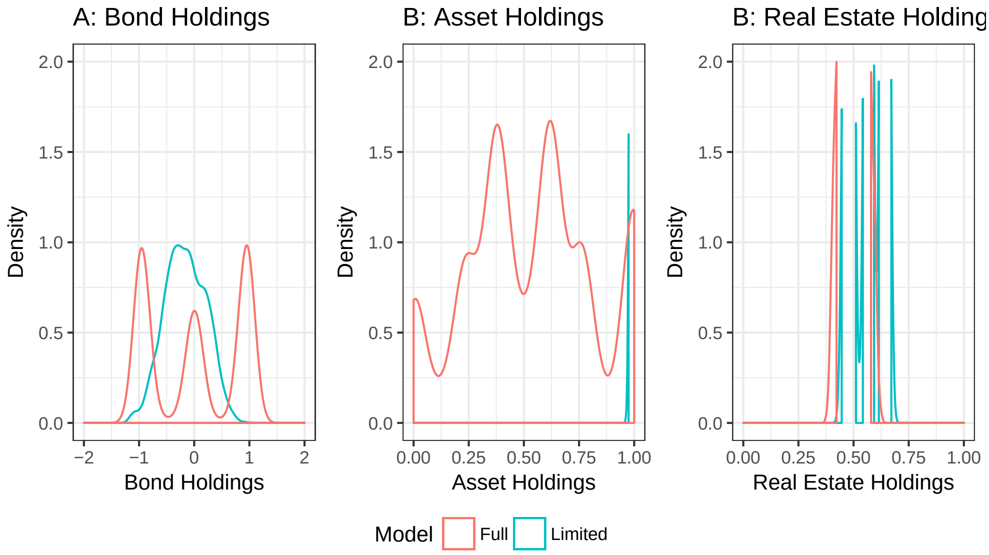

Figure 4: Distribution of bond holdings, stock holdings and real estate holdings after 500 years in economies with limited and full participation and heterogeneous beliefs. Panel A shows bond holdings, panel B shows stock holdings and Panel C shows real estate holdings.

bution of financial wealth share and the wealth share is distributed symmetrically around the median of $\omega^1 = 0.5$ as beliefs are symmetric. And as investors can trade in all three assets the distribution becomes now much wider as before, yet because investors hold housing financial wealth doesn't drop below $\omega^1 = 0.25$ or rises above $\omega^1 = 0.75$.

Figures 3 and 4 show the equilibrium portfolio choices for homogeneous and heterogeneous beliefs. Under homogeneous beliefs and full participation there is no trade in the economy while with limited participation there is some trade in the economy. However, trade in the housing market is very limited and most of the trade happens in the bond market. In addition to intertemporal consumption smoothing, households also have a desire for intratemporal consumption smoothing between the housing and the non-housing. In the baseline case, investors are more willing to substitute overall consumption over time then he is willing to substitute housing with non-housing consumption. Hence, investors trade in the bond market and not the housing market to smooth consumption intertemporally.

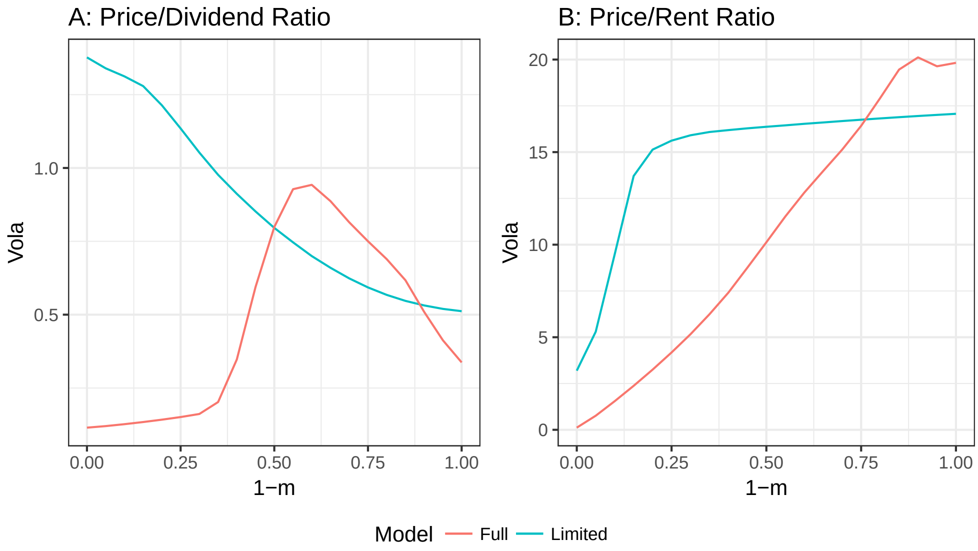

Figure 5: Volatility of Price-Dividend and Price-Rent Ratio as a function of $(1 - m)$

Heterogeneous beliefs add a speculative motif to trading on financial markets which increases the trading volume. However, whether it is limited participation or full partipation trading volume on the housing market is rather small.

3.3 Comparative statics

In this subsection we explore the explanatory contribution to volatility of the three main features of the model: the credit constraint, attitude towards composition risk and heterogeneous beliefs.

3.3.1 The role of credit constraints

Relaxed lending standards have been pointed out as one source of the recent boom-bust cycle in house prices. However, in the literature on asset pricing models with credit constraints whether volatility increases or decreases after relaxing credit constraint depends on agents having homogeneous or heterogeneous beliefs. With homogeneous beliefs a risk-sharing motive for trade dominates the result and relaxing credit standards yields a lower volatility

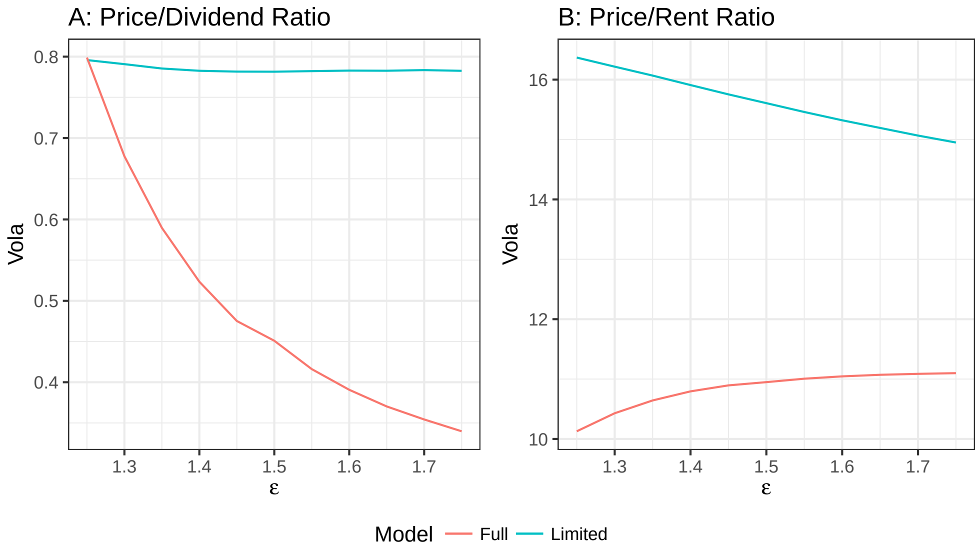

Figure 6: Volatility of Price-Dividend and Price-Rent Ratio as a function of ϵ

(Brumm et al. (2015)) while with heterogeneous beliefs a speculative motive dominates and volatiliy increases (Buss et al. (2016), Cao (2017)) and welfare decreases (Nakata (2013)).

To study the impact of credit constraints we focus on the baseline model with heterogeneous expectations and vary the collateral constraint $(1-m)$.

As can be seen from Figure 5 we see that only the volatility of the price/rent-ratio increases if the collateral constraint is relaxed. Additionally, the volatility of the price/rent ratio under full participation is lower under than under limited participation until $(1-m) = 0.75$. The volatility of the price/dividend.ratio decreases under limited participation yet it increases under full participation until it reaches a maximum and then declines. Or, in other words to create a sharp increase in the volatility of the price/rent ratio not only relaxed credit constraints but we also need a stronger participation in equity markets.

3.3.2 The role of composition risk

We are now studying the impact of ϵ on prices. Recall that ϵ and ψ together determine the attitude towards composition risk. In particular, if intratemporal elasticity is smaller than intertemporal elasticity (i.e. $\epsilon < \psi$) then agents are

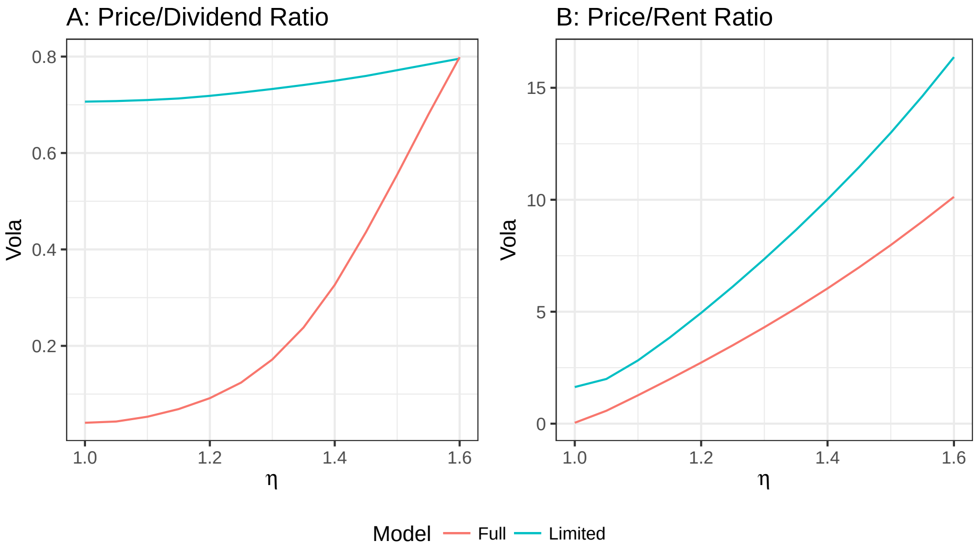

Figure 7: Volatility of Price-Dividend and Price-Rent Ratio as a function of η

more willing to substitute consumption bundles at different points in time then substitute consumption and housing within a period. Hence, housing is valued highly if $\epsilon < \psi$ while consumption of the non-durable consumption good is valued highly if $\epsilon > \psi$.

Figure 6 shows the volatility of the price/dividend ratio and price/rent ratio when we increase ϵ and $\eta = 1.6$. If we increase ϵ households are less willing to substitute consumption over time and more willing to substitute housing and other consumption. Hence, households start to trade in real estate to smooth consumption over time and therefore the price/dividend ratio declines while the price/rent ratio increases for the case of full participation.

3.3.3 The role of beliefs

Figure 7 shows the impact on volatility we change the parameter η. Unsurprisingly, volatility increases if beliefs become more heterogeneous as speculative trading increases.

4 Conclusion

This paper studied a model of limited market participation with heterogeneous beliefs and two consumption goods. Agents were able to trade in up to three assets: Bonds, Stocks and Real estate. Our simulation results showed that in our baseline model full participation reduced volatility of stocks and real estate which is at odds with recent models that study limited participation in economies heterogeneous beliefs and more in line with models of limited participation and homogeneous beliefs.

Additionally, we also studied how relaxing credit constraint affects volatility and we find that the price/rent ratio always increases with the relaxed credit constraints. Furthermore, if credit constraints are loose enough volatility under full participation is increased. Thus indicating that the sharp increase in house prices was not only caused by relaxing credit standards but also by an increased participation in financial markets.

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A Definitions

For the definition of stability and ergodicity use the definitions from Kurz (1994).

Let Ω denote a sample space, $\mathcal{F}$ a σ-field of subsets of Ω, T the shift transformation such that $T(x_t, x_{t+1}, x_{t+2}, ...)$ and Π a probability measure. Define now

$$1_S(x) = \begin{cases} 1 & \text{if x} \in S \\ 0 & \text{if x} \notin S \end{cases} . \tag{42}$$

The relative frequency of the set S visited by the dynamical system given that it start at x as follows

$$m^n(S)(x) = \frac{1}{n} \sum_{k=0}^{n-1} 1_S \left(T^k x \right) . \tag{43}$$

Then we define stability and ergodicity as follows

Definition 2 (Stability)**.** *A dynamical system* $(\Omega, \mathcal{F}, T\Pi)$ *is said to be stochastically stable if for all cylinders* $Z \in \mathcal{F}$ *the limit of* m^n *exist* Π *a.e., and the limit is denoted by*

$$\tilde{m}(S)(x) = \lim_{n \to \infty} m^n(S)(x). \tag{44}$$

Definition 3 (Invariance)**.** $S \in \mathcal{F}$ *is said to be invariant with respect to T if* $T^{-1}S = S$*. A measurable function is said to be invariant with respect to T if for any* $x \in \Omega$, $f(T(x)) = f(x)$.

Definition 4 (Ergodicity)**.** *A dynamical system is said to be ergodic if* $\Pi(S) = 0$ *or* $\Pi(S) = 1$ *for all invariant sets S.*

Definition 5 (Compatibility with the Data)**.** *We say that a probability* $Q \in \mathcal{P}(\Omega)$ *is compatible with the data if*

(a) $(\Omega, \mathcal{F}, Q, T)$ *is stable with a stationary measure m. That is, for all cylinders*

$S \in \mathcal{F}$

$$m_Q(S) \stackrel{d}{=} \lim_{n\infty} \frac{1}{n} \sum_{k=0}^{n-1} Q(T^{-k}S) = m(S)$$

(b) Q satisfies the tightness condition Π.

B Derivation of the first order conditions

For ease of notation, we drop the reference to a household h. The maximization problem of the agent can be written as the following Lagrangian:

$$\begin{aligned} \mathcal{L} &= \left((1-\beta)\,(c_t)^{\frac{1-\gamma}{\rho}} + \beta[E_{Q_t}\left[(U_{t+1}g_{t+1})^{1-\gamma}\,|\mathcal{F}_t\right]^{\frac{1}{\rho}}\right)^{\frac{\rho}{1-\gamma}} \\ &\quad -\mu_t^b\left(c_t + \theta_t q_t + b_t p_t - \theta_{t-1}(q_t + d_t) - \frac{b_{t-1}}{g_t} - e_t\right) - \mu_t^{sh} h_t \mu_t^s s_t \\ &\quad -\mu_t^c\left(b_t + (1-m)h_t \min_{s^{t+1}|s^t}(q_{t+1}^h)\right). \end{aligned} \tag{45}$$

The lagrange multiplier with respect to the budget constraint is denoted byμ_t^b, for the short-sale constraint on housing μ_t^{sh}, the short-sale constraint on stock μ_t^s and for the collateral constraint μ_t^c.

Taking now the derivative with respect to consumption and rearranging yields

$$\frac{\partial \mathcal{L}}{\partial c_t} = (U_t)^{\psi^{-1}}\, c_t^{-\psi^{-1}} \frac{\partial c_t}{\partial \tilde{c}_t} = \mu_t^c. \tag{46}$$

The derivative with respect to asset purchases is

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \theta_t} &= (U_t)^{\psi^{-1}}\, \beta E_{Q_t}\left[(U_{t+1}g_{t+1})^{1-\gamma}\right]^{\frac{1-\rho}{\rho}} E_{Q_t^h}\left[(U_{t+1}g_{t+1})^{-\gamma}\, g_{t+1}\frac{\partial U_{t+1}}{\partial s_t}\right] \\ &\quad -\mu_t^b q_t - \mu_t^s, \end{aligned} \tag{47}$$

and because of the envelope theorem the derivative of U_{t+1} with respect to

s_t is given by

$$\frac{\partial U_{t+1}}{\partial s_t} = \frac{\partial U_{t+1}}{\partial c_{t+1}} \frac{\partial c_t}{\partial \tilde{c}_t} \frac{\partial \tilde{c}_{t+1}}{\partial s_t} = (U_{t+1})^{\psi^{-1}} (1-\beta)(c^h_{t+1})^{-\psi^{-1}} \frac{\partial c_{t+1}}{\partial \tilde{c}_{t+1}} (q^s_{t+1} + d_{t+1}). \tag{48}$$

Combining the last two equations we get

$$\begin{aligned} q_t \mu^b_t &= (U_t)^{\psi^{-1}} \beta E_{Q_t} \left[(U_{t+1} g_{t+1})^{1-\gamma} \right]^{\frac{1-\rho}{\rho}} && (49) \\ & E_{Q_t} \left[(U_{t+1})^{\psi^{-1}-\gamma} g^{1-\gamma}_{t+1} (1-\beta) \left(c^h_{t+1} \right)^{-\psi^{-1}} (q_{t+1} + d_{t+1}) \right] && (50) \\ & + \mu^s + \mu^c (1-m) \min_{s^{t+1}|s^t} (q_{t+1} + d_{t+1}). \end{aligned}$$

The first order conditions for bond holdings can be derived similarly, i.e.

$$\begin{aligned} p_t \mu^b_t &= (U_t)^{\psi^{-1}} \beta E_{Q_t} \left[(U_{t+1} g_{t+1})^{1-\gamma} \right]^{\frac{1-\rho}{\rho}} && (51) \\ & E_{Q_t} \left[(U_{t+1})^{\psi^{-1}-\gamma} g^{-\gamma}_{t+1} (1-\beta) \left(c^h_{t+1} \right)^{-\psi^{-1}} \right] && (52) \\ & + \mu^c. \end{aligned}$$

For the holdings on the housing stock a similar calculation yields

$$\begin{aligned} \mu^b_t q^h_t &= -\mu^c_t \min_{s^{t+1}|s^t} + \left(U^i_t \right)^{\frac{\rho^i}{1-\gamma^i}-1} (1-\beta) \left(c^i_t \right)^{\frac{1-\gamma^i}{\rho^i}-1} \frac{\partial c^i_t}{\partial h^i_t} && (53) \\ & + \left(U^i_t \right)^{\frac{\rho^i}{1-\gamma^i}-1} \beta E_{Q^i_t} \left[\left(U^i_{t+1} g_{t+1} \right)^{1-\gamma^i} \right]^{\frac{1}{\rho^i}-1} E_{Q^i_t} \left[\left(U^i_{t+1} g_{t+1} \right)^{-\gamma^i} g_{t+1} \mu^b_{t+1} q^h_{t+1} \right] && (54) \end{aligned}$$

C Numerical Algorithm

To solve for the stationary equilibrium we use a time-iteration algorithm. The algorithm proceeds as follows:

Step 0: Set an error-tolerance ϵ and form a grid M ove $[0,1]$, Set an initial guess f^0 for policy and price functions.

Step 1: Given a set of policy and price functions f^{n-1}, we obtain a new set of policies and prices f^n by solving the system of equilibrium conditions and the law of motion for the wealth share for each gridpoint $(\omega, s) \in M \times \mathcal{S}$. Do note that the short-sale constraint as well as the margin requirement are not always binding, hence they are not differentiable and thus the system of equations is not differentiable. To circumvent the problem we use the Garcia-Zangwill trick (**zangwill1981pathways**) and replace the lagrange multiplier μ^{hs} and μ^{hc} with $\mu^{hs+} = \max\{0, \mu^{hs}\}^2$, $\mu^{hs-} = \max\{0, -\mu^{hs}\}^2$, $\mu^{hc+} = \max\{0, \mu^{hc}\}^2$, $\mu^{hc-} = \max\{0, -\mu^{hc}\}^2$. Thus, the system of equations is now as follows:

$$
\begin{aligned}
q_n (U^h)^{\psi^{-1}} (c_n^h)^{-\psi^{-1}} &= (U_n^h)^{\psi^{-1}} \beta E_Q^h \left[\left(U_{n-1}^h(\omega^+, s^+) g(s^+) \right)^{1-\gamma^h} \right]^{\frac{1-\rho^h}{\rho^h}} \quad (55) \\
& E_Q^h \left[\left(U_{n-1}^h(\omega_{n-1}^+, s^+) \right)^{(\psi^h)^{-1} - \gamma^h} c_{n-1}^h(\omega^+, s^+)^{-\psi^{-1}} \right. \\
& \left. (1-\beta) g(s^+)^{1-\gamma^h} (q_{n-1}(\omega^+, s^+) + d(s^+)) \right] \\
&+ \mu_n^{hs+} + \mu_n^{hc+} (1-m) \min_{s^+} (q_{n-1}(\omega^+, s^+) + d(s^+))
\end{aligned}
$$

$$
\begin{aligned}
p_n (U_n^h)^{\psi^{-1}} (c_n^h)^{-\psi^{-1}} &= (U_n^h)^{\psi^{-1}} \beta E_Q^h \left[\left(U_{n-1}^h(\omega^+, s^+) g(s^+) \right)^{1-\gamma^h} \right]^{\frac{1-\rho^h}{\rho^h}} \quad (56) \\
& E_Q^h \left[\left(U_{n-1}^h(\omega^+, s^+) \right)^{(\psi^h)^{-1} - \gamma^h} c_{n-1}^h(\omega^+, s^+)^{-\psi^{-1}} \right. \\
& \left. (1-\beta) g(s^+)^{-\gamma^h} \right] + \mu^{hc+}
\end{aligned}
$$

$$
\begin{aligned}
c_n^h &= e^h + \omega^h (q_n + d) - \theta^h q_n - b^h p_n && (57) \\
b_n^1 + b_n^2 &= 0 && (58) \\
\theta^1 + \theta^2 &= 1 && (59) \\
c_n^1 + c_n^2 &= 1 && (60) \\
\omega_n^{h+} &= \frac{\theta^h (q_{n-1}(\omega_n^+, s^+) + d(s^+)) + \frac{b_n^h}{g(s^+)}}{q_{n-1}(\omega^+, s^+) + d(s^+)} && (61) \\
\mu_n^{hs-} &= \theta_n^h && (62) \\
\mu_n^{hc-} &= \left(b_n^h + \theta_n^h (1-m) \min_{s^+} (q_{n-1}(\omega^+, s^+) + d(s^+)) \right) && (63)
\end{aligned}
$$

Here, equations (55) and (56) are the first order conditions for asset and bond holdings respectively. Equation (57) is the budget constraint while equations (58)-(60) are the equilibrium conditions, equation (61) is the dynamics for wealthshare and equations (62) and (63) are the modified complementary slackness conditions.

Do note that to solve for equilibrium prices, in addition to next periods prices, only next periods consumption and Value-function is needed and not portfolio choices. Thus, we do not need to interpolate next periods portfolio choices.

Step 2: Prices and policy functions are updated until $||f^n - f^{n-1}|| < \epsilon$.