Exit expectations and debt crises in currency unions

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Abstract

We study a sovereign debt crisis in a small member state of a currency union. If the country exits the currency union, it may redenominate its liabilities and reduce the real value of debt through depreciation and inflation. We analyse formally how such "exit expectations" impact the dynamics of the sovereign debt crisis. First, we show that public debt accumulates faster and sovereign yields increase more strongly because of redenomination risk. Second, we find that exit expectations induce public debt to be stagflationary. We also contrast the effects of exit expectations to those of outright sovereign default and illustrate that the adverse effects of exit expectations can be quantitatively significant.

Keywords: Currency union, exit, sovereign debt crisis, fiscal policy, redenomination risk, euro crisis, regime-switching model

JEL-Codes: E52, E62, F41

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1 Introduction

Countries which are members of a currency union have limited control over inflation. Public debt, even if issued in nominal terms, is effectively real for such countries, as they cannot reduce the real value of public debt through inflation (Aguiar et al., 2013; De Grauwe, 2011). By exiting the currency union and introducing a new currency, governments regain control of inflation: debt becomes nominal—provided it is issued under domestic law and can be redenominated by fiat. For this reason, expectations of an exit from a currency union may arise in the context of a sovereign debt crisis. The euro area is a case in point, as "fears of a reversibility of the euro" have been considered to be a driver of sovereign yields in the euro area during the recent crisis episode (ECB, 2013).

Figure 1 shows the evolution of the perceived probability that at least one member state left the euro area before end-2014. In the figure we report quarterly probabilities imputed from prices of a corresponding bet offered by the online betting platform "intrade".[1] While exit expectations have been low initially, they rose after 2010 and continued to fluctuate at a higher level during the sovereign debt crisis in the euro area. How do such "exit expectations" impact a sovereign debt crisis? To address this question, we develop a model of a small open economy which is (initially) operating within a currency union. We assume that the country experiences a sovereign debt crisis, as public debt is on an ever-rising, non sustainable trajectory. The crisis can be resolved via a fiscal reform, by outright default, or an exit from the currency union. Exit from the union is inflationary, because the health of public finances can be restored through inflation and depreciation.

Formally, we specify "simple rules" for monetary and fiscal policy as in Schmitt-Grohé and Uribe (2007), and let a Markov chain determine policy changes in a way consistent with agents' expectations. Initially, the country's fiscal policy is "active", as taxes do not systematically adjust to stabilize public debt. Because the country lacks monetary autonomy, public debt is on an explosive trajectory. Equilibrium default can restore public finances, as in the "fiscal theory of sovereign risk" (Uribe, 2006). As an alternative, the country may exit the union and adopt a "passive" monetary policy which accommodates the active fiscal policy. In this case, the price level and the exchange rate after exit are determined by the need to align the real value of public debt and future primary surpluses—an instance of the "fiscal theory of the price level" (Leeper, 1991; Sims, 2013; Woodford, 1995).[2]

[1] The bet was as follows: a "country currently using the Euro [is] to announce [its] intention to drop it before midnight ET 31 Dec 2014." While intrade itself closed down, data is still available from the following website: http://intrade-archive.appspot.com/event.jsp?event=79890. To compute the quarterly exit probability from the share price, we assume a flat yield curve over the horizon of the bet, that is we assume that at any given point in time, agents do not expect the quarterly exit probability to change going forward.

[2] For contributions to the fiscal theory in an open-economy context, see Woodford (1996), Sims (1997),

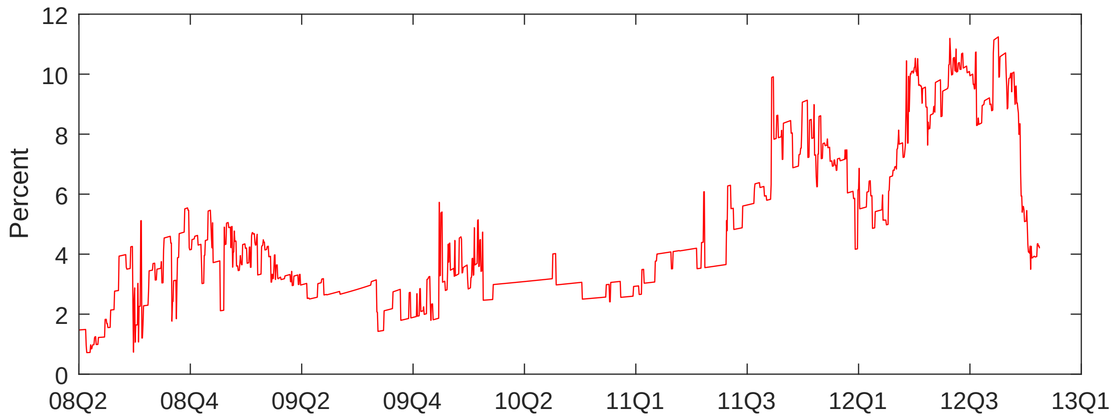

Figure 1: Quarterly exit probability as implied by the price quote of the bet: "Any country currently using the Euro [is] to announce [its] intention to drop it before midnight ET 31 Dec 2014". Source: see footnote 1. The bet closed down in March 2013, hence data is available until this date.

Our methodological contribution is to introduce regime change in a New Keynesian model of a small open economy à la Galí and Monacelli (2005), which we extend to allow for debt dynamics. While New Keynesian models are frequently used to study the properties of alternative exchange rate regimes, the possibility of exchange-rate-regime change as part of the equilibrium process and the expectation thereof are commonly ignored, even though policy regime changes have been analyzed in other contexts (Bianchi, 2013; Davig and Leeper, 2007a).[3] Our analysis is focused on a situation where the country still operates within the currency union, but is subject to spillovers from the events after exit. In this regard, we follow the closed-economy analysis of Bianchi and Melosi (2017).

Because regime change is exogenous in our model, we maintain a high degree of tractability which allows us to derive our main results analytically. Our first result is that, even though active fiscal policy and currency union membership is not sustainable in the long run, this policy mix may be sustained in equilibrium if the probability of regime change is sufficiently large. Yet exit expectations alter the dynamics of the debt crisis because investors anticipate losses due to depreciation and ask for higher sovereign yields prior to exit. Consequently, the refinancing costs of the government rise and the debt crisis is reinforced—as losses are proportional to outstanding debt, yields rise in sync with the debt level. In this respect, exit expectations are no different from expectations of outright sovereign default.

Bergin (2000), Dupor (2000) and Daniel (2001).

[3]See also Bianchi and Ilut (2017); Davig and Leeper (2007b, 2011). These authors put forward models where monetary and fiscal policy rules change over time. Andolfatto and Gomme (2003) consider changes in money-growth rules under imperfect information. All these studies analyze closed-economy models.

Our second result is that if public debt is high, exit expectations drive up interest rates for the sovereign, but also for private borrowers. This is because nominal depreciation after exit affects *all* assets denominated in the (new) domestic currency, not only public debt. This, in turn, has adverse effects on economic activity if prices adjust sluggishly. Moreover, inflation rises already (somewhat) before the exit takes place, due to forward-looking price-setting decisions. As a result, competitiveness deteriorates leading to a further drop in economic activity. Hence, the adverse effect of exit expectations is not limited to public finances. It is felt in the economy at large: in the presence of exit expectations, public debt has a *stagflationary* effect on the economy. Instead, if exit is ruled out, public debt is neutral for the economy. In this respect, exit expectations differ fundamentally from expectations of outright sovereign default.

We show that these results continue to hold once we allow the frequency of price adjustment to change with an exit—a "break" in the Phillips curve—and once we allow for balance sheet effects of depreciation. What is key for the stagflationary effect of exit expectations is that the real exchange rate depreciates alongside the nominal exchange rate after exit, as this raises the real interest rate in the initial regime. This occurs once prices remain somewhat sticky after exit. Empirically, large nominal devaluations tend to be associated with sizeable real depreciations (Burstein et al., 2005).

Finally, we parameterize the model and show that exit expectations can have a quantitatively significant impact on the economy. For this purpose, we assume that a debt level of 90 percent of GDP is sustainable and consider a scenario where debt is initially 100 percent of GDP and the probability of exit is 5 percent per quarter. We then run the model for a period of three years during which the country remains in the currency union and isolate the impact of exit expectations on the dynamics of the crisis. We find that exit expectations account for an increase of public debt by some 20 percentage points of GDP and an increase of (annualized) sovereign yields by some 20 percentage points, too. Output, in turn, declines by about 3 percent whereas (annualized) inflation increases by 1 percent.

In our analysis, we consider outright default and exit as alternative outcomes of a sovereign debt crisis in a currency union. Yet, debt repudiation and devaluation often occur jointly (Reinhart, 2002). Na et al. (2018) rationalize this observation in a model where default and exchange rates are determined optimally. Central to their analysis is the assumption that governments are indebted in foreign currency, the "original sin" of many emerging market economies. As a result, inflation and devaluation are ineffective in reducing the real value of debt. In our analysis, instead, public debt is governed by domestic law, in line with actual practice in the euro area (Chamon et al., 2015), and may be redenominated upon exit.

Our model does not permit self-fulfilling exit as in the stylized models of Drazen and Masson (1994) and Obstfeld (1996), because we assume exogenous transition probabilities. We also abstract from contagion of exit across the members states of a currency union, a possibility which is explored in depth by Eijffinger et al. (2018). Instead, we are able to derive our results analytically within a regime-switching New Keynesian model and obtain quantitative results. Our findings reiterate a theme which features prominently in classic studies of the stability of currency pegs, namely that a non-sustainable policy mix can be maintained only for a limited number of periods (Flood and Garber, 1984; Krugman, 1979). Also, our analysis reestablishes two results of earlier work on currency crises, namely that expected devaluation may raise the refinancing cost of governments, as well as induce a loss in competitiveness due to forward-looking price setting behavior (Obstfeld, 1994, 1997).

Lastly, our paper relates to work which accounts for important aspects of the recent euro-area crisis. Studies with a focus on outright sovereign default include Bi (2012), Daniel and Shiamptanis (2012) and Lorenzoni and Werning (2013), among others. In an influential empirical study Krishnamurthy et al. (2017) decompose yield spreads into a redenomination and a default premium. Gilchrist et al. (2015), Schmitt-Grohé and Uribe (2016), Kuvshinov et al. (2016) and Wolf (2018), in turn, analyze the sluggish adjustment of real exchange rates in the euro area.

The remainder of the paper is organized as follows. Section 2 outlines our model structure, a small open economy with public debt. Section 3 presents our regime switching model. We provide closed-form results in Section 4 and numerical results in Section 5. Here we also discuss model extensions. Section 6 concludes.

2 A small open economy with public debt

Our analysis is based on a New Keynesian model of a small open economy, as in Kollmann (2001), Galí and Monacelli (2005) or Corsetti et al. (2013b). An innovation relative to those studies is that our model features richer dynamics of public debt. More importantly, we allow the conduct of monetary and fiscal policy to change over time. In particular, there is the possibility that a member of currency union exits the union in the future. In this section we outline the economic environment. Section 3 introduces the regime switching model.

In what follows we briefly describe the behavior of households and firms which is standard. We explain in more detail the economy's public sector, which is less standard. As regards fiscal policy, the government may default outright on its liabilities as in Uribe (2006). As regards monetary policy, we model membership in the currency union as an exchange rate peg of unity. This is particularly convenient, as we study an exit from a currency union after which the country will have an independent currency.

2.1 Households and firms

A representative household has preferences over aggregate consumption C_t and hours worked H_t. We let $\beta < 1$ denote the time discount factor, $1/\varphi > 0$ the Frisch elasticity of labor supply, and E_0 the expectation operator. The representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t) - \frac{H_t^{1+\varphi}}{1+\varphi} \right\}$$

subject to a sequence of budget constraints

$$\int_0^1 P_{H,t}(i) C_{H,t}(i) di + \int_0^1 P_{F,t}(i) C_{F,t}(i) di + E_t\{\rho_{t,t+1} D_{t+1}\} = W_t H_t + D_t + \mathcal{Y}_t - P_t \tau_t$$

and a no-Ponzi constraint. In the budget constraint, D_{t+1} is a portfolio of state contingent claims, priced with the stochastic discount factor $\rho_{t,t+1}$, W_t is the nominal wage, $\mathcal{Y}_t$ are (aggregate) firm profits, τ_t are lump-sum taxes, and P_t is the consumer price index to be defined below. In turn, $C_{H,t}(i)$ and $C_{F,t}(i)$ are demand functions for domestically-produced and imported varieties with $i \in [0, 1]$, respectively. $P_{H,t}(i)$ and $P_{F,t}(i)$ is the (domestic-currency) price of each variety. Specifically, aggregate consumption C_t is a composite

$$C_t = \left((1-\omega) \left[\left(\int_0^1 C_{H,t}(i)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\sigma-1}{\sigma}} + \omega \left[\left(\int_0^1 C_{F,t}(i)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $0 < \omega < 1$ is the weight of imported goods in consumption, $\gamma > 1$ the elasticity between varieties and $\sigma > 1$ the elasticity between domestic goods and imports. Minimizing expenditures yields the familiar demand functions

$$C_{H,t}(i) = (1-\omega) \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\gamma} \left(\frac{P_{H,t}}{P_t} \right)^{-\sigma} C_t$$

for domestic varieties with price index $P_{H,t} = (\int_0^1 P_{H,t}(i)^{1-\gamma} di)^{1/(1-\gamma)}$, as well as

$$C_{F,t}(i) = \omega \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\gamma} \left(\frac{P_{F,t}}{P_t} \right)^{-\sigma} C_t$$

for foreign varieties, with price index $P_{F,t} = (\int_0^1 P_{F,t}(i)^{1-\gamma} di)^{1/(1-\gamma)}$. In turn, the consumer price index is $P_t = ((1-\omega) P_{H,t}^{1-\sigma} + \omega P_{F,t}^{1-\sigma})^{1/(1-\sigma)}$.

Optimality requires the following condition to be satisfied:

$$\rho_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{P_t}{P_{t+1}}.$$

We use this expression to define the yield on a nominally riskless, domestic-currency bond $R_t \equiv 1/(E_t \rho_{t,t+1})$. The interest rate R_t is set by domestic monetary policy whenever the country operates outside the currency union.

Households in the rest of the world face a symmetric problem. This yields the condition $\rho_{t,t+1} = \beta(C^*_{t+1}/C^*_t)^{-1}(P^*_t/\mathcal{E}_{t+1})/(P^*_{t+1}/\mathcal{E}_t)$, where $\mathcal{E}_t$ is the price of foreign currency in terms of domestic currency (the nominal exchange rate). P^*_t is the consumer price index in the rest of the world. Combining this and the previous condition yields the risk sharing condition $C_t/C^*_t = \upsilon Q_t$, where $Q_t = (P^*_t \mathcal{E}_t)/P_t$ denotes the price of foreign in terms of domestic consumption (the real exchange rate), and where $\upsilon = (C_{-1}/C^*_{-1})(P_{-1}/(\mathcal{E}_{-1}P^*_{-1}))$ is a constant capturing initial conditions (which we normalize to unity). Finally, first order conditions imply a conventional labor supply curve: $W_t/P_t = C_t N_t^{\varphi}$.

Firms produce varieties. They operate in a monopolistically competitive environment, rely on a linear production technology $Y_t(j) = H_t(j)$, and face price adjustment frictions à la Calvo. Prices are set in the currency of the producer and the law of one price holds at the level of varieties. The demand faced by firm $i \in [0,1]$ at time $t+k$, given that it last reset its price in period t, is given by

$$Y_{t+k|t}(i) = (1-\omega)\left(\frac{P_{H,t}(i)}{P_{H,t+k}}\right)^{-\gamma}\left(\frac{P_{H,t+k}}{P_{t+k}}\right)^{-\sigma} C_{t+k} + \omega\left(\frac{P_{H,t}(i)}{P_{H,t+k}}\right)^{-\gamma}\left(\frac{P_{H,t+k}/\mathcal{E}_{t+k}}{P^*_{t+k}}\right)^{-\sigma} C^*_{t+k}.$$

All firms are owned by domestic households. Let ξ denote the probability of keeping a posted price. A resetting firm at time t therefore faces the following problem

$$\max\ E_t \sum_{k=0}^{\infty} \xi^k \rho_{t,t+k}\left(P_{H,t}(i)Y_{t+k|t}(i) - \mathcal{C}(Y_{t+k|t}(i))\right),$$

subject to the demand function stated above. Here $\mathcal{C}(Y_{t+k|t}(i))$ denotes the (nominal) labor cost of producing $Y_{t+k|t}(i)$, given by $\mathcal{C}(Y_{t+k|t}(i)) = W_{t+k}Y_{t+k|t}(i)$. The first order condition for this problem is given by

$$\frac{\tilde{P}_{H,t}}{P_{H,t}} = \frac{\gamma}{\gamma-1}\frac{K_t}{F_t},$$

where the two auxiliary variables K_t and F_t can be represented in recursive form

$$K_t = C_t^{-1} MC_t Y_t + \xi\beta E_t\left(\frac{P_{H,t+1}}{P_{H,t}}\right)^{\gamma} K_{t+1} \quad \text{and} \quad F_t = C_t^{-1} Y_t + \xi\beta E_t\left(\frac{P_{H,t+1}}{P_{H,t}}\right)^{\gamma-1} F_{t+1},$$

respectively. In the expressions above, real marginal costs are given by $MC_t = W_t/P_{H,t}$ and aggregate output is $Y_t = (\int_0^1 Y_t(i)^{(\gamma-1)/\gamma} di)^{\gamma/(\gamma-1)}$. Furthermore, we have exploited that all resetting firms choose the same reset price, that is, $P_{H,t}(i) = \tilde{P}_{H,t}$ for all resetting firms. This also implies that domestic prices evolve as follows in equilibrium:

$$P_{H,t}^{1-\gamma} = (1-\xi)\tilde{P}_{H,t}^{1-\gamma} + \xi P_{H,t-1}^{1-\gamma}.$$

2.2 Monetary and fiscal policy

In case the country operates outside the currency union, we assume that monetary policy follows a Taylor rule targeting producer price inflation

$$R_t = \beta^{-1}\Pi_{H,t}^{\phi},$$

where $\phi > 0$ captures the feedback of inflation into the policy rate. In this case the nominal exchange rate is fully flexible. Alternatively, to capture the possibility that the country maintains a currency union with the rest of the world we impose:

$$\mathcal{E}_t = 1.$$

Hence, we model membership in a currency union as an exchange rate peg of unity, as, for instance, in Galí and Monacelli (2016). Note that this implies that two currencies co-exist, even as the country is a member of the currency union. As long as the exchange rate is fixed, this model behaves exactly as if there were no independent currencies to begin with. Furthermore, this setup allows us to keep track of the denomination of assets once the country exits the currency union: assets which are "denominated" in "domestic currency" during union membership, can be interpreted as "domestic law" securities—to be *redenominated* into new currency with a currency union exit.[4] As a result, changes in the price of those assets (notably changes in R_t as we will see below) which reflect expected inflation and depreciation after exit from the union, may be interpreted as *redenomination risk*.[5]

The government raises (real) lump sum taxes τ_t and issues short-term nominal bonds B_t at price I_t^{-1}. Government debt is risky, as in each period, the government may renege on a fraction Θ_t of its liabilities. The budget constraint is

$$I_t^{-1}B_{t+1} = B_t(1 - \Theta_t) - P_t\tau_t.$$

Here we assume government debt to be denominated in domestic currency, or equivalently, to be governed by domestic law, as discussed above.[6]

[4] In the recent euro area crisis, market participants expected securities issued under Greek law to be converted into new currency upon exit (Buiter and Rahbari, 2012). Similarly, historical examples of "forcible conversions" of debt issued in foreign currency, but under home law highlights the role of jurisdiction for currency conversions (Reinhart and Rogoff 2011).

[5] One consequence of our modeling two currencies even under union membership is that, with an exit from the union, we implicitly assume conversion at par from common into new currency. However, this assumption does not affect our results. What matters for the effects of exit expectations is the price of the new currency in terms of the old currency in the exit period. Whether this price is the result of an at-par conversion coupled with a large subsequent nominal depreciation of the new currency, or a different conversion rate coupled with a smaller subsequent nominal depreciation, leaves our conclusions unaffected. At the same time, we acknowledge that our formulation abstracts from direct costs of a currency conversion.

[6] This assumption is in line with actual practice in the euro area. During the period 2003–2014, most (many)

For optimizing households to be indifferent between state-contingent claims D_t and government debt B_t, the following no-arbitrage condition must be satisfied:

$$1 = I_t E_t \rho_{t,t+1}(1 - \Theta_{t+1}).$$

Recalling that the risk-free rate on domestic-currency assets equals $R_t = 1/(E_t \rho_{t,t+1})$, this equation shows that the sovereign bond yield exceeds this rate to the extent that the expected haircut is positive. Optimality also requires the following transversality condition to be satisfied (Uribe 2006):

$$\lim_{j \to \infty} \beta^{j+1} E_t \left(\frac{C_{t+j+1}}{C_t} \right)^{-1} (1 - \Theta_{t+j+1}) \frac{B_{t+j+1}}{P_{t+j+1}} = 0.$$

By combining the previous equations and recalling the definition of $\rho_{t,t+1}$, we obtain the government's present value budget constraint (see also Appendix A)

$$(1 - \Theta_t) \frac{B_t}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \left(\frac{C_{t+j}}{C_t} \right)^{-1} \tau_{t+j}.$$

This equation is typically assumed to be satisfied because the government raises sufficient taxes to redeem its debt stock at zero default and at given prices. In other words fiscal policy is assumed to be "passive" (Leeper, 1991). Yet, if the government fails to do so and instead pursues an "active" fiscal policy, the left hand side of the equation may adjust, either the default rate Θ_t or, alternatively, the price level P_t.

Below we study a sovereign debt crisis and consider these possibilities explicitly. First, as in Uribe (2006) we consider outright default: Θ_t adjusts such that the present value budget constraint, post default, is satisfied. Second, while the price level is not free to adjust for as long as the country operates inside the currency union, prices may adjust after exit so as to reduce the real value of government liabilities—a "default by inflation". This is therefore an instance of the "fiscal theory of the price level" (Sims, 2013; Woodford, 1995).[7]

In our model, we assume a specific rule for how the government sets taxes, according to which fiscal policy may be either passive or active (Leeper (1991); see also Schmitt-Grohé and Uribe (2007), and Lorenzoni and Werning (2013) in the context of debt crises):

$$\tau_t - \tau = \psi \left(\frac{B_t}{P_t} - \frac{\tau}{1 - \beta} \right),$$

European countries issued more than 60-70 (90) percent of their public debt under domestic law. Exceptions are the Baltic countries and Cyprus as well as Greece after the restructuring of its debt in 2012 (Buchheit et al., 2013; Chamon et al., 2015).

[7]Notice that neither possibility is priced in the present value of taxes, as taxes are discounted with the real stochastic discount factor—with no reference to either inflation or default. Even though government debt is risky, the fact that it is priced actuarially fair leaves the expected present value of taxes unaffected.

such that fiscal policy adjusts taxes systematically to outstanding (real) public debt.

Therefore, if the government raises an amount $\tau_t = \tau \geq 0$ of taxes in the steady state, it may sustain a steady-state real debt stock $B_t/P_t = \tau/(1-\beta) \geq 0$. Moreover, as debt levels rise above this threshold, taxes adjust with slope coefficient $\psi \geq 0$. For large ψ, debt levels converge back to steady state and fiscal policy is passive. In contrast, if ψ is sufficiently low, fiscal policy is going to be active.

2.3 Market clearing

Aggregate profits are $\int_0^1 \mathcal{Y}_t(i)di = \int_0^1 Y_t(i)(P_{H,t}(i) - W_t)di = P_{H,t}Y_t - W_t H_t$. Goods market clearing is given by

$$Y_t = (1-\omega)\left(\frac{P_{H,t}}{P_t}\right)^{-\sigma} C_t + \omega\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\sigma} C_t^*.$$

where we have used that $P_{F,t}$ equals the domestic-currency price of the foreign consumption basket, $P_{F,t} = \mathcal{E}_t P_t^*$, reflecting that the domestic country is small (De Paoli, 2009). The labor market clears as $H_t = \int_0^1 H_t(i)di = \int_0^1 Y_t(i)di = Y_t \int_0^1 (P_{H,t}(i)/P_{H,t})^{-\gamma} di = Y_t\Delta_t$, where Δ_t is price dispersion which evolves as

$$\Delta_t = (1-\xi)\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\gamma} + \xi\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\gamma} \Delta_{t-1}.$$

Finally, the asset market clears residually:

$$E_t\{\rho_{t,t+1}D_{t+1}\} - D_t = I_t^{-1}B_{t+1} - (1-\Theta_t)B_t + P_{H,t}Y_t - P_t C_t$$

such that household savings (left hand side) equal the newly issued public debt (first term right hand side), plus the accumulation of foreign assets resulting from a trade surplus.[8]

3 A model of changing policy regimes

Our goal is to study the unfolding of a sovereign debt crisis in a small member of a currency union. In particular, we ask how expectations of a future shift in the policy regime—most notably, an exit from the union coupled with a subsequent inflationary policy—impacts the sovereign debt crisis while the country is still part of the currency union. We ask this question in light of the actual developments in the euro area: as public debt started to spiral in Greece

[8]This expression reveals that, for our complete-markets model, it is not pinned down (and therefore immaterial for our results) whether in equilibrium, government debt is held by domestic households or by foreign households. In the incomplete markets version of the model, we assume that government debt is held domestically, see Section 5.2 below.

as of 2009, there was widespread speculation about an exit from the euro zone, alongside speculation about an outright sovereign default.

Specifically, in our analysis we consider the following sequence of regime transitions

$$\text{Sovereign debt crisis} \quad \begin{array}{ll} \nearrow & \text{Fiscal reform} \\ \rightarrow & \text{Default} \\ \searrow & \text{Exit the currency union.} \end{array}$$

Initially, the country is a member of a currency union. Yet, fiscal policy is active, triggering—as we will show below—a sovereign debt crisis. By this we mean that public debt is on an unsustainable trajectory: without a change in policy, public debt grows at an ever faster rate. The change in policy can occur in three different ways. First, there may be a fiscal reform as a result of which fiscal policy turns passive. Second, there may be a default, bringing public debt back on a sustainable level. Third, there may be an exit from the currency union, coupled with an inflationary monetary stance.

To capture this formally, we now introduce our Markov-switching linear rational expectations (MS-LRE) model. In a closed-economy context, this framework has been used extensively to study the implications of discrete shifts in a country's policy regime (e.g., Bianchi, 2013; Davig and Leeper, 2007a). Here we use this framework to model an exit of a country from a currency union. The key benefit from using a Markov-switching framework, where transition probabilities are exogenous, is tractability. Specifically, we are able to derive conditions under which the model has a well defined equilibrium, even as the country experiences a sovereign debt crisis. Moreover, we will derive our key results analytically, allowing us to explore the underlying economic mechanism in depth.

To state our MS-LRE model, we need to linearize the model's equilibrium conditions around a steady state which is independent of policy regimes. The steady state is one of purchasing power parity, zero inflation, and zero default. Lower case letters denote upper case letters in linearized terms.[9] We identify a number of equilibrium conditions which are *invariant* across policy regimes. The economy's private sector consists of a dynamic IS relation and a New Keynesian Phillips curve

$$y_t = E_t y_{t+1} - \varpi(r_t - E_t \pi_{H,t+1}), \tag{3.1}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa \left(\varphi + \varpi^{-1}\right) y_t, \tag{3.2}$$

where we define $\varpi := 1 + \omega(2-\omega)(\sigma-1)$ and $\kappa := (1-\beta\xi)(1-\xi)/\xi$. Furthermore, under

[9] Details on the linearization can be found in Appendix A.

complete financial markets output is tied to the real exchange rate as follows:

$$(1-\omega)y_t = \varpi q_t, \tag{3.3}$$

$$q_t = (1-\omega)(e_t - p_{H,t}). \tag{3.4}$$

In contrast, the equilibrium conditions in the public sector are generally *regime dependent*. First of all, under our assumed fiscal rule, public debt evolves as

$$\beta b_{t+1} = (1-\psi_{\varsigma_t})b_t - \lambda(\beta i_t - \pi_{H,t} - \theta_t), \tag{3.5}$$

where we define $\lambda := \tau/(1-\beta)$ as the level of debt in steady state and where ς_t is an index of the policy regime or, formally, the state of the Markov chain: $\varsigma_t \in \{\text{Crisis}, \text{Reform}, \text{Default}, \text{Exit}\}$. Here, $\psi_{\text{Reform}} > 1-\beta$, whereas $\psi_{\varsigma_t} < 1-\beta$ in all other regimes (active fiscal policy). The sovereign bond yield is not regime dependent. It can be written as

$$i_t = r_t + E_t\theta_{t+1}. \tag{3.6}$$

Monetary policy is characterized by the following equation:

$$\mathbb{1}_{\varsigma_t}(r_t - \phi\pi_{H,t}) + (1-\mathbb{1}_{\varsigma_t})e_t = 0, \tag{3.7}$$

where the indicator variable $\mathbb{1}_{\varsigma_t}$ takes the value of one in regime Exit and the value of zero otherwise. We note that monetary policy must persistently violate the Taylor principle after exit, $\phi < 1$, for it is required to "accommodate" the active fiscal policy. If it refused to do so, the equilibrium after exit would feature hyper-inflation (e.g., Loyo, 2000).

A similar regime-dependent equation governs the dynamics of a sovereign default:

$$\mathbb{1}'_{\varsigma_t}b_{t+1} + (1-\mathbb{1}'_{\varsigma_t})\theta_t = 0. \tag{3.8}$$

Here, a second indicator variable $\mathbb{1}'_{\varsigma_t}$ takes the value of one in regime Default and zero otherwise. Importantly, because fiscal policy remains active in regime Default, the default must be "full" in this regime, in the sense of reducing debt all the way to its sustainable level (the steady state). Hence $b_{t+1} = 0$ in this regime, whereas θ_t is free to adjust.

We define an equilibrium as follows. First, we restate conditions (3.1)-(3.8) and the definition for inflation $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ more compactly as

$$\mathit{\Gamma}_{\varsigma_t}x_t = E_t x_{t+1}, \tag{3.9}$$

where $x_t = (y_t, r_t, i_t, \pi_{H,t}, p_{H,t}, e_t, q_t, b_{t+1})'$. The matrix $\mathit{\Gamma}_{\varsigma_t}$ contains the parameters and ς_t indicates that they may be regime dependent; an overview of the regime dependent parameters

Table 1: Regime-dependent parameters

	Crisis	Reform	Default	Exit
ψ	$\psi_{\text{Crisis}} < 1-\beta$	$\psi_{\text{Reform}} > 1-\beta$	$\psi_{\text{Default}} = \psi_{\text{Crisis}}$	$\psi_{\text{Exit}} = \psi_{\text{Crisis}}$
$\mathbb{1}$	$\mathbb{1}_{\text{Crisis}} = 0$	$\mathbb{1}_{\text{Reform}} = 0$	$\mathbb{1}_{\text{Default}} = 0$	$\mathbb{1}_{\text{Exit}} = 1$
$\mathbb{1}'$	$\mathbb{1}'_{\text{Crisis}} = 0$	$\mathbb{1}'_{\text{Reform}} = 0$	$\mathbb{1}'_{\text{Default}} = 1$	$\mathbb{1}'_{\text{Exit}} = 0$

is given in Table 1. Expectations E_t capture the uncertainty induced by the possibility of regime change. Therefore, second, we specify the Markov chain. In line with our graphical representation above, we specify the following transition matrix

$$\mathcal{P} = \begin{pmatrix} 1-f-\delta-\epsilon & f & \delta & \epsilon \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Hence, the initial regime (the "sovereign debt crisis") persists with probability $1-f-\delta-\epsilon \geq 0$, where $f \geq 0$ denotes the probability of a fiscal reform, $\delta \geq 0$ the probability of default, and $\epsilon \geq 0$ the probability of an exit from the union.[10] Third and last, we state the definition of equilibrium, following Farmer et al. (2011).

Definition 1. A rational expectations equilibrium is a mean square stable (MSS) stochastic process that, given the Markov chain, satisfies (3.9) in all regimes $\{\varsigma_t\}$ at all times.

Definition 2. An $n-$dimensional process $\{x_t\}$ is MSS if there exists an $n-$vector x_∞ and an $n \times n$ matrix Σ_∞ such that in all regimes $\{\varsigma_t\}$ at all times

- $\lim\limits_{n\to\infty} E_t[x_{t+n}] = x_\infty$
- $\lim\limits_{n\to\infty} E_t[x_{t+n}\ x_{t+n}{}'] = \Sigma_\infty$.

Key in this definition is the notion of stability, which is different in a regime switching model from what is commonly used in models where the policy regime is fixed. Intuitively, a spiraling debt level in the initial regime is compatible with equilibrium, if the economy moves sufficiently quickly to a future regime where debt is stabilized. In other words, the expected duration of the initial regime is key for stability.

[10] Assuming absorbing states allows us to keep the analysis tractable. At the same time we acknowledge that reentering a monetary union cannot be ruled out in practice. Yet we abstract from this possibility as its effect on the equilibrium outcome in the initial regime is bound to be small.

4 Results

We now turn to the main results of our analysis. First, we show that for a small open economy, an active fiscal policy coupled with membership in a currency union induces a sovereign debt crisis—that is, public debt follows an explosive trajectory. This policy mix is therefore not sustainable. Moreover, we show that while an exit from the union may resolve the crisis, expectations of an exit reinforce the sovereign debt crisis while the country is still a member of the currency union. In this respect, exit expectations are no different from expectations of an outright sovereign default. Second, we show that exit expectations—unlike expectations of outright default—harm macroeconomic stability more generally: they induce public debt to be stagflationary.

4.1 Exit expectations reinforce sovereign debt crises

To obtain analytical results, throughout the remainder of Section 4 we assume that taxes are invariant to the level of outstanding debt in the initial regime, $\psi_{\text{Crisis}} = 0$. Furthermore, in this subsection, we also assume prices to be perfectly flexible ($\xi = 0$). In this case, public debt is the only state variable in the model, making a closed-form solution particularly tractable.[11] In a first step, we establish a condition for which an equilibrium of the model exists, according to Definitions 1 and 2 above.

Proposition 1. *Assuming that $\psi_{Crisis} = 0$ and that prices are perfectly flexible ($\xi = 0$), an equilibrium of the regime switching model specified in Section 3 exists if and only if the following condition for (mean square) stability is satisfied:*

$$(1 - f - \delta - \epsilon)\left(\frac{1}{\beta(1 - \epsilon - \delta)}\right)^2 < 1. \tag{4.1}$$

Proof. In the Appendix B. □

Intuitively, the way to verify (mean square) stability as detailed in Definition 2, is to make sure that first, variables remain bounded once the exit from the union has occurred (as the three target regimes are absorbing) and that second, the switch to these "stable" regimes occurs sufficiently quickly—which is condition (4.1).[12]

[11] Because the three target regimes are absorbing, we solve the model backwards using the method of undetermined coefficients. All derivations are detailed in the Appendix B.

[12] A criterion for checking mean square stability is provided in Farmer et al. (2009). All eigenvalues of $(\mathcal{P}' \otimes I_{n^2})\text{diag}(F_{\varsigma_1} \otimes F_{\varsigma_1}, ..., F_{\varsigma_h} \otimes F_{\varsigma_h})$ must lie within the unit circle, where h denotes the number of regimes, $\otimes$ is the Kronecker product and the F are solution matrices in the respective regimes, that is, $x_t = F_{\varsigma_h} x_{t-1}$ for all h. This criterion reduces to the criterion for stability in Blanchard and Kahn (1980) in absorbing states of the Markov chain.

Proposition 1 implies that if the first regime lasts forever ($f = \delta = \epsilon = 0$), no equilibrium exists because of $(1/\beta)^2 > 1$. This shows that an active fiscal policy, for a country that is small, is inconsistent with membership in a currency union. Intuitively, for a small union member, public debt, even if issued in nominal terms, is effectively real because the country lacks control of inflation (Aguiar et al., 2013; De Grauwe, 2011). As a result, an explosive path is inevitable for public debt if it is not stabilized by fiscal policy.[13]

It follows that an equilibrium can exist only if there is a possibility of regime change. Expectations of fiscal reform are particularly helpful in this regard: assuming $\epsilon = \delta = 0$, a probability of fiscal reform $f > 1 - \beta^2 > 0$ is sufficient for an equilibrium to exist, see equation (4.1). In contrast, while ϵ and δ enter symmetrically as f in the numerator of (4.1), we also note that their influence is not unambiguous for stability: because both also enter negatively in the denominator, they may actually *reduce* stability.[14]

To understand this result, we characterize explicitly the equilibrium in the initial regime. Note first that, in all regimes, flexible prices imply constant output $y_t = 0$ by equation (3.2). By equation (3.1), this implies a constant real interest rate, $r_t - E_t \pi_{H,t+1} = 0$, and a constant real exchange rate $q_t = 0$ by equation (3.3). The latter, in turn, requires $p_{H,t} = e_t$ by equation (3.4), such that prices move one-for-one with the nominal exchange rate after exit. Prior to exit, debt is on an explosive trajectory. This is because fiscal policy is active, but also because there are expectations of exit and default. Furthermore, sovereign yields evolve in sync with public debt, as stated the following proposition.

Proposition 2. *Assume that $\psi_{Crisis} = 0$ and that prices are perfectly flexible ($\xi = 0$). In this case, the solution for public debt and the sovereign yield in regime Crisis (the initial regime) are given by*

$$b_{t+1} = \frac{1}{\beta(1 - \delta - \epsilon)} b_t, \qquad i_t = \frac{\delta + \epsilon}{\beta\lambda(1 - \delta - \epsilon)} b_t. \tag{4.2}$$

Proof. In the Appendix B. □

The autoregressive root in the solution for debt is larger than unity and increases in ϵ, the probability of exit: exit expectations reinforce crisis dynamics. Observe that the probability of exit enters the solution in exactly the same way as the probability of default δ. This is because investors suffer equally in the event of exit as in the event of default. Ex-ante, they ask for higher yields as they expect these events to materialize. Consequently, for as long

[13] Instead, if the country is large, running an active fiscal policy may impact union-wide inflation (Bergin, 2000; Sims, 1999).

[14] As is easy to verify, the derivative of equation (4.1) with respect to ϵ is positive for ϵ below $\bar{\epsilon} = 1 - 2f - \delta$, but negative above this threshold (and symmetrically for δ). Hence, as ϵ and δ rise stability tends initially to be reduced, whereas once both are large enough, stability tends to be increased.

as the economy continues to operate in the initial regime, higher interest rate expenditures induce public debt to grow faster.

To dig deeper into the mechanics, we decompose the sovereign yield into the premiums which reflect, in turn, default and exit expectations. By equation (3.6), sovereign yields increase directly in expected losses due to default: $i_t = r_t + E_t\theta_{t+1}$. This expression also shows that sovereign yields move one-for-one with the nominal interest rate r_t, the yield on a domestic-currency (equivalently, domestic-law) security. Exit expectations operate at this margin. To see this, combine equations (3.1), (3.3) and (3.4) to obtain for the initial regime:

$$r_t = E_t\Delta e_{t+1} = \epsilon e_{t+1}|\text{Exit}, \tag{4.3}$$

where we have replaced E_t by using the Markov chain. This is the uncovered interest parity (UIP) condition. It reveals that, to the extent that the nominal exchange rate depreciates after exit, r_t must rise in the initial regime. We hence solve for the nominal exchange rate after exit to obtain

$$\Delta e_t|\text{Exit} = \lambda^{-1}b_t, \tag{4.4}$$

which shows that there is depreciation whenever outstanding debt exceeds its sustainable level (scaled by the steady-state level of debt). Finally, the depreciation reflects a monetary stance which is accomodative to inflation, so as to reduce the real value of public liabilities:

$$b_{t+1}|\text{Exit} = \phi b_t. \tag{4.5}$$

If monetary policy after exit does not respond at all to inflation ($\phi = 0$), public debt is reduced to its steady state level immediately upon exit. Otherwise, inflation (and nominal depreciation) are spread over time, and public debt converges back to steady state only slowly ($0 < \phi < 1$). In either case, the level of debt determines the equilibrium price level and the nominal exchange rate after exit.

Turning back to the initial regime, it is now clear why exit and default expectations impact public finances symmetrically: investors suffer losses in both instances. However, there is also an important difference: exit expectations impact sovereign yields by altering interest rates on *all* domestic-law assets whereas default expectations exclusively impact sovereign yields. This is because, after exit, all domestic-law assets—private and public—are affected by nominal depreciation. This has two important implications. First, the nominal interest rate r_t which reflects exit expectations enters the IS curve of the economy (see equation (3.1)). Second, as a result, exit expectations have consequences for the real economy if prices are not fully flexible ($\xi > 0$), a case which we analyze next.

4.2 Exit expectations make public debt stagflationary

If prices are sticky, exit expectations matter for how debt dynamics feed back into the economy. To show this analytically, in this subsection we make the following parametric assumptions: we maintain $\psi_{\text{Crisis}} = 0$ as in the previous subsection; we assume that the dis-utility of labor is linear ($\varphi = 0$); and we set $\kappa = 1 - \beta$, thereby restricting the slope of the Phillips curve. These assumptions are not essential for obtaining our results, but simplify the algebra considerably. However, we make one additional assumption which is critical for obtaining closed-form expressions, namely, we assume that the sovereign debt crisis is expected to last for only one period: $f + \delta + \epsilon = 1$.

If prices are sticky, the real exchange rate is a state variable for as long as the country is a member of the currency union. This gives rise to richer equilibrium dynamics. The following proposition states the solution for output and the price level in regime Crisis.

Proposition 3. *Assume that $\psi_{Crisis} = \varphi = 0$, that $\kappa = 1 - \beta$, and that $f + \delta + \epsilon = 1$. In this case, the solution for output and the domestic price level in regime Crisis (the initial regime) are given by*

$$y_t = \frac{\xi\varpi(1-\epsilon-\delta)}{(1-\omega)\Lambda} q_{t-1} - \frac{\epsilon\xi\varpi\sqrt{\kappa/(1-\beta\phi)}}{\lambda\Lambda} b_t, \tag{4.6}$$

and $p_{H,t} = -\varpi^{-1} y_t$, where $\Lambda =: (1-\epsilon-\delta)(1-\beta\epsilon\xi(1-\xi)) - \epsilon\sqrt{\kappa/(1-\beta\phi)}(1-\xi(1-\beta\epsilon\xi))$ is positive for ϵ sufficiently below unity.[15]

Proof. In the Appendix B. □

Equation (4.6) shows that output increases in the real exchange rate q_{t-1}: a weaker exchange rate boosts output. At the same time, because output and domestic prices are strictly negatively related, a weaker exchange rate puts upward pressure on domestic prices, in order to restore purchasing power parity. Besides these conventional effects, importantly, the expressions in Proposition 3 show that output and domestic prices also depend on public debt—but only to the extent that there are exit expectations ($\epsilon > 0$). In this regard, exit and default expectations differ fundamentally.[16]

While default expectations alter sovereign yields directly, via equation (3.6), they have no bearing on the nominal interest rate r_t which matters for the intertemporal allocation of household expenditures. For this reason default expectations are neutral for the allocation in

[15] We verify numerically that in the region where $\Lambda < 0$, the solution is no longer mean square stable, that is, the model ceases to have an equilibrium. If prices are sticky, the condition for mean square stability can only be checked numerically.

[16] When $\epsilon = 0$, equation (4.6) reduces to $y_t = \xi\varpi(1-\omega)^{-1} q_{t-1}$, where q_t evolves as $q_t = \xi q_{t-1}$ (to obtain the solution for the real exchange rate, we have used equation (3.3)). Clearly, this solution is entirely independent of default expectations.

our model. Intuitively, because in equilibrium, private agents are *indifferent* between saving in state contingent claims and government debt, the *expected* return from saving in government debt is exactly *equal* to the nominally riskless rate (Uribe, 2006).[17] For default expectations to impact economic activity, additional frictions are required, for example a "sovereign risk channel" by which expectations of default impact the riskless rate r_t directly (Bocola, 2016; Corsetti et al., 2013a).

Exit expectations, in contrast, alter the nominal interest rate r_t, see equation (4.3), and, if prices are sticky, also the real interest rate. To see this, we subtract expected inflation on both sides of (4.3) and use equation (3.4):

$$\begin{aligned} r_t - E_t\pi_{H,t+1} &= (1-\omega)^{-1}E_t\Delta q_{t+1} \\ &= (1-\omega)^{-1}[f\Delta q_{t+1}|\text{Reform} + \delta\Delta q_{t+1}|\text{Default} + \epsilon\Delta q_{t+1}|\text{Exit}], \end{aligned} \tag{4.7}$$

where in the second line we have evaluated E_t by using the Markov chain. This expression reveals that the *real* interest rate—the rate which governs intertemporal expenditure decisions—is determined by the expected *real* depreciation of the exchange rate after exit, as well as the *real* depreciation of the exchange rate conditional on staying in the union.

To proceed further, we evaluate the real exchange rate conditional on staying in the union and conditional on exit. Under our assumptions, we find $\Delta q_t|\text{Reform} = \Delta q_t|\text{Default} = -(1-\xi)q_{t-1}$, that is, if the country stays a union member, deviations of the real exchange rate from purchasing power parity correct at the speed of the price stickiness (Calvo) parameter. In the latter case, the result is

$$q_t|\text{Exit} = (1-\omega)\lambda^{-1}b_t. \tag{4.8}$$

such that, upon exit, the real exchange rate is determined by the outstanding debt—a higher debt level implying a more depreciated real exchange rate. In sum, if prices are flexible (the previous subsection), the nominal exchange rate adjusts after exit. This drives up the nominal interest rate in regime Crisis. If prices are sticky, the real exchange rate depreciates alongside the nominal exchange rate after exit. Prior to exit, the real interest rate increases and induces a decline in economic activity.

Lastly, Proposition 3 also shows that in the presence of exit expectations, public debt raises domestic prices. Intuitively, forward looking firms set their prices with an eye on current economic activity as well as in anticipation of future inflation. In turn, future inflation is

[17] In our complete-markets setup, default also leaves the households' balance sheet unaffected. This continues to hold in the incomplete markets model studied in Section 5.2, to the extent that all public debt is held domestically (see the derivation of this model in the Appendix A). This is because in this case, the loss in assets by households due to default is exactly offset by the decline in future taxation.

determined by expectations of future policies. Formally, the New Keynesian Phillips curve (3.2) implies

$$\pi_{H,t} = \beta[f\pi_{H,t+1}|\text{Reform} + \delta\pi_{H,t+1}|\text{Default} + \epsilon\pi_{H,t+1}|\text{Exit}] + \kappa\varpi^{-1}y_t, \tag{4.9}$$

where we have used $\varphi = 0$ and again the Markov chain above. Inflation after exit is positive as debt exceeds its sustainable level. Namely, conditional on Exit we obtain

$$\pi_{H,t}|\text{Exit} = \sqrt{\kappa/(1-\beta\phi)}\ \lambda^{-1}b_t, \quad \phi < 1. \tag{4.10}$$

Therefore, inflation may rise already in the initial regime, reflecting the expected inflation after exit which raises future marginal costs. This is in line with earlier studies, which have emphasized that expected devaluation in fixed exchange rate arrangements tends to erode competitiveness (Obstfeld, 1994, 1997).

We conclude that exit expectations have a recessionary effect to the extent that the real exchange rate is expected to depreciate after exit. At the same time, inflation rises (somewhat) already prior to exit, implying an erosion of competitiveness in the initial regime. Below we assess the robustness of these results by discussing the possibility of a "break" in the Phillips curve in the period of exit. This addresses concerns that price stickiness may not be an invariant parameter vis-à-vis such fundamental policy changes. We will show that exit expectations have a stagflationary impact, to the extent that *some* price stickiness is expected to persists after exit, such that the real exchange rate depreciates upon exit. This is in line with empirical studies, which find that large devaluations tend to be associated with sizeable real depreciations (Burstein et al., 2005).

5 Quantitative analysis

In this section we pursue two objectives. First, we assess the quantitative relevance of exit expectations—how strongly do they impact sovereign debt crises? Second, we explore the robustness of our results as we study two model extensions that appear particularly relevant in the context of our analysis: i) the possibility of balance sheet effects of a currency depreciation, ii) the possibility of a break in the Phillips curve, that is, a change in price rigidity as the country exits the currency union.

5.1 Quantitative relevance of exit expectations

In what follows we present results based on model simulations. Specifically, we employ the algorithm developed in Farmer et al. (2011) to solve our MS-LRE model numerically. To do so we fix parameters at conventional values.

Table 2: Parameters values used in model simulation.

β	φ	ω	γ	ξ	σ	λ	ψ_{Crisis}	ψ_{Reform}	ϕ	f	δ	ϵ
0.99	4	0.3	11	0.75	0.9	3.532	0.0075	0.02	0.8	0.15	0.075	0.05

We assume that a time period in the model corresponds to one quarter and set $\beta = 0.99$. We set $\varphi = 4$. This implies a moderate Frisch elasticity of labor supply (Chetty et al., 2011). For the import share in steady state, ω, we assume a value of 0.3. We set $\gamma = 11$ such that the steady-state mark up is equal to 10 percent. For the trade-price elasticity we assume $\sigma = 0.9$ (Heathcote and Perri, 2002). We assume an average price duration of four quarters and set $\xi = 0.75$. We assume that the sustainable level of debt in steady state is 90 percent of annual GDP (Reinhart and Rogoff, 2011). This implies $\lambda = 3.532$.[18] Regarding active fiscal policy we assume $\psi_{\text{Crisis}} = 0.0075 < 1-\beta$. For the passive regime we assume $\psi_{\text{Reform}} = 0.02 > 1-\beta$. To characterize monetary policy in regime Exit we set $\phi = 0.8$.[19]

We also need to pin down the probabilities of regime change. Start with the most important probability in our context, ϵ. To pin down this probability, we use the evidence from the online betting platform intrade, shown in Figure 1 above. The probability of an exit of at least one country from the euro zone fluctuates somewhat over time, but remains reasonably close to a mean value of about 5 percent. Hence, we set $\epsilon = 0.05$. We assume that the probability of default is somewhat higher at $\delta = 0.075$. Finally, we set $f = 0.15$, a (per-quarter) probability of (successful) fiscal reform of 15 percent.[20] Overall, our choice of probabilities is conservative in the sense that we assume exit to be the least likely scenario. Table 2 summarizes the parameter values used in the model simulations.

We solve the model numerically and study the dynamics of endogenous variables. We assume throughout that the economy is initially off steady state: public debt exceeds the sustainable level by 10 percentage points, such that debt initially equals 100, rather than 90 percent of GDP. Figure 2 shows how public debt and sovereign yields evolve as a result. Here we focus on regime Crisis: assuming that no regime switch occurs, we show the evolution under our baseline parameters (red solid line), as well as for a counterfactual for which we set $\epsilon = 0$ (blue dashed line). The difference between the two lines therefore isolates the effect of exit expectations. We find that the impact of exit expectations on the dynamics of public

[18] Recall that $\lambda = B/P$ corresponds to the (real) stock of debt in steady state. Quarterly output in steady state is $Y = ((\gamma-1)/\gamma)^{1+\varphi}$ and hence $\lambda = 0.9 \times 4 \times Y$.

[19] Recall that, because fiscal policy continues to be active as the country leaves the currency union, it is required that $\phi < 1$, for otherwise the central bank after exit generates a hyper-inflation (Loyo, 2000).

[20] The exact numerical values of δ and f are not crucial for our results. δ does not directly impact economic activity, but only the speed at which public debt accumulates. f, in turn, does not even matter for public finances. A high value is however necessary to ensure the existence of an equilibrium, see Section 4.

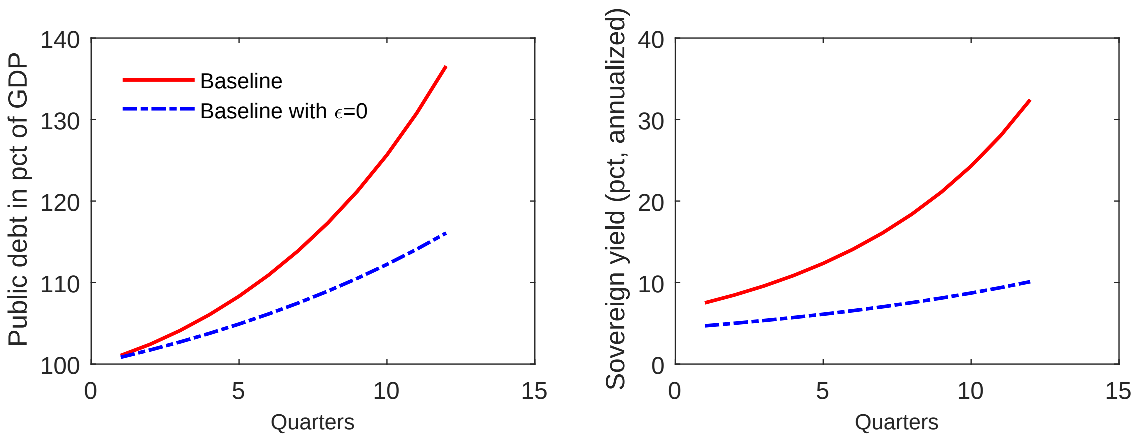

Figure 2: Public debt and sovereign yield in regime Crisis. Initial debt exceeds steady state level by 10 percentage points. Solid line: baseline model; dashed line: no exit expectations.

debt and sovereign yields is strong: over the course of three years, public debt rises by some 35 percentage points with exit expectations, but only by 15 percentage points if there are none. Similarly, exit expectations also account for an increase of (annualized) sovereign yields of some 20 percentage points over the same period.

Figure 3 shows the evolution of output, (producer price) inflation, the nominal exchange rate, and the real exchange rate. Again the red solid line shows the dynamics in regime Crisis, assuming that no regime change takes place even though the probability of regime change is non-zero. Instead, the blue dashed line shows the time path in case exit actually materializes (see Eggertsson and Woodford, 2003).[21]

In line with our theoretical analysis in Section 4, output (upper-left panel) declines in regime Crisis as a result of exit expectations. After three years, it has declined by some 3 percent if the crisis is not resolved. At the same time, inflation (upper-right panel) increases in regime Crisis. The effect is less than one percent annualized and pales in comparison to the effects which are observed in the event of exit. Still, the real exchange rate (lower-right panel) appreciates for as long as regime Crisis lasts. This reflects the increase of inflation because the nominal exchange rate (lower-left panel) is fixed in regime Crisis.

In the event of exit, the nominal exchange rate depreciates strongly, driving domestic inflation. The effect becomes larger, the later the exit materializes, because more debt has been built up and the real exchange rate has appreciated by more in regime Crisis. If exit takes place after three years, the new currency depreciates by some 50 percent on impact and (annualized) inflation is 100 percent. As a result, the real exchange rate depreciates upon

[21] For readability, we consider a possible exit only every second period, even though exit is possible in each period.

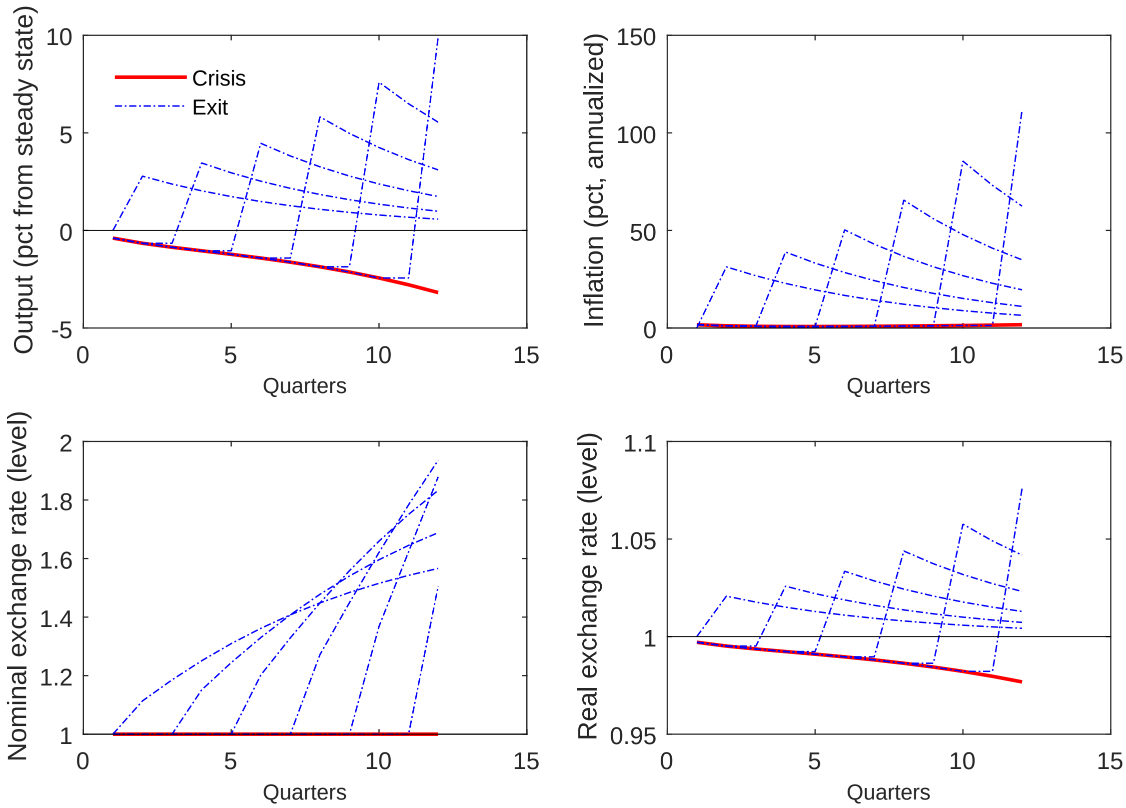

Figure 3: Output, inflation, nominal and real exchange rate in regimes Crisis (red solid) and Exit (blue dashed). Initial debt exceeds steady state level by 10 percentage points.

exit.[22] Yet, under our choice of parameters, the real depreciation is only about 1/10 of the nominal depreciation.[23] The real depreciation, in turn, boosts demand for domestic output, which rebounds and overshoots its steady-state level considerably after exit.

Our main result is robust with respect to variations in parameter values. Exit expectations impact sovereign debt crises profoundly: not only do they reinforce the crisis dynamics, they also induce public debt to have a sizeable stagflationary effect. Our parameter choices turn out to be conservative: the effects become stronger for larger values of σ, ξ and ϕ, and for a fiscal policy coefficient ψ_{Crisis} closer to zero. In what follows we explore the robustness of our results with respect to alternative modeling assumptions.

[22]Recall that inflation rates are annualized. On a quarterly basis, the inflation is 25 percent, clearly lower than the 50 percent nominal depreciation on impact. As a result, the real exchange rate depreciates.

[23]This is a moderate pass through: according to Burstein et al. (2005), real exchange rates tend to mirror up to 90 percent their nominal counterparts during large devaluations.

5.2 Robustness and model extensions

We now consider two modifications of our baseline model which may have a strong bearing on our results.[24] First, we consider the possibility of a structural break in the Phillips curve upon exit. This addresses concerns that the degree of price stickiness may be affected by a currency changeover. Formally, we let price stickiness change across regimes and assume, more specifically, a lower price-stickiness parameter for regime Exit: $\xi_{\text{Exit}} \leq \xi$. In the other regimes the stickiness parameter remains unchanged. In regime Crisis, inflation dynamics are governed by the following "generalized Phillips curve", see the Appendix A:

$$\begin{aligned}\pi_{H,t} &= \beta[\mu\pi_{H,t+1}|\text{Crisis} + f\Omega\pi_{H,t+1}|\text{Reform} + \delta\Omega\pi_{H,t+1}|\text{Default} + \epsilon\Omega'\pi_{H,t+1}|\text{Exit}] \\ &\quad + \kappa\left(\varphi + \varpi^{-1}\right)\Omega y_t. \end{aligned} \tag{5.1}$$

where $\mu := 1 - f - \delta - \epsilon$ measures the persistence of the initial regime. The two correction factors Ω and Ω' are given by

$$\begin{aligned}\Omega &= \frac{(1-\beta\mu\xi)(1-\beta\xi_{\text{Exit}})}{(1-\beta\xi)(1-\beta\xi_{\text{Exit}}) + (1-\beta\xi)\beta\epsilon\xi_{\text{Exit}} + (1-\beta\xi_{\text{Exit}})\beta(f+\delta)\xi} \\ \Omega' &= \frac{\xi_{\text{Exit}}}{\xi}\frac{1-\xi}{1-\xi_{\text{Exit}}}\frac{1-\beta\xi}{1-\beta\xi_{\text{Exit}}}\Omega.\end{aligned}$$

One special case allows for a clear interpretation: in the case of $\xi_{\text{Exit}} = 0$, that is, if prices become fully flexible after exit, we have $\Omega = (1-\beta\mu\xi)/(1-\beta(\mu+\epsilon)\xi)$ and $\Omega' = 0$. Therefore, the Phillips curve in the initial regime becomes steeper, the larger the probability of an exit (Ω increases in ϵ)—as this effectively reduces price stickiness. At the same time, the firms' pricing decisions in the initial regime are entirely unaffected by developments after exit ($\Omega' = 0$). This is because firms anticipate that, once the exit occurs, they will be able to optimally re-adjust their prices.

We simulate the model with the generalized Phillips curve (5.1) instead of the conventional Phillips curve (3.2). All parameters are as in the previous section. We set $\xi_{\text{Exit}} = 0.5$, that is, we assume that 50 percent of firms may adjust prices upon exit (rather than only 25 percent as in the other regimes). We do not allow all firms to adjust prices upon exit since in this case, the real exchange rate depreciation equals zero even as the nominal exchange rate depreciates substantially, which appears counterfactual in light of the results of Burstein et al. (2005). The red lines in Figure 4 show the results. As expected, the quantitative effect of exit expectations in regime Crisis are now reduced relative to the baseline case (see Figure 3). However, they are still sizable.

[24] Details on both extensions are provided in the Appendix A.

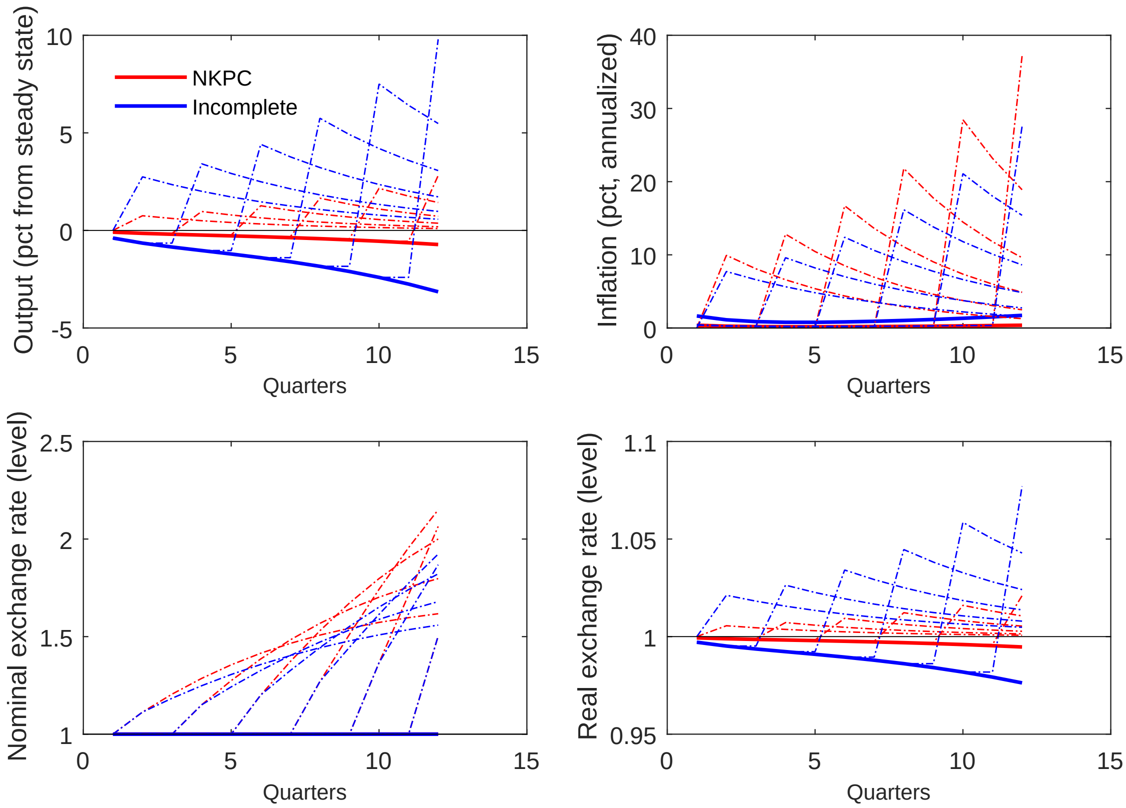

Figure 4: Output, inflation, nominal and real exchange rate, robustness to model extensions. Details as in Figure 3. In case of NKPC, we let $\xi = 0.75$ decline to $\xi_{\text{Exit}} = 0.5$ after exit.

Finally, we relax the assumption that international asset markets are complete. This assumption is useful to obtain analytical expressions.[25] Yet, it rules out balance sheet effects of depreciation which may otherwise obtain in the event of exit. To account for this possibility we assume that households trade only a nominal non-contingent bond internationally. In order to close the model we assume an endogenous discount factor (Schmitt-Grohé and Uribe, 2003).

Moreover, and in contrast to public debt, we assume that private external debt is denominated in foreign currency after exit.[26] As a result, whenever there is a negative net asset position, nominal depreciation entails a negative balance sheet effect, because the real value of external debt increases in terms of domestic currency.

For our simulation, we assume a net foreign asset position of minus 30 percent of (annual) GDP in steady state. We thereby allow for a potentially large balance sheet effect. Because

[25] For, in this case, the stock of net foreign assets is not a relevant state variable.

[26] This was actual practice, for instance, in Greece during 2009–2012, as most of Greek private cross-border debts were issued under foreign law (Buiter and Rahbari, 2012).

this effect is anticipated, it may alter the way exit expectations impact the dynamics in regime Crisis. It turns out, however, that the predictions of the incomplete markets model, shown in blue in Figure 4, are close to those of the baseline model. Intuitively, the balance sheet effect is a purely temporary shock (see also Appendix A). As is well known, whether international financial markets are complete or incomplete is of little consequence in this case (Baxter and Crucini, 1995). Hence, we conclude that balance sheet effects play a limited quantitative role in our environment.

6 Conclusion

Membership in a currency union is not irreversible. Expectations of an exit may arise during sovereign debt crises, because exit allows countries to reduce their liabilities through redenomination, inflation, and depreciation. In this paper, we ask how this possibility impacts on the sovereign debt crisis, and on macroeconomic stability more generally. In our analysis we show that, as market participants anticipate the possibility of exit and depreciation, the debt crisis intensifies, and public debt has a stagflationary impact on the economy.

We build a model of a small open economy with rich debt dynamics and allow for changing policy regimes. Regime change is exogenous, allowing us to derive our results analytically. We focus on a country which operates inside a currency union and experiences a sovereign debt crisis: public debt is on a explosive, non-sustainable path. Through fiscal reform, exiting the union or, alternatively, applying a haircut to government debt, fiscal policy can resolve the debt crisis. Market participants are aware of these possibilities and expectations of exit and default matter for the equilibrium outcome. In particular, exit expectations drive up yields of securities issued under domestic law, both public and private, provided the exchange rate is expected to depreciate after exit. As (real) interest rates rise, economic activity declines. Expectations of outright default, instead, only drive up sovereign yields.

We assess the quantitative relevance of our findings through model simulations. Specifically, we assume empirically plausible parameter values and solve the regime-switching model numerically. We consider a scenario where debt is initially non-sustainable and the probability of exit from the union is 5 percent per quarter. We find that exit expectations account for a rise in public debt of some 20 percentage points of GDP during a three-year period, coupled with a rise in sovereign yields of a similar magnitude. Output, in turn, declines by about 3 percent whereas (annualized) inflation increases by 1 percent. Finally, we assess the robustness of these results by considering a model extension which allows for balance sheet effects of depreciation, and for a "break" in the Phillips curve in the period of exit.

While our analysis is silent on the benefits and costs of an actual exit, it makes transparent how the adverse dynamics of a sovereign debt crisis within a currency union may intensify in the presence of exit expectations. Our findings are thus in line with a more general insight: policy frameworks which lack credibility tend to generate inferior outcomes.

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A Model appendix

A.1 Deriving the present value budget constraint

Here we derive the present value budget constraint, equation

$$(1-\Theta_t)\frac{B_t}{P_t} = \sum_{j=0}^{\infty}\beta^j E_t\left(\frac{C_{t+j}}{C_t}\right)^{-1}\tau_{t+j}. \tag{A.1}$$

from Section 2. The exposition follows closely Uribe (2006). First we define $\tilde{B}_t := I_t^{-1}B_{t+1}$, yielding the flow budget constraint

$$\tilde{B}_t = I_{t-1}\tilde{B}_{t-1}(1-\Theta_t) - P_t\tau_t.$$

Multiplying the left and right-hand side with $I_t(1-\Theta_{t+1})$, and iterating forward j periods, the budget constraint becomes

$$\begin{aligned}
&I_{t+j}\tilde{B}_{t+j}(1-\Theta_{t+j+1})\\
&\quad=\left(\prod_{h=0}^{j} I_{t+h}(1-\Theta_{t+h+1})\right) I_{t-1}\tilde{B}_{t-1}(1-\Theta_t) - \sum_{h=0}^{j}\left(\prod_{k=h}^{j} I_{t+k}(1-\Theta_{t+k+1})\right) P_{t+h}\tau_{t+h}.
\end{aligned}$$

Now divide both sides by P_{t+j+1} and multiply by $(C_{t+j+1}/C_t)^{-1}$

$$\begin{aligned}
&I_{t+j}\frac{\tilde{B}_{t+j}}{P_{t+j+1}}\left(\frac{C_{t+j+1}}{C_t}\right)^{-1}(1-\Theta_{t+j+1})\\
&\qquad=\left(\prod_{h=0}^{j} I_{t+h}\frac{P_{t+h}}{P_{t+h+1}}\left(\frac{C_{t+h+1}}{C_{t+h}}\right)^{-1}(1-\Theta_{t+h+1})\right) I_{t-1}\frac{\tilde{B}_{t-1}}{P_t}(1-\Theta_t)\\
&\qquad\quad-\sum_{h=0}^{j}\left(\prod_{k=h}^{j} I_{t+k}\frac{P_{t+k}}{P_{t+k+1}}\left(\frac{C_{t+k+1}}{C_{t+k}}\right)^{-1}(1-\Theta_{t+k+1})\right)\left(\frac{C_{t+h}}{C_t}\right)^{-1}\tau_{t+h}.
\end{aligned}$$

Now take conditional time$-t$ expectations E_t on both sides, use the law of iterated expectations $E_t(\cdot) = E_t(E_{t+h}(\cdot)),\ h \geq 0$, and exploit that

$$\beta E_t I_t\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-1}(1-\Theta_{t+1}) = 1$$

to arrive at

$$\begin{aligned}
&E_t I_{t+j}\frac{\tilde{B}_{t+j}}{P_{t+j+1}}\left(\frac{C_{t+j+1}}{C_t}\right)^{-1}(1-\Theta_{t+j+1})\\
&\qquad\qquad= \beta^{-j-1}I_{t-1}\frac{\tilde{B}_{t-1}}{P_t}(1-\Theta_t) - \sum_{h=0}^{j}\beta^{-h-j-1}E_t\left(\frac{C_{t+h}}{C_t}\right)^{-1}\tau_{t+h}.
\end{aligned}$$

Finally, multiply both sides by β^{j+1}, take the limit $j \to \infty$ and use the transversality condition

$$\lim_{j\to\infty} \beta^{j+1} E_t I_{t+j} \left(\frac{C_{t+j+1}}{C_t}\right)^{-1} (1-\Theta_{t+j+1}) \frac{\tilde{B}_{t+j}}{P_{t+j+1}} = 0$$

to arrive at

$$(1-\Theta_t) I_{t-1} \frac{\tilde{B}_{t-1}}{P_t} = \sum_{h=0}^{\infty} \beta^h E_t \left(\frac{C_{t+h}}{C_t}\right)^{-1} \tau_{t+h}.$$

Substituting back $B_t = \tilde{B}_{t-1} I_{t-1}$ yields expression (A.1).

A.2 Linearizing the model

Here we provide details on the linearization of our model introduced in Section 2. Lower-case letters denote log deviation of upper case letters from steady state, absolute deviation (scaled by the price level) in case of public debt and taxes. We linearize around purchasing power parity, zero inflation and zero default. Variables in the rest of the world are constant. Public debt can be non-zero in steady state, parameterized by $\lambda = \tau/(1-\beta) \geq 0$.

Log-linearizing the Euler equation $R_t = 1/\{E_t \rho_{t,t+1}\}$, the labor supply curve $W_t/P_t = C_t H_t^{\varphi}$ and the risk sharing condition $C_t/C^* = Q_t$ yields the conditions

$$c_t = E_t c_{t+1} - (r_t - E_t \pi_{t+1}) \tag{A.2}$$

$$w_t^r := w_t - p_t = c_t + \varphi h_t, \tag{A.3}$$

$$c_t = q_t, \tag{A.4}$$

where $\pi_t := p_t - p_{t-1}$ is CPI inflation. We approximate the real exchange rate $Q_t = (P^* \mathcal{E}_t)/P_t$ and the consumer price index $P_t = ((1-\omega) P_{H,t}^{1-\sigma} + \omega(\mathcal{E}_t P^*)^{1-\sigma})^{1/(1-\sigma)}$ as

$$q_t = e_t - p_t \tag{A.5}$$

$$p_t = (1-\omega) p_{H,t} + \omega e_t. \tag{A.6}$$

Aggregate demand $Y_t = (P_{H,t}/P_t)^{-\sigma}[(1-\omega)C_t + \omega Q_t^{\sigma} C^*]$ can be approximated by

$$y_t = -\sigma(p_{H,t} - p_t) + (1-\omega) c_t + \omega \sigma q_t,$$

which, combined with (A.5) and (A.6), can be written as

$$y_t = (1-\omega) c_t + \omega\sigma(2-\omega)/(1-\omega) q_t. \tag{A.7}$$

The aggregate supply block can be written as a New Keynesian Phillips curve

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa m c_t, \tag{A.8}$$

where marginal costs $MC_t = W_t/P_{H,t}$ are approximated as

$$mc_t = w_t - p_{H,t} = w_t^r - (p_{H,t} - p_t). \tag{A.9}$$

Furthermore, production technology $Y_t = H_t$ can be approximated as

$$y_t = h_t. \tag{A.10}$$

The policy rules $\mathcal{E}_t = 1$ and $(R_t^{-1}/\beta) = \Pi_{H,t}^{-\phi}$ can be readily log-linearized as

$$e_t = 0 \tag{A.11}$$

as well as

$$r_t = \phi\pi_{H,t}. \tag{A.12}$$

The government's flow budget constraint can be written as

$$\beta\frac{(I_t)^{-1}}{\beta}\frac{B_{t+1}}{P_{H,t}} = (1-\Theta_t)\frac{B_t}{P_{H,t-1}}\frac{P_{H,t-1}}{P_{H,t}} - \tau_t.$$

We log-linearize around $I = 1/\beta$ as well as $1 - \Theta = 1$, we linearize around $B/P_H = \lambda$ and $T/P_H = (1-\beta)\lambda$ to obtain

$$\beta b_{t+1} = (1-\psi)b_t + \lambda(\beta i_t - \pi_{H,t} - \theta_t), \tag{A.13}$$

where we denote $-\theta_t := \log(1-\Theta_t)$, and where we have used that the tax rule is already in linear form: $\tau_t - \tau = \psi b_t$. Using Euler equation (A.2), the bond price schedule $1 = I_t E_t \rho_{t,t+1}(1-\Theta_{t+1})$ can be log-linearized to

$$i_t = r_t + E_t\theta_{t+1}. \tag{A.14}$$

Finally, the condition $B_{t+1}/P_{H,t} = \lambda$ can be written in linearized terms as

$$b_{t+1} = 0, \tag{A.15}$$

whereas the zero-default condition $1 - \Theta_t = 1$, by using that $-\theta_t = \log(1-\Theta_t)$ as defined above, becomes

$$\theta_t = 0. \tag{A.16}$$

A.3 Equations in Section 3

Here we derive the set of equations that are shown in Section 3. First, equations (A.13)-(A.14) correspond to equations (3.5)-(3.6) from the text. The policy equations (A.11)-(A.12) and (A.15)-(A.16) correspond to (3.7)-(3.8) in the text. Next, equation (3.4) is just the combination of equations (A.5) and (A.6). The three equations (3.1), (3.2) and (3.3) are obtained as follows. Insert risk sharing (A.4) into goods market clearing (A.7) to obtain equation (3.3). Rewrite the Euler equation (A.2) as

$$\begin{aligned} c_t &= E_t c_{t+1} - (r_t - E_t[(1-\omega)\pi_{H.t+1} + \omega \Delta e_{t+1})] \\ &= E_t c_{t+1} - (r_t - E_t \pi_{H,t+1} - \frac{\omega}{\varpi} E_t \Delta y_{t+1}), \end{aligned}$$

where we use (A.6) in the first line and (3.3) and (3.4) from the main text in the second line. Combine (A.4) and (3.3) to obtain

$$c_t = \frac{1-\omega}{\varpi} y_t.$$

Use this expression to substitute for consumption in the Euler equation above to obtain

$$y_t = E_t y_{t+1} - \varpi (r_t - E_t \pi_{H,t+1}),$$

which is equation (3.1). Use equations (A.3), (A.4), (A.5), (A.6) and production technology (A.10) to rewrite marginal cost

$$mc_t^r = w_t^r - (p_{H,t} - p_t) = c_t + \varphi h_t - (p_{H,t} - p_t) = (\varpi^{-1} + \varphi) y_t.$$

Insert this into the Phillips curve to obtain equation (3.2) in the text.

A.4 Model extension I: A break in the Phillips curve

Here we derive the generalized Phillips curve discussed in Section 5. Denote ξ_{Exit} the probability that a firm may not adjust its price in regime Exit, while ξ denotes this probability in all other regimes (as in the baseline model). Denote $\mu := 1 - f - \delta - \epsilon$ the probability of remaining in the initial regime. Denote the k-step-ahead nominal stochastic discount factor $\rho_{t,t+k} := \beta^k (C_{t+k}/C_t)^{-1} (P_t/P_{t+k})$.

The maximization problem of firm j in the initial regime can be written as

$$\begin{aligned} \max \; & \sum_{k=0}^{\infty} (\mu\xi)^k \rho_{t,t+k} \left(P_{H,t}(j) Y_{t+k|t}(j) - \mathcal{C}(Y_{t+k|t}(j)) \right) |\text{Crisis} \\ & + \sum_{i=1}^{\infty} \sum_{k=i}^{\infty} (\mu\xi)^{i-1} f \xi^{k-i+1} \rho_{t,t+k} \left(P_{H,t}(j) Y_{t+k|t}(j) - \mathcal{C}(Y_{t+k|t}(j)) \right) |\text{Reform in } t+i \\ & + \sum_{i=1}^{\infty} \sum_{k=i}^{\infty} (\mu\xi)^{i-1} \delta \xi^{k-i+1} \rho_{t,t+k} \left(P_{H,t}(j) Y_{t+k|t}(j) - \mathcal{C}(Y_{t+k|t}(j)) \right) |\text{Default in } t+i \\ & \quad + \sum_{i=1}^{\infty} \sum_{k=i}^{\infty} (\mu\xi)^{i-1} \varepsilon \xi_{\text{Exit}}^{k-i+1} \rho_{t,t+k} \left(P_{H,t}(j) Y_{t+k|t}(j) - \mathcal{C}(Y_{t+k|t}(j)) \right) |\text{Exit in } t+i, \end{aligned}$$

where the cost and demand functions $\mathcal{C}(Y_{t+k|t}(j))$ and $Y_{t+k|t}(j)$ are as in the baseline model, and where we have split the expectation operator into realizations of the Markov chain (conditional on having switched regimes, as this model has no fundamental shocks, all uncertainty is resolved). Keeping track of the time of the switch is important, since it determines when the shift in rigidity occurs. The first order condition with respect to the reset price $P_{H,t}(j)$ can be written as

$$\begin{aligned}
0 = &\sum_{k=0}^{\infty}(\mu\xi)^k \rho_{t,t+k} Y_{t,t+k}(j)\left(P_{H,t}(j) - \frac{\gamma}{\gamma-1}W_{t+k}\right)|\text{Crisis}\\
&+\sum_{i=1}^{\infty}(\mu\xi)^{i-1} f\xi^{1-i}\sum_{k=i}^{\infty}\xi^k \rho_{t,t+k} Y_{t,t+k}(j)\left(P_{H,t}(j) - \frac{\gamma}{\gamma-1}W_{t+k}\right)|\text{Reform in } t+i\\
&+\sum_{i=1}^{\infty}(\mu\xi)^{i-1} \delta\xi^{1-i}\sum_{k=i}^{\infty}\xi^k \rho_{t,t+k} Y_{t,t+k}(j)\left(P_{H,t}(j) - \frac{\gamma}{\gamma-1}W_{t+k}\right)|\text{Default in } t+i\\
&+\sum_{i=1}^{\infty}(\mu\xi)^{i-1} \varepsilon\xi_{\text{Exit}}^{1-i}\sum_{k=i}^{\infty}\xi_{\text{Exit}}^k \rho_{t,t+k} Y_{t,t+k}(j)\left(P_{H,t}(j) - \frac{\gamma}{\gamma-1}W_{t+k}\right)|\text{Exit in } t+i.
\end{aligned}$$

Using the fact that all resetting firms choose the same reset price we denote $\tilde{P}_{H,t} = P_{H,t}(j)$. We linearize the expressions inside the sums running over k to obtain

$$\begin{aligned}
0 = &\frac{\tilde{p}_{H,t} - p_{H,t-1}}{1-\beta\mu\xi} - \sum_{k=0}^{\infty}(\beta\mu\xi)^k (mc_{t+k} + p_{H,t+k} - p_{H,t-1})|\text{Crisis}\\
&+\sum_{i=1}^{\infty}(\mu\xi)^{i-1} f\xi^{1-i}\left(\frac{(\beta\xi)^i(\tilde{p}_{H,t} - p_{H,t-1})}{1-\beta\xi} - \sum_{k=i}^{\infty}(\beta\xi)^k (mc_{t+k} + p_{H,t+k} - p_{H,t-1})|\text{Reform in } t+i\right)\\
&+\sum_{i=1}^{\infty}(\mu\xi)^{i-1} \delta\xi^{1-i}\left(\frac{(\beta\xi)^i(\tilde{p}_{H,t} - p_{H,t-1})}{1-\beta\xi} - \sum_{k=i}^{\infty}(\beta\xi)^k (mc_{t+k} + p_{H,t+k} - p_{H,t-1})|\text{Default in } t+i\right)\\
&+\sum_{i=1}^{\infty}(\mu\xi)^{i-1} \varepsilon\xi_{\text{Exit}}^{1-i}\left(\frac{(\beta\xi_{\text{Exit}})^i(\tilde{p}_{H,t} - p_{H,t-1})}{1-\beta\xi_{\text{Exit}}} - \sum_{k=i}^{\infty}(\beta\xi_{\text{Exit}})^k (mc_{t+k} + p_{H,t+k} - p_{H,t-1})|\text{Exit in } t+i\right),
\end{aligned}$$

where we have replaced $mc_t = w_t - p_{H,t}$ (equation (A.9)). We note that

$$\begin{aligned}
&\frac{1}{1-\beta\mu\xi} + \frac{1}{1-\beta\xi}\sum_{i=1}^{\infty}(\mu\xi)^{i-1}(f+\delta)\xi^{1-i}(\beta\xi)^i + \frac{1}{1-\beta\xi_{\text{Exit}}}\sum_{i=1}^{\infty}(\mu\xi)^{i-1}\varepsilon\xi_{\text{Exit}}^{1-i}(\beta\xi_{\text{Exit}})^i\\
&\qquad = \frac{(1-\beta\xi)(1-\beta\xi_{\text{Exit}}) + (1-\beta\xi)\beta\varepsilon\xi_{\text{Exit}} + (1-\beta\xi_{\text{Exit}})\beta\delta\xi}{(1-\beta\mu\xi)(1-\beta\xi)(1-\beta\xi_{\text{Exit}})} = \frac{1}{(1-\beta\xi)\Omega},
\end{aligned}$$

where Ω is defined as in the text. This allows us to factorize $\tilde{p}_{H,t} - p_{H,t-1}$ from the previous expression, leading to

$$\tilde{p}_{H,t} - p_{H,t-1} = (1-\beta\xi)\Omega \ \{ \ \sum_{k=0}^{\infty}(\beta\mu\xi)^k (mc_{t+k} + p_{H,t+k} - p_{H,t-1})|\text{Crisis}$$
$$+ \sum_{i=1}^{\infty}(\mu\xi)^{i-1} f\xi^{1-i} \sum_{k=i}^{\infty}(\beta\xi)^k (mc_{t+k} + p_{H,t+k} - p_{H,t-1})|\text{Reform in } t+i$$
$$+ \sum_{i=1}^{\infty}(\mu\xi)^{i-1} \delta\xi^{1-i} \sum_{k=i}^{\infty}(\beta\xi)^k (mc_{t+k} + p_{H,t+k} - p_{H,t-1})|\text{Default in } t+i$$
$$+ \sum_{i=1}^{\infty}(\mu\xi)^{i-1} \varepsilon\xi_{\text{Exit}}^{1-i} \sum_{k=i}^{\infty}(\beta\xi_{\text{Exit}})^k (mc_{t+k} + p_{H,t+k} - p_{H,t-1})|\text{Exit in } t+i \ \}. \quad \text{(A.17)}$$

Equation (A.17) needs to be written recursively. To see how this can be done, assume that regime change occurs at time $t+1$. Consider the example of shifting to regime Exit. In this case, conditional on the regime having changed, we obtain at $t+1$

$$p^*_{H,t+1} - p_{H,t} = (1-\beta\xi_{\text{Exit}}) \sum_{k=0}^{\infty}(\beta\xi_{\text{Exit}})^k (mc_{t+1+k} + p_{H,t+1+k} - p_{H,t})|\text{Exit in } t+1.$$

Similar expressions hold in regimes Reform and Default in period $t+1$. This allows us to rewrite equation (A.17) as

$$\tilde{p}_{H,t} - p_{H,t-1} = \pi_{H,t} + (1-\beta\xi)\Omega \ \{ \ mc_t + \frac{f\beta\xi}{1-\beta\xi}[p^*_{H,t+1} - p_{H,t}|\text{Reform in } t+1]$$
$$+ \frac{\delta\beta\xi}{1-\beta\xi}[p^*_{H,t+1} - p_{H,t}|\text{Default in } t+1] + \frac{\varepsilon\beta\xi_{\text{Exit}}}{1-\beta\xi_{\text{Exit}}}[p^*_{H,t+1} - p_{H,t} \ |\text{Exit in } t+1]$$
$$+ \beta\mu\xi \ \{ \ \sum_{k=0}^{\infty}(\beta\mu\xi)^k (mc_{t+1+k} + p_{H,t+1+k} - p_{H,t})|\text{Crisis}$$
$$+ \sum_{i=1}^{\infty}(\mu\xi)^{i-1} f\xi^{1-i} \sum_{k=i}^{\infty}(\beta\xi)^k (mc_{t+1+k} + p_{H,t+1+k} - p_{H,t})|\text{Reform in } t+1+i$$
$$+ \sum_{i=1}^{\infty}(\mu\xi)^{i-1} \delta\xi^{1-i} \sum_{k=i}^{\infty}(\beta\xi)^k (mc_{t+1+k} + p_{H,t+1+k} - p_{H,t})|\text{Default in } t+1+i$$
$$+ \sum_{i=1}^{\infty}(\mu\xi)^{i-1} \varepsilon\xi_{\text{Exit}}^{1-i} \sum_{k=i}^{\infty}(\beta\xi_{\text{Exit}})^k (mc_{t+1+k} + p_{H,t+1+k} - p_{H,t})|\text{Exit in } t+1+i \ \} \ \}.$$

The sums multiplying $\beta\mu\xi$ correspond to the sums in (A.17), only dated at time $t+1$. Because (A.17) is conditional on being in regime Crisis at time t, we can write

$$\tilde{p}_{H,t} - p_{H,t-1} = \pi_{H,t} + \beta\mu\xi([p^*_{H,t+1} - p_{H,t})|\text{Crisis}] + (1-\beta\xi)\Omega \ \{mc_t$$
$$+ \frac{f\beta\xi}{1-\beta\xi}[(p^*_{H,t+1} - p_{H,t})|\text{Reform}] + \frac{\delta\beta\xi}{1-\beta\xi}[(p^*_{H,t+1} - p_{H,t})|\text{Default}]$$
$$+ \frac{\varepsilon\beta\xi_{\text{Exit}}}{1-\beta\xi_{\text{Exit}}}[(p^*_{H,t+1} - p_{H,t})|\text{Exit}] \ \}, \quad \text{(A.18)}$$

where we have omitted "in $t+i$" because all future variables are now conditional on the shift ocurring (or not ocurring) at time $t+1$. In a last step, we use a standard property of Calvo pricing, which is that

$$\pi_{H,t} = (1-\xi_{\text{Exit}})(\tilde{p}_{H,t} - p_{H,t-1}), \quad \pi_{H,t} = (1-\xi)(\tilde{p}_{H,t} - p_{H,t-1}),$$

the first equation in *Exit*, the second in all other regimes. Insert this into (A.18) and rearrange to obtain the final expression

$$\pi_{H,t} = \beta \left[\mu\pi_{H,t+1}|\text{Crisis} + f\Omega\pi_{H,t+1}|\text{Reform} + \delta\Omega\pi_{H,t+1}|\text{Default} + \epsilon\Omega'\pi_{H,t+1}|\text{Exit}\right] + \frac{(1-\beta\xi)(1-\xi)}{\xi}\Omega mc_t,$$

where we define

$$\Omega' = \frac{\xi_{\text{Exit}}}{\xi}\frac{1-\xi}{1-\xi_{\text{Exit}}}\frac{1-\beta\xi}{1-\beta\xi_{\text{Exit}}}\Omega$$

as in the main text.

The model with the extended Phillips curve satisfies the same set of equations as the baseline model, however with the Phillips curve (3.2) in the initial regime replaced by the extended Phillips curve (5.1).

A.5 Model extension II: Balance-sheet effects of depreciation

Here we discuss the model extension from Section 5 where international financial markets are incomplete, hence giving rise to balance sheet effects of depreciation. As explained in the text, we assume that private external debt is in foreign currency.

There are two changes relative to the baseline model. First, the household budget constraint is replaced by

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di + \int_0^1 P_{F,t}(i)C_{F,t}(i)di + R_t^{-1}D_{t+1} + R_t^{*-1}\mathcal{E}_t D_{t+1}^* + I_t^{-1}B_{t+1} = W_tH_t + D_t + \mathcal{E}_t D_t^* + (1-\Theta_t)B_t + \mathcal{Y}_t - P_t\tau_t.$$

As in the baseline model, R_t denotes the interest rate on a bond in domestic currency. Foreign agents do not trade this bond in equilibrium, such that $D_{t+1} = 0$ at all times. In contrast, foreign agents do trade bonds denominated in their own currency, at price R_t^{*-1}. Finally, domestic households hold risky government debt at price I_t^{-1}.

Second, as is well understood, incomplete assets markets induce non-stationarity (a unit root) to small open economy models—the steady state level of D_{t+1}^* is indeterminate. To avoid this property, we follow Schmitt-Grohé and Uribe (2003) and introduce an endogenous discount factor as ($\alpha > 0$ a small positive number)

$$\beta_{t+1} = \beta\left(1 + \alpha(\tilde{D}^*_{t+1} - \lambda^*)\right)^{-1}\beta_t, \quad \beta_0 = 1.$$

The discount factor depends on the country's (aggregate) net foreign asset position, which in equilibrium equals the net foreign asset position at the individual level (that is, $\tilde{D}^*_{t+1} = D^*_{t+1}$). By the arguments put forward in Schmitt-Grohé and Uribe (2003), this discount factor guarantees a net foreign asset position of λ^* in steady state.

Household maximization implies the following set of Euler equations

$$\begin{aligned}
1 &= \beta_{t+1} R_t E_t M_{t,t+1} \\
1 &= \beta_{t+1} R^*_t E_t M_{t,t+1}(\mathcal{E}_{t+1}/\mathcal{E}_t) \\
1 &= \beta_{t+1} I_t E_t M_{t,t+1}(1 - \Theta_{t+1}).
\end{aligned}$$

where $M_{t,t+1} := (C_{t+1}/C_t)^{-1}(P_t/P_{t+1})$. Using similar steps as in Section A.2 these can be approximated as

$$c_t = E_t c_{t+1} - (r_t - E_t \pi_{t+1} - \alpha \hat{d}^*_{t+1}) \tag{A.19}$$

$$r_t = E_t e_{t+1} - e_t \tag{A.20}$$

$$i_t = r_t + E_t \theta_{t+1}, \tag{A.21}$$

where in equation (A.20) we use that $R^*_t = \beta^{-1}$ remains in steady state throughout. Up to this point, the incomplete markets and the complete markets model coincide except for the endogenous discount factor in equation (A.19) (in contrast, equations (A.20) and (A.21) are also part of the complete markets model—see equations (4.3) and (3.6)).

The key difference between the two models arises because the risk sharing condition (A.4) and hence condition (3.3) are not part of the equilibrium. Instead, we keep track of net foreign assets via the aggregate resource constraint

$$P_t C_t + (R^*_t)^{-1}\mathcal{E}_t D^*_{t+1} = P_{H,t} Y_t + \mathcal{E}_t D^*_t,$$

where we have used the price indexes and consumption-demand functions from the main text to rewrite $\int_0^1 P_{H,t}(i) C_{H,t}(i) di + \int_0^1 P_{F,t}(i) C_{F,t}(i) di = P_t C_t$, where we have used the equilibrium expressions for profits $\mathcal{Y}_t = P_{H,t} Y_t - W_t H_t$ and replaced the government budget constraint $I_t^{-1} B_{t+1} = B_t(1 - \Theta_t) - P_t \tau_t$. Note that, as government debt and taxes drop out of the household budget constraint, Ricardian equivalence always obtains in this model. We divide both sides by P_t to re-write this as

$$C_t + (R_t^*)^{-1} Q_t D_{t+1}^* = \frac{P_{H,t}}{P_t} Y_t + Q_t D_t^*,$$

where we have used that the real exchange rate $Q_t = \mathcal{E}_t/P_t$ by definition (recall that $P_t^* = 1$ by our normalization). This expressions shows how a real depreciation (a rise in Q_t) harms consumption to the extent that net foreign assets are negative ($D_t^* < 0$)—the balance sheet effect of depreciation.

If net foreign assets are zero in steady state, the balance sheet effect drops out up to first order. Hence, we must linearize around some $\lambda^* \neq 0$. From the last equation, this generally implies $C \neq Y$, such that the linearization is around a different steady state than in the complete markets model. To make results comparable, we keep the assumption of linearizing around a steady state of purchasing power parity: $Q = 1$, implying that also $P_H/P = 1$. The previous budget constraint then implies that $C = Y + (1 - \beta)\lambda^*$, where we use that $R_t^* = 1/\beta$.[1] We therefore obtain

$$\beta(d_{t+1}^* + q_t\lambda^*) + Cc_t = Y(y_t + p_{H,t} - p_t) + d_t^* + q_t\lambda^*, \tag{A.22}$$

where $Y = ((\gamma-1)/\gamma)^{1+\varphi}$ is pinned down by the supply side of the model, and where C was given above. Finally, the demand side of the economy must also be adjusted, such that goods market clearing (A.7) needs to be replaced by

$$Yy_t = (1-\omega)Cc_t - ((1-\omega)C + \omega C^*)\sigma(p_{H,t} - p_t) + C^*\omega\sigma q_t, \tag{A.23}$$

where C^* is pinned down by $Y = (1-\omega)C + \omega C^*$. All remaining equations are unchanged from the baseline model. The incomplete markets model can be summarized as a set of variables $\{y_t, \pi_{H,t}, p_{H,t}, \pi_t, p_t, e_t, q_t, i_t, r_t, d_{t+1}^*, b_{t+1}, \theta_t, w_t, h_t, mc_t, c_t\}$ that satisfy the sixteen equations (A.3), (A.5)-(A.6), equation (A.23), equations (A.8)-(A.10), one of the two policy rules (A.11)-(A.12), equations (A.13)-(A.14), one of the two policy rules (A.15)-(A.16), equations (A.19)-(A.20) and (A.22), and the definitions for inflation $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ and $\pi_t = p_t - p_{t-1}$, for given initial b_0 and d_0^*.

[1] The reader may wonder how we may simply assume that purchsing power parity holds in this steady state. From the demand function in steady state, $Y = (1-\omega)C + \omega C^*$, the underlying assumption is that C^* must adjust, that is, foreign consumption is the "tail of the dog".

B Propositions Appendix

B.1 Propositions 1-2

Here we provide the proof of Propositions 1-2. Consider the model in Section 3, but impose flexible prices $\xi = 0$. In this case the model collapses to

$$r_t = E_t \pi_{H,t+1} \tag{B.1}$$

$$e_t = p_{H,t} \tag{B.2}$$

$$\beta b_{t+1} = (1-\psi) b_t + \lambda(\beta i_t - \pi_{H,t} - \theta_t) \tag{B.3}$$

$$i_t = r_t + E_t \theta_{t+1} \tag{B.4}$$

as well as $y_t = q_t = 0$ and policy $r_t = \phi \pi_{H,t}$ or $e_t = 0$, and $b_{t+1} = 0$ or $\theta_t = 0$. We solve the model backwards by using the method of undetermined coefficients, thereafter we show that the solution derived is mean square stable whenever condition (4.1) holds.

In regime **Reform**, $e_t = 0$, such that from (B.2) $p_{H,t} = 0$ and therefore $\pi_{H,t} = 0$. Since $\theta_t = 0$ in this regime, and further regime change is ruled out, $r_t = i_t = 0$ from (B.1) and (B.4). Public debt hence evolves according to

$$\beta b_{t+1} = (1 - \psi_{\text{Reform}}) b_t,$$

and is mean-reverting by assumption ($\psi_{\text{Reform}} > 1 - \beta$).

Regime **Default** is identical to regime Reform, expect that fiscal policy is “active” such that debt has an explosive root ($\psi < 1-\beta$ by assumption). As a result, default θ_t must adjust such that $b_{t+1} = 0$ at all times, as argued in the main text. Hence the solution is $e_t = 0$, $p_{H,t} = 0$ and therefore $\pi_{H,t} = 0$ as before, as well as $\theta_t = ((1-\psi)/\lambda) b_t$ in the period upon entering the regime, as well as $b_{t+1} = 0$ and $\theta_{t+1} = 0$ in all periods thereafter.

There is no default in regime **Exit**, thus $i_t = r_t$ from equation (B.4). By contrast, generally $e_t = p_{H,t} \neq 0$ in this regime. The system (B.1)-(B.4) can be re-written as

$$\phi \pi_{H,t} = E_t \pi_{H,t+1}$$
$$\beta b_{t+1} = (1-\psi) b_t + \lambda(\beta\phi - 1)\pi_{H,t}.$$

It features one forward looking ($\pi_{H,t}$), one backward looking variable (b_{t+1}). As can be easily checked, in spite of $\psi < 1 - \beta$, the system exhibits bounded dynamics if the Taylor principle does not hold $\phi < 1$ (Leeper, 1991). A guess and verify approach yields

$$(\Delta e_t =)\ \pi_{H,t} = \frac{1-\psi-\beta\phi}{\lambda(1-\beta\phi)} b_t, \qquad b_{t+1} = \phi b_t.$$

In the main text, we show a special case of these equations when $\psi = 0$.

In the initial regime **Crisis**, $e_t = 0$ and hence $p_{H,t} = 0$ and $\pi_{H,t} = 0$ from equation (B.2). However, generally $r_t \neq 0$ because of expected changes in inflation and nominal depreciation (equations (B.1) and (B.2)), and $i_t \neq 0$ because of (in addition to the variation in r_t) expected outright default (equation (B.4)). Moreover, movements in the bond yield i_t feed back into b_{t+1} through equation (B.3).

By using the Markov chain we can write equation (B.3) as

$$i_t = \left[\varepsilon \frac{1-\psi-\beta\phi}{\lambda(1-\beta\phi)} + \delta \frac{1-\psi}{\lambda}\right] b_{t+1}, \tag{B.5}$$

where we have used the equilibrium default and nominal depreciation rates from regimes Default and Exit. Insert this into (B.3)

$$\beta b_{t+1} = (1-\psi) b_t + \lambda\beta i_t$$

and rearrange for b_{t+1} to obtain

$$b_{t+1} = (1-\psi)\frac{1}{\beta}\left(1 - \epsilon\left(\frac{1-\psi-\beta\phi}{1-\beta\phi}\right) - \delta(1-\psi)\right)^{-1} b_t =: (1-\psi)\Theta^b b_t$$

for public debt, and

$$i_t = (1-\psi)(\Theta^\theta + \Theta^r) b_t,$$
$$\text{where} \quad \Theta^\theta = \delta(1-\psi)\frac{\Theta^b}{\lambda} > 0 \quad \text{and} \quad \Theta^r = \epsilon\left(\frac{1-\psi-\beta\phi}{1-\beta\phi}\right)\frac{\Theta^b}{\lambda} > 0,$$

for the sovereign yield. In the main text, we show the special case of these two equations as $\psi = 0$. This completes the proof of Proposition 2.

Now turn to stability, which is Proposition 1. It is clear that the stability of the overall system hinges on the stability of its endogenous states, which are only b_t when prices are flexible. In this case, the general condition characterizing mean square stability, which is that all eigenvalues of

$$(\mathcal{P}' \otimes I_{n^2})\mathrm{diag}(F_{\varsigma_1} \otimes F_{\varsigma_1}, ..., F_{\varsigma_h} \otimes F_{\varsigma_h})$$

must lie within the unit circle (n denoting the number of endogenous variables, h the number of regimes, F the solution matrices in the respective regimes, see the main text), reduces to the simple condition that all eigenvalues of

$$\begin{pmatrix} 1-f-\delta-\epsilon & 0 & 0 & 0 \\ f & 1 & 0 & 0 \\ \delta & 0 & 1 & 0 \\ \epsilon & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \left(\frac{1}{\beta(1-\delta-\epsilon)}\right)^2 & 0 & 0 & 0 \\ 0 & \left(\frac{1-\psi_{\text{Reform}}}{\beta}\right)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi^2 \end{pmatrix} \tag{B.6}$$

must lie within the unit circle. Because the target regimes are absorbing and the matrix on the ride hand side is diagonal, a sufficient condition for this is that

$$(1-f-\delta-\epsilon)\left(\frac{1}{\beta(1-\epsilon-\delta)}\right)^2 < 1.$$

which is equation (4.1) in the main text. This completes the proof of Proposition 1.

B.2 Proposition 3

Here we derive the solution of the sticky price model shown in Section 4. To do so, we make a number of parametric assumptions. First and most importantly, we require that $f+\delta+\epsilon = 1$, that is, agents expect the first regime to persist with probability zero. Second, we set $\varphi = 0$, that is, we impose a linear disutility of labor. Third, we impose $\psi = 0$ such that taxes do not systematically respond to debt in the crisis regime, as well as $\kappa = 1-\beta$, i.e. that the slope of the Phillips curve relates in a particular way to the discount factor. The first assumption is strictly needed for a derivation of closed form results to be feasible. In contrast, the last three assumptions simplify the exposition considerably.

We solve the model backwards using the method of undetermined coefficients. For convenience, we repeat the relevant system of equations

$$y_t = E_t y_{t+1} - \varpi(r_t - E_t \pi_{H,t+1}) \tag{B.7}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa(\varphi + \varpi^{-1}) y_t, \tag{B.8}$$

$$(1-\omega) y_t = \varpi q_t, \tag{B.9}$$

$$q_t = (1-\omega)(e_t - p_{H,t}) \tag{B.10}$$

$$\beta b_{t+1} = (1-\psi) b_t + \lambda(\beta i_t - \pi_{H,t} - \theta_t), \tag{B.11}$$

$$i_t = r_t + E_t \theta_{t+1} \tag{B.12}$$

along with the definition for inflation $\pi_{H,t} = p_{H,t} - p_{H,t-1}$. Furthermore, differing across regimes are the conduct of monetary policy

$$e_t = 0 \qquad \text{or} \qquad r_t = \phi \pi_{H,t} \tag{B.13}$$

and the rate of equilibrium default

$$\theta_t = 0 \qquad \text{or} \qquad b_{t+1} = 0. \tag{B.14}$$

Starting with regime **Reform**, it holds that $e_t = \theta_t = 0$ and that $\psi_{\text{Reform}} > 1 - \beta$. We also derive that $y_t = -\varpi p_{H,t}$ from (B.12) and (B.9)-(B.10). Inserting this in (B.8) allows us to derive a second order difference equation in the price level

$$\beta p_{H,t+1} = (1 + \beta + \kappa\varpi(\varphi + \varpi^{-1}))p_{H,t} - p_{H,t-1}.$$

Guessing that $p_{H,t} = G_{pp}p_{H,t-1}$ for some unknown coefficient G_{pp} we obtain the restriction $1 = G_{pp}(1 + (1 - G_{pp})\beta + \kappa\varpi(\varphi + \varpi^{-1}))$. This is a quadratic equation in G_{pp} with strictly one root below unity. To obtain this root we rewrite this as

$$\frac{(1 - \beta G_{pp})(1 - G_{pp})}{G_{pp}} = \kappa\varpi(\varphi + \varpi^{-1}).$$

Recognizing that $\kappa \equiv (1 - \beta\xi)(1 - \xi)/\xi$ reveals that, once we impose our assumption $\varphi = 0$, the solution is $G_{pp} = \xi < 1$. We now determine the equilibrium behavior of interest rates and public debt. First, combining (B.7), (B.9)-(B.10) yields $r_t = \Delta e_{t+1}$, such that $r_t = 0$ in this regime. Second, it follows that $i_t = 0$ from (B.12) as there is no possibility of default. The equilibrium behavior of debt can now be derived from (B.11)

$$b_{t+1} = \frac{1 - \psi_{\text{Reform}}}{\beta} b_t + \frac{\lambda(1 - \xi)}{\beta} p_{H,t-1},$$

where we have inserted the solution for $p_{H,t}$. This is a stable difference equation, because of our assumption $\psi_{\text{Reform}} > 1 - \beta$ above.

The solution for the price level is the same in regime **Default**, given that $e_t = 0$ holds in this regime, too. As a result, it also holds that $r_t = 0$. Instead, θ_t is generally non-zero. Because $b_{t+1} = 0$ at all times in this regime, it must be that

$$0 = (1 - \psi)b_t + \lambda(\beta\theta_{t+1} + (1 - \xi)p_{H,t-1} - \theta_t),$$

where we have used that $i_t = \theta_{t+1}$ under $r_t = 0$, see equation (B.12). This is a first order difference equation in θ_t, for given states b_t and $p_{H,t-1}$. To solve it, we guess that $\theta_t = G_{\theta b}b_t + G_{\theta p}p_{H,t-1}$ for coefficients $G_{\theta b}$ and $G_{\theta p}$ to be determined. Note that, at time $t+1$, the guess reduces to $\theta_{t+1} = G_{\theta p}\xi p_{H,t-1}$, where we have used that $b_{t+1} = 0$ and that $p_{H,t} = \xi p_{H,t-1}$. Inserting this in the previous equation yields

$$0 = (1 - \psi)b_t + \lambda(\beta G_{\theta p}\xi p_{H,t-1} + (1 - \xi)p_{H,t-1} - (G_{\theta b}b_t + G_{\theta p}p_{H,t-1})),$$

which reveals that $G_{\theta b} = (1-\psi)/\lambda$ and that $G_{\theta p} = (1-\xi)/(1-\beta\xi)$.

In regime **Exit** we use that $r_t = \phi\pi_{H,t}$ and that $\theta_t = 0$. We solve the two by two system

$$\begin{aligned} y_t &= y_{t+1} - \varpi(\phi\pi_{H,t} - \pi_{H,t+1}) \\ \pi_{H,t} &= \beta\pi_{H,t+1} + \kappa\varpi^{-1}y_t, \end{aligned}$$

along with the evolution of debt

$$b_{t+1} = \frac{1-\psi}{\beta}b_t + \frac{\lambda(\beta\phi-1)}{\beta}\pi_{H,t}.$$

Guess that $\pi_{H,t} = G_{\pi b}b_t$ and $y_t = G_{yb}b_t$ for coefficients $G_{\pi b}$ and G_{yb} to be determined. We obtain the two restrictions

$$\begin{aligned} &G_{yb}[1-(1-\psi)/\beta - (\lambda(\beta\phi-1)/\beta)G_{\pi b}] + G_{\pi b}\varpi[\phi - (1-\psi)/\beta - (\lambda(\beta\phi-1)/\beta)G_{\pi b}] = 0 \\ &G_{\pi b}[1-(1-\psi) - \lambda(\beta\phi-1)G_{\pi b}] - \kappa\varpi^{-1}G_{yb} = 0. \end{aligned}$$

Solving the second equation for $G_{\pi b}\varpi[-(1-\psi)/\beta-(\lambda(\beta\phi-1)/\beta)G_{\pi b}] = (\kappa/\beta)G_{yb}-(\varpi/\beta)G_{\pi b}$ and replacing in the first equation yields

$$\begin{aligned} &G_{yb}[1-(1-\psi)/\beta - (\lambda(\beta\phi-1)/\beta)G_{\pi b}] + G_{\pi b}\varpi(\phi - 1/\beta) + (\kappa/\beta)G_{yb} = 0 \\ \Leftrightarrow\ &G_{yb}[\kappa - (1-\beta-\psi) - \lambda(\beta\phi-1)G_{\pi b}] + G_{\pi b}\varpi(\beta\phi-1) = 0. \end{aligned}$$

Because we assume that $\kappa = 1-\beta-\psi$ (which is implied by our assumptions $\kappa = 1-\beta$ and $\psi = 0$), this equation reduces to

$$G_{\pi b}(\beta\phi-1)[\varpi - \lambda G_{yb}] = 0$$

which reveals that $G_{yb} = \varpi/\lambda$. Using this information we can use the second restriction above to obtain a quadratic equation for $G_{\pi b}$ as

$$G_{\pi b}^2 + \frac{\psi}{\lambda(1-\beta\phi)}G_{\pi b} - \frac{\kappa}{\lambda^2(1-\beta\phi)}.$$

The two roots of this equation are

$$G_{\pi b} = -\frac{\psi}{2\lambda(1-\beta\phi)} \pm \sqrt{\frac{\psi^2}{4\lambda^2(1-\beta\phi)^2} + \frac{\kappa}{\lambda^2(1-\beta\phi)}}$$

Under our assumption $\psi = 0$ the single positive root is $G_{\pi b} = \sqrt{\kappa/(1-\beta\phi)}/\lambda$. Hence we have verified that equilibrium output and inflation evolve as $\pi_{H,t} = (\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_t$ and $y_t = (\varpi/\lambda)b_t$. Inserting this into the equation for debt above

$$\begin{aligned} b_{t+1} &= (1/\beta)b_t + (1/\beta)\lambda(\beta\phi-1)(\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_t \\ &= (1/\beta)(1-(1-\beta\phi)\sqrt{\kappa/(1-\beta\phi)})b_t. \end{aligned}$$

Because of $\kappa = 1-\beta$, it holds that $1-\sqrt{\kappa} < \beta$. Furthermore, necessarily $1-\beta\phi < 1$. From this follows that the coefficient on debt is smaller than one, such that debt is indeed mean reverting after exit.

Finally, in the initial regime **Crisis** we use our assumption on the transition probabilities. Because $e_t = 0$ in this regime we again write the difference equation

$$\begin{aligned} \beta E_t p_{H,t+1} &= (1+\beta+\kappa)p_{H,t} - p_{H,t-1} \\ \beta[(f+\delta)p_{H,t+1}|\text{Reform} + \epsilon p_{H,t+1}|\text{Exit}] &= (1+\beta+\kappa)p_{H,t} - p_{H,t-1}, \end{aligned}$$

where we have evaluated the expectations operator using that $f+\delta+\epsilon = 1$, used that $p_{H,t+1}$ is the same in both Reform and Default (see the earlier derivation), and where we have imposed our assumption $\varphi = 0$. Inserting the solutions $p_{H,t+1}|\text{Reform} = \xi p_{H,t}$ and $p_{H,t+1}|\text{Exit} = p_{H,t} + (\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_{t+1}$ we rewrite this as

$$\beta[(f+\delta)\xi p_{H,t} + \epsilon(p_{H,t} + (\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_{t+1})] = (1+\beta+\kappa)p_{H,t} - p_{H,t-1}. \tag{*}$$

To evaluate this further, we require the equilibrium behavior of b_{t+1}. Public debt is given by

$$\beta b_{t+1} = b_t + \lambda(\beta i_t - (p_{H,t} - p_{H,t-1})),$$

where we have used our assumption $\psi = 0$ and the fact that $\theta_t = 0$. The evolution of i_t is obtained from (B.12). Using that $r_t = E_t \Delta e_{t+1}$ we have

$$\begin{aligned} i_t &= r_t + E_t\theta_{t+1} = E_t(\Delta e_{t+1} + \theta_{t+1}) \\ &= \epsilon e_{t+1}|\text{Exit} + \delta\theta_{t+1}|\text{Default} \\ &= \epsilon[(1/\varpi)y_{t+1}|\text{Exit} + p_{H,t+1}|\text{Exit}] + \delta\theta_{t+1}|\text{Default}. \end{aligned}$$

Here we have used that $e_t = 0$ in the initial regime and combined equations (B.9)-(B.10) to replace e_{t+1}. Inserting our solutions $p_{H,t+1}|\text{Exit} = p_{H,t} + (\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_{t+1}$, $y_{t+1}|\text{Exit} = (\varpi/\lambda)b_{t+1}$ as well as $\theta_{t+1}|\text{Default} = (1/\lambda)b_{t+1}$ we rewrite this further as

$$i_t = \epsilon\{[(1/\lambda) + (\sqrt{\kappa/(1-\beta\phi)}/\lambda)]b_{t+1} + p_{H,t}\} + \delta(1/\lambda)b_{t+1}$$

such that public debt can be written as

$$\begin{aligned} & \beta b_{t+1} = b_t + \beta[\epsilon\{[1+\sqrt{\kappa/(1-\beta\phi)}]b_{t+1} + \lambda p_{H,t}\} + \delta b_{t+1}] - \lambda(p_{H,t} - p_{H,t-1}) \\ \Leftrightarrow \;\; & (\beta[1-\epsilon(1+\sqrt{\kappa/(1-\beta\phi)}) - \delta])b_{t+1} = b_t - \lambda(1-\beta\epsilon)p_{H,t} + \lambda p_{H,t-1}. \end{aligned} \tag{**}$$

Combining equations (*) and (**) yields the solution for $p_{H,t}$ and b_{t+1} in the initial regime. Here we merely state the solution for the price level

$$\begin{aligned}([1-\epsilon(1+\sqrt{\kappa/(1-\beta\phi)})-\delta][\xi^{-1}-\beta\epsilon(1-\xi)]+\epsilon(1-\beta\epsilon)\sqrt{\kappa/(1-\beta\phi)})p_{H,t}\\ =\epsilon(\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_t+(1-\epsilon-\delta)p_{H,t-1},\end{aligned}$$

where we have used that $\xi(1+\beta(1-\xi)+\kappa)=1$. We simplify this a bit further to obtain

$$\begin{aligned}((1-\epsilon-\delta)(1-\beta\epsilon\xi(1-\xi))-\epsilon\sqrt{\kappa/(1-\beta\phi)}(1-\xi(1-\beta\epsilon\xi))p_{H,t}\\ =\epsilon\xi(\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_t+(1-\epsilon-\delta)\xi p_{H,t-1}.\end{aligned}$$

Note how this nests the case of $\epsilon=0$: in this case, the price level out reverts at the speed of the Calvo parameter $p_{H,t}=\xi p_{H,t-1}$ (as in regimes Reform and Default). The solutions for the real exchange rate and for output follow directly from this equation, from the fact that $e_t=0$ in the initial regime, hence $q_t=-(1-\omega)p_{H,t}$, then from the risk sharing condition $(1-\omega)y_t=\varpi q_t$,, which is equation (B.9). This completes the proof of Proposition 3.