Idiosyncratic Firm Risk, Cash Holdings and Lumpy Investment *

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Abstract

In this work I explore the hypothesis that the trend increase in firm-level risk, measured as the volatility of cash-flow growth rate, is a main driver of the long-run increase in cash holdings by firms over the last decades. I document a novel stylized fact: the impact of firm level uncertainty on cash holdings is stronger for firms that belong to sectors with greater skewness and kurtosis in investment rates. To rationalize this empirical finding I build a dynamic model in which financially constrained firms face uncertainty about future revenues and are subject to fixed adjustment costs of investment. In the model the degree of investment lumpiness is key to amplify the effects of volatility on cash holdings. The model is estimated on the Compustat data and used in a number of counterfactual experiments. I find that the increase in firm level volatility can explain almost 90% of the observed increase in cash holdings in the period 1980-2010. Furthermore, if non-convex investment frictions are absent, the increase in cash generated by higher uncertainty is reduced by half.

JEL Classification: E22, D22, E32, G31.

Keywords: Financing constraints, cash holdings, heterogeneous firms, lumpy investment, idiosyncratic shocks.

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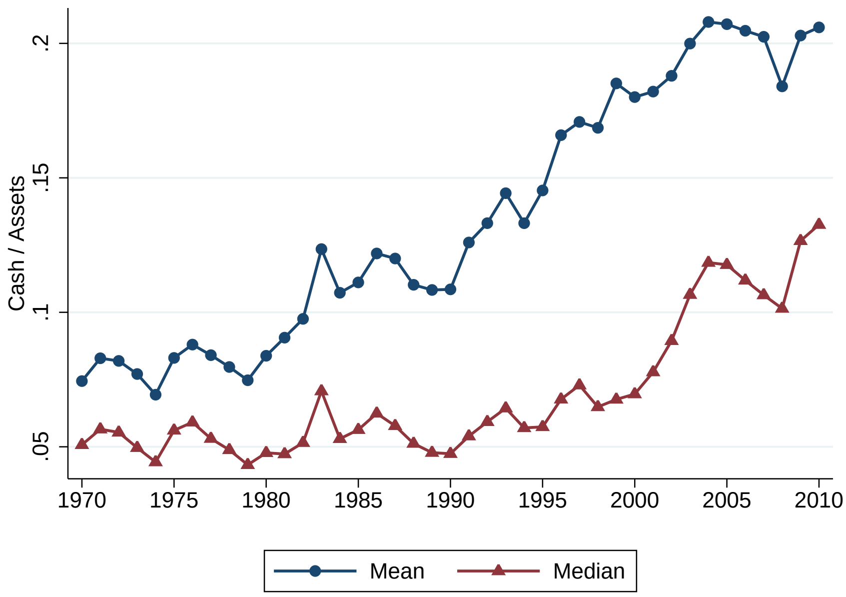
1 Introduction

The US corporate sector is holding record-high amounts of cash. Publicly traded US firms have increased dramatically their cash holdings since the 1980s, except for a slowdown during the recent financial crisis, as it can be seen in Figure 1. In 2011 these cash holdings amounted to nearly 5 trillion \$. Over the time period 1980-2010, the share of cash holdings in the balance sheet of U.S. firms has increased from 10% to 22%. A similar picture can be drawn for the evolution of net leverage at the firm level[1], showing that net leverage (measured as total debt minus liquid assets) dropped from 17% into negative territory in the first decade of 2000s. These stylized facts on firms cash management challenge the traditional wisdom that views the corporate sector as a net borrower from the rest of the economy. Understanding this phenomenon can also shed some light on the reasons for the slow recovery from the Great Recession.

Figure 1: Corporate Cash Holdings, U.S. Economy

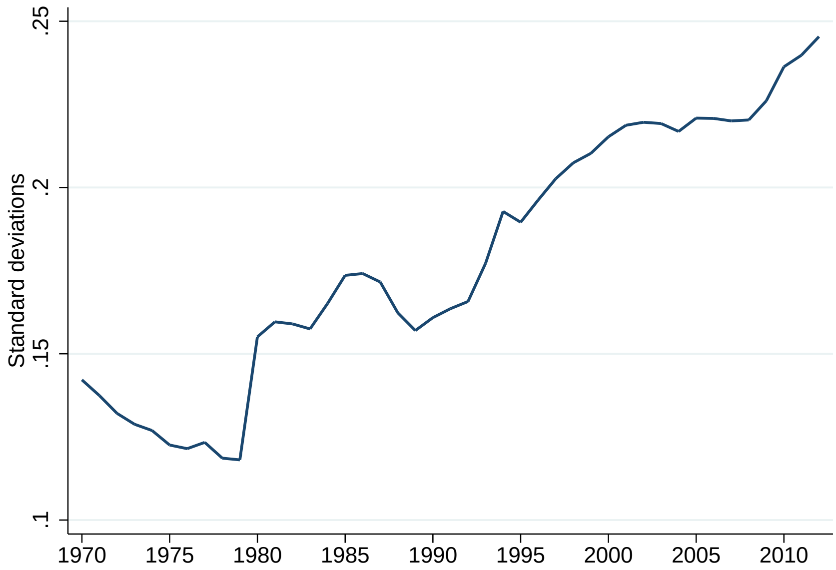


Note: The figure shows the evolution of the average cash holdings for U.S. public firms. Cash holdings are defined as cash and marketable securities (Compustat item CHE) divided by total assets (item AT)

Over the same time period, firm-level uncertainty has increased substantially, as documented by Comin and Philippon (2005) and Davis et al. (2007). Figure 2 shows a measure

[1] For a more detailed analysis, see Section 2 of the paper.

Figure 2: Firm Level Uncertainty



Note: The figure shows the volatility of cash flow growth at the firm level over 1970-2010. Cash flow is measured by operating income (Compustat item OIBDP). The volatility is computed as the standard deviation of cash flow growth over a ten-year rolling window and then averaged across firms.

of firm level uncertainty: the volatility of the cash flow growth rate. This measure of cash flow volatility has doubled over the period 1980-2010.

One explanation most frequently put forward for the secular growth in cash relates to firm level uncertainty. The common story is that financially constrained firms accumulate more cash in order to protect themselves from negative profit shocks, and this effect can increase if the volatility of the underlying stochastic process increases. This precautionary behaviour has been extensively studied in the consumption literature and it has been recently extended to the finance literature in the important contribution of Riddick and Whited (2009).

In this paper I document a novel empirical finding: the positive effect of firm level uncertainty on cash holdings is stronger for firms that belong to sector with higher *investment lumpiness*. I start by documenting the stylized facts of the link between the rise in firm level risk and the secular trend in corporate cash holdings. To this end I build on Comin and Philippon's approach to construct a new set of firm level measures of uncertainty for all non-financial firms in Compustat between 1970 and 2010. I measure uncertainty at the firm level by looking at the cross-sectional dispersion of firm growth rate of sales, profit and TFP.

In the financial literature it is common to measure stock market volatility by computing the standard deviation of stock returns in a given time span. For example, Campbell, Lettau, Malkiel and Xu (2001) document an increase in the volatility of idiosyncratic stock returns. Since this measure can be plagued with compositional biases I also build an alternative measure of firm level uncertainty based on computing the standard deviation in the time series with a five year rolling window. Using this firm-level measure of uncertainty, I document several new empirical regularities on the link between firm financing and uncertainty.

In particular, there is a strong positive relation between firm risk and corporate cash both in the cross-section and in the time-series, with firm level shocks volatility emerging as the most important determinant of cash holdings. The key empirical fact that I subsequently discover is that the link between cash and firm level risk is especially strong for firms that are financially constrained and those that belong to sectors with greater investment inflexibility. Overall my empirical results suggest that firm level volatility has a pervasive impact on corporate cash holding decisions and that the impact is due to both financial and real frictions.

In order to better understand the economic forces that drive the empirical link between firm level volatility, cash holdings and investment lumpiness, I next develop a dynamic model of the joint determination of firm's financing, risk management, and capital accumulation decisions, extending the basic framework of Riddick and Whited (2009). The model is cast in an infinite horizon, discrete-time stochastic environment (without aggragate shocks) where firms make value maximizing investment decisions in real (capital) and financial (cash holdings, debt and equity) assets. There are two key frictions: first financial frictions arise since debt financing is subject to a credit limit, and equity financing involves issuance costs; second, there are real frictions that arise since investment in real assets is subject to non-convex adjustment costs, which make it infrequent and lumpy (see Caballero (1999) for a standard treatment).

In this setting I show that the interplay of external financing costs (in the form of costly equity issuances) and non-convex adjustment costs generates a quantitatively large precautionary demand for cash hoarding because it makes difficult for firms to generate funds by raising external finance when it has to pay a large fixed cost to invest. As a result firms with positive invesment opportunities tend to hold more cash in anticipation of Ss type adjustments in phisical capital to avoid raising external finance. To assess the quantitative importance of this *precautionary saving channel*, I simulate the model and perform a comparative statics exercise by increasing the degree of volatility faced by firms: all else equal, a technological transformation that increases the turbolence of the economic environment to levels observed in the US economy during the last decade can account for almost 90 percent of the increase in corporate cash holdings that are observed among US public firms for the same period. Non-convex investment frictions prove to be crucial in generating this effect: a counterfactual experiment where such real investment frictions are shut down shows that the increase in cash generated by higher uncertainty is reduced by almost half.

My paper is related to existing work along different lines of the macroeconomic literature. First, it contributes to the recently growing literature on the so-called *cash puzzle* in the US economy. The role of uncertainty as a major determinant of corporate cash holdings was first analyzed in Opler et al. (1999) and subsequentely by Bates et al. (2009) in various empirical settings. Turning to more theoretical analysis, my modelling framework builds on Whited (2006), Riddick and Whited (2009) and DeAngelo et al. (2011). These works introduce corporate saving behaviour into the standard Tobin's Q investment model, showing that financing costs and uncertainty play a crucial role in determining the optimal saving policy of firms. However they do not consider the issue of the rising corporate cash over time, focusing only on the cross-section. Moreover Whited (2006) and Riddick and Whited (2009) do not perform any structural estimation of their dynamic models, whereas I bring my model to the data using the simulated method of moments. Falato et al. (2013) propose the increase in asset intangibility as a plausible explanation behind the increase in cash holdings. Regarding the relationship between uncertainty shocks and firm investment the seminal contribution is Bloom (2009) who constructs a volatility index based on stock market returns and measures the impact of sudden jumps in such uncertainty on real economic activity. Bloom then builds a structural model with non-convex adjustment costs and shows that an increase in uncertainty leads to a drop in employment and output through a "freeze" in hiring and investment. My focus is however different since I investigate the relationship between uncertainty and cash (rather than investment) in the long-run: in particular I relate the long-run increase in uncertainty to the long-run increase in firm-level cash-holdings without considering the respose of cash to shocks to uncertainty. Furthermore Bloom (2009) considers uncertainty at the aggregate level whereas I focus on uncertainty at the firm level.

My paper is also connected to an important strand of the literature in macroeconomics dealing with firm level volatility and its implications. Comin and Philippon (2005) and Comin and Mulani (2006) were the first to document the increasing trend in volatility among U.S. Compustat firms, from 1970s to 2000. Their finding was put into a broader perspective by Davis et al. (2007), who showed that private U.S. experienced instead a decrease in volatility over the same time period. The coexistence of increasing uncertainty among public firm and decreasing uncertainty among private firms has been further studied by Thesmar and Thoenig (2011). Whereas I do not focus explicitely on the ultimate reasons behind such long-run increase in uncertainty[2], I add to these studies by first extending the evidence on the increasing volatility of listed firms after 2000, and, second and more importantly, relating the uncertainty trend to the high levels of corporate cash.

The paper proceeds as follows. In section 2 I document the empirical evidence regarding the relation between corporate cash holdings, firm-level risk and investment lumpiness. In section 3 I set up a structural model of heterogeneous firms with saving in cash and costly equity financing to rationalize the empirical findings detailed in the previous section. In section 4 I explain the calibration of the model and perform a number of counterfactual experiments, measuring the impact of the increase in firm-level uncertainty on cash holdings. Section 5 concludes the paper.

2 Stylized facts and Empirical Evidence

I begin my empirical analysis by summarizing the stylized facts of the evolution of corporate cash holdings and firm level uncertainty over time. To this aim I assemble a large (unbalanced) panel of US firms over the 1970 to 2010 period. My sample includes all Compustat firm-year observations from 1980 to 2010 with positive values for (book) total assets and sales revenue for firms incorporated in the US. Financial firms (SIC code 6000-6999) and utilities (SIC codes 4900-4999) are excluded from the sample. This sample selection criteria are standard (see for example Bates et al. (2009)). In the end the sample comprises 19,065 firms for a total of 209,265 firm-year observations.

[2]One prominent explanation scrutinized by Comin and Philippon (2005) is higher competition in the goods market.

In the remainder of this section, I document the positive link between cash holdings and risk, and the fact that such impact is stronger for firms that belong to sectors with higher investment lumpiness and for firms that are more credit constrained.

Empirical Fact 1: *Firm cash holdings show a secular upward trend in 1970-2010, whereas net leverage (or net debt) show a decreasing trend over the same time span.*

I begin my analysis by considering basic stylized facts about the evolution of corporate cash holdings in the US economy. It is widely recognized that US firms have been accumulating large piles of cash during the last three decades. Figure 3 plots the average ratio of cash to book assets over the sample period. It is evident from the figure that cash holdings display a pronounced upward trend increasing from 8 percent in 1970 to more than 20 percent in 2010. This evidence is consistent with the findings in Bates et al. (2009). There might be concerns that such rise in cash is driven only by a few but very large companies: however even if one considers the median level of cash ratio by year, or the aggregate cash to aggregate assets (which is average cash ratio weighted by assets[3]) the increasing trend is still there, as it is apparent from figure 4 and 5. More in detail, the fact that cash ratio weighted by firm size is lower than unweighted cash ratio suggests that larger firms hold on average *less* cash than small ones.

It is worth to notice that such increasing trend in cash comes together with a decreasing trend in net leverage (see Figure 6). When I consider leverage there is no clear trend. Considering net leverage instead we see a decrease from 19 percent in 1970 to negative values after 2004. For my purposes considering *net* leverage as opposed to leverage is of

[3]Figure 1 shows the evolutions of the average cash ratio, computed as:

$$Cash_t = \frac{1}{N}\sum_i \frac{Cash_{it}}{AT_{it}}$$

Another way is to compute aggregate cash to aggregate asset ratio:

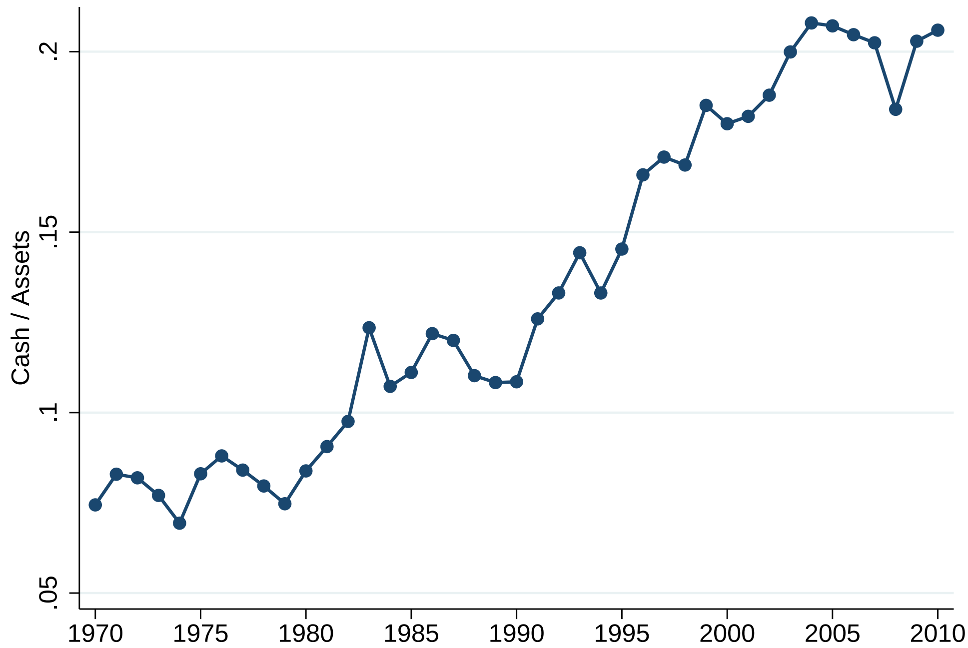
$$Agg.Cash_t = \frac{\sum_i Cash_{it}}{\sum_i AT_{it}}$$

The latter measure is just a weighted version of the former:

$$\begin{aligned} Weighted.CashRatio &= \sum_i \frac{Cash_{it}}{AT_{it}} \cdot \frac{AT_{it}}{\sum_j AT_{jt}} \\ &= \sum_i \frac{Cash_{it}}{\sum_j AT_{jt}} = Agg.Cash_t \end{aligned}$$

fundamental importance for the following reason. Net leverage is computed at the firm level as (LT debt + current debt – Cash)/Book Assets, whereas leverage is computed as (LT debt + current debt)/Book Assets. It is more meaningful to consider net leverage as a measure of firm level financial leverage, and indeed by considering net leverage instead of leverage I get dramatically different conclusions. The net leverage series shows important business cycle fluctuations, but still the downward trend is apparent. Around 2004 net leverage becomes negative, which is a surprising fact given the conventional wisdom that considers the corporate sector as typically a net borrower.

Figure 3: Rising Corporate Cash Holdings



I estimate a simple regression of the cash ratio on a constant and time to see whether there is a statistically significant trend in the cash ratio (see Figure 7).

The coefficient for the time trend for the average cash ratio corresponds to a yearly increase of 0.39% and is highly statistically significant. The R^2 of the regression is 94%. For the median, the slope coefficient represents a 0.20% yearly increase. The R^2 is 71%. This evidence is consistent with a positive time trend in cash holdings during the sample period. When I regress instead net leverage (mean), which substracts cash from debt, on a constant and a time trend I get a decrease of 0.54%, again significative at the 99 percent

Figure 4: Rising Corporate Cash Holdings, Robustness

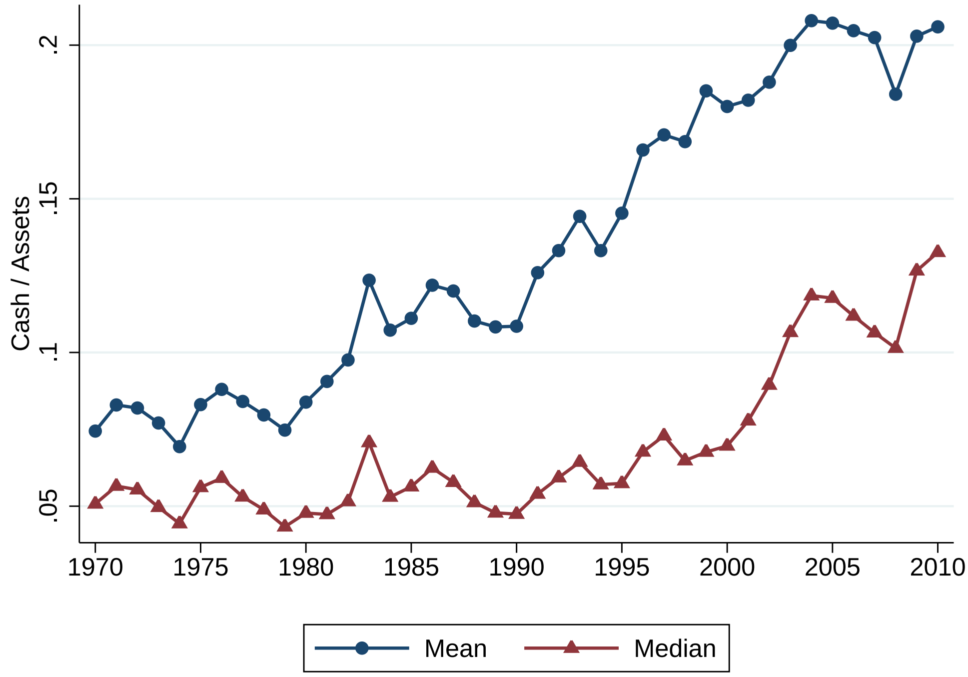


Figure 5: Rising Corporate Cash Holdings, Robustness

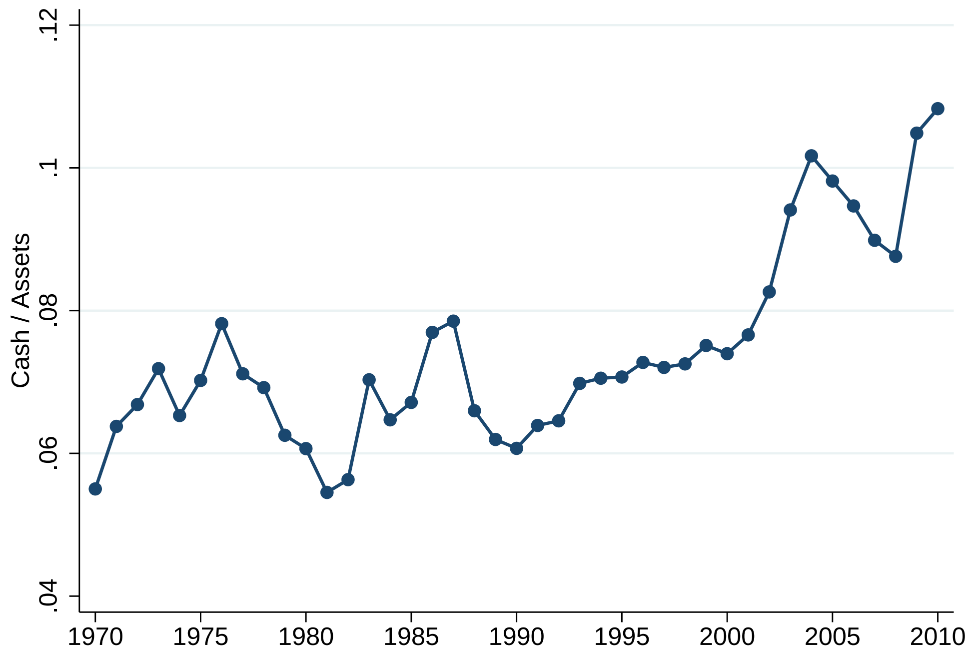


Figure 6: Net Leverage over the sample period. Net leverage = (long-term debt + current debt - cash) / Total Assets)

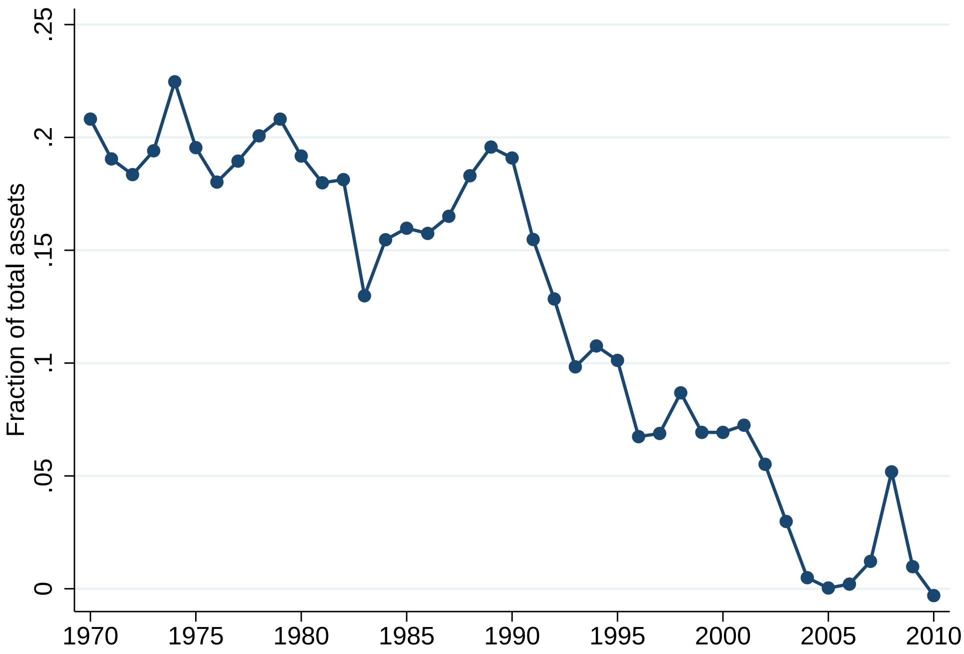
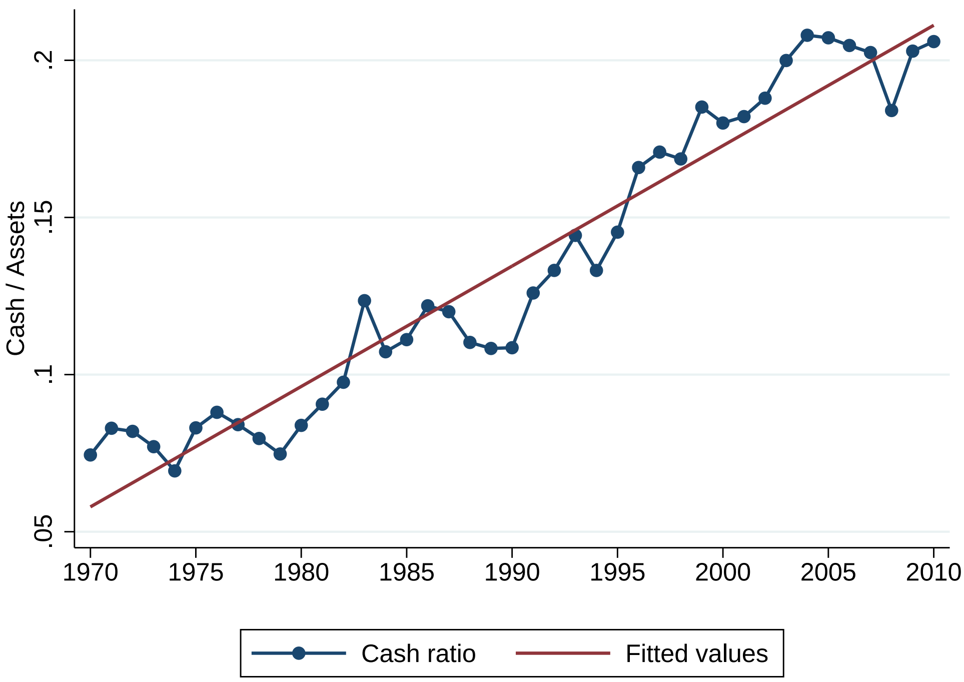


Figure 7: Linear Trend

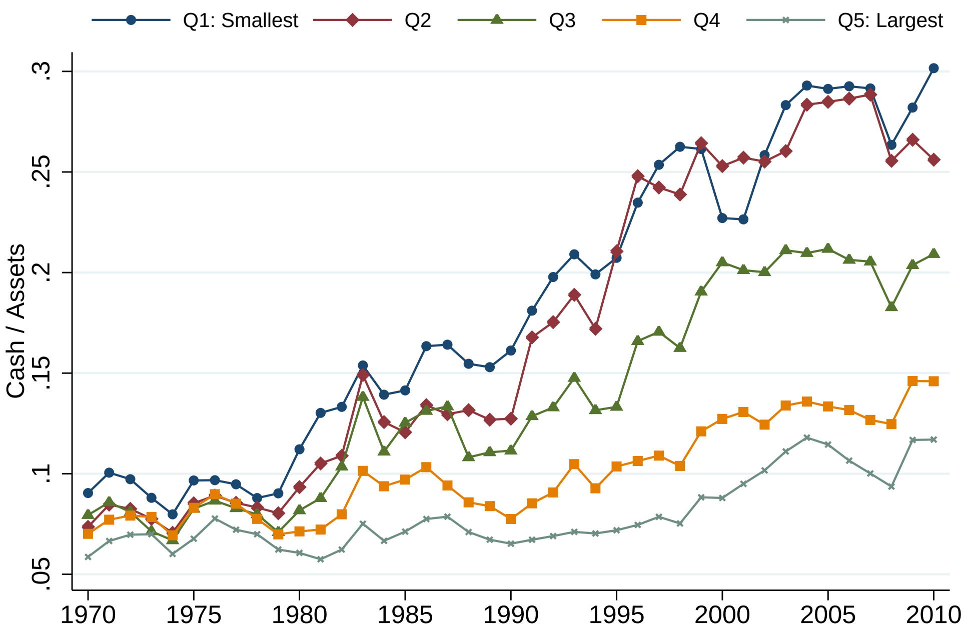


level. Of course such simple regressions are only useful to characterize the evolution of the cash holdings during the sample period, and it would not make sense to extrapolate the in-sample trend to future years.

Empirical Fact 2: *The increasing trend in cash is not driven by the largest firms or by a particular sizeclass of firms.*

In the figure below I break the firms into quintiles of size (assets). Considering that small firms may find it harder to access credit markets, I would expect smaller firms to have higher cash-to-assets ratios. And indeed this intuition is confirmed in the data: smallest firms (those in the bottom quintiles of total assets, Q1/Q2) have on average higher cash-to-assets ratios. Moreover the increasing trend in cash ratios is more pronounced for smaller firms.

Figure 8: Average Cash Ratios by Firm Size



Empirical Fact 3: *The increasing trend in cash is significant only for* ***non-paying dividend*** *firms.*

Figure 9: Cash Holdings by Dividend Paying Status

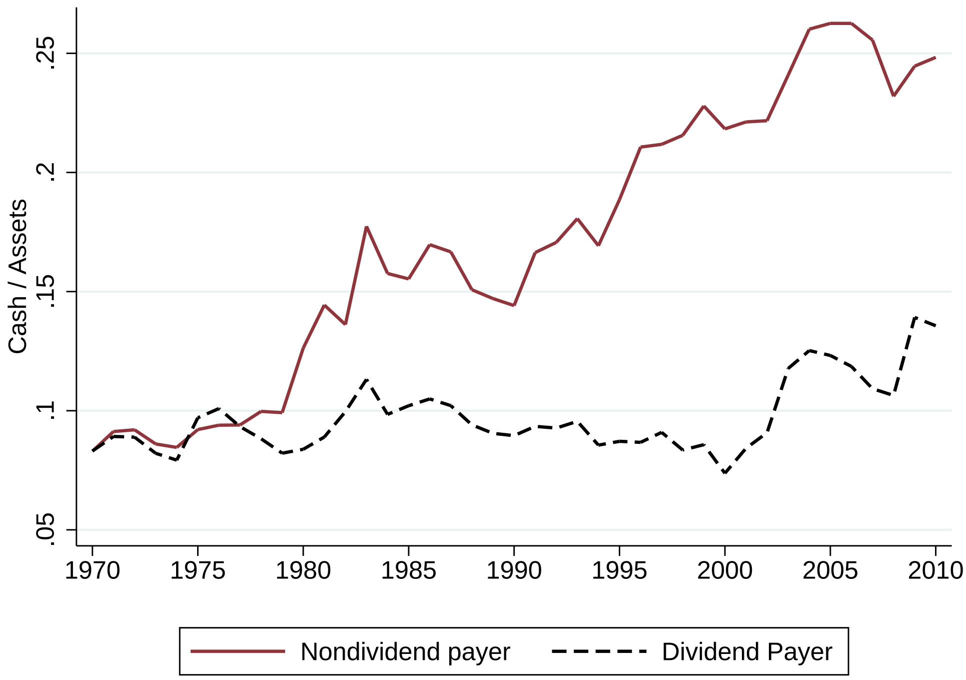
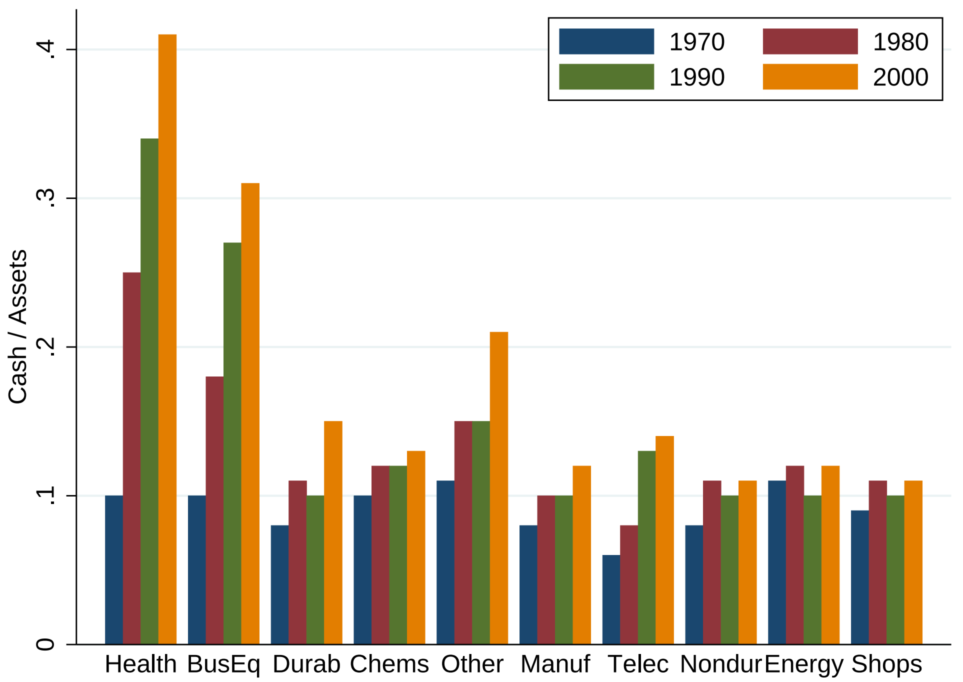


Figure 10: Cash Holdings by Sectors



Note: Sectors are sorted by the frequency of investment inaction in the 2000s.

Many papers in the corporate finance literature (for example, Almeida et al. (2004)) consider nondividend paying firms as firms that are more likely to be credit constrained. The evidence above suggests that the increase in cash holdings occurred mostly in financially constrained firms.

From Figure 10 we can see that there is a dramatic increase in the cash ratio among the nondividend payers, but is much less so among the dividend payers. Under the aforementioned assumption that nondividend paying firms are more likely to be financially constrained, this evidence means that the increase in cash holdings occurred mostly in financially constrained firms.

Therefore if we consider *size* and *dividend payer status* as proxies for being financially constrained, my evidence supports the precautionary motive for holding cash.

Empirical Fact 4a: *Firm level risk showed an upward trend in last decades*

As a first proxy to measure uncertainty at the firm level, I consider the cross-sectional dispersion of sales growth rate. The main finding is that the growth rate of sales has become more volatile over the sample period.

To measure the volatility inherent in the firm's environment I thus focus on the cross-section. Specifically I compute the standard deviations of growth rates across all the firms in a given year. As illustrated in Figure 11, volatility at the firm level exhibits a significant upward trend. In order to build a more representative measure of volatility, firm-level volatility measures are weighted using the firm's share of sales in total sales in a given year. Figure 11 also shows the persistent upward trend after the use of these weights.

The source of this increase in volatility, however, may be subject to question. The upward trend may reflect a feature specific to sample in use, rather than reflecting a true change in the economy. Therefore I perform several robustness checks to validate this finding. First I adopt a different definition for the volatility of the growth rate of firm level sales. Instead of computing the dispersion of growth rate across all firms in a year (as I did before), I define volatility as the time series of standard deviations of five-year rolling windows. In particular, for each firm i with more than five year of data I compute the rolling volatility as a moving-average:

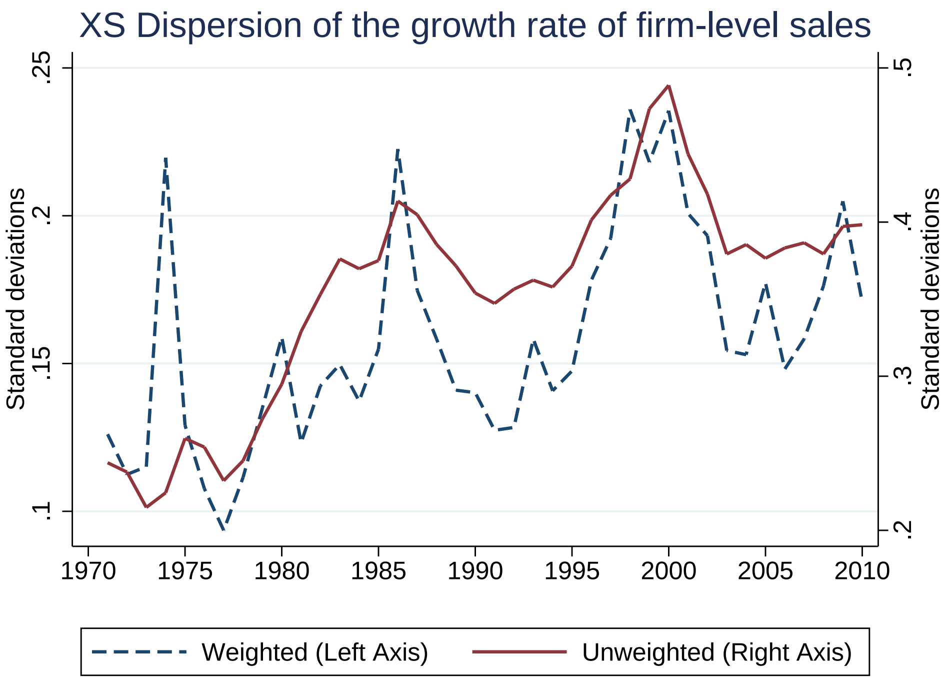
$$\sigma_{it} = \left[\frac{1}{5} \sum_{r=1}^{5} \left(\gamma_{i,t+r} - \overline{\gamma}_{it} \right)^2 \right]^{1/2}$$

Then, to aggregate at the year level, I take the median[4] across all firms:

$$\sigma_t = median_i \{\sigma_{it}\}$$

This alternative measure of volatility is less likely to be affected by compositional biases. When computing the standard deviation of the window in the time series, I remove the average growth rate for the firm in the window, and in effect control for firm-specific aspects that affect the growth rate of sales. These aspects, however, potentially show up in the cross-sectional measure. Figure 12b reports the finding using this second definition. As a robustness check I re-compute the rolling standard deviation of the sales growth rate using windows of shorter (3 year) and longer (5 year) size. Results are displayed in figures (12a) and (12c), respectively. The results are similar: an increasing trend in the dispersion is evident, even though there seems to be a partial reversion in the trend after 2000.

Figure 11: Cross-sectional Dispersion of the Growth rate of firm-level Sales

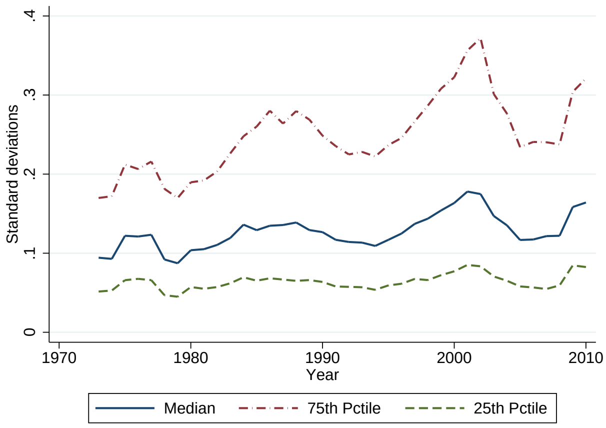


Note: Weight is firm's share of sales in total sales in a given year

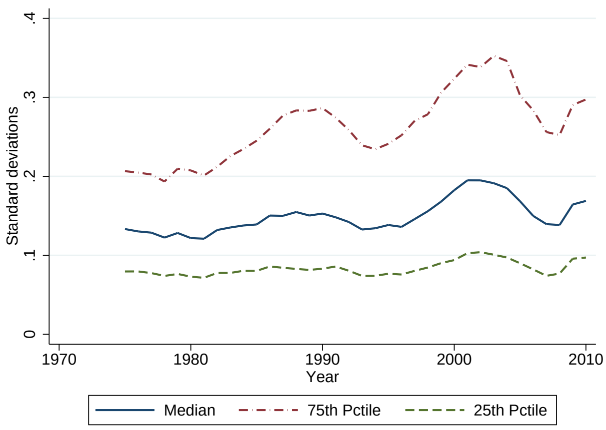
Robustness checks. It would be interesting to establish whether the increasing trend for

[4]In a first exercise I considered the average, but it turned out to be more sensitive to extreme values.

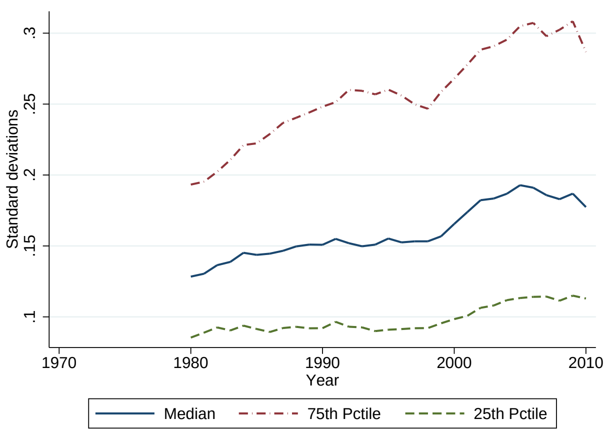
Figure 12: Time series Dispersion of the Growth rate of firm-level Sales



(a) 3-year rolling window



(b) 5-year rolling window



(c) 10-year rolling window

Note: The panels shows the evolution of the median, 25th and 75th percentiles of firm volatility. Volatility is computed on a rolling window base with different widths (3,5 and 10 years).

the dispersion in the growth rate of sales charaterizes also other firm-level variables that proxy profitability. The figures below report the time evolution of cross-sectional standard deviation, for a bunch of different variables at the firm level. We use sales, pretax profit and operating income before depreciation; notice that all these variables are highly correlated (Actually the correlations are significant at the 5% but the values are low).

The figure below include also the subset of firms that have been continuously listed since 1970 (see Acemoglu's discussion in Comin and Philippon (2005). It seems that the trend in the cross-sectional standard deviation is not affected by the changing composition of the sample, although it is true that older firms that are continuously listed are on average less risky than younger firms that become listed in the recent past.

2.1 Panel Evidence

In the remainder of this section, I confirm the stylized facts I presented above using panel data analysis. To this end I regress cash holdings on my preferred measure of firm-level uncertainty, while controlling for a set of standard determinants of cash holdings (see for example Bates et al. (2009) and Opler et al. (1999)). I consider several different variations of the following baseline specification:

$$Cash_{it} = \beta_0 + \beta_1 Risk_{it} + \beta_2' X_{it} + \delta_i + \lambda_t + \varepsilon_{it}$$

where $Cash_{it}$ is the ratio of firm i cash holdings to total assets in year t. The main explanatory variable is firm level uncertainty (measured as the rolling standard deviation of firm's growth rate of sales) and X_{it} is a vector of time-varying firm level controls that include market-to-book ratio, firm size, cash flow, capital expenditures, and a dummy for whether the firm pays dividend in a given year. I include year fixed-effects to control for time variation in cash holdings. The null hypothesis of my analysis is that β_1 equals zero. From Table 1 we can see that the coefficient on uncertainty is robustly positive and statistically significant in both OLS and FE specifications.

When I replicate the test for net leverage, which is the ratio of total debt net of cash to book assets, the coefficient on firm level risk is robustly negative and statistically significant across both specifications. These results suggest that firm level uncertainty is not only an important determinant of firms' cash holdings decisions, but also of their capital structure and net indebtness.

In the second part of this panel analysis (please refer to Tables 2 and 3) I use sample-split analysis to better understand why firm level volatility is an economically important determinant of corporate cash holdings. In particular, I examine both financial and real investment frictions, which are the key elements of my model (that I will explain in the next section).

Financial Frictions. If firms which operate in more volatile environment hold more cash for precautionary reasons, I would expect that the relation between uncertainty and cash is stronger among firms for which financing frictions are more severe.

Real Frictions. It is well know, since at least Bertola and Caballero (1994), that fixed adjustment costs lead firms to make large, lumpy investments. Therefore these real frictions may lead firms with higher uncertainty to accumulate even more cash to finance their large investments.

Indeed Table 2 shows evidence supporting the role of financial frictions. Here I follow the standard approach in the literature (see for example Whited and Wu (2006)) and in every year I rank firms based on two ex-ante indicators of their financial constraint status, which include firm size and dividend payer status. I assign to the financially contrained groups those firm in the bottom quartile of the annual distribution of each of these measures. Remember that the tables report the coefficient of firm level uncertainty on cash. Irrespective of which indicator of financing status is chosen, I find that the effect of uncertainty on cash is much stronger for firms that are more likely to face financial frictions.

In Table 3 instead I split the sample between bottom and top quartiles of the following two proxies of investment frictions: industry frequency of investment inaction and an indicator for whether there are investment spikes in the industry. For constructing these two indicators I follow closely Cooper and Haltiwanger (2006).

Consistently across the specifications and the type of indicator, the economic significance of the coefficient on uncertainty is much stronger for firms that are more likely to face investment frictions.

Table 1: Panel evidence on firm level uncertainty and cash holdings

	Cash		**Net Leverage**	
	OLS	FE	OLS	FE
	(1)	(2)	(3)	(4)
Firm level risk σ_{it}^F	0.056***	0.048***	-0.122***	-0.039***
	(0.000)	(0.000)	(0.000)	(0.000)
Year fixed effects	Yes	Yes	Yes	Yes
Firm controls	Yes	Yes	Yes	Yes
Adjusted R^2	0.299	0.672	0.325	0.601

Note: The regression includes time dummies and standard controls for firm-level determinants of cash holdings. p-values are in parenthesis and are clustered at the firm level.

Table 2: Sample split analysis: Financial Frictions

	Panel A: Financial Frictions	
	OLS	FE
	(1)	(2)
By Firm Size		
Q1	0.088	0.036
	(0.009)**	(0.008)**
Q4	0.065	-0.001
	(0.010)**	(0.007)
By Dividend Payer Status		
No	0.114	0.026
	(0.007)**	(0.005)**
yes	0.041	0.001
	(0.009)**	(0.006)

Note: The effect of uncertainty on cash holdings is stronger for financially constrained firms. The sample is split between bottom and top quartiles of the (yearly) distribution of: firm size and dividend payment status. The table reports point estimates of the coefficient of cash holdings on firm level uncertainty. The regression includes time dummies and standard controls for firm-level determinants of cash holdings. Standard errors are in parenthesis and are clustered at the firm level.

Table 3: Sample split analysis: Real Frictions

	Panel B: Real Frictions	
	OLS	FE
	(5)	(6)
By Frequency of Invest. Inaction		
Q1	0.075	0.011
	(0.007)**	(0.005)*
Q4	0.159	0.030
	(0.011)**	(0.009)**
By investment Spikes		
No	0.102	0.012
	(0.008)**	(0.006)*
yes	0.132	0.024
	(0.008)**	(0.005)**

Note: The effect of uncertainty on cash holdings is stronger for firms with higher Investment Lumpiness. The sample is split between first and last quartile of (year): frequency of investment inaction - —Capex/Assets—¡0.01, and whether in the industry there are investment spikes - —Capex/Assets—¿0.2, based on Cooper and Haltiwanger (2006). The table reports point estimates of the coefficient of cash holdings on firm level uncertainty. The regression includes time dummies and standard controls for firm-level determinants of cash holdings. Standard errors are in parenthesis and are clustered at the firm level.

In summary, the empirical regularities discovered by the panel analysis presented above are:

Empirical Fact 5: *Firm level risk is a major determinant of the increase in corporate cash holdings in the cross section.*

Empirical Fact 6: *The positive impact of firm level uncertainty on corporate cash holdings is higher for financially constrained firms and for firms belonging to sectors with higher lumpiness in investment.*

Why does a higher level of uncertainty lead firms to accumulate more cash? In this section I highlighted the role of financial and real frictions. As I summarized in the introduction, higher uncertainty increase the need to protect against negative shocks, but this effect is relevant as long as firm face credit constraints. Moreover, firms that are subject to large fixed costs when investing, are more likely to need external finance, since their cash flow from operations may not be enough to cover large capital expenditures. When the volatility of firm level shocks increases, firm become more uncertain about the timing of future investment opportunities, and anticipating the possibility of financial frictions, accumulate more cash and financial assets.

3 The Model

In this section I develop a dynamic model of the joint determination of firm's financing, cash management, and capital accumulation decisions. The aim is to use the model to assess the contribution of the increase in firm level risk observed in the 1980-2010 period on the increase in corporate cash holdings over the same time span. The economic mechanism (as explained in Section 1) relies on firms precautionary saving: firms accumulate cash as a buffer to finance future investment opportunities that come infrequently due to non-convex capital adjustment costs. The dynamic model contains therefore the following features:

- Firms with *heterogeneous* productivity levels.
- *Financial frictions*: I assume that external funds are more costly than internal funds. If firms could simple borrow at no additional cost whenever funds become necessary, they would not need to hold cash for precautionary reasons.
- *Real frictions*: I assume that firms face non-convex costs of capital adjustment. Lumpy investment would reinforce the precautionary motive of holding cash. In the data the increase in cash holdings is stronger for firms that are more likely to face real frictions. I ask then if the model is able to match also this related stylized fact.

3.1 Technology

I begin with describing the economic problem of the firms. Firms are ex-ante identical and are subject to an idiosyncratic productivity shock denoted by z_{it}[5]. I assume that these shocks are generated by a first-order Markov process with transition probability $Q(z', z)$, where a prime indicates next period variables. More specifically I assume an autoregressive process with persistence ρ:

$$\log z' = \rho \log z + \varepsilon' \tag{1}$$

[5] The shocks z_{it} could in principle capture any shock affecting firm's revenues, hence not only shocks to technical efficiency but also (idiosyncratic) demand shocks.

where $\varepsilon' \sim N(0, \sigma_\varepsilon^2)$. As it is standard in the literature I discretize the continuous time process described in (1) as a first-order Markov chain with transition matrix Q using the method in Tauchen (1986). The transition matrix Q is such that $\Pr\{z' = z_j | z = z_i\} = Q_{ij} \geq 0$ and $\sum_j Q_{ij} = 1$ for each $i = 1, \ldots, N_z$. Even though firms are ex-ante identical they differ ex-post since they experience different histories of idiosyncratic productivity shocks. Later in my empirical analysis, in order to make the actual data comparable with the simulated data generated from the model, I use firm and time fixed effects: the idea is to eliminate from the actual data all the heterogeneity that is not accounted by the model[6].

The firm's per period profits are denoted with $\pi(k, z)$. It is not restrictive to assume that the profit function is simply a function of (k, z) since it is understood that variable factors of production, such as labor, have already been maximized out. In other words,

$$\pi(k, z) = \max_n \{f(k, n, z) - wn\}.$$

where $f(k, n, z)$ is the production function w is the wage rate and n is labor (or any other variable production factor). I assume the profit function is strictly increasing in each argument and strictly concave in capital, i.e. $\pi_k(k, z) > 0$, $\pi_z(k, z) > 0$ and $\pi_{kk}(k, z) > 0$. An example that satisfies these properties is given by $\pi(k, z) = zk^\alpha$, with $0 < \alpha < 1$. The empirical counterpart of $\pi(k, z)$ in Compustat is *operating cash flows* (OIBDP).

Investment in capital is defined as:

$$I(k, k') = k' - (1 - \delta) k$$

where $\delta \in (0, 1)$ is the depreciation rate of the capital stock. When a firm buys or sells capital it also incurs adjustment costs:

$$\psi(I, k) = \frac{\psi_s}{2} \left(\frac{I}{k}\right)^2 k + F \cdot \mathbb{1}_{\{I \neq 0\}} k \tag{2}$$

where the first term on the right-hand side of (2) denotes *convex* adjustment costs whereas the second term denotes *fixed* adjustment costs. The functional form in (2) is standard in the structural investment literature (see for example Cooper and Haltiwanger (2006)). Without

[6]In fact the simulations of the model produce indipendent and identically distributed firms where the only heterogeneity comes from the realizations of the shocks. When matching the model to the data, it is therefore important to extract as much heterogeneity as possible.

convex adjustment costs the firm would react too quickly to productivity shocks. The presence of fixed costs capture any instances in which the firm bears costs that are independent on the size of the adjustment. It is needed to account for infrequent adjustment and the presence of many firm-year observations with zero investment in the real firm level data. The presence of fixed cost of adjusting capital is also crucial for explaining the cross-sectional dynamics of cash holdings over time, namely the empirical finding that firms belonging to sectors with more investment lumpiness accumulated more cash in response to an increase in volatility. Indeed the presence of fixed costs make the firm investment policy lumpy (i.e. the firm invests or disinvests only for large enough shocks, and stay inactive otherwise); large and infrequent investments typically require large debt or equity issuances. If these external financing is costly, then firm holds cash for precautionary reasons.

3.2 Financing

I now turn to the firm's financing and saving choices. Crucially capital is *not* the only way to transfer resources across time periods. The firm can save in the form of cash holdings p. Saving in cash is modelled as investing in a riskless one-period bond that earns taxable interest at a rate r. Firms are also subject to corporate taxation in the form of a flat tax τ_c. I assume that both profits and interest on cash holdings are taxed at rate τ_c, but depreciation expenses are tax deductible.

The firm taxable income can then be written as

$$y = \pi(k, z) - \delta k + rp$$

and cash flow is defined as the firm's after-tax income $(1 - \tau_c)\, y$ plus depreciation:

$$CF = (1 - \tau_c)\, \pi(k, z) + \tau_c \delta k + r(1 - \tau_c)\, p \quad (3)$$

The cash flow together with the choices for capital and cash in the next period pin down the gross equity payout[7] distributed (if positive) to shareholders:

$$e = CF - [k' - (1-\delta)\,k] - \psi(I,k) - [p' - p] \tag{4}$$

Firms can finance investment either with internal funds or borrowing from the financial market (by raising new equity or issuing debt). By raising external finance the firm incurs a variety of additional costs going from flotation costs to adverse selection premia. As in Gomes (2001) and Hennessy and Whited (2007) I do not model explicitly a setting with asymmetric information but I attempt to capture the simple fact that external funds are more costly than internal funds in a reduced form way. In particular, I assume that the additional cost of raising external finance is given by:

$$\lambda(e) = \begin{cases} \lambda_0 + \lambda_1\,|e| + \lambda_2 e^2, & \text{if } e < 0 \\ 0, & \text{if } e \geq 0. \end{cases} \tag{5}$$

The interpretation is the following: if $e \geq 0$ the firm is making distributions to shareholders, and if instead $e < 0$ the firm is issuing new equity. In this latter case the firm has to pay an additional cost given by a fixed cost λ_0, a per unit linear cost λ_1 and a per unit quadratic cost λ_2. A large body of empirical research provides detailed evidence regarding underwriting fees (see, among others, Altinkilic and Hansen (2000)) finding that there are significant economies of scale for small to medium issuances but then costs increase; therefore a fomulation with convex cost function that generates a U-shaped average cost seems most appropriate. In any case the simulated method of moments is used to estimate the three parameters of (5).

Finally the net equity payout is given by:

$$(1 - \tau_d \mathbb{1}_{e\geq 0})e - \lambda(e)$$

This captures the fact that if the gross equity payout e is positive then the firm pays out

[7]Substituting 3 into 4 shows that the gross equity payout can be rewritten as:

$$\begin{aligned} e \;=\; & (1-\tau_c)\,\pi(k,z) + \tau_c \delta k \\ & - [k' - (1-\delta)\,k] - \psi(I,k) \\ & - p' + [1 + r\,(1-\tau_c)\,p]\,. \end{aligned}$$

dividends, subject to the dividend tax

$$d = (1 - \tau_d)\, e$$

If instead the gross equity payout is negative, the firm has to raise external finance and pays the extra borrowing cost associated to it:

$$eqiss = e - \left(\lambda_0 + \lambda_1 |e| + \lambda_2 e^2\right) < 0$$

Absent any form of financial market imperfections, cash would have no value for the firm; in this framework cash derives value because it is an alternative to costly external finance (see the subsequent analysis for more on this point).

The firm problem is to choose investment and financial policy to maximize net payments to its shareholders, taking as given the interest rate r (which is the opportunity cost of funds):

$$V(k, p, z) = \max_{k', p'} \left\{ \left(1 - \tau_d \mathbb{1}_{\{e>0\}}\right) e - \lambda(e) + \frac{1}{1+r} E_{z'|z} V(k', p', z') \right\} \tag{6}$$

s.t.

$$e = (1 - \tau_c)\, \pi(k, z) + \tau_c \delta k - I - \psi(I, k) - p' + (1 + r(1 - \tau_c))\, p, \tag{7}$$

$$k' = (1 - \delta)\, k + I, \tag{8}$$

$$\psi(I, k) = \frac{\psi_0}{2} \left(\frac{I}{k}\right)^2 k + F \cdot \mathbb{1}_{\{I \neq 0\}}$$

$$\lambda(e) = \left(\lambda_0 + \lambda_1 |e| + \frac{1}{2} \lambda_2 e^2\right) \mathbb{1}_{\{e<0\}}, \tag{9}$$

$$0 \leq p' \leq \overline{p}. \tag{10}$$

The term $E_{z'|z} V(k', p', z')$ is the continuation value associated to a given choice of future capital and future cash and can be written more extensively as

$$E_{z'|z} V(k', p', z') = \int_{z'} V(k', p', z')\, dQ(z', z).$$

Notice that the upper bound in (10) makes the choice set for p compact but is never binding because of the tax penalty associated with holding cash. Problem (6) admits a unique solution given the existence of upper bounds $\overline{p}$ and $\overline{k}$ for cash balances and capital[8].

Equation (7) represents a sources and uses of funds identity: dividends/issuances are equal to cash flow from operations minus capital expenditures (the term $I(k,k') + AC(k,k')$) minus net increases in cash (the term $p' - (1 + r(1-\tau_c))p$). Please notice that the gross equity payout e can be either positive or negative. If positive it represents distribution of internal cash flows to shareholders (i.e. dividends), and if negative it represents infusions of cash flows from shareholders into the firm. Hence if capital expenditures exceed internal cash flows the firm can either raise costly external finance (incurring the additional cost) or decrease the stock of cash holdings.

3.3 Economic mechanism

In this section I develop the economic intuition behind the model by examining its optimality conditions (although the model has no analytical form solution). The value function is in general not differentiable due to the presence of the cost of issuing equity, but for the sake of exposition I assume that V is concave and once differentiable[9]. See Appendix A for all derivations.

Denoting with μ the multiplier attached to the non-negativity constraint for p, the lagrangian is:

$$\mathcal{L} = \left(1 - \tau_d \mathbb{1}_{\{e>0\}}\right) e - \lambda(e) + \frac{1}{1+r} E_{z'|z} V(k', p', z') + \mu' p'$$

with

$$e(k, k', p, p', z) \equiv (1-\tau_c)\pi(k,z) + \tau_c \delta k - I(k',k) - \psi(k',k) - p' + (1 + r(1-\tau_c))p$$

[8]Indeed if we define the current return function as $F(k,k',p,p',z) = \left(1 - \tau_d \mathbb{1}_{\{e(k,k',p,p',z)>0\}}\right) e - \lambda(e(k,k',p,p',z))$ then problem 6 satisfies the assumptions of Theorem 9.6 in Stokey, Lucas and Prescott (1989). In fact the existence of upper bounds for capital and cash and the assumption that the shock z is discrete ensure that F is defined over the compact set $\left[0,\overline{k}\right]^2 \times [0,\overline{p}]^2 \times \mathcal{Z}$, and therefore since it is continuous it is bounded.

[9]The optimality conditions for the choice of cash balances are obtained under the further assumptions that $\tau_d = 0$ and $\lambda_0 = 0$.

and

$$\begin{aligned}\lambda(e) &= \left(\lambda_0 + \lambda_1 |e| + \frac{1}{2}\lambda_2 e^2\right) \mathbb{1}_{\{e<0\}} \\ &= \left(\lambda_0 - \lambda_1 e + \frac{1}{2}\lambda_2 e^2\right) \mathbb{1}_{\{e<0\}}\end{aligned}$$

The first order condition for p' is:

$$1 + (\lambda_1 - \lambda_2 e) \mathbb{1}_{\{e<0\}} = \frac{1}{1+r} E_{z'|z} \left[V_p(k', p', z')\right] + \mu' \tag{11}$$

The left-hand side of (11) are the costs of holding more cash balances into the next period, which include the equity issuance costs if the current period payoff is negative. The right-hand-side of (11) are instead the benefits related to additional cash.

The envelope condition reads as:

$$V_p(k, p, z) = (1 + r(1 - \tau_c)) \left[1 + (\lambda_1 - \lambda_2 e) \mathbb{1}_{\{e<0\}}\right]$$

The above condition summarizes the benefits of entering the current period with more cash holdings carried over from the last period. In particular the benefits take into account the tax penalty (i.e. interests on cash are taxed at rate τ_c) but also the savings on issuance costs if this period payoff e turns out to be negative.

Updating the envelope condition and plugging it into the first order condition (11) delivers:

$$1 + (\lambda_1 - \lambda_2 e) \mathbb{1}_{\{e<0\}} = \left(\frac{1 + r(1 - \tau_c)}{1+r}\right) E_{z'|z} \left[1 + (\lambda_1 - \lambda_2 e') \mathbb{1}_{\{e'<0\}}\right] + \mu' \tag{12}$$

If external finance is costless (i.e. $\lambda_0 = \lambda_1 = \lambda_2$) then the optimality condition (12) becomes

$$1 - \frac{1 + r(1 - \tau_c)}{1+r} = \mu'$$

which implies $\mu' > 0$ and hence $p' = 0$ by complementary slackness. Hence the firm optimally holds no cash if external finance is free. However, in presence of credit frictions, holding cash increases the *financial flexibility* of the firm; by increasing saving by a marginal amount today, the firm entails the tax penalty but also reduces the probability of having to issue external finance tomorrow. The firm invests in cash up to the point where the marginal gain from reducing external finance costs equates the tax penalty on saving.

Figures 13 and 14 help visualize the optimal financial policy of the firm. Figure 13 shows the marginal cost and marginal benefit of a marginal increase in next-period cash balances, p', under the hypothesis that financing frictions are absent. In such a case the marginal benefit is given by the term $\frac{1+r(1-\tau_c)}{1+r} < 1$ (see also equation 12) whereas the marginal cost is simply 1. Trivially since the marginal cost is always higher than the marginal benefit, the optimal choice of p' and hence p will be zero. Figure 14 shows instead the more interesting case of costly external finance: now the marginal cost is a weakly increasing and piecewise differentiable curve whereas the marginal benefit is decreasing. It is useful to split the financing situation of the firm in three regions[10]:

- *Dividend distribution regime*: for low values of p' the firm is able to pay dividends to its shareholders and does not need to raise any extra equity. In this case the marginal cost of cash is 1 (or $1 - \tau_d$ if dividend taxes are positive).

- *Inaction regime*: for intermediate values of p' the firms retains all internal funds and does not distribute dividends nor issue new equity. In this case when $e = 0$ the marginal cost belongs to the compact set $[1, 1 + \lambda_1]$ (or $[1 - \tau_d, 1 + \lambda_1]$ if dividend taxation is considered).

- *Equity issuance regime*: for high values of p' the firm is likely to run a financing deficit (i.e. $e < 0$) and has to raise additional equity. Since equity issuances are not free, the marginal cost is now higher and equal to $1 + \lambda_1 - \lambda_2 e$ (this is represented by the increasing line in Figure 14).

The marginal benefit is represented instead as a decreasing curve in Figure 14. In fact, from equation (12) it can be seen that the marginal benefit of cash is given by

$$\left(\frac{1 + r\,(1 - \tau_c)}{1 + r}\right) E_{z'|z}\left[1 + (\lambda_1 - \lambda_2 e')\,\mathbb{1}_{\{e'<0\}}\right]$$

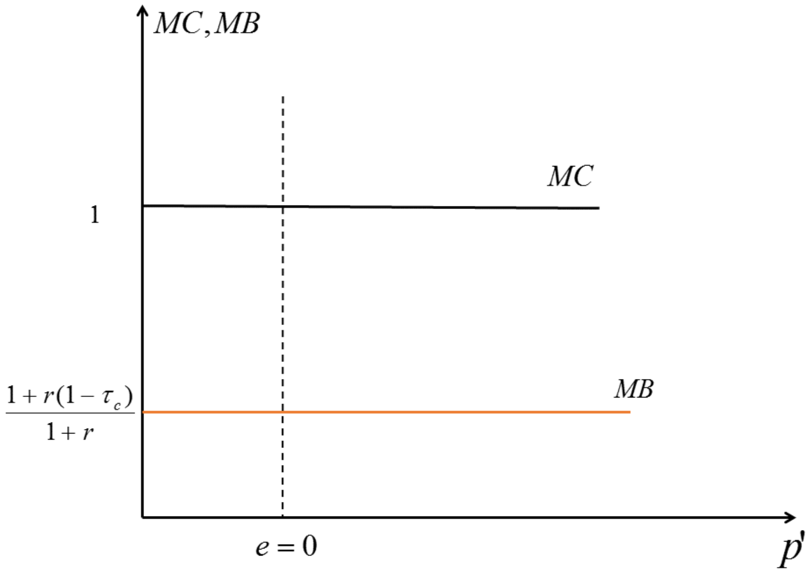
A higher level of next-period cash balances reduces the likelihood of incurring into a negative payoff in the next period (in other words, the probability of the event $\{e' < 0\}$ decreases)

[10]See Appendix A for the formal derivations.

and having to tap costly external finance. Thus the financial flexibility benefit of cash is decreasing in p'.

The optimal choice for an interior p' occurs at the intersection between the marginal cost and the marginal benefit schedule. Depending on the particular parametrization, a corner solution at $p' = 0$ may also occur.

Figure 13: Cash holdings: optimal choice. Hp: no financial frictions



To further inspect the economic incentives for investment and cash in the model, Figure 15 shows the policy functions for investment and next-period cash balances along the productivity dimension (i.e. varying z, for given values of current capital k and cash p). The message here is that investment in physical capital and in cash are closely substitutes. As current productivity z rises, the firm optimally invests a greater deal of capital and brings down cash holdings to finance these investment opportunities. The firm chooses instead to increase the level of cash balances when the productivity of capital is relatively low.

Before presenting the calibration and the results in the next section, I delve more into the economics behind the model by exploring the simulated moments as a function of a subset of parameters. In particular I examine the sensitivity of average cash holdings, defined as $p/\left(k+p\right)$, to some model parameters (please see Figure 16). The key intuition delivered from this exercise is that any model feature that raises the probability of needing external

Figure 14: Cash holdings: optimal choice. Hp: costly equity finance

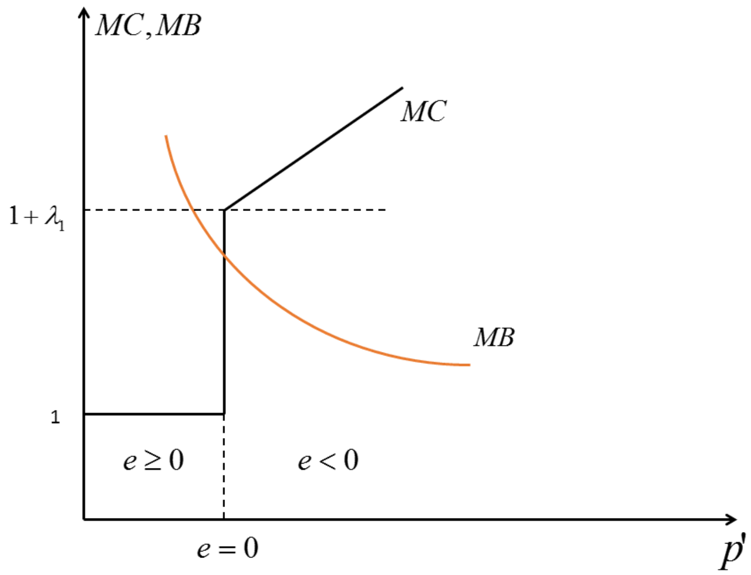
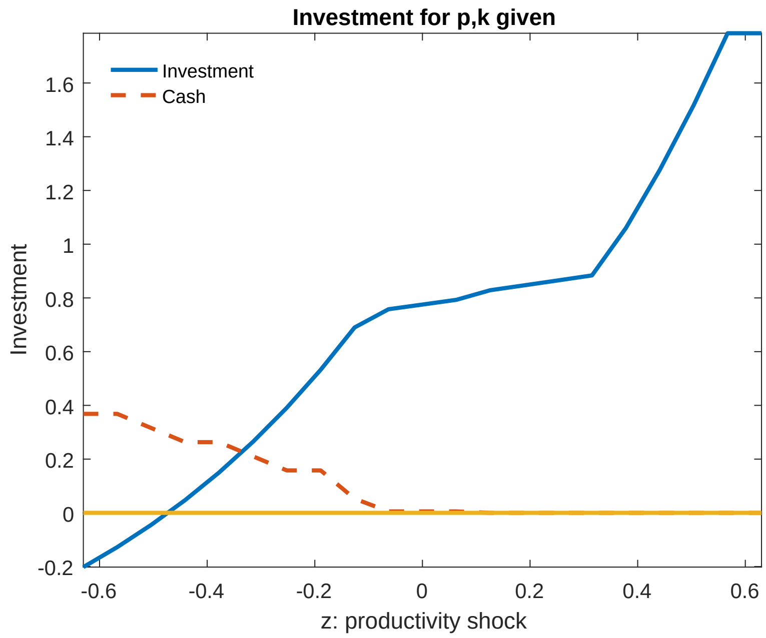


Figure 15: Cash and Investment policy functions along the z dimension



financing will also raise the precautionary motive of holding cash.

The top panels of Figure 16 illustrate the sensitivity of cash holdings to each of the external finance parameters[11]. It is evident that cash increases with the fixed and linear component of the external finance function λ_0 and λ_1, because the value of financial flexibility increases as external finance becomes more costly. When either λ_0 or λ_1 is zero then the optimal level of cash balances is also zero. Then as the cost parameters increase, optimal cash holdings increase sharply and then flatten out.

Turning our attention to the second row of the figure, we see how the profit function curvature (i.e. the capital coefficient α in the production function) and the depreciation rate δ affect cash balances. A higher depreciation rate monotonically increases cash holdings, whereas the relation between the capital coeffcient α and cash is hump-shaped, revealing the presence of two contrasting effects. First, as α rises, the average size of investment rises and hence the firm has to hold more cash to finance greater investment needs. Second, a higher profit curvature α implies that for given capital the firm generates larger internal revenues (i.e. the term $\pi(z, k)$ is larger) which reduces the likelihood of needing external finance, and hence the firm needs to hold less cash.

Focusing now on the third row of Figure 16, it is shown that cash holdings increase in both the variance σ_ε^2 and the serial correlation ρ_ε of the idiosyncratic shocks: firms facing more volatile or persistent shocks are more likely to need external funds for investment, and hence optimally accumulate more cash as a buffer.

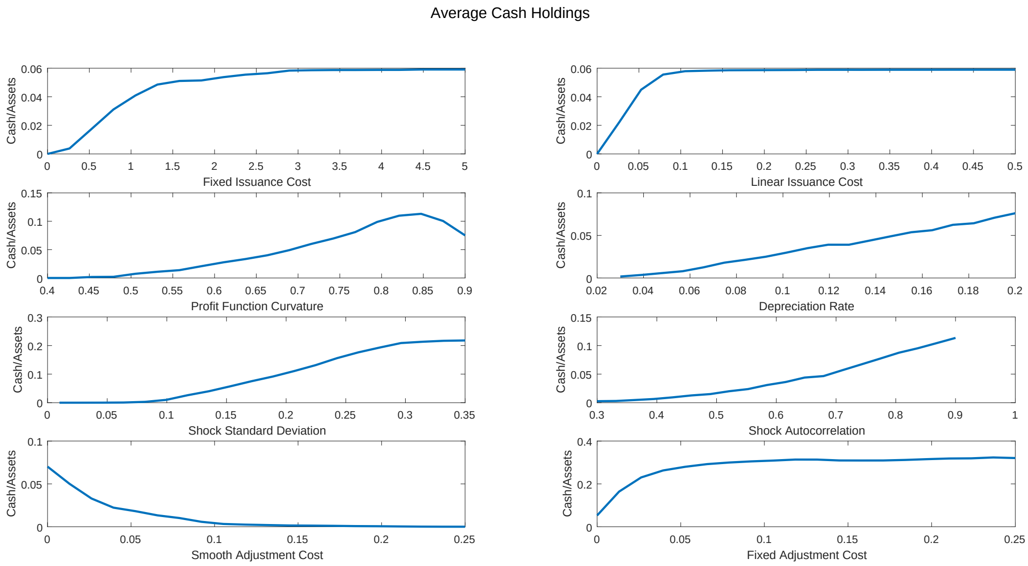
Finally, the bottom panels in the figure show that smooth capital adjustment costs and fixed adjustment costs affect cash holdings in opposite directions[12]. With high convex adjustment costs the firm invests only to replace depreciated capital, never expects to need outside financing and therefore holds no cash, as it is shown in the left bottom panel. The right panel shows instead that larger fixed costs of adjusting capital induce optimally higher cash holdings. This effect occurs because higher fixed adjustment costs lead to larger investment that occur less frequently. The firm then uses episodes of inaction to accumulate cash, which acts to lower the probability of the firm having to raise external equity when it does

[11] However it seems that the effect of the quadratic component λ_2 (not reported in the figure) is much less clear-cut: the relation between average cash holdings and λ_2 is almost flat: with λ_0 and λ_1 set at their baseline levels, the effect of λ_2 is of second-order importance.

[12] This feature turns out to be very useful in the identification of the adjustment cost parameters, when the model is calibrated to the firm level data.

invest.

Figure 16: The Cash/Assets ratio as a function of parameters. Sensisitivity Analysis.



Note: To construct each graph, I solve and simulate the model for each parameter value and then I compute the average ratio of cash across simulations.

4 Results

Since the model admits no closed form solution, I solve and simulate it with the help of numerical techniques. In order to find a numerical solution I need to resort to value function iteration on a discrete grid with interpolation. The numerical solution cannot be accomplished with either policy function iteration since the policy function is not continuous (due to the presence of fixed costs) or methods based on a low level polynomial approximation since the value function needs to be approximated well on a wide range of values of k. Therefore I implement the less efficient but robust method of value function iteration on a discrete state space. I specify a grid with a finite number of values for the capital stock k and the cash stock p, together with upper bounds $\overline{k}$ and $\overline{p}$. I choose the upper bound for capital $\overline{k}$ to be such that

$$\pi\left(\overline{k}, z\right) - \delta\overline{k} = 0$$

Because $k > \overline{k}$ is not economically profitable, k lies in the interval $(0, \overline{k}]$. The grid for capital is made of n_k points and is defined as follows:

$$K = \left\{\overline{k}, \overline{k}\,(1-\delta)\,, \overline{k}\,(1-\delta)^2\,, \ldots, \overline{k}\,(1-\delta)^{n_k-1}\right\}$$

Given the presence of fixed adjustment costs $F \cdot \mathbb{1}_{\{k'-(1-\delta)k \neq 0\}} k$, it is crucial that the capital grid includes the point $k' = k\,(1-\delta)$ for every k to account for investment inaction exactly.[13]

The upper bound for cash holdings is chosen so that $\overline{p} = \overline{k}$. The grid for cash holdings is made of $n_p = \frac{n_k}{2}$ points and it is equispaced. Please remember that in the model without costly external finance the tax penalty of cash would induce the firm to choose $p = 0$. With financial frictions the firm engages in precautionary saving behaviour letting $p > 0$ but nonetheless the optimal choice for p never hits the upper bound $\overline{p}$ in the simulations.

The unknown parameters of the model are estimated using the simulated method of moments (SMM). This procedure selects model parameters in order to minimize the distance between the a given number of data moments and the corresponding model-generated moments. With the estimated version of the structural model at hand, I subsequently perform a counterfactual simulation feeding into the model the exogenous increase in firm-level uncertainty observed over the 1980-2010 period, but keeping everything else constant. In this way I can quantify the relative contribution of increasing uncertainty to the increasing cash holdings at the corporate level.

4.1 Data

The data used for the estimation are based on the Compustat sample described in Section 2. The sample is an unbalanced panel of U.S. firms covering from 1980 to 2010. I exclude firm-year observations for which total assets, sales or capital are either negative or missing. I also drop observations with total assets less than 10 million USD. Finally I delete firms belonging to SIC codes 6000-6999, 4900-4999 or greater than 9000 since my model is ill-suited to study regulated, financial firms or utilities. I trim the top and bottom 2% of the variables

[13] In the computation the capital grid is actually constructed in a slightly different form as:

$$K = \left\{\overline{k}, \overline{k}(1-\delta)^{\chi}, \overline{k}(1-\delta)^{2\chi}, \ldots, \overline{k}(1-\delta)^{(n_k-1)\chi}\right\}$$

where $\chi > 0$. The computational parameter χ governs the width of the grid: if $\chi > 1$ then the grid is streched out, if $\chi < 1$ then the grid becomes more concentrated.

following Hennessy and Whited (2007). After the data cleaning criteria are applied, there are a total of 133803 firm-year observations with 14256 unique firms.

Data variables are defined as follows: book assets is Compustat item AT; capital stock is item PPENT; investment is CAPX; equity issuance is defined as equity issuances net of repurchases (SSTK-PRSTKC) and cash flow is defined as income before extraordinary items plus depreciation (IB+DP in Compustat).

The cross-sectional statistics are determined by considering each firm-year as a data point. See Appendix B for more details on the variables definition both in the Compustat data and in the model.

4.2 Selection of Targets and Identification

The model is estimated over a cross-section of firms comprising 5 years, from 1980 to 1984. I parametrize the model at an annual frequency using the SMM as estimation method, with the exception of three parameters that are instead taken from other studies or directly from the data. Regarding these parameters set outside the model, I fix the interest rate r at 4% by taking the average of the real interest rate[14] over the period 1980-1984. The dividend tax rate τ_d and the corporate tax rate τ_c are instead taken from Hennessy and Whited (2007). These choices are summarized in Table 4.

Table 4: External Parameters

Parameter	**Description**	**Value**
r	Real interest rate	0.04
τ_c	Corporate tax rate	0.46
τ_d	Dividend tax rate	0.12

After fixing the parameters in Table 4, there are 9 parameters that need to be estimated inside the model: the production function curvature α, the depreciation rate of capital δ, the standard deviation and the autocorrelation of the productivity shock $\{\sigma_\varepsilon, \rho\}$, the adjustment cost parameters $\{\psi, F\}$ and the parameters related to the external finance costs $\{\lambda_0, \lambda_1, \lambda_2\}$. The model is overidentified in the sense that these 9 parameters are chosen to match 13 data targets (discussed below).

[14]The real interest rate in the data is computed as the difference between the 3 months interest rate on government bonds and the rate of inflation.

Regarding the data targets, a discussion of the identification strategy is in order. It is well understood that changes to any of the parameters considered in estimation affect the model outcomes for all of the targeted moments. However, some moments are more responsive to certain parameters. Heuristically a moment target is informative about an unknown parameter if that target is sensitive to changes in the parameter value. Before starting the discussion, let me cathegorize the parameters estimated inside the model as either real parameters or financial parameters.

Real parameters. To pin down the returns to scale and the depreciation rate (α, δ) I match the variance of investment and the average annual investment rate defined as the ratio of total investment to total capital stock.

The autocorrelation of investment and the autocorrelation of EBIT help identify ρ, the persistence of the firm profit shock. The standard deviation of investment and the volatility of cash flow provide instead information about σ_ε, the dispersion of the firm profit shock. As an additional restriction to identify the shocks parameters $(\rho, \sigma_\varepsilon)$, I run a panel autoregression of operating income zk^α on lagged operating income, and match the autoregressive coefficient (i.e. the coefficient on lagged operating income) and the standard deviation of the regression residual (see Hennessy and Whited 2007).

To pin down the parameters of the capital adjustment cost function (ψ_0, F) , I match the frequency of investment inaction and the average cash/assets. Furthermore, the quadratic adjustment cost affects also the volatility of investment rate and the autocorrelation: a higher ψ_0 makes investment less volatile and more autocorrelated. Higher ψ_0 also implies a lower mean equity issuances and lower cash holdings. The opposite happens with F. A higher F makes the firm invest less frequently. The episodes of investment become though more pronounced so the firm is more likely to tap external equity finance, and in turn increases the precautionary motive for holding cash.

Financial parameters. In order to pin down the three parameters of the external finance cost function $(\lambda_0, \lambda_1, \lambda_2)$, I use targets that are informative about the financial choices of the firm. In particular I ask the model to match the mean of equity issuances[15], the standard deviation of equity issuances and the frequency of equity issuances. This latter moment is very infomative about the size of the fixed cost of raising new equity, λ_0. The

[15]All moments are defined as ratio to total assets.

variance of equity issuances is instead very sensitive to changes in λ_2.

The informal discussion about identification is summarized in Table 5.

Table 5: How Identification Works: an heurstic approach. The first line, for example, is to be read in the following way: An increase in α leads to an increase in the volatility of investment and cash flow, and to more equity issuances.

Parameter		Target
α	$\Longrightarrow$	+std(invest), + std(cash flow), +eqiss
δ	$\Longrightarrow$	+average annual investment rate
σ_ε	$\Longrightarrow$	+std(invest), + std(cash flow), +eqiss, +E(cash/assets)
ρ	$\Longrightarrow$	+autocorr(ebit), +eqiss, +E(cash/assets)
ψ_0	$\Longrightarrow$	- std(invest), –eqiss, - E(cash/assets)
F	$\Longrightarrow$	*+Freq of INV=0, +*E(cash/assets)
λ_0	$\Longrightarrow$	*- frequency equity issuances*
λ_1	$\Longrightarrow$	*- mean equity issuances, +*E(cash/assets)
λ_2	$\Longrightarrow$	*- variance equity issuances*

4.3 Estimation Results

Turning now to the results from the estimation exercise, Table 6 presents the point estimates and the standard errors, whereas Table 7 shows the goodness of fit of the model comparing the real targets to the model-simulated targets.

The estimated parameters together with their standard errors are presented in Table 6. The point estimates of the profit function curvature, the depreciation rate and the serial correlation of the profit shock are qualitatively in line with Hennessy and Whited (2007) and DeAngelo et al. (2011). The estimates of the adjustment costs of capital are also comparable to those in the literature using Compustat data, even though they differ significantly from Cooper and Haltiwanger (2006). With respect to the latter study my estimates for the fixed cost are lower and the estimate for the quadratic cost are higher. Two reasons can explain such discrepancy: First, the data used here are based on publicly traded firms whose investment behaviour is typically smoother than in plant-level observations from the Longitudinal Research Database used in Cooper and Haltiwanger (2006). Second, the adjustment costs parameters in my model influence greatly also the ratio of cash to assets, as it was shown in Figure 16.

Turning to the financial side of the model economy, the parameters λ_0 and λ_1 are both statistically significant at the 5% level. Although λ_2 is positive, it is statistically insignificant. Overall the estimates of the external equity cost function are somewhat smaller than Hennessy and Whited (2007). However, given that the effect of increasing uncertainty on cash holdings is larger the higher the external finance costs, my result is on the conservative side.

As shown in Table 7 my model fits the data reasonably well[16]. Most simulated moments in Table 7 match the corresponding data moments well, with the relevant exceptions of (i) the average size of equity issuances and (ii) the standard deviation of investment. Regarding (i), the model significantly underpredicts mean equity issuances: lowering the cost of external finance would of course raise equity issuances but at the same time would worse the model fit along other dimensions. Finally, on (ii), adding some extra friction to the capital adjustment process would help match the volatility of the investment rate more closely.

Overall the model is able to reproduce key features of the data. This result strenghtens the reliability of using the estimated models to examine the relationship between firm level uncertainty and firm's cash holdings.

Table 6: Estimated parameters: results of SMM procedure

Parameter	**Description**	**Value**	**Standard Error**
Real parameters			
α	Profit function curvature	0.849	0.273
δ	Depreciation rate	0.133	0.033
σ_ε	Shock volatility	0.300	0.085
ρ	Shock autocorrelation	0.758	0.210
ψ_0	Quadratic adjustment cost	0.236	0.107
F	Fixed adjustment cost	0.0012	0.00048
Financial parameters			
λ_0	Fixed cost of external finance	0.589	0.302
λ_1	Linear cost of external finance	0.053	0.024
λ_2	Quadratic cost of external finance	0.0004	0.0005

[16]The reader should bear in mind that the model is highly nonlinear and overidentified.

Table 7: Model Fit: Comparing Real Moments vs Simulated Moments

Target	**Data (1980-1985)**	**Model**
Aggregate I/K	0.1334	0.1334
Average Cash	0.1037	0.0996
Average Equity Iss.	0.0371	0.0150
Average Q	1.3764	2.1609
Average EBIT	0.0978	0.0813
StDev Cash	0.1259	0.1284
StDev Invest	0.1155	0.1763
StDev Equity Iss.	0.1082	0.0979
StDev EBIT	0.1044	0.0912
StDev Cash Flow	0.0784	0.0794
Average (invest=0)	0.0519	0.0659
Average (eqiss¿0)	0.2509	0.2438
Autocorr EBIT	0.6932	0.6905

4.4 Counterfactual Experiments: explaining the corporate cash increase

I can now use the model economy estimated on the time span 1980-1984 for counterfactual analysis. I perform a set of counterfactual experiments in order to quantify the relative importance of increasing firm-level volatility on the size of rising cash holdings over the 1980-2010 period. Table 8 summarizes the main quantitative results. In the main experiment reported in panel (a) of Table 8 I increase the firm shock volatility σ_ε from the estimated value of 0.3 to 0.45 in order to generate the exact same increase in cash flow voaltility that is observed in the data, from the 1980-1984 to the 2006-2010 time span. For the sake of clarity all other model parameters are kept at their initial values (estimated on the 1980-1984 data): in this way the effect of rising uncertainty is completely isolated from other shocks that may have hit the U.S. economy over the same time horizon. As a further robustness check I also perfom another exercise in which I recalibrate also the other parameters to match the new steady state and the results do not change dramatically[17].

First consider the baseline economy with both real and financial frictions (non-convex adjustment costs of capital plus costly equity financing) in panel (a) of table 8. The first row shows that, as firm-level uncertainty increases from 0.3 to 0.45, the cash/assets ratio in the model increases from 9% to 19%. This result shows that an increase in the firm

[17] Results are available upon request.

precautionary demand for cash can explain around 87% of the increase in cash observed in the data. In fact, over the 1980-2010 period, the cash ratio in the data increased by 12 percentage points, from 9% to 21%, suggesting that the estimated model with real and financial frictions can account for the main share of the increase in cash holdings. As a validation of these results, Table 9 shows how all the other data moments change after the shock standard deviation is increased from 0.3 to 0.45 to match the increase in cash flow volatility in the data. The increase in cash flow volatility is matched exactly by construction (as explained above) but also most of the other moments change in the correct direction. This provides further reassurance about the validity of the model.

The empirical analysis in Section 2 pointed out the significant heterogeneity between industries in the rise in cash holdings: sectors that vary a great deal in terms of lumpiness and investment irreversibility may have a different reaction to the economy-wide increase in uncertainty. In order to explore the interplay between the increase in CFV and investment lumpiness I perform a second counterfactual experiment, presented in panel (b) of Table 8, In this second experiment I again increase the dispersion parameter in the model to generate the same increase in uncertainty that is observed in the data, but this time I completely shut down the non-convex adjustment costs channel, by setting $F = 0$. It is clear that financial frictions (or, to be more precise, the precautionary motive coming from financial frictions) alone are not sufficient to match the magnitude of the increase in corporate cash holdings observed in the data: as firm level risk increases from 0.15 to 0.45, the cash-to-assets ratio increases by only 5 percentage points. This result shows that the interaction between financial and real frictions is key to generate a sufficient motive for cash hoarding.

Table 8: Firm-Level Risk and Cash Holdings: Results from the Model

	(a) Fin.Frictions + Fixed cost			**(b) Fin.Frictions only**		
Firm Volatility σ_ε	0.15	0.45		0.15	0.45	
StDev Cash Flow	0.078	0.259		0.078	0.259	
Cash/Assets (US data)	0.09	0.21	+133%	0.09	0.21	+133%
Cash/Assets (model)	0.09	0.19	+116%	0.09	0.14	+55%
Perc. explained by model			87.22%			48.67%

Table 9: Model validation. How all model moments change after shock volatility σ_ϵ is increased to match the cash flow volatility increase in the data (**StDev Cash Flow** is matched by construction)

Target	**Model experiment**	**Data 2010**
Aggregate I/K	0.105	0.096
Average Cash	0.19	0.22
Average Equity Iss.	0.062	0.06
Average Q	2.371	1.84
Average EBIT	0.025	0.017
StDev Cash	0.340	0.238
StDev Invest	0.323	0.101
StDev Equity Iss.	0.132	0.174
StDev EBIT	0.182	0.219
StDev Cash Flow	0.258	0.258
Average (invest=0)	0.073	0.10
Average (eqiss¿0)	0.385	0.296
Autocorr EBIT	0.791	0.773

5 Conclusions

In this paper I have presented new evidence which supports the hypothesis that the rise in firm levell volatility can explain the secular increase in US corporate cash holdings over the last four decades. The empirical analysis shows that firm level uncertainty is a key empirical determinant of firm level cash holdings. In addition, the available evidence suggests that both financial and real frictions contribute substantially to explain why firm level uncertainty matters so much. Regarding real frictions, I document a novel stylized fact: the impact of firm level uncertainty on cash holdings is stronger for firms that belong to sectors with greater investment lumpiness (measured by the industry frequency of investment inaction and by the industry investment spikes indicators).

Next, I build a structural dynamic model where uncertainty induces a precautionary behaviour: firms retain cash to avoid raising costly external finance when positive investment opportunities arrive. I show quantitatively that such precautionary effect depends strongly on the presence of fixed capital adjustment costs. Indeed in the baseline simulation, when I increase firm volatility to match the corresponding increase in firm volatility in the US data, the model is able to generate a substantial increase in cash-to-asset ratios only when non-convexities in adjustment costs are present.

I conclude that firm level uncertainty is a crucial ingredient to provide a satisfactory account of key stylized facts in corporate finance.

References

Heitor Almeida, Murillo Campello, and Michael S. Weisbach. The Cash Flow Sensitivity of Cash. *Journal of Finance*, 59(4):1777–1804, 08 2004. URL http://ideas.repec.org/a/bla/jfinan/v59y2004i4p1777-1804.html.

Oya Altinkilic and Robert S Hansen. Are there economies of scale in underwriting fees? evidence of rising external financing costs. *Review of Financial Studies*, 13(1):191–218, 2000. URL http://ideas.repec.org/a/oup/rfinst/v13y2000i1p191-218.html.

Thomas W. Bates, Kathleen M. Kahle, and Rene M. Stulz. Why do u.s. firms hold so much more cash than they used to? *Journal of Finance*, 64(5):1985–2021, October 2009. URL http://ideas.repec.org/a/bla/jfinan/v64y2009i5p1985-2021.html.

Guiseppe Bertola and Ricardo J Caballero. Irreversibility and Aggregate Investment. *Review of Economic Studies*, 61(2):223–46, April 1994. URL http://ideas.repec.org/a/bla/restud/v61y1994i2p223-46.html.

Nicholas Bloom. The impact of uncertainty shocks. *Econometrica*, 77(3):623–685, 05 2009. URL http://ideas.repec.org/a/ecm/emetrp/v77y2009i3p623-685.html.

Ricardo J. Caballero. Aggregate investment. In J. B. Taylor and M. Woodford, editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 12, pages 813–862. Elsevier, 1999. URL http://ideas.repec.org/h/eee/macchp/1-12.html.

Diego Comin and Sunil Mulani. Diverging trends in aggregate and firm volatility. *The Review of Economics and Statistics*, 88(2):374–383, May 2006. URL http://ideas.repec.org/a/tpr/restat/v88y2006i2p374-383.html.

Diego Comin and Thomas Philippon. The rise in firm-level volatility: Causes and consequences. NBER Working Papers 11388, National Bureau of Economic Research, Inc, May 2005. URL http://ideas.repec.org/p/nbr/nberwo/11388.html.

Russell W. Cooper and John C. Haltiwanger. On the nature of capital adjustment costs. *Review of Economic Studies*, 73(3):611–633, 2006. URL http://ideas.repec.org/a/oup/restud/v73y2006i3p611-633.html.

Steven J. Davis, John Haltiwanger, Ron Jarmin, and Javier Miranda. Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms. In *NBER Macroeconomics Annual 2006, Volume 21*, NBER Chapters, pages 107–180. National Bureau of Economic Research, Inc, October 2007. URL https://ideas.repec.org/h/nbr/nberch/11178.html.

Harry DeAngelo, Linda DeAngelo, and Toni M. Whited. Capital structure dynamics and transitory debt. *Journal of Financial Economics*, 99(2):235–261, February 2011. URL http://ideas.repec.org/a/eee/jfinec/v99y2011i2p235-261.html.

Antonio Falato, Dalida Kadyrzhanova, and Jae W. Sim. Rising intangible capital, shrinking debt capacity, and the us corporate savings glut. Finance and Economics Discussion Series 2013-67, Board of Governors of the Federal Reserve System (U.S.), 2013. URL http://ideas.repec.org/p/fip/fedgfe/2013-67.html.

Joao F. Gomes. Financing investment. *American Economic Review*, 91(5):1263–1285, December 2001. URL http://ideas.repec.org/a/aea/aecrev/v91y2001i5p1263-1285.html.

Christopher A. Hennessy and Toni M. Whited. How costly is external financing? evidence from a structural estimation. *Journal of Finance*, 62(4):1705–1745, 08 2007. URL http://ideas.repec.org/a/bla/jfinan/v62y2007i4p1705-1745.html.

Tim Opler, Lee Pinkowitz, Rene Stulz, and Rohan Williamson. The determinants and implications of corporate cash holdings. *Journal of Financial Economics*, 52(1):3–46, April 1999. URL http://ideas.repec.org/a/eee/jfinec/v52y1999i1p3-46.html.

Leigh A. Riddick and Toni M. Whited. The corporate propensity to save. *Journal of Finance*, 64(4):1729–1766, 08 2009. URL http://ideas.repec.org/a/bla/jfinan/v64y2009i4p1729-1766.html.

George Tauchen. Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177–181, 1986. URL http://ideas.repec.org/a/eee/ecolet/v20y1986i2p177-181.html.

David Thesmar and Mathias Thoenig. Contrasting Trends in Firm Volatility. *American Economic Journal: Macroeconomics*, 3(4):143–180, October 2011. URL https://ideas.repec.org/a/aea/aejmac/v3y2011i4p143-80.html.

Toni M. Whited. External finance constraints and the intertemporal pattern of intermittent investment. *Journal of Financial Economics*, 81(3):467–502, September 2006. URL https://ideas.repec.org/a/eee/jfinec/v81y2006i3p467-502.html.

Toni M. Whited and Guojun Wu. Financial constraints risk. *Review of Financial Studies*, 19(2):531–559, 2006. URL http://ideas.repec.org/a/oup/rfinst/v19y2006i2p531-559.html.

Appendix

A Optimal Cash Policy

In this appendix I present a detailed derivation of the analytical results shown in Section 3.3. For convenience I define:

$$e\left(k,k',p,p',z\right)\equiv\left(1-\tau_c\right)\pi\left(k,z\right)+\tau_c\delta k-\left[k'-\left(1-\delta\right)k\right]-\psi\left(k',k\right)-\left[p'-\left(1+r\left(1-\tau_c\right)\right)p\right]$$

and

$$e\left(k',k'',p',p'',z'\right)\equiv\left(1-\tau_c\right)\pi\left(k',z'\right)+\tau_c\delta k'-\left[k''-\left(1-\delta\right)k'\right]-\psi\left(k'',k'\right)-\left[p''-\left(1+r\left(1-\tau_c\right)\right)p'\right]$$

A.0.1 Case $\tau_d = 0$

Case NO financial frictions (i.e. $\lambda_1 = \lambda_2 = 0$)

In this case we have

$$MB=\frac{1+r\left(1-\tau_c\right)}{1+r}<1=MC$$

Hence $p' = p = 0$.

Case $\lambda_1 > 0$ and $\lambda_2 > 0$

In this case the FOC wrt p' is

$$1+\left(\lambda_1-\lambda_2 e\right)\mathbb{1}_{\{e<0\}}\geq\frac{1}{1+r}E_{z'|z}\left[V_p\left(k',p',z'\right)\right]$$

with equality if $p' > 0$. The envelope condition is

$$V_p\left(k,p,z\right)=\left(1+r\left(1-\tau_c\right)\right)\left[1+\left(\lambda_1-\lambda_2 e\right)\mathbb{1}_{\{e<0\}}\right]$$

Updating the envelope condition and plugging it into the FOC gives:

$$1+\left(\lambda_1-\lambda_2 e\right)\mathbb{1}_{\{e<0\}}\geq\left(\frac{1+r\left(1-\tau_c\right)}{1+r}\right)E_{z'|z}\left[1+\left(\lambda_1-\lambda_2 e'\right)1_{\{e'<0\}}\right]$$

with equality if $p' > 0$. Hence the marginal values are:

$$MC=\begin{cases}1 & \text{if } e\geq 0\\ 1+\lambda_1-\lambda_2 e>1 & \text{if } e<0\end{cases}$$

and

$$MB=\begin{cases}\frac{1+r(1-\tau_c)}{1+r}, & \text{if } e'\geq 0\\ \left(\frac{1+r(1-\tau_c)}{1+r}\right)E_{z'|z}\left[1+\left(\lambda_1-\lambda_2 e'\right)\right] & \text{if } e'<0\end{cases}$$

Comments/Results: MC is weakly increasing in p'. Proof: If p' increases, the payoff e decreases (and becomes negative). In particular there exists a threshold $\widetilde{p}'$ such that $e\left(k,k',p,p',z\right)\geq 0$ if $p'\leq\widetilde{p}'$ and $e\left(k,k',p,p',z\right)<0$ if $p'>\widetilde{p}'$. Indeed

$$\begin{aligned} e &\geq 0 \implies (1-\tau_c)\,\pi\left(k,z\right)+\tau_c\delta k-I-\psi\left(k',k\right)-\left[p'-\left(1+r\left(1-\tau_c\right)\right)p\right]\geq 0\\ &\implies p'\leq (1-\tau_c)\,\pi\left(k,z\right)+\tau_c\delta k-I-\psi\left(k',k\right)+\left(1+r\left(1-\tau_c\right)\right)p\equiv\widetilde{p}' \end{aligned}$$

MB is decreasing in p'. Proof: A higher value for future cash implies that the probability of $\{e'<0\}$ is lower, hence MB goes down.

Higher z implies higher investment, which leads to a lower $\widetilde{p}'$. This increases the MC of cash, hence the optimal choice for p' decreases.

A.0.2 Case $\tau_d>0$

In this case the FOC wrt p' is

$$1-\tau_d\mathbb{1}_{\{e\geq 0\}}+\left(\lambda_1-\lambda_2 e\right)\mathbb{1}_{\{e<0\}}\geq\frac{1}{1+r}E_{z'|z}\left[V_p\left(k',p',z'\right)\right]$$

with equality if $p'>0$. The envelope condition is

$$V_p\left(k,p,z\right)=\left(1+r\left(1-\tau_c\right)\right)\left[1-\tau_d\mathbb{1}_{\{e\geq 0\}}+\left(\lambda_1-\lambda_2 e\right)\mathbb{1}_{\{e<0\}}\right]$$

Updating the envelope condition and plugging it into the FOC gives:

$$1-\tau_d 1_{\{e\geq 0\}}+\left(\lambda_1-\lambda_2 e\right)1_{\{e<0\}}\geq\left(\frac{1+r\left(1-\tau_c\right)}{1+r}\right)E_{z'|z}\left[1-\tau_d\mathbb{1}_{\{e'\geq 0\}}+\left(\lambda_1-\lambda_2 e'\right)1_{\{e'<0\}}\right]$$

with equality if $p'>0$. Hence the marginal values are:

$$MC=\begin{cases}1-\tau_d<1 & \text{if } e\geq 0\\ 1+\lambda_1-\lambda_2 e>1 & \text{if } e<0\end{cases}$$

and

$$MB = \begin{cases} \left(\frac{1+r(1-\tau_c)}{1+r}\right)(1-\tau_d) < 1-\tau_d, & \text{if } e' \geq 0 \\ \left(\frac{1+r(1-\tau_c)}{1+r}\right) E_{z'|z}\left[1+(\lambda_1-\lambda_2 e')\right] & \text{if } e' < 0 \end{cases}$$

B Data

The data used in the paper are from Compustat annual database for the period 1980-2010. A detailed description of variables definition follows.

Total Assets: Total assets are defined as Compustat variable AT (data6). In the model, total assets are obtained by summing physical assets k and cash p.

Cash Holdings: Cash holdings are computed as cash and marketable securities (data item `CHE`) divided by total assets (data item `AT`). In the model cash holdings are:

$$\frac{p}{k+p}$$

where total assets = capital + cash holdings.

Investment: It is computed as the ratio between capital expenditures (data item `CAPX`) and total assets (data item `AT`). Investment in the model is defined as:

$$ik = \frac{I}{k+p} = \frac{k' - (1-\delta)\,k}{k+p}$$

Sometimes it is better to define the investment rate as I/k, excluding the cash stock p from the denominator. Average investment can be computed in two ways (both in the data and in the model). Average cross-sectional investment:

$$E\,(ik) = \frac{1}{T}\sum_t \frac{1}{N}\sum_i \frac{I_{it}}{k_{it}} = \frac{1}{TN}\sum_{i,t}\frac{I_{it}}{k_{it}}$$

or average investment ratio:

$$E\,(IK) = \frac{1}{T}\sum_t \left[\frac{I_t}{K_t}\right] = \frac{1}{T}\sum_t \left[\frac{\sum_i I_{it}}{\sum_t k_{it}}\right]$$

In the current version of the paper, the investment standard deviation is computed as follows:

$$InvSTDEV = \frac{1}{TN}\sum_{i,t}\left(\frac{I_{it}}{k_{it}}\right)^2 - \left[E\left(\frac{I_{it}}{k_{it}}\right)\right]^2$$

Table 10: Firm's balance sheet in Compustat

+**SALE**	Revenues	
−COGS	cost of goods sold	
−XSGA	Selling, General, and Admin Expense	
= **OIBDP**	operating income before depreciation (EBITDA)	zk^{α}
−DP	depreciation	
= **OIADP**	operating income after depreciation.EBIT	$zk^{\alpha} - \delta k$
−XINT	interest expenses	
−TXT	taxes paid	
=**IB**	income before extraord items	

so basically is the standard deviation of the pooled data.

EBITDA: Earnings before interests taxes and depreciation are defined as the data item OIBDP. The Compustat variable OIBDP is obtained as Sales (SALE, data12) - Cost of Goods Sold (COGS, data41) - Selling,General and Administrative Expense (XSGA, data189). See Table 10 for more details. The model counterpart of OIBDP (and, hence, EBITDA) is simply:

$$EBITDA = \frac{zk^{\alpha}}{k+p}$$

EBIT: Earnings before interest expenses and taxes are defined as operating income minus depreciation. In the model it is as follows:

$$EBIT = \frac{zk^{\alpha} - \delta k}{k+p}$$

Cash Flow: It is defined as operating income (sales minus operating costs) after interest and taxes but before depreciation. In Compustat we have Cash Flow = Operating Income (OIBDP, data13) - Interest Expense (XINT, data15) - Income Taxes (TXT, data16). By construction this is equal to Income before extra items (IB, data18) + Depreciation (DP, data14)

Dividend: Dividends are measured as commond dividends (data item `DVC`).

Distributions. Total distributions are the sum of item 19 (DVP), item 21 (DVC), item 115 (PRSTKC).

Equity issuances. Equity issuances are defined as item 108 (SSTK) or net as SSTK-PRSTKC

To sum up, Table 10 should help clarify all these definitions.