Divergent Risk-Attitudes and Endogenous Collateral Constraints*

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Abstract

The booms preceding financial crises typically feature high exposure to risky assets, high leverage, asset price growth and low debt margins, which are then followed by sharp de-leveraging after the crisis. We build a model that endogenously generates such heightened leverage/de-leverage cycle and asset price boom/bust with three elements. First, borrowers exhibit gain-loss preferences, being increasingly risk-tolerant at the upper tails and loss-averse on the lower tails. Second, they are subject to occasionally binding collateral constraints. The latter interacted with borrowers' valuation of collateral, which varies between the upper and the lower tails and affects the extent of the constraint region, contribute to explain debt pro-cyclicality, despite endogenous supply. Third, there is heterogeneity in risk-attitudes between borrowers and lenders. The latter implies that the debt margin varies endogenously and countercyclically to close the gap between lenders/borrowers evaluations (namely debt demand and supply). We solve the model analytically and numerically, through a global method (policy function iterations with endogenously Markov-switching regimes) and show that the model matches well moments for asset prices, returns, equity premia and Sharpe ratio, the volatility of leverage, its procyclicality and the counter-cyclicality of the debt margins.

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1 Introduction

There is by now consensus on the narrative of the recent financial crisis as well as of its unfolding[1]. The boom phase preceding the crisis was characterized by a build-up of leverage, low debt margins (borrowers' down-payment), lax debt constraints and a boom in asset prices. The post-crisis was characterized by de-leverage with binding constraints, increasing debt margins and a burst of the asset price. This pattern of events, featuring debt and asset price heightened pro-cyclicality and margins counter-cyclicality, is actually common to many debt driven financial crises. We build a parsimonious lenders/borrowers model of debt that endogenously generates the above described leverage/de-leverage cycle and the boom/bust in asset prices. Beyond that the model is able to match debt moments and main asset price facts.

Three are the crucial ingredients. First, borrowers investing in risky assets exhibit increasing risk-tolerance at the upper tail and at large gambles and loss aversion at the lower tail. This implies that in face of booms or credit expansions shocks, borrowers' evaluation of collateral is high. The latter contributes to relax the borrowing constraint and to expand credit availability. Second, debt markets shall be disciplined by occasionally binding collateral constraints. This ties debt availability to fluctuations in collateral values, which are in turn linked to time-varying changes in borrowers' stochastic discount factors. Hence, at the upper tail, or after a sequence of positive shocks, borrowers become increasingly risk-tolerant, their collateral valuation raises and the debt constraint is relaxed. The opposite is true in recessions or at the lower tails of the shock distribution. The interaction between the time-varying risk-attitudes and the occasionally binding nature of the constraint contributes to explain debt pro-cyclicality and the heightened dynamic of asset prices, with the first fact being at odd with most models of credit and financial frictions. Third, supply of debt is endogenous in our model and divergence in risk-attitudes, hence in collateral valuations, between borrowers, who demand debt, and lenders, who supply debt, induces scope for trading in debt and determines margins making them behave counter-cyclically. Margins are indeed akin to compensation premia which borrowers transfer to lenders to compensate them for bearing the risk of collateral fluctuations. In recessions, for instance, when collateral values decline, larger premia

[1] See Morris and Shin [71], Hanson, Kashyap and Stein [47], Gorton and Metrick [41] among many others.

shall be transferred and margins increase. Importantly in our model, S-shaped preferences do not trivially induce debt and asset pro-cyclicality per se, as the equilibrium level of debt depends upon the balance between demand and supply forces and upon the balance of traditional precautionary saving motives and more novel risk-tolerance motives.

The idea that divergence in risk-attitudes and time-varying fluctuations of risk attitudes and/or subjective beliefs are key drivers of the financial cycles pro-cyclicality and of the amplification ensuing even small credit or income shocks, is not new. It dates back to Minsky[70] and it has been recently rationalized through pioneering models such as those in Geanakoplos[40] or Simsek[81] and subsequent others. Both the last cited contributions focus on elegant static models with borrowers/lenders and binomial distribution of beliefs. Our contribution is to develop these ideas, both analytically and numerically, into a fully dynamic context. In terms of asset valuations and stochastic discount factor, in our paper we focus on risk-attitudes rather than ambiguity attitudes or subjective beliefs[2]: we do so since the S-shaped preferences, that we use to model risk-attitudes, have strong empirical support and are compatible with risk-behaviour at large and small gambles, as we further explain below. With the support of most modern global methods we can also quantify effects and show that our model can contemporaneously explains facts on debt dynamic and asset price dynamic.

In our model lenders and borrowers have heterogenous preferences and trade debt. Borrowers, who lever up to invest in risky assets, have gain-loss and reference-dependent preferences[3]. This implies that they exhibit increasing risk-tolerance on the upper tails, or on the right of the reference level[4], and increasing risk-sensitivity on the lower tails[5]. Risk-tolerance at the upper tails implies that borrowers have high and increasing demand for risky assets, for which they need to leverage.

[2] Noe that from the point of view of stochastic discount factor estimations the two are often indistinguishable.

[3] Those preferences were first introduced within the prospect theory by Kahneman and Tversky[54]. They have found strong ground in experimental evidence and have been used to explain important puzzles in the asset price literature and in household portfolio decisions. See for instance Benartzi and Thaler[9], Barberis, Huang and Santos [7] and Barberis and Huang [8]. Our formulation is closer in the spirit to Kõszegi and Rabin[58] and Yogo[84] who embed time-varying reference levels.

[4] Note that in the immediate neighborhood of the kink marginal utilities actually go to infinity, as common in all kinked preferences. However in our gain-loss preferences, as the marginal utility becomes finite again, it is declining at gains and increasing at losses.

[5] For traders in risky assets loss averse preferences and varying risk-sensitivities on the tails are well documented in experimental evidence . See Haigh and List [44] and other references cited below.

Asset demand, and its price, as well as leverage, raise by more than in the corresponding case with constant risk-aversion. The opposite is true in recessions. Lenders are instead risk-averse, equally so at small and large gambles and along the full distribution of shocks. When trading in debt markets lenders require a collateral guarantee which can mitigate asymmetric information or discipline lack of commitment. Because of this borrowers are then subject to endogenous and occasionally binding collateral constraints[6], with time-varying margins. The shadow price of debt in the market, a proxy for the premium transferred from borrowers to lenders or the margin, is endogenous and depends upon the difference in evaluations between borrowers and lenders, namely debt demand and supply. The lenders/borrowers evaluations are proxied by their respective stochastic discount factors. Lenders always require a debt return which covers for the certainty equivalent and for their precautionary saving motives. However, borrowers have valuations which change along the shock distribution. In booms borrowers are increasingly risk-tolerant and their evaluation of collateral is high and increasing. Hence, the premia that they transfer to lenders, or the margin, declines relatively to their increasing valuations of the risky collateral asset. As the margin declines the constraint becomes slack and leverage builds up. The opposite happens in recessions or when shocks bring borrowers' consumption below the reference level. As borrowers' wealth declines and since losses resonate more than gains, de-leverage materializes, risky asset demand falls and so does its price. In this region the premia that borrowers transfer to lenders increase, relatively to their declining valuation of the collateral asset.

We discuss and prove the above-described dynamic through analytical derivations and numerical simulations[7]. Analytically we show three things. First, debt margins or the shadow price of debt depend endogenously on the difference between lenders/borrowers risk attitudes as captured by their stochastic discount factor (SDF hereafter). Second the asset price growth is enhanced by fluctuations in borrowers' SDF, which vary around the reference level, and in the debt margins. Third, we show that the Sharpe ratio is counter-cyclical in our model. Intuitively on the lower tail

[6] See also Mendoza[66] or Bianchi and Mendoza[12] for models with occasionally binding collateral constraints. Favilukis, Ludvigson and Van Nieuwerburgh[34] is a recent contributions that studies the asset pricing implications of collateral constraints.

[7] In the main text we solve the general case with infinite horizon. This is our benchmark since we are interested to confront the dynamic properties of the model with the ones in the data. In Appendix A however we also present a three period version of the model which allows us to discuss the economic intuition in a simplified environment.

borrowers/investors are increasingly risk-sensitive, hence they require higher excess returns on risky assets for given volatility. The fact that the conditional (to negative shocks) Sharpe ratio raises contributes to bring the unconditional one closer to the data, relatively to the case with standard preferences.

To assess the quantitative relevance of our model we also solve it numerically. To account for the role of non-linearities and regime changes (binding versus non binding constraint and kinked utility) we employ a global method. The algorithm is a based on a policy function iteration in which expectations shift across regimes endogenously in the spirit of Coleman[20][8]. We perform event study analysis and show through a crisis exercise that the model can generate endogenously the leverage/de-leverage cycle and asset price boom/bust even in face of standard shocks. The declining risk-tolerance at the upper tails is crucial in generating appetite for risky assets heightening the asset price growth and fostering the build-up of leverage in booms, that is when consumption is above the reference level. De-leverage emerges instead since losses resonate more than gains. A simpler reference dependent preference, in absence of the loss aversion, would not be able to account for the full dynamic of debt. Our simulations confirm that the model can reproduce the volatility and pro-cyclicality of debt, the counter-cyclical margins and can match numerous asset price statistics, all of which we compare to the data-equivalent for the US and the UK[9]. Equity premia, their volatility and cyclicality are matched, despite low debt returns. As hinted earlier the divergence in risk-attitudes between borrowers and lenders allows our model to induce margins' countercyclicality. Consider a boom. Lenders require the risk-free and a premium for the variance of consumption due to precautionary motives. But borrowers have declining risk-tolerance and increasing valuation of the risky collateral asset. This relaxes the debt constraint, hence the margin, as borrowers are willing to transfer to lenders higher premia relatively to their valuation of the collateral. At last, the model non-linearities, namely occasionally binding constraint and the kinked utility, are crucial in generating high volatility of equity premia and assets prices and in bringing the Sharpe ratio closer to the Hansen and Jagannathan[45] bounds. The fact that marginal utility raises exponentially

[8] See also Davig and Leeper [23], Farmer, Waggoner and Zha[32] and [33].

[9] We tested our numerical results with respect to changes in other relevant parameters, but also to alternative reference levels. They remain robust.

around the kink and that borrowers' SDF vary, both contribute to heightened asset price volatility. High and volatile premia emerge when investors become very afraid of bad events[10], something which in our model is due to the fact that losses resonate more. This last channel also contributes to bring the unconditional Sharpe ratio closer to the data.

The rest of the paper is divided as follows. Section 2 discusses related literature. Section 3 describes the model. Section 4 describes the channels in the model and is articulated in two parts. The first part uses a simple two-state example to highlight the role of gain-loss preferences for the pro-cyclical dynamic of leverage. The second part derives analytical results for the full model to highlight the joint role of borrowers' gain-loss preferences and divergence in risk-attitudes for the full dynamic of leverage and of its margin. Section 5 describe the simulation method, the calibration and presents numerical results. Section 6 concludes. Appendices follow.

2 Literature Review

The idea that the build-up of leverage and asset price boom-bust dynamics are determined by market forces in presence of heterogenous agents' risk-tolerance dates back to Minsky[70]. At large agents holding optimistic beliefs or high risk-tolerance lever up to invest in high-return/high-risk assets. As the distance in risk-valuations between borrowers and lenders increases compensating risk-premia emerge to close this gap. Such compensating risk-premia endogenously determine the market price of debt and of the risky asset, hence the tightening of the collateral constraint.

The idea that margins and leverage are endogenously determined in debt markets with heterogenous preferences/beliefs had been modeled recently in static general equilibrium models by Geanakoplos[40]. The paper which is closer to our in the spirit is Simsek[81], who builds a static and elegant model featuring beliefs dis-agreement between optimistic borrowers, who acquire funds to invest in a risky asset by engaging in collateralized debt contracts, and pessimistic lenders, who need to be compensated for the risk of changes in re-sale collateral values[11]. In his model endogenous leverage and margins emerge due to beliefs heterogeneity. In our model the divergence in risk-valuation between borrowers and lenders determines the tightness of the borrowing limit, as

[10] See Cochrane [18].

[11] See also Miller[69] for role of beliefs dis-agreement.

captured by the shadow price of debt, the borrowers' increasing risk-tolerance under gains determines the build-up of leverage, while de-leverage is explained by the fact that losses resonate more than gains. Moreover we work with a fully dynamic quantitative model, something which allows us to take into account the role of non-linearities, namely kinked utility and occasionally binding constraints, and to quantify the implications of our model through moment matching.

In terms of preference specifications two are the key ingredients in our analysis. First, heterogeneity in risk preferences between borrowers and lenders is crucial to obtain cyclical margins and leverage amplification. There is a past literature, focusing on different questions, that stresses the importance and relevance of heterogeneity in risk-preferences: examples include among others Dumas[28], Garlenau and Pedersen[39], Guvenen[43] and Santos and Veronesi[78]. Second, our borrowers-speculators are increasingly risk-tolerant at gains and increasingly risk-sensitive at losses. This induces the increasing exposure to risky assets and the build-up of leverage in booms, namely when a sequence of positive shocks brings consumption to the right of the reference level, and the sharp de-leverage in recessions. Haig and List[50] presents experimental evidence that borrowers-speculators exhibit the kind of behavior described and that the latter is normally associated with gain-loss preferences. Consistently Fafchamps et. al.[30] also show through experimental evidence that winnings in previous rounds increases risk-tolerance.

Our paper contributes to the expanding literature on the role of demand and pecuniary externalities for financial markets. It is also well understood that collateral constraints shall be modelled as occasionally binding. Beyond being realistic, such modelling set-up generates anticipatory effects that contribute to amplify the dynamic of leverage and asset prices. In face of negative shocks for instance borrowers anticipate the upcoming strains in credit and start to de-leverage in anticipation. The literature on this is large and expanding. A non-exhaustive list includes Greenwald and Stiglitz[42], Mendoza[66], Bianchi and Mendoza[12], Korinek and Simsek[57], Dávila and Korinek[24]. One difference between our analysis and the above-mentioned papers is that they focus on set-ups with risk-neutral lenders populating the rest of the world. This renders debt supply fully exogenous and most of the movements in leverage are determined by demand. In our case the dynamic of debt and margins is determined endogenously in a market with demand and supply. A second difference is that in our work we also consider the role of kinked borrowers' utility and preference heterogeneity and how they interact with collateral constraints. Finally, we provide strong empirical and quantitative ground to our results showing that the model can match several asset price facts and cyclical properties of leverage.

Our model is applicable to all debt markets in which collateral-margin constraints are operative[12]. The economic rationale behind collateral constraints is the presence of some limited enforcement or agency problems that can be disciplined through debt contracts. Collateralized debt is superior to equity since it obviates the need for price discovery. Lenders are assigned the right to redeem collateral based on a bargaining agreement with the borrowers (see Holmstrom[52]). We contribute to the understanding of how collateralized debt affects the cycle and its link to crises development.

Finally, the gain-loss reference dependent preferences, which we employ, were pioneered by Kahneman and Tversky[54], and have been recently embedded into consumers' optimization problems or in other dynamic models (see Kõszegi and Rabin[58], Dubin, Grishchenko and Kartashov [27] or Yogo[84] among others). Recent works show how loss-averse preferences, namely gain-loss utilities in which losses weight more than gains, can contribute to explain asset price facts (see Benartzi and Thaler[9], Barberis, Huang and Santos[7], Pagel[73]). Other authors have introduced those preferences into macro settings (see Gaffeo et. al.[36]) or have used them to explain the dynamic of house prices financed through collateralized mortgages (see Bracke and Tenreyro[15]). Extensive experimental evidence established that those preferences are better suited at capturing behavior in presence of risk as they embed all gradations of risk-attitudes at large and small gambles (see Yogo[84]). Hence, there is already strong ground for employing them into models with risky assets. Beyond their general validity, they happen to be particularly well suited for our case for two reasons. First, as explained extensively so far, having borrowers' valuations of collateral that change at the upper and the lower tail provides a powerful and convincing mechanism to rationalize the build-up of leverage and of risky assets in booms and the sharp de-leveraging in recessions. Second, the changing valuation of collateral enhances fluctuations in the tightness of

[12] See Adrian and Shin [3] for empirical studies on the role of margin constraints.

the collateral limit and explains its counter-cyclicality.

At last our paper bears some relation to papers that look at the implications of preferences encoding peer-effects (see Duesenberry[26], Abel[2], Campbell and Cochrane[17], Gali[38], De Giorgi, Frederiksen and Pistaferri[21] or Ljungqvist and Uhlig[63]). Following this literature we employ aggregate past consumption as our reference level in the S-shaped utility. This specification captures both relative concerns and past-dependence. Note however that all of the above-cited papers focus on preferences over gains, while neglecting loss-aversion. The latter is crucial in explaining sharp de-leveraging. The kinked nature of our utility is also crucial in accounting for asset price facts. Another distinctive feature of our work is that we examine the role of relative concerns and reference-dependence for debt dynamic and in models with occasionally binding constraints.

In sum our model combines elements from institutional finance, as captured by the collateral constraint, and from the behavioral literature to give an account of the dynamic of leverage, the dynamic of asset prices and the emergence and unfolding of crises. In our work we also devote significant attention to validate our model against a number of empirical asset price and leverage facts.

3 The Model

We consider an economy with lenders and borrowers. The latter acquire debt in order to invest in a risky asset. Lenders and borrowers receive labor income and, at each date, choose consumption and investment. Lenders invest in a liquid asset, B^l, which pays a gross return R^d. Borrowers lever up to invest in a risky asset, S^b, that pays a net dividend of d and has ex-dividend price p[13]. Borrowers' demand for debt is limited by an occasionally binding collateral constraint, which serves the purpose of disciplining limited enforcement. We assume that both the dividend and the labour income follow random Markov-stationary processes. Borrowers and lenders feature heterogenous income processes and also discount the future differently. Borrowers are relatively more impatient, hence in equilibrium their precautionary savings won't be enough to ease up the collateral constraint at all times. Furthermore, borrowers exhibit time0varying and state-contingent

[13]The risky asset can be interpreted as equities.

risk-sensitivity. Relatively to a reference consumption level gains induce decreasing risk sensitivity or increasing risk-tolerance. On the contrary, losses resonate more than gains and induce increasing risk-sensitivity. Overall on the upper tails borrowers are relatively more risk-tolerant than lenders, hence they increase their demand for risky assets, for which they need to leverage. This results also in higher leverage growth. In recessions, on the contrary they have a tendency to de-leverage. Increasing risk-sensitivity to losses reduces their demand for risky assets, hence demand for leverage. In face of negative shocks also lenders reduce debt supply due to precautionary saving motives.

In the main text we focus on the dynamic infinite horizon model. This allows us also to discuss the quantitative implications of the model relatively to data. In Appendix A we lay down and solve a three period and binomial state model. In that context we also assume rsk-neutral lenders, an assumption which allows us to highlight the role of borrowers' gain-loss preferences and time-varying risk-sensitivity for inducing pro-cyclical dynamic of leverage and asset prices, relatively to the case with standard preferences.

3.1 Unconstrained Households

The economy is populated by a fraction ν of households who are endowed with labor income w_t^l and maximize a bounded[14] utility of consumption:

$$\max_{\{C_t^l, B_t^l, S_t^l\}_0^\infty} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U^l \left(C_t^l \right) \right\} \tag{1}$$

subject to the budget constraint:

$$C_t^l = w_t^l + R_t^d B_{t-1}^l - B_t^l \tag{2}$$

where $\mathbb{E}_0$ denotes the expectation operator conditional on information at time $t = 0$, C_t^l and B_t^l denotes lenders' consumption and debt at time t and β is their subjective discount factor. The first order condition of the above optimization problem with respect B_t^l reads as follows:

$$U_C^l \left(C_t^l \right) = \beta \mathbb{E}_t \left\{ U_C^l \left(C_{t+1}^l \right) R_{t+1}^d \right\} \tag{3}$$

Equation 3 is the Euler condition on bonds. For optimality a No-Ponzi condition on both assets shall hold, so that $\lim_{k \to \infty} \mathbb{E}_t \left\{ \beta^k U_C^l \left(C_{t+k}^l \right) B_{t+k}^l \right\} = 0$.

[14] This is the minimal requirement that we impose at the moment on lenders' preferences.

3.1.1 Preference Heterogeneity

We will discuss further below the exact borrowers' and lenders' utility specifications. At this stage it is worth introducing a few considerations. Consider debt as akin to a commodity transferred from lenders to borrowers. First, note that the scope for debt trading between the two materializes only to the extent that the two hold different evaluations of the risky collateral. The latter can be convincingly captured by differences in risk-attitudes.

Second, in presence of borrowing constraints, the shadow price of debt, hence the tightness of the borrowing limit, is determined endogenously by equilibrating the lenders' marginal propensity to supply debt and the borrowers' marginal propensity to acquire debt. As distance in the risk-tolerance between the two raises, larger fluctuations in the shadow price of debt, and generally in asset returns, are required to equilibrate the market. In our model the distance in risk-tolerance is time-varying. While lenders hold risk-averse attitudes, which hold equally at large and small gambles and along the full distribution of shocks, borrowers have varying ones. Specifically, borrowers become relatively and increasingly more risk-tolerant on the upper tails or on the right of the reference level, while they become increasingly more-sensitive to risk when negative shocks bring consumption below the reference level. Hence, while lenders require a return which covers for the certainty equivalent and embeds a traditional premium for precautionary saving, borrowers' propensity to compensate lenders for the risk of collateral varies along the shock distribution.

3.2 Borrowed Constrained Households

A fraction $1-\nu$ of the population is composed by households who face a collateral-based borrowing constraint. They consume and invest in a risky asset. To do so they need to acquire external funds. Borrowers choose consumption, C_t^b, debt, B_t^b, and investment in risky assets, S_t^b, to maximize:

$$\max_{\{C_t^b, B_t^b, S_t^b\}_0^\infty} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \rho^t U^b(C_t^b, X_t) \right\} \tag{4}$$

subject to the following budget constraint:

$$C_t^b = w_t^b - R_t^d B_{t-1}^b + B_t^b + S_{t-1}^b(p_t + d_t) - S_t^b p_t \tag{5}$$

where w_t^b is the borrowers' labor income and the variable X_t in the utility function is the time and state dependent consumption reference level. We will return on the preferences' specification later on. B_t^b and S_t^b denote the holdings of debt and of the risky asset at time t.

Borrowers' optimization is also limited by the following collateral constraint (in Appendix B we provide a microfoundation of the collateral constraint based on a no-default enforcement constraint):

$$R_{t+1}^d B_t^b \leq \phi S_t^b \mathbb{E}_t \{p_{t+1}\}. \tag{6}$$

where $\mathbb{E}_t$ is the expectation operator given the information set at time t and where R_{t+1}^d is known at time t since it is based on a contractual agreement . Collateral constraints provide a discipline device in presence of an underlying agency problem. Because of borrowers' moral hazard or because of asymmetric information, the lender ensures repayment by pledging collateral in case of borrowers' bankruptcy. Under this perspective the collateral constraint can be rationalized as arising from the incentive compatibility condition of a standard debt contract which ensures that default never occurs in equilibrium (see Hart and Moore[49] and more recently Simsek[81]). The Lagrangian for the borrowers' optimization problem is:

$$\mathbf{L} = \mathbb{E}_0 \left\{ \sum_{j=0}^{\infty} \rho^j U^b(w_t^b - R_t^d B_{t-1}^b + B_t^b + S_{t-1}^b(p_t + d_t) - S_t^b p_t, X_t) - \lambda_t \left(R_{t+1}^d B_t^b - \phi S_t^b \mathbb{E}_t \{p_{t+1}\} \right) \right\} \tag{7}$$

Define λ_t as the lagrange multiplier on the collateral constraint or the shadow price of debt. In equilibrium this variable also proxies the margin that borrowers transfer to lenders. The first order conditions, with respect to B_t^b and S_t^b, of the above Lagrangian problem read as follows:

$$U_C^b(C_t^b, X_t) = \rho \mathbb{E}_t \left\{ U_C^b(C_{t+1}^b, X_{t+1}) R_{t+1}^d \right\} + \lambda_t R_{t+1}^d \tag{8}$$

$$p_t U_C^b(C_t^b, X_t) = \rho \mathbb{E}_t \left\{ U_C^b(C_{t+1}^b, X_{t+1})(p_{t+1} + d_{t+1}) \right\} + \lambda_t \phi \mathbb{E}_t \{p_{t+1}\} \tag{9}$$

Equation 8 above is the borrowers' Euler condition with respect to debt, B_t^b, while equation 9 is the borrowers' Euler condition with respect to the risky asset, S_t^b. The above optimality conditions will hold alongside with No-Ponzi conditions on accumulated debt and asset wealth (at the final date). Hence, $\lim_{k\to\infty} \mathbb{E}_t \left\{ \rho^k U_C^b(C_{t+k}^b, X_{t+k}) B_{t+k}^b \right\} = 0$ and $\lim_{k\to\infty} \mathbb{E}_t \left\{ \rho^k U_C^b(C_{t+k}^b, X_{t+k}) p_{t+k} S_{t+k}^b \right\} =$

0. Note that we have assumed $\beta > \rho$, which implies that households are more patient than borrowers. This prevents the borrower from doing enough precautionary saving so as to ease up the collateral constraint permanently, hence it provides a sufficient condition for the collateral constraint to bind at least in some states.

3.3 General preferences specification

Preferences are modelled based on a loss averse reference-dependent utility of consumption a' la Kõszegi and Rabin[58]. The gain' loss utility has acquired significant empirical standing since first introduced by Kahneman and Tversky[54]. Among other things this utility specification is able to capture changes in risk-attitudes at the upper versus the lower tails and at small and large gambles. Those properties make those preferences an ideal tool to study variations in risky asset demand[15], and in leverage, which in our model is tied to the first through the collateral constraint. Following Kõszegi and Rabin[58] the general functional form reads as follows:

$$U^i(C_t^i, X_t) = \alpha W^i(C_t^i) + (1-\alpha)\mathcal{W}^i(C_t^i, X_t) \tag{10}$$

$$\mathcal{W}^i(C_t^i, X_t) = \begin{cases} -\Lambda \frac{\left|\frac{(C_t^i)^{(1-\gamma)}}{1-\gamma} - \frac{(X_t)^{(1-\gamma)}}{1-\gamma}\right|^{1-\theta}}{1-\theta}, & \text{if } C_t^i < X_t, \\ \frac{\left(\frac{(C_t^i)^{(1-\gamma)}}{1-\gamma} - \frac{(X_t)^{(1-\gamma)}}{1-\gamma}\right)^{1-\theta}}{1-\theta}, & \text{if } C_t^i \geq X_t, \end{cases} \tag{11}$$

where C_t^i is consumption and the index $i = l, b$. First it is useful to introduce a few general considerations on gain-loss utilities. In each period t the parameter Λ captures the degree of loss aversion. Define $\Delta = C^i - X$ as the gain-loss. The function above satisfies the standard loss-gain utility assumptions, namely $\mathcal{W}(\Delta)$ is continuous and strictly increasing for all $\Delta \in R$, is twice differentiable for all $\Delta \neq 0$, and $\mathcal{W}''(\Delta) \leq 0$ for all $\Delta > 0$, $\mathcal{W}''(\Delta) \geq 0$ for all $\Delta < 0$ and $\mathcal{W}(\Delta^1)+$ $\mathcal{W}(-\Delta^1) < \mathcal{W}(\Delta^2)+ \mathcal{W}(-\Delta^2)$ for $\Delta^1 > \Delta^2 > 0$ and $\lim_{\Delta\to 0} \mathcal{W}'(-\Delta^2)/\mathcal{W}'(\Delta^2) = \Lambda > 1$. Those assumptions imply that the utility is monotonic with respect to gains, that there is diminishing sensitivity to gains (the more so the higher the θ) and that losses resonate more than gains (the more so the higher the Λ).

[15] In fact several authors have shown the ability of those preferences in explaining asset price as well as portfolio allocation puzzles (see for instance Barberis, Huang and Santos[7]).

Notice that we choose to embed loss-aversion in consumption rather than wealth. Some past literature in behavioral finance has chosen to embed loss aversion in wealth[16]. While this choice has a valid rationale for the study of wealth dynamic in presence of idiosyncratic shocks, it does not serve well our purpose, which is that of assessing the response of risky asset demand to changes in the shock distribution. To be modelled properly, the latter requires indeed an optimizing-saving decision.

Another important choice to be made concerns the modelling of the reference point. To fully exploit the role of non-linearities we choose a time-varying and state-dependent reference point, namely past consumption. This implies that risk sensitivity changes depending on whether income and asset shocks bring borrowers' consumption above or below the reference point[17]. In the next section we return on the role of the reference-level in capturing both relative concerns and path-dependence.

As noted earlier we will assume that borrowers and lenders differ in their risk-attitudes. Borrowers/investors exhibit state contingent risk-attitudes, acting as increasing risk-takers at gains and on the upper tails and increasing risk-fearing at losses. Lenders' preferences exhibit traditional channels such as inter-temporal substitution and precautionary savings. For the purpose of generality in the analytical derivations we start by assuming that borrowers and lenders have the same utility specification. We then vary their risk-attitudes by assuming that lenders have higher α relatively to borrowers.

3.3.1 The Reference Level

Following Dubin, Grishchenko and Kartashov [27] or Yogo[84][18], we assume that the reference level of consumption X_t is given by a fraction of the per-capita consumption in the previous period.

$$X_{t+1} = (C_t)^b \tag{12}$$

[16] See Benartzi and Thaler [9] or Barberis and Huang[8].

[17] Note that an alternative specification would be to model the reference point as past consistent expectations of consumption (see Kõszegi and Rabin[58]). This second alternative assigns a role to future expectations in the consumer's decision problem. In our case anticipatory effects already emerge from the occasionally binding nature of the debt constraint. Furthermore, future expectations enter the borrowers' saddle point optimization through the future value of the collateral in the borrowing constraint. On the other side, we find that the external habit component is needed to best capture debt pro-cyclicality and/or debt cross-sectional behavior.

[18] Gaffeo et al. [36] also use similar preference specifications in a monetary model.

where $C_t = \nu C_t^l + (1-\nu)C_t^b$ is the per-capita consumption in the economy and $0 < b < 1$. For analytical convenience we define $G_{t+1}^i = \frac{C_{t+1}^i}{C_t^i}$ and $S_t^i = \frac{\upsilon C_t^i}{\nu C_t^l + (1-\nu)C_t^b}$ be growth rate and the share of borrowers' consumption, respectively. Finally, denote the consumption-habit ratio as $Y_t^i = \frac{C_t^i}{X_t}$.

A few considerations are worth in relation to the choice of the reference level. First, there is extensive experimental evidence showing that reference points are time-varying and that the past has an important role (see Schmidt[80] or Arkes et. al.[5] among others).

Second, an important component in the reference point is given by status concerns and social comparison, which are embedded in the choice of external habits. Peer-effects have a long tradition in economics starting with Duesenberry[26]. Abel[2], Campbell and Cochrane[17], Gali[38], De Giorgi, Frederiksen and Pistaferri[21] are some recent contributions on the role of external habits and peer effects, albeit none examines the leverage cycle. Given this evidence we decided to employ as our benchmark specification the case with past aggregate consumption as reference level. This specification indeed includes both the past and the peer-effects. See also Della Vigna et. al. [22] for the importance of past reference levels. While this specification of the reference level bears some relation to the literature on habits, there are actually important differences. First, our preference specification includes an aversion to losses vis-a-vis the reference. As we explain further below this part of the utility is important to account for de-leveraging when bad shocks bring the borrower to the left of the reference levels. Furthermore, while the presence of the kink in the utility contributes per se to explain asset price volatility[19], it won't suffice to explain the dynamic of the unconditional Sharpe ratio. Indeed for the latter to become closer to the empirically relevant Hansen and Jagannathan[45] bound, we need again the assumption that losses resonate more than gains. In fact at the lower tails borrowers/investors require higher excess returns to hold the risky asset and this raises the unconditional Sharpe ratio.

Two additional considerations are worth in terms of the timing of the reference level.

First, it is useful to note that the qualitative channels of our model, are unaffected by whether we choose past or current aggregate consumption as reference level. Both are time-varying and pro-cyclical. And in both cases it remains true that the increasing risk-tolerance with respect

[19] This is so since marginal utility tend to infinity in the neighborhood of the kink.

to *gains* induces the build-up of leverage, while loss-aversion induces de-leveraging, and that the difference in lenders-borrowers risk-attitudes determines the endogenous fluctuations in the margins[20]. Quantitatively however, a reference level featuring path-dependence, helps to bring the model-based moments closer to the data-equivalent as it enhances procyclicality. This is another reason for which we have chosen this as our benchmark specification.

Second, it would be possible to consider a forward-looking specification of the reference level[21]. This specification adds an additional channel, namely a first-order precautionary saving motive, through which agents are averse to news and tend to post-pone consumption if they foresee adverse shocks[22]. While again this specification would not alter our main qualitatively channels as described so far, the additional first-order precautionary saving component would make it difficult to generate plausible build up of leverage in booms, as borrowers would tend to post-pone consumption and save. Second, as shown by Pagel[73] the forward-looking component tends to generate high levels and high volatility of the risk-free rate[23]. Indeed the expected risk-free rate is increased in the event of adverse shocks since the agent dislikes immediate reductions in consumption and is unwilling to substitute inter-temporally. Pagel[73] also shows that more plausible values of the risk-free rate volatility can be recovered by assuming sluggish beliefs-updating, thereby confirming that some persistence in consumption is needed.

3.3.2 The Arrow-Pratt Metric

At this stage it is also useful to derive the Arrow-Pratt measure of absolute risk aversion for the gain-loss part of the utility (namely $\mathcal{W}^i(C^i_t, X_t)$):

$$r^A(C^i_t, X_t) = \begin{cases} \Lambda \left| \frac{(C^i_t)^{(1-\gamma)}}{1-\gamma} - \frac{(X_t)^{(1-\gamma)}}{1-\gamma} \right|^{-\theta}, & \text{if } C^i_t < X_t, \\ \left(\frac{(C^i_t)^{(1-\gamma)}}{1-\gamma} - \frac{(X_t)^{(1-\gamma)}}{1-\gamma} \right)^{-\theta}, & \text{if } C^i_t \geq X_t, \end{cases} \tag{13}$$

[20] This finding is corroborated by the fact that we also checked robustness of our simulations by using as reference level current consumption.

[21] In this case the reference level is stochastic and corresponds to the fully probabilistic rational belief about current and future consumption that the agent formed in previous periods (see Kõszegi and Rabin[59] or Pagel[74]).

[22] Preferences based on this specification of the reference-level have proven very successful in explaining facts related to the dynamic, the cross-section and the life-cycle pattern of the consumption and of the consumption-wealth ratios.

[23] In Pagel[73] paper the volatility of the risk-free rate ranges from 11.3 to 16.9 against a data reference between 1 and 4, the latter being consistent with the case of corporate debt. In our simulations with past aggregate consumption as reference the volatility of the debt rate is at around 4.

The above metric will be used to characterize the role of gain-loss preferences. At this stage it is worth noting that the relative risk-aversion is different on the two sides of the references point. Specifically, since Λ is larger than one, it is higher on the left than on the right. In other words the agent becomes more risk-sensitive when consumption is below the reference point.

3.3.3 Decentralized Equilibrium

Market equilibrium is defined as interest rates on debt, asset prices and portfolio choices of lenders and borrowers $\{R_t^d, p_t, B_t^l, B_t^b, S_t^b\}_{t=0}^{\infty}$ such that: *i*) households choose B_t^l to maximize 1 subject to 2, *ii*) borrowers choose B_t^b, and S_t^b to maximize 13 subject to 5 and 6, *iii*) and the markets for equity, debt and consumption clear, that is:

$$S_t^b(1-\nu) = 1 \tag{14}$$

$$\nu B_t^l + (1-\nu)B_t^b = 0 \tag{15}$$

$$\nu C_t^l + (1-\nu)C_t^b = \nu w_t^l + (1-\nu)w_t^b + d_t \tag{16}$$

Note that conditions 15 and 16 automatically imply that 14 holds for any t. This is so by Walras law.

4 The Channels in the Model

In what follows we characterize the main channels of the model. We devote particular attention to explain how each of the model features contribute in explaining the dynamic of leverage and its margin, and of the asset price. The section is articulated in two parts. In the first and next we construct a simplified version of the model with risk-neutral lenders and a binomial shock distribution to highlight the role of borrowers' gain-loss utility for the pro-cyclicality of debt. In the second part we derive analytical results for the full model to highlight the role of each of the model features, namely borrowers' gain-loss preferences, divergence in risk-attitudes and occasionally binding constraints, in explaining the leverage and asset price cycle.

4.1 A Simple Intuition with Binomial State Space

Prior to solving the model with a general recursive shock specification, it is useful to depict a simple case with a binomial state space and risk-neutral lenders. This simple example allows us to highlight the role of borrowers' gain-loss preferences in explaining leverage and asset price procyclicality, relatively to the case with standard preferences A more extensive three period version of this example is presented in Appendix A.

First, to isolate only the role of borrowers' gain-loss utilities, we assume that lenders are risk-neutral and that they supply debt at a risk-free rate, R^d. This implies that debt demand by borrowers plays a larger role in determining leverage and asset prices and that differences in borrowers-lenders evaluations are due simply to changes in borrowers' SDF over states. Second, we assume a binomial shock distribution with a high and a low realization of labour income, w_t^H and w_t^L, taking place respectively with probability π and $(1-\pi)$. To reduce the dimensionality of the state space we tie dividend and labour through the following relation $d_t = \alpha w_t$. To fix ideas we assume that the high state brings consumption above the reference level and relaxes the borrowing constraint. The opposite is true for the low state. Furthermore, we assume that lenders are risk-neutral and that $\beta R = 1$. This allows us to tie fluctuations in consumption across periods to fluctuations in the exogenous states. At last, to work out analytically this case we keep the reference level constant at X. This can be easily relaxed by defining a mapping between the reference level and the consumption policy function.

Starting in period t with a boom (high) state, namely one in which $\lambda_t = 0$, we can write the Euler condition on debt as follows:

$$U_C(C_t) = \pi U_C(C_{t+1}^H(B_t, w_{t+1}^H)) + (1-\pi) U_C(C_{t+1}^L(B_t, w_{t+1}^L)) \tag{17}$$

where $C_{t+1}^H(B_t, w_{t+1}^H)$ and $C_{t+1}^L(B_t, w_{t+1}^L)$ are the policy functions linking consumption to the exogenous and pre-determined states respectively for the case of the high and low future states. For ease of notation we skip denoting the dependence of the utility upon X. Consider now the case where the boom is expected to remain in the future so that $\pi \geq (1-\pi)$. Under gain-loss utility the marginal utility, call it $U_C^{GL}(C_{t+1}^H(B_t, w_{t+1}^H))$, declines under gains relatively to the case of risk-averse preferences, $U_C^{RA}(C_{t+1}^H(B_t, w_{t+1}^H))$. But this implies that:

$$\pi U_C^{GL}(C_{t+1}^H(B_t, w_{t+1}^H))+(1-\pi)U_C^{GL}(C_{t+1}^L(B_t, w_{t+1}^L)) < \pi U_C^{RA}(C_{t+1}^H(B_t, w_{t+1}^H))+(1-\pi)U_C^{RA}(C_{t+1}^L(B_t, w_{t+1}^L)) \tag{18}$$

hence that $U_C^{GL}(C_t) < U_C^{RA}(C_t)$ and that $C_t^{GL} > C_t^{RA}$. For given income shocks and asset price, this implies that in booms borrowers tend to leverage more to fund higher consumption under gain-loss utilities than under standard preferences. The opposite would be true if recessions are more likely in the future.

Let's now examine the determinants of asset prices. To this purpose we can write the Euler equation on risky assets which, in the depicted environment, reads as follows:

$$q_t = \left[\pi m_{t,t+1}^H(q_{t+1}^U + w_{t+1}^H) + (1-\pi)m_{t,t+1}^L(q_{t+1}^B + w_{t+1}^L)\right] \tag{19}$$

where $m_{t,t+1}^H = \frac{\rho U_C(C_{t+1}^H(B_t, w_{t+1}^H))}{U_C(C_t)}$ and $m_{t,t+1}^L = \frac{\rho U_C(C_{t+1}^L(B_t, w_{t+1}^L))}{U_C(C_t)}$ are the borrowers' SDFs from above and below the reference point, where q_{t+1}^U indicates the asset price prevailing at time $t+1$ when the debt constraint is not binding, while q_{t+1}^B indicates the asset price prevailing at time $t+1$ when the constraint binds. Under gain-loss preferences the stochastic discount factor, which is the inverse of the price of risk, is typically higher on the right of the reference level and lower below it relatively to the SDF under standard preferences. Intuitively, above the reference level borrowers are more risk-tolerant, hence they assign lower price of risk to future contingencies, which results in higher SDF. This relation is proven formally in Proposition 2 below.

This implies that if a boom is foreseen, that is $\pi \geq (1-\pi)$, the following holds:

$$\pi m_{t,t+1}^{H,GL}(q_{t+1}^U+w_{t+1}^H)+(1-\pi)m_{t,t+1}^{L,GL}(q_{t+1}^B+w_{t+1}^L) \geq \pi m_{t,t+1}^{H,RS}(q_{t+1}^U+w_{t+1}^H)+(1-\pi)m_{t,t+1}^{L,RS}(q_{t+1}^B+w_{t+1}^L) \tag{20}$$

where $m_{t,t+1}^{H,GL}$ and $m_{t,t+1}^{L,GL}$ are the SDFs under gain-loss preferences for the high and the low state, while $m_{t,t+1}^{H,RA}$ and $m_{t,t+1}^{H,RA}$ are the SDFs for risk-averse preferences under high and low states respectively. Condition 20 above implies that $q_t^{GL} \geq q_t^{RA}$ or that asset price increases in booms will be higher under gain-loss utilities than under standard preferences. The opposite is true for asset price falls in face of recessions.

In the above example for reasons of tractability we kept lenders' preferences, hence debt supply, as exogenous and time-invariant. This did not allows us to discuss the role of lenders/borrowers preferences' heterogeneity, which is the second crucial ingredient to fully dissect the dynamic of debt. We will do so in the next sections, where we show that the difference between lenders and borrowers SDF determines the debt margins and through it the binding nature of the constraint. The latter in turn will determine how likely is for the borrower to be on the right or the left of the reference level, hence his propensity to unload or off-load debt and risky assets.

4.2 Analytical Results

In this section we derive a number of analytical results that allows us to gain intuition on the channels operating in our model. We start by discussing results related to the dynamic of debt and then we move to the dynamic of asset prices and premia, including derivations of the Sharpe ratio. In each case we discuss how each of the model features contribute to a realistic and economically relevant dynamic of leverage and asset prices.

4.2.1 Endogeneity of the Borrowing Limit

Before proceeding with the model solution it is useful to characterize the conditions under which the collateral constraint binds in equilibrium.

Lemma. *The collateral constraint binds if and only if* $\beta\frac{\mathbb{E}_t\{U_C^l(C_{t+1}^l)\}}{U_C^l(C_t^l)} \geq \rho\frac{\mathbb{E}_t\{U_C^b(C_{t+1}^b)\}}{U_C^b(C_t^b)}$.

By merging together equations 3 and 8 we obtain the following condition:

$$\beta\frac{\mathbb{E}_t\left\{U_C^l\left(C_{t+1}^l\right)\right\}}{U_C^l\left(C_t^l\right)} = \rho\frac{\mathbb{E}_t\left\{U_C^b\left(C_{t+1}^b\right)\right\}}{U_C^b\left(C_t^b\right)} + \lambda_t^{'} \tag{21}$$

where $\lambda_t^{'} = \frac{\lambda_t}{U_C^b(C_t^b)}$ is the shadow price of debt adjusted for the borrowers' marginal utility. Condition 21 states that that, in expected terms, the shadow price of debt is given by the difference between the marginal evaluation of the borrowers, who demand debt, and of the lenders, who supply debt. Consider debt as a commodity. Its shadow price will be positive, hence the collateral constraint will bind, to the extent that borrowers and lenders marginal evaluation differ. More specifically the constraint binds to the extent that:

$$\beta\frac{\mathbb{E}_t\left\{U_C^l\left(C_{t+1}^l\right)\right\}}{U_C^l\left(C_t^l\right)} \geq \rho\frac{\mathbb{E}_t\left\{U_C^b\left(C_{t+1}^b\right)\right\}}{U_C^b\left(C_t^b\right)} \tag{22}$$

The above condition intuitively states that to the extent that the lenders price risk more than the borrowers, they will require a risk premium $\lambda_t^{'}$. In this respect the shadow price of debt represents the premium that borrowers shall transfer to lenders to convince them to supply funds. Indeed when the constraint binds lenders bear the risk of fluctuations in the collateral value for which they need to be compensated. The higher is the differences in the evaluation of risk and of the value of collateral, the higher is the compensation that lenders require.

In the short run differences in risk-assessment are due to differences in the borrowers/lenders SDF, namely $\frac{\beta \mathbb{E}_t\{U_C^l(C_{t+1}^l)\}}{U_C^l(C_t^l)}$ and $\frac{\rho \mathbb{E}_t\{U_C^b(C_{t+1}^b)\}}{U_C^b(C_t^b)}$. Preference heterogeneity in our model materializes since borrowers hold higher or positive α in 10. Indeed in this case borrowers' valuation of the risky collateral asset changes depending on whether shocks bring their consumption above or below the reference level. This in turn induces the time-varying differences between borrowers' and lenders' valuations. In the long run differences emerge to the extent that $\beta > \rho$[24].

Proposition 1. *The borrowers' SDF between periods t and $t+s$ takes the form:*

$$m_{t,t+s}^b = \rho^s e^{-\gamma \hat{g}{}_{t+s}^b} \frac{k(\hat{y}{}_{t+s}^b)}{k(\hat{y}{}_t^b)} \tag{23}$$

where $\hat{y}{}_{t+1}^b = \log\left(\frac{C_{t+1}^b}{X_{t+1}}\right)$ and where variables with a hat are in logs, $\hat{g}{}_{t+1}^b$ is log consumption growth. Logs are used for convenience in this derivation.

The exact functional form of $k(\hat{y}{}_t^b)$ depends on borrowers' preferences. Under power reference-dependent utility (i.e., $\theta \in [0,1)$ and $\Lambda > 1$):

$$k(\hat{y}{}_t^b) = \begin{cases} \Lambda \left| \frac{(C_t^b)^{(1-\gamma)}}{1-\gamma} - \frac{(X_t)^{(1-\gamma)}}{1-\gamma} \right|^{-\theta}, & \text{if } \hat{y}{}_t^b < 0, \\ \left(\frac{(C_t^b)^{(1-\gamma)}}{1-\gamma} - \frac{(X_t)^{(1-\gamma)}}{1-\gamma} \right)^{-\theta}, & \text{if } \hat{y}{}_t^b \geq 0, \end{cases} \tag{24}$$

Under linear reference-dependent utility (i.e.,$\theta = 0$ and $\Lambda > 1$):

$$k(\hat{y}{}_t^b) = \begin{cases} \Lambda, & \text{if } \hat{y}{}_t^b < 0, \\ 1, & \text{if } \hat{y}{}_t^b \geq 0, \end{cases} \tag{25}$$

Under power utility (i.e., $\theta = 0$ and $\Lambda = 1$), $k(\hat{y}{}_t^b) = 1 \ \forall \hat{y}{}_t^b$.

Proof. *See Appendix C.*

[24] Recall that in the long run the term $\mathcal{W}^i(C_t^i, X_t)$ in 10 is nil since consumption is at the steady state, hence preference heterogeneity vanishes.

4.2.2 Debt Margins and Leverage

Given the above derivation we are now interested in analyzing the link between the shadow price of debt, which determines the tightness of the collateral constraint or margins, and the specific agents' SDFs, which measure their risk-attitudes in our model. Specifically we show how our borrowers/lenders preference specification delivers realistic dynamic of debt and of its margin.

Debt Margins The margins, or the down-payment requested to borrowers, typically decline in booms and increase in recessions (see also Simsek[81]). In our model margins are given the λ_t. As argued earlier the latter is determined endogenously in equilibrium so as to close the gap between the risk-assessment of lenders and borrowers. When the constraint is tight, hence the λ_t is positive, lenders bear the risk of fluctuations in collateral values. The shadow price of risk also serves as compensation for this risk. If the difference in lenders/borrowers risk-assessment widens, lenders require higher premia.

We start by discussing how borrowers' SDF and their time-varying risk attitudes affect the margin, λ_t. To isolate this channel we work again with a notional economy in which lenders are risk-neutral and are willing to fund borrowers at the market risk-free rate, R_t^d. In this case it is primarily the valuation of the borrowers that affects the margins. This situation is not entirely unrealistic. On the contrary, it captures well the dynamic of leverage in booms or periods with low output volatility, in which the constraint is close to slack, the margins are low and lenders supply funds at low rates. If lenders are risk-neutral by merging equations 8 and 9 we obtain the shadow price of debt which reads as follows:

$$\lambda_t^{'} = \frac{\lambda_t}{U_C^b(C_t^b, X_t)} = \left[\frac{1}{R_{t+1}^d} - \mathbb{E}_t\left\{m_{t,t+1}^b\right\}\right] \tag{26}$$

Note that in this case the collateral constraint would bind only to the extent that $\frac{1}{R_{t+1}^d} > \frac{1}{\mathbb{E}_t\left\{m_{t,t+1}^b\right\}}$, hence under the rare case in which the risk-free rate would be lower than the return offered by borrowers. From 26 the margin is linked solely to borrowers' SDF, it is therefore useful to obtain an analytical expression of it, which we do next.

Proposition 2. *If lenders are willing to offer debt at the risk-free return and assuming that*

$\hat{g}{}_t^b$ *follows a normal distribution,* $N(\mu, \sigma^2)$*, the shadow price of debt,* λ_t*, is lower in states to the right of the reference point and higher in the opposite case.*

Proof. In the interest of analytical tractability we place the shock at the level of consumption rather than of income. Following Tallarini[82] we assume that the consumption growth, $\hat{g}{}_t^b$, follows a normal distribution, $N(\mu, \sigma^2)$. In this case the expected value of the borrowers' SDF can be written as (see Appendix D):

$$\mathbb{E}_t \left\{ m^b_{t,t+1} \right\} = \rho \exp \left\{ -\gamma\mu + \frac{(\gamma\sigma)^2}{2} \right\} \Xi_t(\gamma, \Lambda) \tag{27}$$

where:

$$\Xi_t(\gamma, \Lambda) = \begin{cases} \left[\frac{1}{\Lambda} + (1 - \frac{1}{\Lambda}) F(\gamma\sigma + \frac{\overset{\wedge}{\kappa}_{t+1} - \mu}{\sigma}) \right] & \text{if } \hat{y}{}_t^b < 0 \\ \left[1 + (\Lambda - 1) F(\gamma\sigma + \frac{\overset{\wedge}{\kappa}_{t+1} - \mu}{\sigma}) \right] & \text{if } \hat{y}{}_t^b \geq 0 \end{cases} \tag{28}$$

and where F is the cumulative distribution function of the log-normal and $\overset{\wedge}{\kappa}_{t+1} = \hat{x}_{t+1} - \hat{c}_t$. Equation 26 establishes a negative relation between the shadow price of debt and the borrowers SDF. From equation 27 and 28 we can determine that when consumption is above the reference level, hence $\hat{y}{}_t^b > 0$, the borrowers' SDF is higher, hence the constraint's tightness, λ_t', declines, as per equation 26. *Q.E.D.*

Intuitively, in the region to the right of the reference level borrowers are increasingly risk-tolerant, and their valuation of collateral is high and increasing. The more so the higher the θ. Borrowers wish to invest more in the risky asset and this in turn increases the market value of the collateral guarantee. To invest in the risky asset borrowers also demand more debt. Since lenders are willing to accept the risk-free rate, the high value of collateral translates into low margins or low down-payments. The opposite happens on the left of the reference point. Borrowers become increasingly risk-sensitive when they experience losses. $\mathbb{E}_t \left\{ m^b_{t,t+1} \right\}$ declines, the tightness of the constraint and the margins increase. The more so as the higher the Λ. Overall margins behave counter-cyclically.

To sum up when borrowers' consumption is above the reference level, debt grows, the constraint tends to be slack and the margins are low. If negative shocks bring borrowers to the left of the reference level, de-leverage takes place. This happens for two reasons. First, the borrower expects the constraint to bind in the future, hence precautionary saving motives, captured by the component $F(\gamma\sigma + \frac{\hat{\kappa}_{t+1}-\mu}{\sigma})$ in equation 28, reduce debt demand. Second and most important, as the borrower moves to the left of the reference level loss aversion increases his/her risk-sensitivity, thereby inducing him/her to off-load risky assets and de-leverage.

Let's now examine the role of differences in agents' risk-attitudes for the endogenous tightness of the borrowing limit. To do this we consider the general case in which lenders and borrowers both exhibit preferences as in 10. By merging equations 3 and 8, we obtain the shadow price of debt for the general case:

$$\lambda_t^{'} = \mathbb{E}_t \left\{ m_{t,t+1}^l \right\} - \mathbb{E}_t \left\{ m_{t,t+1}^b \right\} \tag{29}$$

The link between the margin, $\lambda_t^{'}$, and the divergence in risk-attitudes emerges neatly. To fix ideas consider the case in which lenders are risk-averse, both at low and large gambles and along the whole shock distribution. Using Tallarini[82] formula again, we can write their SDF as follows:

$$\mathbb{E}_t \left\{ m_{t,t+1}^l \right\} = \beta \exp \left\{ -\gamma\mu + \frac{(\gamma\sigma)^2}{2} \right\} \tag{30}$$

The borrower's SDF is still given by equations 27 and 28. To isolate the effects of gain-loss preferences we assume that γ is the same for borrowers and lenders. Hence, technically in this case the difference between lenders and borrowers SDF is captured by the term in 28, which as explained above is increasing to the right of the reference and decreasing to the left. Consider a boom. For given consumption volatility, σ, lenders accept the risk-free rate and a premium proportional to their precautionary saving motives, as captured by the term $\frac{(\gamma\sigma)^2}{2}$. However, borrowers have declining risk-tolerance and increasing SDF, $m_{t,t+1}^b$, hence they value risky collateral more as they approach the upper tail. This relaxes the constraint, as $\lambda_t^{'}$ falls. The margin counter-cyclicality follows. In terms of economic rationale this implies that borrowers transfer to lenders lower premia relatively to their high and increasing collateral asset valuation.

Consider now a large negative shock that pushes borrowers on the left of the reference point. Their risk-sensitivity raises, the more so as they approach the lower tail. The decline in $m_{t,t+1}^b$ implies that borrowers are more sensitive to asset risk, hence in expectations they value collateral less. Several things happen simultaneously. First, the collateral constraint tightens. Second, borrowers de-leverage. This the margin, $\lambda_t^{'}$, raises as $m_{t,t+1}^b$ falls. The latter implies that borrowers have to transfer increasing premia to the lenders relatively to their valuation of collateral.

The above results show that our model is compatible with evidence of countercyclical debt margins.

Leverage We shall now discuss the link between leverage dynamic and risk-attitudes. We do so by deriving the equilibrium level of leverage. The latter results by combining the debt constraint, 6, with the borrowers' Euler condition on debt, 8. Substituting $R_{t+1}^d = \frac{U_C^b(C_t^b, X_t)}{\left[\rho \mathbb{E}_t\left\{U_C^b(C_{t+1}^b, X_{t+1})R_{t+1}^d\right\}+\lambda_t\right]} = \lambda_t^{'} + \mathbb{E}_t\left\{m_{t,t+1}^b\right\}$ into the collateral constraint delivers:

$$B_t \leq \phi S_t^b \mathbb{E}_t\left\{p_{t+1}\right\}\left[\lambda_t^{'} + \mathbb{E}_t\left\{m_{t,t+1}^b\right\}\right] \tag{31}$$

To discuss the dynamic of debt we shall consider four regimes, depending on whether the constraint is binding or not and depending on whether aggregate consumption is on the right or the left of the reference point.

We start with the case in which the constraint does not bind, hence $\lambda_t^{'} = 0$. From equation 31 leverage is bounded above by the borrowers' valuation of risk, $\mathbb{E}_t\left\{m_{t,t+1}^b\right\}$. In general when the constraint is lax the dynamic of debt is mainly driven by the borrower's evaluation of assets. The latter increases when the price of risk decreases, namely on the right of the reference level (gains), and falls otherwise. This in turn implies, as per equation 31, that leverage raises when gains materialize and falls otherwise. Note that de-leverage in our model takes place since asset losses resonate more than gains[25].

We now examine the case of a binding constraint, $\lambda_t^{'} > 0$. Recall that $\lambda_t^{'} = \mathbb{E}_t\left\{m_{t,t+1}^l\right\} - \mathbb{E}_t\left\{m_{t,t+1}^b\right\}$, hence:

$$B_t = \phi S_t^b \mathbb{E}_t\left\{p_{t+1}\right\}\left[\mathbb{E}_t\left\{m_{t,t+1}^l\right\}\right] \tag{32}$$

When the constraint is tight the dynamic of debt is driven primarily by the evaluation of the lenders. In this case the margin or down-payment shall raise if lenders require higher premia to

[25] In models with occasionally binding debt constraints de-leverage materializes when debt is bounded above by the current value of collateral (see the examples in Mendoza[66] among others).

supply the same amount of debt. Consider as in the previous section a sequence of negative shocks that raises the (conditional to negative shocks) consumption volatility, σ^2. Lenders' SDF raises as per equation 30 due to precautionary saving motives. This implies that lenders supply debt, but at increasing margins. In equilibrium whether leverage increases or falls depends upon borrowers' demand for the risky asset, which in turn depends upon the balance between their precautionary saving motives and their risk-taking attitudes. On the far right of the reference point, borrowers' declining risk-tolerance prevails over their precautionary saving motives (this can also be seen from equation28). When this happens borrowers demand more risky assets and leverage more. This effect is also consistent with cross-sectional evidence showing that wealthy agents also leverage significantly (see Albanesi, De Giorgi and Nosal[4]). On the left of the reference point, losses resonate more than gains. This effect coupled with the precautionary saving induces de-leverage. This second case rationalizes evidence presented in Mian and Sufi[68] that sharp de-leveraging takes place at low income levels.

4.2.3 Asset Prices, Premia and the Sharpe Ratio

Leverage cycles and boom/busts in asset prices are tightly linked phenomena. In our model too they can be studied in combination as they affect each other through feedback loops.

Excess Returns We start by examining the link between excess returns of risky assets over debt and risk-attitudes. To isolate again the effects of borrowers' risk-attitudes alone we consider again the limiting case with risk-neutral lenders. From 26 we can derive an expression for the borrowers' SDF which reads as follows:

$$\mathbb{E}_t\left\{m^b_{t,t+1}\right\} = \frac{1 - R^d_{t+1}\lambda^{'}_t}{R^d_{t+1}} \tag{33}$$

To link asset prices with borrowers' risk-attitudes we substitute equation 33 into the borrowers' first order condition for the risky asset. First we re-arrange equation 9 to obtain:

$$1 = \rho\mathbb{E}_t\left\{m^b_{t,t+1}R^s_{t+1}\right\} + \lambda^{'}_t\phi\mathbb{E}_t\left\{\frac{p_{t+1}}{p_t}\right\} \tag{34}$$

where we set $R^s_{t+1} = \frac{p_{t+1}+d_{t+1}}{p_t}$. Equation 34 determines the pricing kernel[26]. Since $\mathbb{E}_t\left\{m^b_{t,t+1}R^s_{t+1}\right\} =$

[26] See Harrison and Kreps[48].

$\mathbb{E}_t\left\{m^b_{t,t+1}\right\}\mathbb{E}_t\left\{R^s_{t+1}\right\}+Cov(m^b_{t,t+1},R^s_{t+1})$ and upon substituting $\mathbb{E}_t\left\{m^b_{t,t+1}\right\}=\frac{1-R^d_{t+1}\lambda^{'}_t}{R^d_{t+1}}$, we get the following expression linking the return on the risky asset and the shadow price of debt:

$$\mathbb{E}_t\left\{R^s_{t+1}\right\}=\frac{R^d_{t+1}\left[1-\rho Cov(m^b_{t,t+1},R^s_{t+1})-\lambda^{'}_t\phi\mathbb{E}_t\left\{\frac{p_{t+1}}{p_t}\right\}\right]}{\rho(1-R^d_{t+1}\lambda^{'}_t)} \tag{35}$$

Using 35 we obtain the following expression for the premium between the return on the risky asset and the debt rate:

$$\Psi_t=\frac{\left[1-\rho Cov(m^b_{t,t+1},R^s_{t+1})-\lambda^{'}_t\phi\mathbb{E}_t\left\{\frac{p_{t+1}}{p_t}\right\}\right]}{\rho(1-R^d_{t+1}\lambda^{'}_t)} \tag{36}$$

The expression 36 shows unequivocally the dependence of the premia over the borrowers' SDF, m^b_{t+1}, and upon the shadow price of debt, $\lambda^{'}_t$. The exact dynamic of the premium depends on the solution of the full-model and upon its general equilibrium effects, but we can already draw some general conclusions on the determinants of it. Later on in our simulations we show that our model is successful in explaining both the long run equity premium and its high variance. We can now show analytically how risk-attitudes and collateral constraints raise the long run premium and its variance.

First, the premium raises if there is a negative covariance between the SDF and the risky asset returns. The latter implies that borrowers are less hedged, hence they require a premium to hold the risky asset. The higher is the covariance, the higher is the required premium. Borrowers' risk-attitudes do increase the upper bound for this covariance. Indeed by the Cauchy–Schwarz inequality we know that $cov(m^b_{t,t+1},R^s_{t+1})\leq\sqrt{Var(m^b_{t,t+1})Var(R^s_{t+1})}$. Given the kinked nature and the changing risk-attitude (around the reference level), it goes by construction that the borrowers' SDF, $m^b_{t,t+1}$, exhibits higher variance under S-shaped preferences than under standard CRRA preferences. Hence, the long run equity premium shall be larger for borrowers with S-shaped preferences. Fluctuations under those preferences are also higher since the term $\Xi_t(\gamma,\Lambda)$ in the borrowers' SDF (equation 27) features additional variations around the reference level.

Second, the premium also depends upon the shadow price of debt. The conditions for this dependence are however trickier. Upon computing the derivative of $\frac{\partial\Psi_t}{\partial\lambda^{'}_t}$ it is possible to show that the latter is positive when $R^d_{t+1}\geq\frac{\phi\mathbb{E}_t\left\{\frac{p_{t+1}}{p_t}\right\}}{1-Cov(m^b_{t,t+1},R^s_{t+1})}$. This implies that the borrower is demanding a higher premium on equity when the return that he has to pay on debt is higher than the expected growth on the value of equity, weighted by the loan to value ratio and discounted by the premium for hedging. Hence and to fix ideas, if the borrower expects the growth on asset values, $\mathbb{E}_t\left\{\frac{p_{t+1}}{p_t}\right\}$, to outpace the return that he has to pay on debt, he will invest in risky assets that pay lower returns. Equally if the borrower is badly hedged, that is $1 - Cov(m^b_{t,t+1}, R^s_{t+1})$ is higher, he is more likely to demand a high equity premium.

Asset Prices Leverage and asset price growth often go together. In this section we examine the link between asset prices, the shadow price of debt and borrowers' risk-attitudes. We start by deriving the recursive expression for the asset price in our model.

Proposition 3. *Let's denote by $\lambda'_t = \frac{\lambda_t}{U^b_C(C^b_t, X_t)}$ as the normalized Lagrange multiplier (i.e., the shadow price of the borrowing constraint expresses in unit of the marginal utility of consumption). Then, the price of the risky asset can be expressed as:*

$$\begin{aligned} p_t &= \mathbb{E}_t\left\{m^b_{t,t+1}d_{t+1}\right\} + \\ &+\mathbb{E}_t\left\{\sum_{i=1}^{T} m^b_{t+i,t+i+1}d_{t+i+1}\prod_{j=1}^{i} K_{t+j-1,t+j}\right\} + \\ &+\mathbb{E}_t\left\{\prod_{i=0}^{T} K_{t+i,t+i+1}p_{t+T}\right\} \end{aligned} \tag{37}$$

where $K_{t,t+i} = m^b_{t,t+i} + \phi\lambda'_{t+i-1}$.

Proof. *See Appendix E.*

It is worth discussing the economic channels unveiled by equation 37. First, the price includes the warrant value of the asset, second and third component on the right hand side of Eq 37, where the general term $K_{t+i,t+i+1}$ includes the λ'_{t+i-1}. In this economy the asset has a dual role, it is used for consumption and for collateral. The value of the asset as a consumption claim is captured primarily by the first and second component of Eq 37. In addition, the asset can be pledged as collateral to secure loans. The second and third terms on the right hand side of Eq 37 include the collateral value of the asset, $\phi\lambda'_{t+i-1}$. As the latter increases the investor can lever up more, hence invest more in risky asset. This in turn raises the asset price by a factor

$\lambda_{t+i-1}\phi\mathbb{E}_t\{p_{t+i}\}$ for every period $t+i$ in which the collateral constraint remains binding. These terms capture the *leverage multiplier*[27]. Note that if the borrowing constraint were slack at all future dates the warrant component would be zero and the asset price reduces to the familiar discounted expected value of future dividends. Importantly however the discounted sum of future dividends is weighted by borrowers' SDF, as shown by the term $\mathbb{E}_t\left\{\sum_{i=1}^{T} m^b_{t+i,t+i+1} d_{t+i+1} \prod_{j=1}^{i} K_{t+j-1,t+j}\right\}$. Hence borrowers' risk-attitudes affect prices too. As usual we start by considering the case in which borrowers' consumption is on the right of the reference point. Borrowers' SDF is higher and increasing in this region, thereby the asset price increases in this region too. This is so since $m^b_{t+i,t+i+1}$ scales up all terms of the asset price equation. Intuitively borrowers are risk-takers, they demand more of the risky assets and this increases its price. This channel capture the *risk multiplier.*

The Sharpe Ratio To complete the characterization of the link between asset prices, risk-attitudes and debt margins we now derive and examine the Sharpe ratio. We follow Hansen and Jagannathan[45] who obtain model-based volatility bounds for the Sharpe ratio.

Proposition 4. *The Sharpe ratio for the excess return $R^s_t - R^d_t$ reads as follows:*

$$\frac{\mathbb{E}_t\{Z_{t+1}\}}{\sigma_z} = \left[\frac{Var(m^b_{t,t+1})}{(\mathbb{E}_t\left\{m^b_{t,t+1}\right\})^2} + 2\lambda'_t \frac{\mathbb{E}_t\{Z_{t+1}\}}{\sigma^2_z}\frac{\mathbb{E}_t\{\Sigma_{t+1}\}}{(\mathbb{E}_t\left\{m^b_{t,t+1}\right\})^2} + \frac{(\lambda'_t)^2}{(\mathbb{E}_t\left\{m^b_{t,t+1}\right\})^2}\frac{(\mathbb{E}_t\{\Sigma_{t+1}\})^2}{\sigma^2_z}\right]^{\frac{1}{2}} \tag{38}$$

where $Z_t = R^s_t - R^d_t, \Sigma_t = \phi\mathbb{E}_t\left\{\frac{p_{t+1}}{p_t}\right\} - R^d_t, \sigma^2_z = Var(R^s_t - R^d_t)$.

Proof. *See Appendix F.*

The Sharpe ratio in our model, equation 38, depends upon the borrowers' SDF, the shadow price of debt and upon the difference between the capital gains, weighted by the loan to value ratio, $\phi\mathbb{E}_t\left\{\frac{p_{t+1}}{p_t}\right\}$, and the rate of return on debt. Recall that the Sharpe ratio characterizes the slope

[27] Note that even when imposing the No-Ponzi condition, $lim_{T\to\infty}\mathbb{E}_t\left\{\prod_{i=0}^{T} m^b_{t+i,t+i+1}p_{t+T}\right\}$, the asset price would feature a drift if the borrowing constraint were to bind at any period between t and T. This is due to the presence of the term $\prod_{i=0}^{T}\lambda'_{t+i-1,t+i}p_{t+T}$ which is positive to the extent that the $\lambda'_{t+i-1,t+i}$ are positive.

of the efficient portfolio frontier of an investor, hence it gives the expected excess return needed to bear the amount of risk associated with a given asset returns' volatility. Several considerations arise. First, recall that $m^b_{t,t+1}$ and its expected value, are higher on the right of the reference level. This means that the Sharpe ratio is lower in this region. When consumption is on the right of the reference level, namely in booms, the borrower requires lower expected returns for given variance of the excess return, since his/her risk-tolerance is higher in this exuberant state. Such channel accounts well for the empirically observed counter-cyclicality of the Sharpe ratio. Second, for the same arguments the Sharpe ratio is higher on the left of the reference level or at the lower tails. The increase in the conditional (to negative shocks) Sharpe ratio helps to raise the unconditional Sharpe ratio. In our numerical simulations below indeed we show that in our model the unconditional Sharpe ratio gets closer to its empirical counterpart. At last, note that the third term of 38 depends positively upon $\lambda_t^{'}$. When the constraint binds, namely λ_t becomes positive and starts raising, the Sharpe ratio shall raise to compensate the borrower for the increased risk of asset expropriation. The increase is proportional to the distance between the expected recovery value of collateral and the rate of return on debt.

5 Quantitative Results

In this section we examine the quantitative implications of the model using a simulation method that allows us to take into account the role of non-linearities and anticipatory effects linked to the occasionally binding nature of the collateral constraint and to the kink in the utility. In the next sub-section we detail the benchmark calibration used throughout. We then present quantitative results in two steps. First, through graphical analysis we give indications on the transmission mechanism of the model and its ability to re-produce leverage cycles. To this purpose we show policy functions and the transmission emerging under a crisis event. Second, to assess the empirical validity of our model we simulate the model in response to a large sequence of shocks and compute model based moments for selected variables, specifically asset prices and returns, equity premia, Sharpe ratios, volatility and cyclicality of debt. We compare the latter to the data equivalent from the US and the UK over the period 1980:Q1-2016:Q3. We focus on this sample since this was the period of rapid growth in debt.

5.1 Calibration, Estimation and Numerical Method

We employ a simulation method that allows to deal with both, the occasionally binding nature of the collateral constraint as well as the non-linearities emerging from the kinked utility. The method is based on a policy function with a Markov-switching regime structure, which allows agents to form ex ante expectations about the future regimes based on current state variables. Specifically, at any time t agents form expectations with regard to the binding nature of the constraint and to the kinked nature of the utility at all future periods. To this purpose we insert Markov-switching rational expectation techniques (see Davig and Leeper [23], Farmer, Waggoner and Zha[32] and [33]) into a policy function iteration algorithm. In such a numerical environment the agents' expectation formation process and the related switching mechanism are endogenous, as they depend upon the state space. To solve for endogenous regime switching we rely on the monotone mapping algorithm that finds the fixed point in decisions rules, namely a policy function iteration (Coleman[20]). In Appendix G we provide more details on the simulation method and on the combination of regimes considered.

Calibration is based on micro data and in some cases on estimation. More specifically preference parameters are based on a GMM estimation of the model-implied Euler equation. For more details on this estimation methodology, the regression specification and on the estimation results see Appendix L.

Preferences parameters. Lenders' discount factor is set at a canonical value of $\beta = 0.98$. This value ensures that in a deterministic steady state with no binding constraint the annual risk-free rate is at 4%. This strategy for setting the discount factor is compatible with papers employing occasionally binding collateral constraints (see Mendoza[66]) as well as with papers studying the implications of S-shaped and loss averse preferences (see for instance Yogo[84] or Gaffeo et. al.[36]). The discount factor of the borrowers, ρ, shall be set at a slightly lower value since borrowers' impatience is needed to guarantee an occasionally binding borrowing constraint (see Krussell and Smith[61]). We choose a value of 0.97. We experiment with different values. Lowering the value of β tends to decrease the level of the risk-free rate and to increase the likelihood of the constraint being binding. The rest of the results are largely unaffected by this parameter.

The risk-aversion parameter, γ, is set equal to 1.5 for both borrowers and lenders. This value is common in RBC and macro literature and is in line with both quantitative studies with occasionally binding constraints and with S-shaped preferences. It is useful to note that our model can reproduce the equity premium and its high volatility (see section 4.3 below) without the need to assume an implausibly large value for the risk-aversion parameter[28]. We set the $\alpha = 0.75$ for borrowers and equal to 1 for lenders, implying that state-varying risk-aversion is associated to borrowers' investment behaviour[29]. The loss-aversion parameter, Λ, is set to 2.5. There are several concurrent reasons for this choice. First, a value larger or equal to 2 is obtained by Kahneman and Tversky[54] in their experimental study. Several other experimental studies confirm values around or above 2[30]. A positive Λ is responsible for larger sensitivity of the borrowers' SDF to losses. Second, a value above 2 is also obtained in other studies that run GMM estimation onto those preferences (see Yogo[84] who finds values ranging between 2 and above 3 or Berkelaar, Kouwenberg and Post[10] who find a value of 2.71). To be fully sure that loss averse preferences have a significant role in the data we also run GMM estimate on the non-linear SDF resulting from the borrowers' optimality conditions of our model (details are in Appendix L). On our more recent data sample our estimation delivers a value for Λ of 2.04, which is compatible with previous studies. For our calibration we then choose a value Λ slightly above 2. Our GMM estimation also delivers a value for the inter-temporal elasticity of substitution, θ, of 0.3 (see Appendix L for details), which we employ in our calibration. A value larger than zero for this parameter generates diminishing risk-sensitivity. Kandel and Stambaugh[55] and Yogo[84] also show that a positive value for this parameter is crucial for generating the smaller insurance premium required by consumers at small gambles relatively to the one required at large gambles. Finally, the parameter b, which determines

[28] Using models with standard utility and no collateral constraint requires a γ of about 50 in order to match the mean and volatility of the equity premium and a value of 250 to reach the Hansen and Jagannathan[45] bounds. See Cochrane[19] chapter 21.

[29] The assumption that borrowers exhibit varying risk-sensitivity on the tails is also compatible with evidence presented in Haigh and List [44]. They perform an experiment with professional traders and students. they find that professional traders (borrowers in our case) exhibit more loss aversion, hence diminishing risk-sensitivity on the tails, than students.

[30] Abdellaoui, Bleichrodt, and Paraschiv [1] find median coefficients between 1.53 and 2.52, Booij and van de Kuilen [14] find coefficients between 1.73 and 2.00. Lower values are found by Pennings and Smidts [76] and Schmidt and Traub[79]. Using surveys in finance Merkle[67] finds a value around 2.

the dependence of the reference point from past consumption, is set to 0.35. This parameter governs the moments for the return on debt (see also Yogo[84] and Gaffeo et. al. [36]).

Credit parameters. The parameter ν, namely the fraction of lenders/savers in the economy. To set this value we examine the Survey of Consumer Finance (SCF hereafter) and the debts reported to Equifax using the FRBNY Consumer Credit Panel (CCP hereafter). From there we can gather the fraction of households who declare to own collateralized debt. For the 2007 for instance we find that the fraction of households with collateralized debt is 53.1 in SCF and 42.3 for CCP; for the 2010 the SCF reports 52.6 and CCP reports 42.6. The figures are pretty stable over all other the years. Based on this we choose a benchmark calibration for the fraction of borrowers of 0.5. This implies a $\nu = 0.5$. Importantly note that since we have a model with occasionally binding constraints our borrowers include not only the liquidity or credit constraints (which are between 30% and 40% from the SCF Wealth Supplement), but also any other household with collateralized debt.

To set the loan-to-value ratio (LTV here-after), ϕ, we check several sources of information. First, note that the loan to value ratio is normally heterogenous among borrowers, hence we shall choose a value that captures a significant moment in the distribution and that helps to match empirical facts about credit constrained households. First, we check aggregate data which are available at the Flow of Funds, the quarterly macro-level Financial Accounts of the United States. Based on those data a report by Krimmel, Moore and Smith[60] shows that the LTV has remained at below 40% until 2007 and jumped to 60% afterwards. Hence if one targets aggregate data a choice around 50% shall capture well the average over time. Second, one can examine data at household levels. This is done in a paper by Fuster et. al.[35] who compute LTV ratio using newly available data from Equifax's Credit Risk Insight Servicing McDash (CRISM), which combines the mortgage-servicing records of about two-thirds of outstanding U.S. first-line mortgages with credit record information on the respective mortgage holders. The availability of a credit record allows the author to observe second lines of credit associated with a first mortgage, so that they can construct an updated combined loan-to-value (CLTV). Their data show that the average and median LTV was around 50%. Third, the choice of the LTV affects most directly the probability that the constraint binds in the model. The fraction of liquidity constraint households, both wealthy and low-income ones, ranges between 30% and 40% (see Kaplan, Violante and Weidner[56] and the Wealth Supplement of the PSID survey). By solving the model numerically we find that a value of $\phi = 0.45$ generates a probability that the constraint binds at around 40%, compatibly with the fraction of liquidity constrained households discussed above. We therefore use this value as our benchmark calibration.

Note that for robustness in section 5.3 we also discuss results for an alternative calibration with $\phi = 0.55$ and $\nu = 0.4$. More generally we experiment in our robustness checks with values for ϕ ranging from 0.4 to 0.7. While the general results remain unaffected, we shall note that high values of the LTV tend to increase the probability of binding constraints and slightly also the average returns on risky assets.

Income and dividend processes. All shocks follow AR(1) processes. Specifically the lenders' income process (all in logarithms) reads as: $w_t^l = \alpha^l + \rho^l w_{t-1}^l + \sigma_t^l \varepsilon_t^l$; the borrowers' income process reads as follows: $w_t^b = \alpha^b + \rho^b w_{t-1}^b + \sigma_t^b \varepsilon_t^b$; finally the dividend income process reads as: $d_t = \alpha^d + \rho^d d_{t-1} + \sigma_t^d \varepsilon_t^d$. Details on the choice of the volatilities and persistence of those processes are explained in Appendix I. In brief, the dividend process is estimated fitting an AR(1) process through OLS estimation and using NIPA data from 1960. Second, we estimate the persistence of the income processes using PSID data for the time span 1968 until now. We collect information on family income including transfers. We then divide the families into borrowers and non-borrowers using the information in the Wealth supplement. Finally we fit the AR(1) processes detailed above through OLS estimation. The parameters of the aggregate income processes are obtained by taking sample averages of the household estimated income processes. As for the volatility of the error component we take values well established in the literature (Deaton[25]), which warns against measurement errors of income surveys. Our benchmark calibration is as follows. $\alpha^l = 0.1, \alpha^b = 0.77, \alpha^d = 0.1, \rho^l = 0.65, \rho^b = 0.6, \rho^d = 0.6, \sigma_t^l = 0.03, \sigma_t^b = 0.03, \sigma_t^d = 0.05$.We then perform robustness checks by changing the standard deviations in the interval 0.02 to 0.05. Note that the constant parameter for the borrower is higher than the one for the lender since they have higher wealth in the steady state.

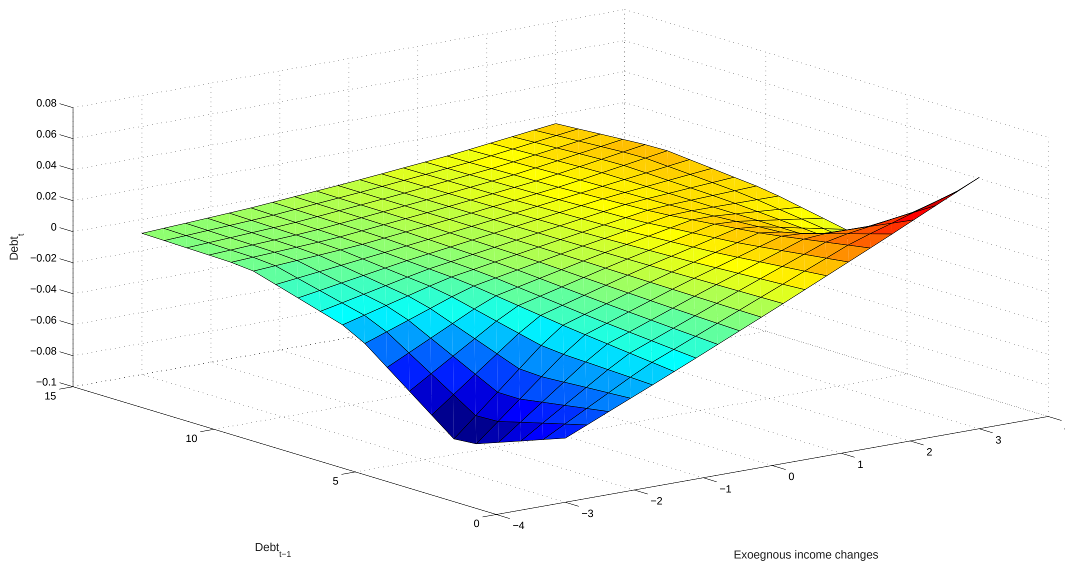

Figure 1: **Policy function of debt with respect to past debt and exogenous changes in debt supply.**

5.2 Policy Functions and Crisis Event

Since the transmission mechanism of our model is made up of a number of channels, it is useful to dissect some of them by examining the shapes of the debt policy function. Of course, since the policy function provides changes of debt in responses to changes in one exogenous variable at the time, information stemming from therein is meant to be indicative and it won't exhaust all the channels present in the model. Most importantly we simulate and show graphically a crisis event exercise. This one shows clearly the emergence of endogenous debt cycles in our model also under different type of shocks.

For the debt policy function we focus on the response of debt to changes in lenders' wealth. The latter captures changes in debt supply. Indeed as lenders' income increases, his supply of debt tends to increase too. Figure 1 shows the pattern of changes of debt from the initial level as a function of past debt and shocks to lenders' income. This pattern gives a first indication on the emergence of the leverage cycle in our model.

Two main observations emerge. First, increases in lenders' income (on the far right), presumably occurring in booms, by expanding debt supply can facilitate the build-up of leverage. Second, negative changes in income (on the far left) induce de-leveraging as each level of past debt is followed by a negative change of debt with respect to its steady state level.

It is also interesting to examine whether, around a crisis event, our model produces a realistic dynamic of leverage. To this purpose we perform event analysis. Specifically starting from the steady state we simulate a series of good states, followed by a sequence of bad states (a crisis). In our model there are three different exogenous states, for this reason we repeat the exercise varying the source of expansion and crisis. For instance in figure 2 below starting from the steady state we simulate a sequence of positive shocks to dividends. Borrowers experience a positive wealth shock, thereby moving toward the upper tail. Herein their risk-aversion declines, their demand for risky assets increases and so does leverage. The latter raises for some periods. A crisis is triggered by the arrival of negative shocks in lenders' income, something which tightens the supply of debt. Margins (not shown for brevity) raise, leading to a decrease in the demand for leverage. The latter indeed decreases.

A similar pattern is observed in figure 3, where the source of expansion is given by positive shocks to borrowers' income. Once again borrowers move toward the upper tail and increase the demand for risky assets and debt. A crisis in this case is triggered by falls in dividend income, which by eroding the value of collateral reduce borrowers' wealth and their debt taking capacity. Overall, under different shock scenarios, the model endogenously generates the growth in debt and asset price which normally anticipates financial crises (including the 2007-2008 one), as well as the fall in both variables which normally follows crises.

5.3 Model-Data Moments Comparison

So far we have shown that our model can generate the patterns of debt and asset prices which are in line with the narrative of debt based financial crises. Loss averse preferences coupled with collateral constraints can explain the surge in debt and asset prices typically observed prior to a crisis, as well as the sharp de-leverage and the asset price burst following the financial shock. However to assess the empirical relevance of the model we go further and perform quantitative simulations to compare model-based statistics for asset prices, returns and debt with the data

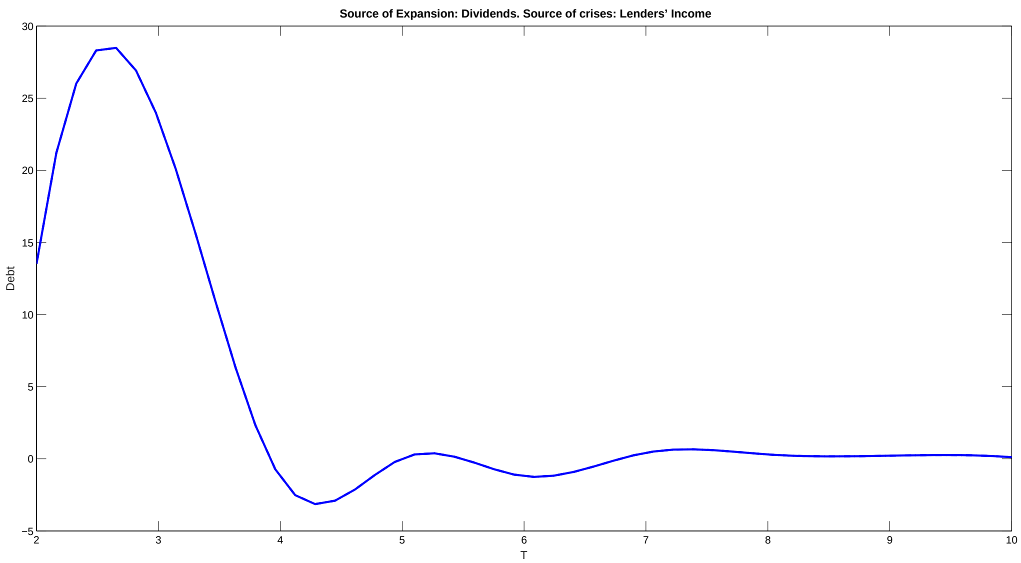

Figure 2: **Crisis event. Following a sequence of positive shocks to dividends, a crisis is triggered by negative shocks to lenders' income. Response of debt growth as percentage changes with respect to the initial state and shown starting from period T=2.**

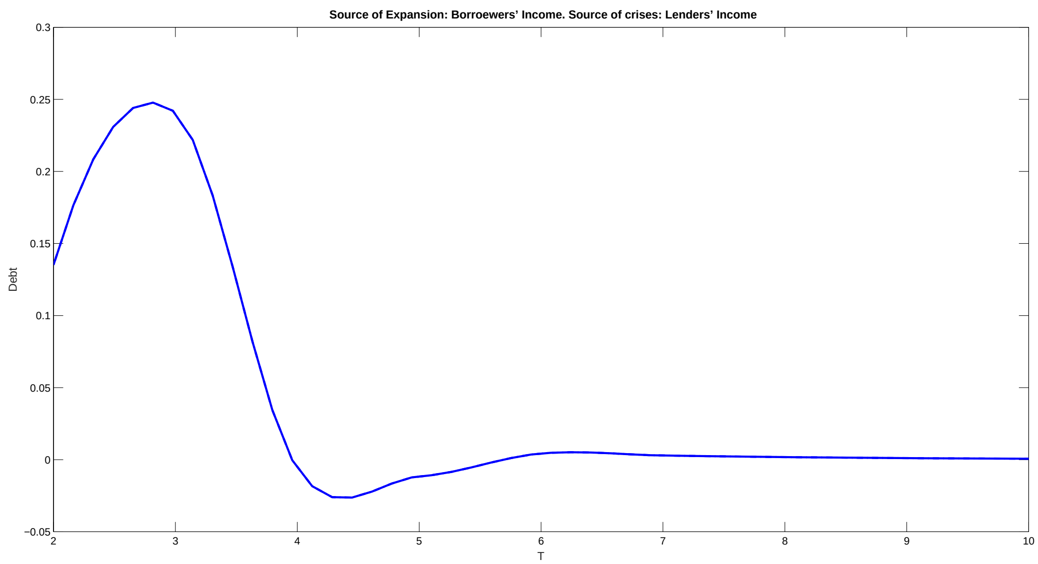

Figure 3: **Crisis event. Following a sequence of positive shocks to borrowers' income, a crisis is triggered by negative shocks to dividends, which affect the value of collateral. Crisis event. Response of debt growth as percentage changes with respect to the initial state and shown starting from period T=2.**

equivalent for two countries, the US and the UK. Table 1 presents the results. Specifically table 1 shows in the third column model-based moments for the benchmark calibration. Specifically we show expected values and volatilities for the return on debt and for the rate on the risky asset, expected values and volatilities of the equity premium, correlations between returns on the risky asset and consumption growth, the Sharpe ratio, the volatility of debt growth and its correlation with consumption growth. In the fourth and fifth columns the Table shows the same data statistics for the U.S. and the U.K. over the sample 1980-2016. We have chosen this sample period for two reasons. First, we get more consistent data for both countries[31]. Second, this sample is of particular interest since it corresponds to the period of rapid expansion in leverage and of asset prices boom and bust. To compute the equity premium for the US we collected data for the stock market index with dividends[32] and for corporate debt returns[33], following the definition in the literature (see chapter 21 of Cochrane[19]). Note that we take data for corporate debt returns[34] as this variables fits better our model-based debt returns. For the UK we compute debt returns using money market rates[35]. Data for collateralized debt are taken from the BIS total non-financial debt data[36], but for the US we also check with Equifax data. For full details on the data description and sources see Appendix H.

The comparison emerging from table 1 shows that the model-based statistics match well the data-equivalent. First, the model reproduces well the excess volatility of the risky asset returns, with the model exhibiting a 14.06% in the benchmark calibration (column 3) against a data equivalent of 16.57% in the US and 15.55% in the UK. Second, the average or long run equity premium generated by the model is 6.62% against a US value of 5.05% and a UK value of 5.83%. The

[31] We also computed the second moments for the longer sample period starting from the 1960, which is only available for the US. We got similar second moments for all asset price and debt statistics. We could only detect a difference in the mean of the equity premium which raises in the more recent sample.

[32] S&P500 for the U.S. and the Moody's Seasoned Aaa Corporate Bond Yield for the U.K.

[33] Those are taken from the CRSP database: http://www.crsp.com/products/research-products/crsp-historical-indexes.

[34] Those are taken from the FRED database available at https://fred.stlouisfed.org.

[35] Note that money market rates are less of a favoured variable for us, since they are closer to a true risk-free rate than to corporate debt. However we have chosen this for two reasons. First, for the UK we do not find collateralized corporate debt rates, but can only find rates for commercial paper, which are short term, uncollateralized and not widely used in Europe. Second, we thought that it was useful to have as term of comparison for our model also a rate which is closer to a risk-free.

[36] Source: http://www.bis.org/statistics/totcredit.htm.

volatility of the equity premium is high and in line with the data equivalent, albeit slightly below. The model can therefore replicate fairly well the dynamic of the equity premium, despite the fact that we use a low value for the risk-aversion parameter ($\gamma = 1.5$). Typically macro model with standard utility and no financial frictions require implausibly high values of γ of about 50 in order to replicate the mean and volatility of the equity premium and a value of 250 to reach the Hansen and Jagannathan[45] bounds[37]. Also, matching the pattern of the equity premium is often achieved by inducing implausibly high values of the debt rates. In our case those are are not too high compared to the literature and are well in line with the US data (4.81% in the model against 4.30% in the US data). Debt returns volatilities are also not too far from the data. By running robustness checks we noted that certain parameter combinations can lower debt returns even further. For instance for an alternative calibration in which $\phi = 0.55$ and $\nu = 0.4$ we obtain a debt return slightly above 2%.

As noted also in Cochrane[18] the ability to match contemporaneously the long run equity premia and asset returns and their cyclical properties is related to the agents' attitude toward losses. As agents become very afraid of bad times, they tend to shy away from risky investment and tend to over-react to the possibility of future bad events. In our model the loss averse reference dependent utility achieves this outcome.

Other cyclical properties of the asset prices are also matched well. For instance, returns on the risky asset are pro-cyclical both in the data and in the model, with similar values for the correlations. Finally, the Sharpe ratio is 0.51 in the model against a 0.30 for the US and a 0.36 for the UK. The ability of the model to match the Sharpe ratio well, despite the low value for the risk-aversion parameter, is due to two reasons. the first is the presence of two kinks, namely the ones in the S-shaped utility and the one stemming from the occasionally binding constraint. At around the kinks marginal utilities raise sharply and this raises the premia that investors require for each level of variance in excess returns. Second, since losses resonate more than gains Sharpe ratios raise at the lower tail and this increases its unconditional mean.

Regarding the debt statistics, the volatility of debt growth is very well in line with the empirical

[37] See Cochrane[19], chapter 21.

counterparts. Most, importantly the model replicates jointly the pro-cyclicality of debt and the counter-cyclicality of margins. Let us first discuss the debt pro-cyclicality. In the model the correlation between debt growth and the cycle[38] is 0.14[39] against empirical values for US and UK that are in the range of 0.2 to 0.4 (see Table1)[40]. Models with always binding constraint induce counter-cyclicality of debt. Leverage indeed falls in booms, or when net wealth goes up, and goes up in recessions, or when net wealth declines. On the contrary, data show that debt grows in booms and de-leverage takes place in recessions. Occasionally binding constraints are not enough to achieve this pattern since precautionary saving motives would induce high income borrowers to reduce leverage. In our model however positive shocks to income and wealth bring borrowers' consumption to the right of the reference level. In this region risk-aversion is declining, or risk-tolerance is increasing. This brings about an increase in the demand of the risky asset and of debt. This pattern is also consistent with cross-sectional data. Albanesi, De Giorgi and Nosal[4] examine Equifax data and show that high income and wealthy agents also tend to lever up significantly. Furthermore, in recessions (shocks that bring the agents on the left of the reference level) our model induces de-leverage. In this region indeed loss aversion increases borrowers' risk-sensitivity and reduces their demand for risky assets, hence leverage. Debt supply also falls in this region since lenders increase their precautionary saving in face of negative shocks. As a consequence collateral constraints become tight and margins increase. This allows our model to reproduce the margins counter-cyclicality (see also Simsek[81]). In Table 1 the correlation between λ_t and the distance of consumption from the reference level, our metric for the identifying booms and recessions, is negative and equal to -0.183. The counter-cyclical nature of the margins is well documented[41].

At last, note that we performed numerous robustness checks by changing parameters. Results remain generally robust and parameters' changes induce sensible changes in the moments. For instance, as one might expect an increase in risk-aversion increases the volatility of all returns and of the equity premium. An increase in the volatility of income shocks increases the volatility of

[38] Cycle is proxied with consumption growth since income is exogenous in our model.

[39] Correlation between debt and the cycle in the model is even higher in the order of 0.20.

[40] The difference in the correlation between the model and the data can be well explained by the fact that in the model we use borrowers' consumption, while the data are based on aggregate consumption.

[41] Unfortunately since we lack a reliable synthetic measure of the loan to value ratio, which is available only at individual level and it is often proprietary information, we cannot compute the equivalent correlation in the data.

Table 1: **Comparison model-based versus empirical moments of selected variables. All values are in percentage terms.**

Statistics	Mnemonics	Model	U.S.	U.K.
		Benchmark calibration	1980-2016	1980-2016
Exp return risky asset	$E_t\left\{R_t^s\right\}$	11.40	9.35	8.23
Volatility risky asset	$\sigma(R_t^s)$	14.06	16.57	15.55
Exp retrun risk-free	$E_t\left\{R_t^f\right\}$	4.85	4.30	2.62
Volatility risk-free	$\sigma(\ R_t^f)$	6.81	2.22	3.04
Equity premium	$E_t\left\{R_t^s - R_t^f\right\}$	6.62	5.05	5.62
Volatility equity premium	$\sigma(R_t^s - R_t^f)$	12.77	16.49	15.10
Volatility debt growth	$\sigma(\Delta B_t)$	1.44	1.03* - 1.65**	1.90
Debt cyclicality	$Corr(\Delta B_t, C_t - X_t)$	0.14	0.46* - 0.301**	0.27
Risky return cyclicality	$Corr(R_t^s, \frac{C_t}{C_{t-1}})$	0.40	0.47	0.29
Sharpe ratio	$\frac{E_t\left\{R_t^s - R_t^f\right\}}{\sigma(R_t^s - R_t^f)}$	0.51	0.30	0.36
Margin cyclicality	$Corr(\lambda_t, C_t - X_t)$	-0.18		
Corr SDF/asset price growth	$Corr(\Delta p_t, \Delta m_t)$	0.87		
Corr SDF/asset price growth	$Corr(\Delta B_t, \Delta m_t)$	0.36		

Values in percentages; *Based on BIS data, **Based on Equifax data.

returns and asset prices, but leaves the ranking of moments unaltered compared to the benchmark calibration. Finally, an increase of the loan to value ratio, ϕ, increases debt procyclicality and makes the constraint bind more often. Borrowers can take up more debt, hence they de-leverage more sharply when their consumption falls below the reference level. Recall indeed that our policy function shows that de-leverage is stronger the higher the level of past debt.

So far we have assessed the empirical validity of the model. We shall now quantify the link, discussed in the analytical part, between leverage growth and asset price growth on the one side and the agents' risk-attitudes on the other. In section 4.2.1 we have shown analytically that the shadow price of debt, or the margin, in the model depends upon the distance in borrowers/lenders risk-attitudes. Intuitively, the higher is such distance the higher is the premium required by lenders to supply funds. Furthermore in section 4.3.2 we have shown analytically that asset price growth is affected by the shadow price of debt, which in turn depends on the distance in SDFs between borrowers and lenders. We wish to verify that these results survive in the numerical simulations of the full model. Table 1 (last two rows) reports the correlation between leverage growth and the (absolute) distance in lenders/borrowers SDF, labelled Δm, and between asset price growth and Δm. They are indeed positive as discussed.

Before closing it is important to note that we tested the robustness of our results by modifying the reference level. Specifically we maintained the assumption of external habits, but inserted aggregate current consumption instead of past aggregate consumption as reference level. Results (not reported for brevity, but available upon request) show that moments for asset prices and leverage remain largely the same with the only exception of debt cyclicality, which becomes less positive under the case of current consumption. This is understandable since current consumption brings about less persistence, which in turn leads to lower pro-cyclicality. Since leverage pro-cyclicality is a prominent aspect of the data, we decided to maintain in the main text the specification with past aggregate consumption, which overall seems best suited to capture the data. The empirical validity of the past reference level is actually well in line with results from previous papers using reference dependent preferences with a loss-averse part (see Della Vigna et. al. [22]).

6 Conclusions

Prior to the emergence of a debt-driven financial crisis we typically observe leverage and asset price growth, high exposure to risky assets or high risk-tolerance at large gambles and low and declining margins. Following a financial crisis we typically observe sharp de-leverage, fire sales of risky assets and increasing debt margins. We show that this dynamic of leverage, asset prices and margins can be rationalized through a model which combines three elements. First, borrowers exhibit gain–loss reference-dependent preferences, so that they act a risk-tolerant on the upper tails, but fear losses more than gains. Their risk-tolerant behavior can explain leverage and asset price growth at large gains, while loss aversion explains de-leveraging and off-load of assets in recessions. Second, occasionally binding collateral constraints are needed since they introduce anticipatory effects and are responsible for amplification in debt and asset markets. Third, heterogeneity in risk-attitudes between borrowers and lenders justifies the emergence of a time-varying and counter-cyclical margin. The latter captures the premium that borrowers shall transfer to lenders to entice them into providing funds.

We embed those features in a dynamic infinite horizon model, which also allows us to quantify the mechanisms described and to test their empirical relevance. The model is indeed capable of matching numerous asset price and leverage statistics.

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7 Appendix A. Three Period Model

In this section we detail and solve a three period and binomial state model. For this case we assume that lenders are risk-neutral. The sole purpose of this simple case is that of highlighting the role of gain-loss borrowers' preferences for the pro-cyclicality of leverage and asset prices.

Define the state space for the exogenous shocks as Σ. There is a continuum of borrowers (we skip the index b since we focus only on borrowers), who live for three periods: $t = 0, 1, 2$. Preferences are given by:

$$U = U(C_0) + \mathbb{E}_X \left[\beta U(C_1) + \beta^2 U(C_2) \right] \tag{39}$$

where we assume that the instantaneous utility is given by the gain-loss specification detailed in the main text, equation 11. We avoid reporting the dependence on the reference level, X, only to keep a more parsimonious notation. Borrowers start period zero at their reference level of consumption. Hence $U(C_0)$ is given. We also assume that $\beta R = 1$, where R is the exogenously given risk-free rate. Borrowers receive endowment income in period 1 and 2, but not in period 0. In period 1 the endowment is stochastic and depends on the realization of the event $\sigma \in \Sigma$. Dividends are subject to the same source of volatility. To gain analytical tractability we reduce the dimension of the state space by assuming the following relation: in each period a fraction $(1 - \alpha)w_t$ is given by the labor income and a fraction $d_t = \alpha w_t$ is given by the financial income or the dividends' realization. We devise a simple binomial structure for the shocks. They can take a high value, which we label with a sup-index H, which occurs with probability π or a low value, which we label with sup-index L and which occurs with probability $(1 - \pi)$. We assume that wealth in the high state is large enough that the borrower finds himself on the right of the reference level and that the debt constraint is not binding. The opposite is true for the low state. Such structure allows us to focus on two regimes, namely one with non-binding constraint and consumption to the right of the reference level and one with binding constraint and consumption to the left of the reference level, rather than on the four regimes considered in the full model of the main text. The set of budget constraints in each period reads as follows:

$$C_0 + q_0 S_0 + \frac{B_0}{R} = 0 \tag{40}$$

$$C_1^x + q_1^x S_1^x + \frac{B_1^x}{R} = (1-\alpha)w_1^x + S_0(q_1^x + \alpha w_1^x) + B_0 \tag{41}$$

$$C_2^x = (1-\alpha)w_2 + S_1 \alpha w_2 + B_1^x \tag{42}$$

The definition of all variables and of the budget constraint follows exactly the one of the main text, hence we do not repeat it here. The sup-index x indicates variables whose realization is uncertain, hence they are indexed by the event realization. We have assumed that $B_{-1} = B_2 = 0$, $S_{-1} = 0$ and $d_{-1} = 0$. This is reasonable. Borrowers are borne without past debt and cannot ask for new debt at the end of life. In period $t = 1$ a collateral constraint limits the amount of debt as follows:

$$-\frac{B_1^x}{R} \leq \phi q_2 S_2^x \tag{43}$$

The de-centralized equilibrium can be summarized as follows. The debt Euler equations in periods 1 and 2 are the following:

$$1 = \beta R \mathbb{E}_x \left[U_C(C_1^x) \right] \tag{44}$$

$$U_C(C_1^x) = \beta R U_C(C_2^x) + \lambda_1^x \tag{45}$$

where λ_1^x is as usual the Lagrange multiplier on the debt constraint. The asset price conditions between periods 0 and 1 and between periods 1 and 2 read as follows:

$$q_0 = \beta \mathbb{E}_x \left[U_C(C_1^x)(q_1^x + \alpha w_1^x) \right] \tag{46}$$

$$q_1^x = \beta \frac{U_C(C_2^x)(\alpha w_2 + q_2)}{U_C(C_1^x)} + \phi \lambda_1^x q_2 \tag{47}$$

The complementarity slackness condition reads as follows:

$$\lambda_1^x \left[\frac{B_1^x}{R} + \phi q_2 \right] = 0 \tag{48}$$

Finally, we have the following market clearing conditions:

$$C_0 + q_0 + \frac{B_0^b}{R} = 0; C_1^x + \frac{B_1^x}{R} = w_1^x + B_0; C_2^x = w_2 + B_1^x \tag{49}$$

where we impose the stock market clearing condition $S_t = 1$. We solve the model by backward induction.

7.0.1 Time 1 Continuation Value

Between time $t = 1$ and time $t = 2$, two cases are possible, either the debt constraint binds or it does not. We start to examine the case with non-binding constraints, whose solution we label with a sup-index U. For this case the system of equilibrium conditions reads as follows (we can use $\beta = R^{-1}$ and $\lambda_1 = 0$):

$$U_C(C_1^x) = U_C(C_2^x) \qquad C_1^x = C_2^x = C^U \tag{50}$$

$$q_1^x = \beta \frac{U_C(C_2^x)}{U_C(C_1^x)}(\alpha w_2 + q_2) \tag{51}$$

$$C_1^x + \frac{B_1^x}{R} = w_1^x + B_0; C_2^x = w_2 + B_1^x \tag{52}$$

Solving for the endogenous variables as function of the exogenous states, we have:

$$C^U = \frac{1}{1+\beta}\left(w_1^x + B_0 + \frac{w_2}{R}\right) \tag{53}$$

The optimal consumption depends on the lifetime wealth. Using 53 and 50 we obtain the equilibrium level of debt:

$$B_1^U = \frac{\beta}{1+\beta}\left(w_1^x + B_0 - \frac{w_2}{R}\right) \tag{54}$$

Finally, the asset price reads as follows: $q_1^U = \beta(\alpha w_2 + q_2)$.

Next, we shall solve for the case in which the constraint binds ($\lambda_1 > 0$). For this case the system of equilibrium conditions reads as follows:

$$\lambda_1^x = U_C(C_1^x) - U_C(C_2^x) \qquad C_1^x < C_2^x \tag{55}$$

$$\hat{q}{}_1^x = \beta \frac{U_C(C_2^x)(\alpha w_2 + q_2)}{U_C(C_1^x)} + \phi \lambda_1^x q_2 \tag{56}$$

$$C_1^x + \frac{B_1^x}{R} = w_1^x + B_0; C_2^x = w_2 + B_1^x; \frac{B_1^x}{R} = -\phi q_2 \tag{57}$$

The above equations already allows us to highlight one result. The asset price is higher under binding constraint, $\hat{q}_1 > q_1^U$. This is due to the fact that the warrant value of the risky asset increases the demand for it, hence its price.

7.0.2 Time Zero Continuation Value

By taking into account the state space binomial structure, at time $t = 0$ we can write the Euler on debt and on the risky asset as follows:

$$U_C(C_0) = \pi U_C(C_1^H(B_0, w_1^H, w_2)) + (1-\pi)U_C(C_1^L(B_0, w_1^L, w_2)) \tag{58}$$

$$q_0 U_C(C_0) = \beta\left[\pi U_C(C_1^H(B_0, w_1^H, w_2))(q_1^U + w_1^H) + (1-\pi)U_C(C_1^L(B_0, w_1^L, w_2))(\hat{q}_1 + w_1^L)\right] \tag{59}$$

$$C_0 + q_0 S_0 + \frac{B_0}{R} = 0 \tag{60}$$

where we use the use the sup-index H to indicate the high state realization and the sup-index L to indicate the low realization. Under loss averse preferences the marginal utility is lower, relatively to the standard risk-averse case, for high states and higher, relatively to the standard utility, on the low states. To the extent that $\pi \geq (1-\pi)$ this implies that $U_C(C_0)$ is lower, hence C_0 is higher, under our gain-loss preferences, relatively to the case with standard preference. In other words if booms are expected as more likely in the future, that is $\pi \geq (1-\pi)$, and considering that borrowers' risk-aversion declines along the good states, borrowers tend to consume more already today. For given q_0, by equation 60, the above result implies that leverage is higher under the economy with gain-loss preferences and under expected booms. If recessions are more likely, that is if $\pi < (1-\pi)$, then the opposite result is true and de-leverage will be larger with gain-loss preferences than under standard preferences.

We shall now solve for the asset price, q_0. For this it is useful to re-write condition59 as follows:

$$q_0 = \beta\left[\pi m^H(q_1^U + w_1^H) + (1-\pi)m^L(\hat{q}_1 + w_1^L)\right] \tag{61}$$

where $m^H = \frac{U_C(C_1^H(B_0, w_1^H, w_2))}{U_C(C_0)}$ and $m^L = \frac{U_C(C_1^L(B_0, w_1^L, w_2))}{U_C(C_0)}$ are the SDF in high and low states. Define, $m^{H,LA}$ as the SDF under the high state and gain-loss preferences, and $m^{H,RA}$ as the SDF under high states and standard risk-averse preferences. Define correspondingly also $m^{L,LA}$ and $m^{L,RA}$. Again, as before, consider the case in which booms are going to be more likely in the future, so that $\pi \geq (1-\pi)$.The asset price under gain-loss preferences, q_0^{LA}, would be higher than the one under standard preferences, q_0^{RA}, if:

$$\pi m^{H,LA}(q_1^U + w_1^H) + (1-\pi)m^{L,LA}(\hat{q}_1 + w_1^L) > \pi m^{H,RA}(q_1^U + w_1^H) + (1-\pi)m^{L,RA}(\hat{q}_1 + w_1^L) \tag{62}$$

The latter also implies that:

$$\pi m^{H,LA}(q_1^U + w_1^H) - \pi m^{H,RA}(q_1^U + w_1^H) > (1-\pi)m^{L,LA}(\hat{q}_1 + w_1^L) - (1-\pi)m^{L,RA}(\hat{q}_1 + w_1^L) \quad (63)$$

Recall from Proposition 2 in the main text that the SDF in good states under gain-loss preferences, call it $m^{H,LA}$, is larger than the corresponding one under standard risk-averse preferences, $m^{H,RA}$. The opposite is true for $m^{L,LA}$ relatively to $m^{L,RA}$. This therefore implies that 63 is satisfied. It also follows that in case in which recessions are more likely, that is when $\pi < (1-\pi)$, the opposite is true, namely the reduction in asset prices will be larger in the economy with gain-loss preferences.

At last, if consumption at time zero and asset price at time zero are larger under the economy with gain-loss preferences, by equation 60 the same shall be true for leverage.

8 Appendix B. Enforcement constraint

Debt contracts emerge as result of bargaining agreements between lenders and borrowers. The agency problems are mitigated by imposing a borrower's enforcement constraint that prevents rational default. As it is common in the standard debt contract literature[42] we assign all the bargaining power to the borrower. This implies that the lender receives only the threat value of collateral, the liquidation value in case of default. Given this all that remains to characterize is the functional form of the enforcement constraint on the borrower, which, as we show below, once manipulated results in a collateral constraint[43].

Typical of debt contracts is a double-ignorance (see Holmstrom [52]), such that neither the lender nor the borrower observe the liquidation value of collateral before a default event. Consider a debt contract that runs over several periods. Once the contract starts the borrower must service the debt in every period and repay the full amount at the final date. To enforce repayment the loan contract shall be written to accommodate an enforcement constraint, for which the value from servicing the debt in every period and from repaying the final stock is higher than the value of defaulting, hence of appropriating current asset returns, but loosing the collateral.

[42] See Gale and Hellwig[37] and Townsend[83].
[43] See also Jermann and Quadrini[53] for a similar characterization.

To incorporate the double-ignorance we assume that at the time in which the contract is stipulated, both the lender and the borrower are uncertain about the future value of collateral, hence they assign a probability ϕ that the value can be recovered through a market sale and a probability $(1-\phi)$ that the recovery value will be zero. Let's denote by $\mathbb{E}_t\left\{m^b_{t,t+1}V^b_{t+1}\right\}$ the expected future discounted value of the period $t+1$ or the borrowers' value function in case he services the debt in every period. If the borrower decides not to service the debt in the current period, he would be able to retain $R^d_{t+1}B^b_t$, but collateral would be liquidated. Hence the borrower expects a loss in value of $\phi S^b_t\mathbb{E}_t\left\{p_{t+1}\right\}$. After that the borrower can enter a new debt contract and enjoy the same discounted value function from that point onward[44]. In this context the enforcement constraint, or no-rational default constraint, reads as follows:

$$\mathbb{E}_t\left\{m^b_{t,t+1}V^b_{t+1}\right\} \geq R^d_{t+1}B^b_t + \mathbb{E}_t\left\{m^b_{t,t+1}V^b_{t+1}\right\} - \phi S^b_t\mathbb{E}_t\left\{p_{t+1}\right\} \tag{64}$$

By re-arranging equation 64 we obtain our collateral constraint:

$$R^d_{t+1}B^b_t \leq \phi S^b_t\mathbb{E}_t\left\{p_{t+1}\right\} \tag{65}$$

9 Appendix C. Derivations of the gain-loss utility IMRS

Given the utility:

$$U^i(C^i_t, X_t) = \begin{cases} \Lambda\frac{\left|\frac{(C^i_t)^{(1-\gamma)}}{1-\gamma} - \frac{(X_t)^{(1-\gamma)}}{1-\gamma}\right|^{1-\theta}}{1-\theta}, & \text{if } C^i_t < X_t, \\ \frac{\left(\frac{(C^i_t)^{(1-\gamma)}}{1-\gamma} - \frac{(X_t)^{(1-\gamma)}}{1-\gamma}\right)^{1-\theta}}{1-\theta}, & \text{if } C^i_t \geq X_t, \end{cases} \tag{66}$$

It follows that:

$$IMRS_{t,t+s} = \rho^s\frac{U_C(C^i_{t+1}, X_{t+1})}{U_C(C^i_t, X_t)} = \begin{cases} \Lambda\frac{\left|\frac{(C^i_{t+s})^{(1-\gamma)}}{1-\gamma} - \frac{(X_{t+s})^{(1-\gamma)}}{1-\gamma}\right|^{-\theta}(C^i_{t+s})^{-\gamma}}{\left|\frac{(C^i_{t+s})^{(1-\gamma)}}{1-\gamma} - \frac{(X_{t+s})^{(1-\gamma)}}{1-\gamma}\right|^{-\theta}(C^i_t)^{-\gamma}} & \text{if } C^i_t < X_t, \\ \frac{(\frac{(C^i_{t+s})^{(1-\gamma)}}{1-\gamma} - \frac{(X_{t+s})^{(1-\gamma)}}{1-\gamma})^{-\theta}(C^i_{t+s})^{-\gamma}}{(\frac{(C^i_{t+s})^{(1-\gamma)}}{1-\gamma} - \frac{(X_{t+s})^{(1-\gamma)}}{1-\gamma})^{-\theta}(C^i_t)^{-\gamma}} & \text{if } C^i_t \geq X_t, \end{cases} \tag{67}$$

Recall that $G^i_{t+s} = \frac{C^i_{t+s}}{C^i_t}, \hat{g}{}^i_{t+s} = \log(G^i_{t+s})$ hence $G^i_{t+s} = \exp(\hat{g}{}^i_{t+s})$. Working with logarithms and recalling that $\hat{y}_{t+s} = \log\left(\frac{C^i_{t+s}}{X_{t+s}}\right)$ we obtain the expression for the $IMRS_{t,t+s}$ as given by equations 23,24, 25.

[44]We are assuming no exclusion from the market. This does not impair the constraint, as adding the punishment of the market exclusion would only make more likely that the constraint is satisfied.

10 Appendix D. Borrowers' SDF

Following Tallarini[82] and Yogo[84] we assume that consumption growth, $\hat{g}{}^{b}_{t}$, follows a normal distribution $N(\mu, \sigma^2)$ at any date t. Imposing the random structure already on consumption rather than on income allows us to obtain analytically tractable expressions for the computation of the SDF. Such approximation does not affect the main channels of our model compared to the situation in which the ultimate source of randomness lies in income or wealth and is consistent with empirical evidence (Lettau and Ludvigson[62]) showing that consumption and wealth are cointegrated, albeit the volatility of consumption falls short that of wealth.

We define $\overset{\wedge}{\kappa}_{t+1} = \hat{x}_{t+1} - \hat{c}_{t} = -\log(b) + \hat{s}{}^{b}_{t}$. Under this assumption it holds that:

$$\mathbb{E}_t \left[\exp(\hat{g}{}^{b}_{t+1}) \mid \hat{g}{}^{b}_{t+1} > \overset{\wedge}{\kappa}_{t+1} \right] = \exp\left\{ \mu + \frac{\sigma^2}{2} \right\} \frac{F(-(\overset{\wedge}{\kappa}_{t+1} - \mu - \sigma^2)/\sigma)}{F(-(\overset{\wedge}{\kappa}_{t+1} - \mu)/\sigma)} \tag{68}$$

$$\mathbb{E}_t \left[\exp(\hat{g}{}^{b}_{t+1}) \mid \hat{g}{}^{b}_{t+1} < \overset{\wedge}{\kappa}_{t+1} \right] = \exp\left\{ \mu + \frac{\sigma^2}{2} \right\} \frac{F((\overset{\wedge}{\kappa}_{t+1} - \mu - \sigma^2)/\sigma)}{F((\overset{\wedge}{\kappa}_{t+1} - \mu)/\sigma)} \tag{69}$$

where F is the cumulative conditional distribution of the standard normal. The borrowers' *IMRS* given by equation 23 and 24 can be written as:

$$m^{b}_{t,t+1} = \begin{cases} \frac{\Lambda \rho \exp\left\{ -\gamma \hat{g}{}^{b}_{t+1} \right\}}{k(\hat{y}_{t})} & \text{if } \hat{g}{}^{b}_{t+1} < \overset{\wedge}{\kappa}_{t+1}, \\ \frac{\rho \exp\left\{ -\gamma \hat{g}{}^{b}_{t+1} \right\}}{k(\hat{y}_{t})} & \text{if } \hat{g}{}^{b}_{t+1} > \overset{\wedge}{\kappa}_{t+1}, \end{cases} \tag{70}$$

Given the above we can compute the first moment of $m^{b}_{t,t+1}$ as follows:

$$\begin{aligned} \mathbb{E}_t \left\{ m^{b}_{t,t+1} \right\} \quad = \quad & \frac{\rho}{k(\hat{y}_{t})} F(-\frac{(\overset{\wedge}{\kappa}_{t+1} - \mu)}{\sigma}) \mathbb{E}_t \left[\exp\left\{ -\gamma \hat{g}{}^{b}_{t+1} \right\} \mid \hat{g}{}^{b}_{t+1} > \overset{\wedge}{\kappa}_{t+1} \right] \times \\ & \times \Lambda F(\frac{(\overset{\wedge}{\kappa}_{t+1} - \mu)}{\sigma}) \mathbb{E}_t \left[\exp\left\{ -\gamma \hat{g}{}^{b}_{t+1} \right\} \mid \hat{g}{}^{b}_{t+1} < \overset{\wedge}{\kappa}_{t+1} \right] \end{aligned} \tag{71}$$

Using formulas in 68 and 69 we can re-write 71 as follows:

$$\begin{aligned} \mathbb{E}_t \left\{ m^{b}_{t,t+1} \right\} \quad = \quad & \frac{\rho}{k(\hat{y}_{t})} \exp\left\{ \gamma \mu + \frac{(\gamma\sigma)^2}{2} \right\} \times \\ & \times \left[1 + (\Lambda - 1) F(\gamma \sigma + \frac{(\overset{\wedge}{\kappa}_{t+1} - \mu)}{\sigma} \right] \end{aligned} \tag{72}$$

11 Appendix E. Derivation of asset prices. Proposition 3.

We start by re-arranging borrowers' optimality condition on risky assets as follows:

$$p_t = \mathbb{E}_t \left\{ m^b_{t,t+1}(p_{t+1} + d_{t+1}) \right\} + \lambda^{'}_t \phi \mathbb{E}_t \left\{ p_{t+1} \right\} \tag{73}$$

Next we can rearrange equation 73 as follows:

$$p_t = \mathbb{E}_t \left\{ m^b_{t,t+1} d_{t+1} \right\} + \mathbb{E}_t \left\{ K_{t,t+1} p_{t+1} \right\} \tag{74}$$

where $K_{t,t+1} = m^b_{t,t+1} + \phi \lambda^{'}_t$. We now iterate forward equation 74:

$$\begin{aligned}
p_t &= \mathbb{E}_t \left\{ m^b_{t,t+1} d_{t+1} \right\} + \mathbb{E}_t \left\{ K_{t,t+1} p_{t+1} \right\} = \\
&= \mathbb{E}_t \left\{ m^b_{t,t+1} d_{t+1} \right\} + \mathbb{E}_t \left\{ K_{t,t+1} \left[m^b_{t+1,t+2}(p_{t+2} + d_{t+2}) + \lambda^{'}_{t+1} \phi \mathbb{E}_t \left\{ p_{t+2} \right\} \right] \right\} = \\
&= \mathbb{E}_t \left\{ m^b_{t,t+1} d_{t+1} + K_{t,t+1} m^b_{t+1,t+2} d_{t+2} \right\} + \mathbb{E}_t \left\{ K_{t,t+1} \left[m^b_{t+1,t+2} + \lambda^{'}_{t+1} \phi \right] p_{t+2} \right\} = \\
&= \mathbb{E}_t \left\{ m^b_{t,t+1} d_{t+1} + K_{t,t+1} m^b_{t+1,t+2} d_{t+2} \right\} + \mathbb{E}_t \left\{ K_{t,t+1} K_{t+1,t+2} p_{t+2} \right\} = \\
&= \mathbb{E}_t \left\{ m^b_{t,t+1} d_{t+1} + K_{t,t+1} m^b_{t+1,t+2} d_{t+2} \right\} + \mathbb{E}_t \left\{ K_{t,t+1} K_{t+1,t+2} \left[m^b_{t+2,t+3} d_{t+3} + (m^b_{t+2,t+3} + \lambda^{'}_{t+2} \phi) p_{t+3} \right] \right\} = \\
&= \mathbb{E}_t \left\{ m^b_{t,t+1} d_{t+1} + K_{t,t+1} m^b_{t+1,t+2} d_{t+2} + K_{t,t+1} K_{t+1,t+2} m^b_{t+2,t+3} d_{t+3} \right\} + \\
&\quad + \mathbb{E}_t \left\{ K_{t,t+1} K_{t+1,t+2} K_{t+2,t+3} p_{t+3} \right\}
\end{aligned} \tag{75}$$

After merging terms we obtain:

$$\begin{aligned}
p_t &= \mathbb{E}_t \left\{ m^b_{t,t+1} d_{t+1} \right\} + \mathbb{E}_t \left\{ \sum_{i=1}^{T} m^b_{t+i,t+i+1} d_{t+i+1} \prod_{j=1}^{i} K_{t+j-1,t+j} \right\} + \\
&\quad + \left[\prod_{i=0}^{T} K_{t+i,t+i} p_{t+T} \right]
\end{aligned} \tag{76}$$

12 Appendix F. Sharpe Ratio. Proposition 4.

In this section we derive the Sharpe ratio, namely the slope of the portfolio frontier, for the borrower. We start from re-arranging the borrower's first order conditions, namely equations 8 and 9, as follows:

$$1 = \mathbb{E}_t \left\{ m^b_{t,t+1} R^d_{t+1} \right\} + \lambda^{'}_t R^d_{t+1} \tag{77}$$

$$1 = \mathbb{E}_t \left\{ m^b_{t,t+1} R^s_{t+1} \right\} + \lambda^{'}_t \phi \mathbb{E}_t \left\{ \frac{p_{t+1}}{p_t} \right\} \tag{78}$$

where as in the text $\lambda^{'}_t = \frac{\lambda_t}{U_c(C^b_t, X_t)}$and $R^s_{t+1} = \frac{p_{t+1}+d_{t+1}}{p_t}$. We now subtract 77 from 78. The goal is that of deriving the Hansen and Jagannathan [45] bounds on the excess return between the risky asset and the debt rate. We obtain:

$$0 = \mathbb{E}_t \left\{ m^b_{t,t+1} Z_{t+1} \right\} + \lambda^{'}_t \mathbb{E}_t \left\{ \Sigma_{t+1} \right\} \tag{79}$$

where $Z_{t+1} = (R^s_{t+1} - R^d_{t+1})$ and $\mathbb{E}_t \left\{ \Sigma_{t+1} \right\} = \mathbb{E}_t \left\{ \phi \frac{p_{t+1}}{p_t} \right\} - R^d_{t+1}$. Let's assume that the SDF takes a general linear functional form as: follows:

$$m^b_{t,t+1} = \mathbb{E}_t \left\{ m^b_{t,t+1} \right\} + \beta_M (Z_{t+1} - \mathbb{E}_t \left\{ Z_{t+1} \right\}) \tag{80}$$

The functional form above is compatible with a log-linear approximation of the Euler equation. If the above is a valid SDF it must satisfy equation 79, which once expanded delivers the following expression:

$$\begin{aligned} 0 &= \mathbb{E}_t \left\{ m^b_{t,t+1} Z_{t+1} \right\} + \lambda^{'}_t \mathbb{E}_t \left\{ \Sigma_{t+1} \right\} = \\ &= \mathbb{E}_t \left\{ m^b_{t,t+1} \right\} \mathbb{E}_t \left\{ Z_{t+1} \right\} + Cov(m^b_{t,t+1} Z_{t+1}) + \lambda^{'}_t \mathbb{E}_t \left\{ \Sigma_{t+1} \right\} = \\ &= \mathbb{E}_t \left\{ m^b_{t,t+1} \right\} \mathbb{E}_t \left\{ Z_{t+1} \right\} + \mathbb{E}_t \left\{ (m^b_{t,t+1} - \mathbb{E}_t \left\{ m^b_{t,t+1} \right\})(Z_{t+1} - \mathbb{E}_t \left\{ Z_{t+1} \right\}) \right\} + \lambda^{'}_t \mathbb{E}_t \left\{ \Sigma_{t+1} \right\} = \\ &= \mathbb{E}_t \left\{ m^b_{t,t+1} \right\} \mathbb{E}_t \left\{ Z_{t+1} \right\} + \mathbb{E}_t \left\{ (Z_{t+1} - \mathbb{E}_t \left\{ Z_{t+1} \right\})(Z_{t+1} - \mathbb{E}_t \left\{ Z_{t+1} \right\}) \beta_M \right\} + \lambda^{'}_t \mathbb{E}_t \left\{ \Sigma_{t+1} \right\} = \\ &= \mathbb{E}_t \left\{ m^b_{t,t+1} \right\} \mathbb{E}_t \left\{ Z_{t+1} \right\} + \sigma^2_z \beta_M + \lambda^{'}_t \mathbb{E}_t \left\{ \Sigma_{t+1} \right\} \end{aligned} \tag{81}$$

Hence inverting 81 we obtain:

$$\beta_M = -(\sigma^2_z)^{-1} \mathbb{E}_t \left\{ m^b_{t,t+1} \right\} \mathbb{E}_t \left\{ Z_{t+1} \right\} - (\sigma^2_z)^{-1} \lambda^{'}_t \mathbb{E}_t \left\{ \Sigma_{t+1} \right\} \tag{82}$$

Given the above we can compute the variance of the SDF as follows:

$$\begin{aligned} Var(m^b_{t,t+1}) &= Var((Z_{t+1} - \mathbb{E}_t \left\{ Z_{t+1} \right\})^{'} \beta_M) = \\ &= \beta^{'}_M \sigma^2_z \beta_M = \\ &= \left[-(\sigma^2_z)^{-1} \mathbb{E}_t \left\{ m^b_{t,t+1} \right\} \mathbb{E}_t \left\{ Z_{t+1} \right\} - (\sigma^2_z)^{-1} \lambda^{'}_t \mathbb{E}_t \left\{ \Sigma_{t+1} \right\} \right]^{'} \sigma^2_z \times \\ &\quad \times \left[-(\sigma^2_z)^{-1} \mathbb{E}_t \left\{ m^b_{t,t+1} \right\} \mathbb{E}_t \left\{ Z_{t+1} \right\} - (\sigma^2_z)^{-1} \lambda^{'}_t \mathbb{E}_t \left\{ \Sigma_{t+1} \right\} \right] \end{aligned} \tag{83}$$

Once re-arranged the last delivers:

$$\begin{aligned} Var(m^b_{t,t+1}) &= -(\sigma^2_z)^{-1}(\mathbb{E}_t\left\{m^b_{t,t+1}\right\})^2(\mathbb{E}_t\left\{Z_{t+1}\right\})^2 + \\ &+\mathbb{E}_t\left\{\Sigma_{t+1}\right\}2\lambda'_t\mathbb{E}_t\left\{m^b_{t,t+1}\right\}(\sigma^2_z)^{-1} + (\sigma^2_z)^{-1}(\lambda'_t)^2(\mathbb{E}_t\left\{\Sigma_{t+1}\right\})^2 \end{aligned} \tag{84}$$

After re-arranging 84 we obtain the Sharpe ratio in Proposition 4.

13 Appendix G. Numerical Method

Our simulation method is based on a policy function iteration augmented with a Markov-switching structure. We start with a guess on the consumption decision rule as a policy function of the pre-determined variables, namely the three exogenous shocks, debt and the reference point, and we iterate until convergence. The policy function shall arise as solution to the system of competitive equilibrium conditions. The latter however takes different functional forms depending on the economy's regime, namely on whether the constraint binds or not ($\lambda_t > 0$ or $\lambda_t = 0$) and on whether consumption is above or below the reference level. Therefore in each period we verify if the policy function is such that the constraint binds or not and if consumption is above or below the reference point. Based on that we update expectations. This induces an endogenous regime switching, since in every period agents form expectation on the consumption policy function, which in turn depend upon the state variables. The latter summarize information about the relevant economy's regime. Methodologically the procedure is based on a monotone mapping algorithm that finds the fixed point in decisions rules (Coleman[20]). The list of equilibrium conditions describing the economy can be summarized as follows:

$$U^l_C\left(C^l_t\right) = \beta\mathbb{E}_t\left\{U^l_C\left(C^l_{t+1}\right)R^d_{t+1}\right\} \tag{85}$$

$$U^b_C(C^b_t, X_t) = \rho\mathbb{E}_t\left\{U^b_C(C^b_{t+1}, X_{t+1})R^d_{t+1}\right\} + \lambda_t R^d_{t+1} \tag{86}$$

$$p_t U^b_C(C^b_t, X_t) = \rho\mathbb{E}_t\left\{U^b_C(C^b_{t+1}, X_{t+1})(p_{t+1} + d_{t+1})\right\} + \lambda_t\phi\mathbb{E}_t\left\{p_{t+1}\right\} \tag{87}$$

$$(1-\nu)S_t^b = 1 \tag{88}$$
$$\nu B_t^l + (1-\nu)B_t^b = 0$$
$$\nu C_t^l + (1-\nu)C_t^b = \nu w_t^l + (1-\nu)w_t^b + d_t$$

Notice that the lenders and borrowers budget constraints are not needed. Indeed by Walras' law their linear combination results in the resource constraint. In each period t the policy function is computed conditional on expectations that in the next period λ_t can be either positive or zero and that $U_C^b(C_{t+s}^b, X_{t+s})$ could be either $\Lambda\frac{|\frac{(C_t^b)^{(1-\gamma)}}{1-\gamma}-\frac{(X_t)^{(1-\gamma)}}{1-\gamma}|^{1-\theta}}{1-\theta}$ or $\frac{|\frac{(C_t^b)^{(1-\gamma)}}{1-\gamma}-\frac{(X_t)^{(1-\gamma)}}{1-\gamma}|^{1-\theta}}{1-\theta}$. The exact steps of the algorithm are detailed below.

Our model features 5 state variables: two are predetermined (the reference level of the borrower (X_t) and the previous level of debt of the borrower (B_{t-1})) and 3 are truly exogenous (the income of the borrower (w_t^b), the income of the lender (w_t^l) and the dividend paid by the risky asset (d_t)). To obtain a solution to the system of Euler equations, market clearing conditions and budget constrains we follow the step listed below:

1. The map algorithm begins with a guess for the optimal consumption policy of the lender and the borrower, $\hat{c}^l(B_{t-1}, X_t, w_t^l, w_t^b, d_t)_0$ and $\hat{c}^h(B_{t-1}, X_t, w_t^l, w_t^b, d_t)_0$ and an initial guess for the equilibrium price of the risky asset $\hat{P}(B_{t-1}, X_t, w_t^l, w_t^b, d_t)_0$.

   For any combination of possible state variables over a discrete partition of the state space we proceed as follows:

   (a) The guess $\hat{c}_0^l$, $\hat{c}_0^b$ and $\hat{P}_0$, in conjunction with the log-normal distribution of the exogenous state variables allow us to compute the expected values on the right hand side of the Euler equations. Expectations are evaluated using trapezoid integration and linear interpolation is used to evaluate the policy functions between the grid points.

   (b) After obtaining a numerical value for the right hand side of the Euler equations we solve the system given by the 2 Euler equations for the risk-less assets and the market clearing conditions of the consumption market to find the risk-free rate and a new guess for the optimal consumption policy $\hat{c}_1^l$ and $\hat{c}_1^b$. The optimal debt of the borrower follows from her budget constraint and the debt of the lender from the clearing condition of the debt market. Using the consumption of the borrower and the guess $\hat{c}_1^b$ we determine the equilibrium price of the risky asset that solves the corresponding Euler equation of the borrower. This produces the new guess for the stock price $\hat{P}_1$.

(c) The new guess $\hat{c}_1^l$, $\hat{c}_1^b$ and $\hat{P}_1$ have to be consistent with the borrowing constraint. If the constraint is satisfied we retrieve $\hat{c}_1^l$, $\hat{c}_1^b$ and $\hat{P}_1$ as new candidate solutions to our problems.

(d) If the constrains is not satisfied we set the optimal debt of the borrower to its limit (the debt of the lender automatically follows from the clearing condition of the debt market) and solve the system given by the 2 Euler equations of the borrower (augmented to include the Lagrange multiplier), the Euler equation of the lender, the market clearing condition of the consumption market and the borrower's budget constraint to find c^l, c^b, R_f, the Lagrange multiplier λ_t and the price of the risky asset P. This procedure yields the new candidate solutions $\hat{c}_1^l$, $\hat{c}_1^b$ and $\hat{P}_1$.

2. We repeat steps (a)-(d) until convergence which occurs when the Euclidean distance between current and new guess is less than $1e^{-2}$ for $\hat{c}^l$, $\hat{c}^b$ and $\hat{P}$, simultaneously. In this way we obtain the solutions to our problem $c^l(B_{t-1}, X_t, w_t^l, w_t^b, d_t)$, $c^h(B_{t-1}, X_t, w_t^l, w_t^b, d_t)$, $P(B_{t-1}, X_t, w_t^l, w_t^b, d_t)$ as well as the equilibrium risk free rate and optimal debt of the agent as a function of the sate variables of our economy.

3. Finally, we generate 100100 realizations of income and dividend processes and compute asset pricing and macroeconomic moments using the policy functions obtained from the algorithm described above. Burning period: first 100 observations.

14 Appendix H. Data description debt and asset prices

In this appendix we detail data sources, samples and definitions for asset prices, returns and debt. All data are quarterly series and cover the sample period 1980Q1:2016Q3.

Debt is from private non-financial sectors. The source is BIS total non-financial debt data[45] for both the U.S. and the U.K.. Statistics on debt data are computed on differences from HP-filtered series. For the US we also check with Equifax data[46].

Data for asset prices and returns are as follows. For stock market indices in the U.S. we take the S&P500 from Bob Shiller database[47]. For the UK the stock market indices are from Moody's Seasoned Aaa Corporate Bond Yield, obtained from Morgan Stanley via Datastream. We choose the stock market index that includes dividends as this is the definition which is closer to the one employed in our model. To compute the equity premium we also uses data for debt rates taken from the FRED database, available at Link: https://fred.stlouisfed.org. For the US debt rates are proxied by Moody's Seasoned Aaa Corporate Bond Yield, while for the UK we use the money market rates (Effective Federal Funds Rate), which is taken from the FRED database.

Note that all equity premia are calculated in real terms, deflated by the CPI series. Specifically the premium (in logs) has been calculated as $EP_t = RER_t - RRFR_t$,where RER_t is the real equity premium and $RRFR_t$ is the real return on debt. The latter is calculated as $RRFR_t = RFR_t - \pi_t^c$, where π_t^c is CPI inflation (data on CPI are taken from the OECD dataset). The real equity premium (inclusive of dividends) is calculated as follows:

$$RER_t^{Div} = \frac{p_t + d_t - p_{t-1}}{p_{t-1}} \tag{89}$$

where p_t is the real share price index (here S&P500 for the US) and d_t are real dividends. When using data from Moody's Seasoned Aaa Corporate Bond Yield the premium is calculated as follows:

$$RER_t^{Div} = \frac{T_t - T_{t-1}}{T_{t-1}} \tag{90}$$

where T_t is a real total return index (i.e., including dividends) based on MSCI data[48].

[45] Source: http://www.bis.org/statistics/totcredit.htm.
[46] Those are obtained from https://www.newyorkfed.org/microeconomics/databank.html..
[47] See http://www.econ.yale.edu/~shiller/data.htm.
[48] See https://www.msci.com/real-time-index-data-search.

15 Appendix I. Estimation of dividend and income processes

To estimate the dividend process we use data for net corporate dividends from NIPA Tables[49]. Regarding the data sample we use the longest available, which in this case is 1960Q1-2014Q3, to gain maximum efficiency. We compute real dividend per capita by deflating for CPI, again taken from the OECD dataset, and by dividing for population in a given year, the latter is taken from U.S. Census Bureau. We also de-trend the series through HP-filter. The estimated persistence is 0.56. In our calibration we set persistence to 0.6 and volatility to 0.05.

To estimate the income process we use PSID data[50]. The PSID is a longitudinal study of a sample of the US population. It has been conducted annually since 1968 until 1997 and biannually afterwards. It consists of two independent samples. One of about 2,000 households drawn from the Survey of Economic Opportunity respondents, the SEO sample, which represents low-income families and one drawn by the Survey Research Center, the SRC sample, that includes about 3,000 households representative of the US population. We only consider the SRC sample since we are interested in a representative sample. From this sample we excluded families that stopped responding to the survey at some point in time or families which reported no income for one or more years. This leaves us with a sample of 375 families that cover a time-series sample from 1968 to 2013.

For those families we construct income as follows. The survey follows both the original families as well as their split-offs. Total income of the family is obtained by adding together the reported taxable income and transfer income from all sources for the family head, the wife and all other earners in the family. Taxable income includes labor income (wages and salaries, bonuses, overtime, tips, commissions, professional practice or trade, and market gardening) and income from other sources. Transfer income includes social security income, unemployment and workers compensation, child support, retirement income as well as other welfare transfers to the head and wife. Total family income (constructed as per sum above) is weighted by the number of family members and deflated by the CPI.

[49] See https://www.bea.gov/iTable/index_nipa.cfm.

[50] See also Heatcote, Perri and Violante[51].

We divide the entire group of the families into two, borrowers and non-borrowers, by using the Wealth supplement search. Families sometimes switch back and forth from the borrower to the non-borrower status. We defined a borrower as the family that has debt in at least 50% of all the years considered. For each household income series we fit a AR(1) process through an OLS estimation (see Heaton and Lucas [50]). We then obtain the coefficient for the aggregate income process of borrowers and lenders by computing sample averages. The estimation delivers persistence for the lenders' income process of 0.62 and for the borrowers' income process of 0.65. As for the standard deviations of the error component, consistently with the literature using PSID data (see Heaton and Lucas [50] and MaCurdy[64]) we find implausibly high values (of 0.38 for borrower and 0.49 for lenders). As argued in Deaton [25] those volatilities over-state the true innovation to the income process due to measurement error, hence the values should be scaled down. Values for the volatilities can then be calibrated indirectly by matching empirical moments. In our calibration we use a benchmark value of 0.03 as this contribute us to match the volatility of the risky asset return. We then experiment with values between 0.02 to 0.06.

16 Appendix L. Estimating the Stochastic Discount Factor

Preferences parameters, θ and Λ, have been estimated through a GMM procedure along the lines of Yogo[84] and Berkelaar, Kouwenberg and Post[10], but with a more recent data sample, namely 1970Q2:2017Q1.

We obtain the moment conditions by substituting the borrowers' Euler equation on debt into the one for risky assets. We then estimate the resulting equation in non-linear form through GMM methods[51]. The resulting SDF is computed assuming a risk-aversion coefficient of 1[52]. We use data at quarterly frequency for the US. Specifically we take real per capita consumption as non-durable and services[53] from the NIPA tables[54]. Data for risky assets are the S&P500 equity returns, with

[51] See Hansen and Singleton[46].

[52] This assumption follows Yogo[84] who estimates an SDF for loss averse agents and argues that a unitary risk-aversion is needed to maintain scale-invariance. Although our SDF also features term related to the collateral constraint, scale invariance remains important.

[53] We lag this variable of one quarter following the convention in Campbell [16]. This convention is widely used in the literature to match consumption and returns data.

[54] See https://www.bea.gov/iTable/index_nipa.cfm.

and without dividends for risky assets. Data for debt returns are the 3months T-bill. All returns are nominal and deflated with CPI deflator. The sources for the financial market data is the Shiller Database[55] for the risky assets and the CRSP Indices database for 3mT-Bill[56]. As instruments we use the excess market return (this is the first factor in Fama-French[57]), consumption growth, the dividend-price ratio, the size of the spread, the value of the spread, the yield spread and two lags of the asset returns plus a constant. The source for those data is the Kenneth French's database[58].

The GMM estimate is done in two-steps. We estimate the preference parameters based on time-series tests conditional on the information given by the instruments[59]. For our sample period the estimated value for θ is 0.3. This value is compatible with values used in Yogo[84], who argues that a positive value induces increasing risk-tolerance on the upper tail. Indeed this value is important in generating the declining risk-aversion or higher risk-tolerance at upper tails. The estimated value for Λ is 1.98. In our calibration we use a somewhat higher value since values in the GMM literature tend to be generally higher and since this improves the matching for the debt statistics.

[55] Shiller Database: http://www.econ.yale.edu/~shiller/data.htm.

[56] CRSP Indeces Database: http://www.crsp.com/products/research-products/crsp-us-treasury-database.

[57] See Fama and French[31].

[58] Kenneth French's Database: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[59] The system is over-identified. For this reason we define a weighted loss function on the moments, where the weights are estimated with a kernel-based approach (Newey and West [72]).