

# STRUCTURAL STRESS TESTS\*

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## Abstract

We develop a structural banking model for microprudential stress testing. The balance sheet structure, dividend policy and exit policy of the bank are endogenous. Banks face idiosyncratic and aggregate uncertainty, and are subject to regulatory constraints. As in the data, banks hold a capital buffer to protect its charter value. We calibrate the model using U.S. data on Bank Holding Companies. We explore how banks respond to a stress scenario that resembles the stress scenario performed by the Federal Reserve. In contrast to reduced form (top down) stress test models, our structural approach offers a framework to evaluate possibly nonlinear dynamics in margins, endogenous changes in the asset structure of the bank, dividend payments and bank failure. We find that the reduced form model underpredicts the drop in profits and dividend payments during the stress scenario and the strong adjustment in banks' asset composition. Since we can solve our model under different policy regimes (i.e., after adjusting capital regulation or liquidity regulation), we also evaluate how the model comparisons changes when we alter regulation.

**JEL classifications:** C63, G11, G17, G21, G28

**Key words:** bank, stress testing, structural model, microprudential, top-down

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# 1 Introduction

State-of-the-art models for micro-prudential stress testing rely on exogenous rules and reduced-form relationships that are linearly estimated from historical data susceptible to misspecification and the Lucas critique. This paper takes a step towards a quantitative structural prudential stress testing framework.<sup>1</sup>

As in [De Nicolo, Gamba, and Lucchetta \(2014\)](#), banks in our model face a maturity transformation problem between demandable external funding and long-term loans. Unlike that paper, a bank makes an explicit loan portfolio decision which is closed by a loan demand equation that is derived from an estimated discrete choice model and determines the equilibrium loan interest rate. We consider a rich balance sheet where banks can extend more than one type of loan.<sup>2</sup> We calibrate the model using balance sheet and income statement data for the top 35 bank holding companies included in the stress test performed by the Federal Reserve.<sup>3</sup> We then track bank behavior, including the endogenous exit decision, during different stress scenarios and compare it to behavior predicted using a reduced form approach.

We show that the bank has an incentive to hold a capital buffer above the minimum to reduce the likelihood of exit and protect its charter value. This endogenous nonlinear buffer stock implies that linear rules can lead to incorrect inference about bank exit decisions. In particular, we contrast our structural stress test results with those of a stylized non-structural stress test. Following the CLASS approach [Hirtle, Kovner, Vickery, and Bhanot \(2014\)](#), we show that stress tests that are based on the extrapolation of historical correlations underestimates the decline in profits and dividend payments. We also show that it underpredicts the sharp decline in risk-weighted assets. The combination of a different path of profits and risk-weighted assets results in a very different path of risk-weighted capital (lower in the reduced form model than in the structural model). The structural model predicts a shifts to safe assets that increases the risk-weighted capital ratio and keeps failure rates at very low levels even during the stress scenario.

We perform a series of decompositions to show how important each of the features of the reduced form model are. In particular, we relax one by one the main assumptions of the reduced form model: linear predictions for interest margins, charge-offs and costs; a dividend rule, constant asset shares and no exit rule. While relaxing each assumption at a time brings the reduced form model closer to the structural model, introducing variation in the evolution

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<sup>1</sup>Our model and the CLASS model can be classified as a “top-down” approach. This approach is intended to complement the more detailed supervisory models of components of bank revenues and expenses, such as those used in the DFAST, CCAR and European Stress Test that are classified as using a “bottom-up” approach. A benefit of the “top-down” approach is the use of publicly available data. CCAR evaluates the capital planning processes and capital adequacy of bank holding companies with \$50 billion or more in total consolidated assets. The top-down approach and the bottom-up approach share some of the assumptions regarding the asset structure of the bank and dividend payments during the stress scenario.

<sup>2</sup>In banking, [Dick \(2008\)](#) and [Egan, Hortacsu, and Matvos \(2015\)](#) estimate firm-specific demand using a discrete choice approach in the deposit market while here we apply the approach to the loan market. Our approach is also related to the work of [Elizalde and Repullo \(2007\)](#) by quantifying the wedge between regulatory and economic bank capital.

<sup>3</sup>This corresponds to all bank holding companies or U.S. intermediate holding companies with \$50 billion or more in total consolidated assets. We exclude Deutsche Bank Trust corporation since it is a subsidiary of foreign bank.

of asset shares goes a long way in explaining the differences in the asset side of the bank, the evolution of profits and the risk-weighted capital ratio between the structural model and the reduced form model. We also introduce an exit rule based on a given level of capital (the minimum required level) to the reduced form model but the model over-predicts the number of banks that fail during stress since this simple rule does not capture the dynamics of the charter value of the bank.

**Related Literature.** State-of-the-art stress testing frameworks use a combination of reduced-form dependencies (Acharya, Engle, and Pierret, 2014; Covas, Rump, and Zakrajcek, 2014) and exogenous behavioral rules (Burrows, Learmonth, and McKeown, 2012; Board of Governors of the Federal Reserve System, 2013; Hirtle, Kovner, Vickery, and Bhanot, 2014; European Banking Authority, 2011, 2014) to map aggregate economic conditions to bank-specific variables.<sup>4</sup> These frameworks do not identify structural parameters of the bank, which makes them prone to the Lucas critique. In that case, these frameworks cannot conduct stress tests under counterfactual capital requirements or risk weights, since the estimated parameters are only implicit functions of these parameters. Our model replaces backward looking and exogenous rules by optimizing forward looking behavior. Thus, the policy functions that describe bank behavior become explicit functions of exogenous states and structural parameters. This offers a flexible laboratory for stress testing as a battery of counterfactual scenarios can be considered without having to extrapolate from observed conditions.

The remainder of the paper is structured as follows: Section 2 lays out the model, Section 3 presents the dynamic program of the bank, Section 4 shows the calibration to U.S. data and Section 5 provides intuition about bank behavior. Section 6 conducts stress testing exercises and compares structural with reduced-form stress test outcomes.

## 2 Model

Time is discrete and infinite. Each period is divided into two subperiods. In the first subperiod a bank enters with risk free securities, a sector specific loan portfolio, and deposits. Then there is a realization of an aggregate shock  $z_t$  with Markov transition matrix  $\mathcal{F}(z_{t+1}, z_t)$  that determines the fraction of its non-performing loans and hence its cash-flows. The bank then decides whether to stay or exit and receive a salvage value. If it decides to stay it enters the second subperiod where it receives an idiosyncratic funding shock  $\delta_{bt+1}$  which follows a Markov process with transition matrix  $G(\delta_{t+1}, \delta_t)$ . The bank then supplies long term loans to sector  $s \in \mathcal{S}$ , chooses how many safe securities to hold, and how many dividends to pay, retain earnings, or issue costly seasoned equity before entering the next time period.

### 2.1 Loan Demand

To derive bank  $b$ - and sector  $s$ -specific loan demand we employ a discrete choice model. Following Egan, Hortacsu, and Matvos (2015), a new unit loan with discounted price  $q_{bst}^n$  from bank  $b$  in sector  $s$  ( $s \in \{\text{real estate, C\&I}\}$ ) in period  $t$ , generates utility  $\alpha_s q_{bst}^n$  for a

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<sup>4</sup>For a survey on state-of-the-art stress testing models see for example Foglia (2009); Borio, Drehmann, and Tsatsaronis (2012).

potential borrower  $j$ . In addition,  $j$  also receives non-interest utility  $\gamma_{bs} + \varepsilon_{jbst}$  when borrowing from bank  $b$ , where  $\gamma_{bs}$  captures time-invariant but bank-specific factors and  $\varepsilon_{jbst}$  captures any borrower-specific bank preferences. We assume that  $\varepsilon_{jbst}$  are drawn i.i.d. from a Type 1 Extreme Value distribution. When not investing in a risky project, potential borrower  $j$ 's utility is given by the stochastic realization of an outside option  $\omega_{jt}$  that is distributed according to a cdf  $\Omega(\omega, z_t)$  itself a function of the aggregate shock  $z_t$ . One can think of  $\omega_{jt}$  as representing alternative (non-bank) funding possibilities.

Loans are long term contracts that mature probabilistically.<sup>5</sup> Specifically, if an individual from sector  $s$  takes out a loan in period  $t$ , the loan matures next period with probability  $m_s$ ; if the loan does not mature, it pays out coupon  $c_s$ . Borrowers make the agreed upon payments as long as the project they invest in does not fail. In case of failure, the borrower returns only  $(1 - \lambda_{st})$  units. The failure probability is denoted by  $(1 - p_s(z_t))$ , a function of the aggregate state of the world.

Potential borrower  $j$ 's total utility conditional on receiving a new loan from bank  $b$  in sector  $s$  and period  $t$  is given by

$$u(\varepsilon_{jbst}) = \alpha_s q_{bst}^n + \gamma_{bs} + \varepsilon_{jbst}$$

Let  $U_{st}$  denote the expected utility of  $j$  when choosing optimally to take a loan from bank  $b$

$$U_{st} = \int_{-\infty}^{+\infty} \max_b \{u(\varepsilon_{jbst})\} dG(\varepsilon)$$

It can be shown that by properties of the Extreme Value distribution, this can be rearranged to

$$U_{st}(\mathbf{q}_{st}^n) = \iota + \log \left( \sum_{b=0}^{B_s} \exp(\alpha_s q_{bst}^n + \gamma_{bs}) \right),$$

where  $\iota$  is the Euler constant,  $\mathbf{q}_{st}^n$  is the vector of new loan prices and  $B_s$  is the set of banks operating in sector  $s$ .

Given that the potential borrower  $j$  can also choose the outside option, his first-stage problem is given by

$$\max_{x \in \{0,1\}} x U_{st}(\mathbf{q}_{st}^n) + (1 - x) \omega_{jt}$$

where  $x$  is the choice of taking a loan ( $x = 1$ ) or not taking a loan ( $x = 0$ ). Integrating over the mass of potential borrowers, we obtain a measure of borrowers in sector  $s$  and period  $t$  (i.e. aggregate loan demand):

$$L_{st}^d(\mathbf{q}_{st}^n, z_t) = \int_{\underline{\omega}}^{\bar{\omega}} \mathbb{I}[U_{st}(\mathbf{q}_{st}^n) > \omega] d\Omega(\omega, z_t). \quad (1)$$

With the assumption of the extreme value distribution for  $\varepsilon_{jbst}$ , bank  $i$ 's market share  $\sigma_{ist}(\mathbf{q}_{st}^L)$  is given by

$$\sigma_{bst}(\mathbf{q}_{st}^n) = \frac{\exp(\alpha_s q_{bst}^n + \gamma_{bs})}{\sum_{b=0}^{B_s} \exp(\alpha_s q_{bst}^n + \gamma_{bs})}. \quad (2)$$

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<sup>5</sup>As in, for instance, [Chatterjee and Eyigungor \(2012\)](#)

As a result, bank  $b$ -specific loan demand can be written as

$$L_{bst}^d(\mathbf{q}_{st}^n, z_t) = \sigma_{bst}(\mathbf{q}_{st}^n) \times M_s(z_t), \quad (3)$$

where  $M_s(z_t)$  captures changes in the total demand for loans due to aggregate conditions.

## 2.2 Bank Environment

Bank  $b$  maximizes its expected discounted dividends:

$$\mathbb{E}_t \sum_{t=0}^{+\infty} \tilde{M}_{t,t+1} \mathcal{D}_{bt}, \quad (4)$$

where  $\tilde{M}_{t,t+1}$  is a stochastic discount factor.

Bank  $b$  can invest in long term loans, whose stock in sector  $s$  is denoted  $\ell_{bst}$ , and risk-free securities  $a_{bt}$ , and takes on deposits  $\delta_{bt}$ . We assume securities have a return equal to  $r_t^a$ . Deposits are assumed to be covered by deposit insurance and pay interest equal to  $r_t^d$ . The discounted price of new long-term loans is determined endogenously and denoted by  $q_{st}^n$ . We denote the value of loans that were issued in the past, that have not matured, and are in good standing by  $q_{st}^o$ . We assume that  $z_t$  follows an AR(1) process. Performing bank loans generate cash flow of  $[c_s(1 - m_s) + m_s]$ . Non-performing loans pay no interest.

Given a stock of loans, securities, deposits at hand, and the aggregate shock (that determines the loan price schedule), banks are matched with a random number of depositors  $\delta_{bt}$  drawn from  $G(\delta_t, \delta_{t+1})$  which is i.i.d across banks. These shocks capture liquidity variation derived from changes in the inflow of deposits and other short term funding. The bank then chooses how many new loans to extend  $L_{bst}^s$ , how many securities to hold  $a_{bt+1}$ , whether to pay dividends  $\mathcal{D}_{bt}$ , issue equity and/or retain earnings.

At beginning of the first subperiod, after the realization of  $z_t$ , profits on previous period's investments are

$$\pi_{bt} = \sum_s \{ [p_s(z_t)(1 - m_s)c_s - (1 - p_s(z_t))\lambda_{st}] \ell_{bst} \} - r^d \delta_t + r^a a_t - \kappa, \quad (5)$$

where  $\kappa$  is a fixed operating cost. Once profits are determined we can define bank cash-at-hand  $n_t$  as

$$n_{bt} = \pi_{bt} + \sum_s \{ [p_s(z_t)m_s + (1 - p_s(z_t))] \ell_{bst} \} + a_{bt} - \delta_{bt}. \quad (6)$$

At the beginning of the second subperiod, the bank then draws its new deposit funding ( $\delta_{bt+1}$ ) and makes portfolio choices. The cost of issuing new loans is given by  $\phi(L_{bst}^s)$ . After choosing the amount of new loans to extend  $L_{bst}^s$  (we allow  $L_{bst}^s$  to be negative in which case the bank is liquidating part of its portfolio), securities  $a_{bt+1}$  and given deposits  $\delta_{bt+1}$ , the cash flow for the bank is

$$\mathcal{F}_{bt} = n_{bt} + \delta_{bt+1} - a_{bt+1} - \sum_s \{ [I_+ q_{st}^n + I_- q_{st}^o] L_{bst}^s - I_+ \phi(L_{bst}^s) \}, \quad (7)$$

where  $I_+$  takes value 1 if  $L_{bst}^s$  is positive and  $I_-$  is 1 if  $L_{bst}^s$  is negative.

The law of motion for the stock of loans is

$$\ell_{bst+1} = p_s(z_t)(1 - m_s)\ell_{bst} + L_{bst}^s. \quad (8)$$

The value of cash-on-hand  $n_t$  and the choice of loans and securities determine whether the bank distributes dividends, retains earnings or issues new equity. The net-payoff to shareholders is

$$\mathcal{D}_{bt} = \begin{cases} \mathcal{F}_{bt} & \text{if } \mathcal{F}_{bt} \geq 0 \\ \mathcal{F}_{bt} - \nu(\mathcal{F}_{bt}, z_t) & \text{if } \mathcal{F}_{bt} < 0 \end{cases}. \quad (9)$$

where  $\nu(\mathcal{F}_{bt}, z_t)$  denote flotation costs per-unit of new funds. After loan and securities decisions have been made, we can define the present value of bank book equity capital  $e_t$  as

$$e_{bt+1} \equiv \underbrace{\sum_s \{p_s(z_t)(1 - m_s)\ell_{bst} + [I_{[L_{bst}^s \geq 0]}q_{st}^n + I_{[L_{bst}^s < 0]}q_{st}^o]L_{bst}^s\}}_{\text{assets}} + a_{bt+1} - \underbrace{\delta_{bt+1}}_{\text{liabilities}}. \quad (10)$$

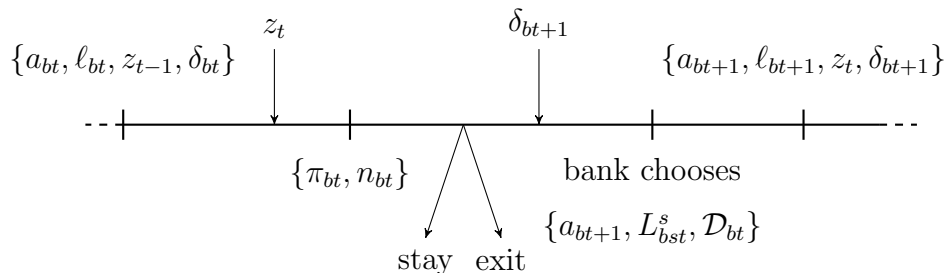
The bank's portfolio choice is subject to a regulatory minimum capital constraint

$$e_{bt+1} \geq \varphi \left( \sum_s \{w_{\ell_s} (p_s(z_t)(1 - m_s)\ell_{bst} + [I_{[L_{bst}^s \geq 0]}q_{st}^n + I_{[L_{bst}^s < 0]}q_{st}^o]L_{bst}^s)\} + w_A a_{bt+1} \right) \quad (11)$$

where  $\varphi$  is the minimum regulatory common equity Tier 1 capital ratio requirement and  $w_k$ ,  $k \in \{\ell, A\}$ , are regulatory risk-weights.

The timing of events are summarized in Figure 1.

**Figure 1:** Timing



## 2.3 Loan Market

Corbae and D'Erasmus (2014) consider an imperfectly competitive loan market. In order to simplify the analysis here, we assume that banks operate in a perfectly competitive environment (which can be used to generate initial conditions for the imperfect competition case that we are in the process of solving). In this case, the problem of bank  $b$  presented in the previous section corresponds to a representative bank  $b$  from a unit measure of banks of the same type. Perfect competition leads banks intermediaries to charge a new loan price that in

expectation earns zero profits.<sup>6</sup> This implies that the price of new loans for  $L_{st}^s > 0$  satisfies the following price equation:

$$q_{st}^n(L_{st}^s) = E_{z_{t+1}|z_t} \left[ M_{t,t+1} \tilde{\left\{ p_s(z_{t+1}) [m_s + (1 - m_s)(c + q_{st+1}^o)] + (1 - p(z_{t+1}))(1 - \lambda_{st+1}) \right\}} \right] - \frac{\phi(L_{st}^s)}{L_{st}^s}, \quad (12)$$

where  $q_{st+1}^o$  satisfies

$$q_{st}^o = E_{z_{t+1}|z_t} \left[ M_{t,t+1} \left\{ p_s(z_{t+1}) [m_s + (1 - m_s)(c_s + q_{st+1}^o)] + (1 - p_s(z_{t+1}))(1 - \lambda_{st+1}) \right\} \right] \quad (13)$$

The difference between  $q_{st}^n(L_{st}^s)$  and  $q_{st}^o$  arises from the cost of extending a new loan  $\phi(L_{st}^s)$ . The price function is independent of the exit probability of the bank because we assume that there are no liquidation costs (i.e. if the bank fails, other banks will bid the price of the existing loan portfolio down to the one that satisfies the expected zero profit condition). For given  $p_s$  functions and parameters  $m_s$  and  $c_s$ , solving  $q_{st}^o$  implies only solving a system of  $n_z$  equations in  $n_z$  unknowns (where  $n_z$  is the number of grid points for  $z$ ). Once we obtain  $q_{st}^o$  from (13), it is straightforward to compute  $q_{st}^n(L_{st}^s)$  in (12) given  $L_{st}^s = L_{st}^d$  from equation (3) for the representative bank.

Having specified the environment, we can now explain fundamental differences from the existing literature on structural banking models. While [Egan, Hortacsu, and Matvos \(2015\)](#) also use a logit approach to estimating demand, they focus on the deposit market while we focus on the loan market. By modeling the loan supply decision conditional on the estimated demand for a given bank's loans we take a deep approach to determining cash flows as opposed to the reduced form approach in [De Nicolo, Gamba, and Lucchetta \(2014\)](#).

### 3 Recursive Formulation of Bank Problem

Due to the recursive nature of the bank's problem, we can drop time subscripts. Let  $x_t = x$  and  $x_{t+1} = x'$ . Letting  $\ell$  denote the vector of the stock of loans for both sectors, the value of the bank at the beginning of the period is given by

$$V(a, \ell, \delta, z) = \max_{x \in \{0,1\}} \{V^{x=0}(a, \ell, \delta, z), V^{x=1}(a, \ell, \delta, z)\} \quad (14)$$

where  $x \in \{0, 1\}$  denotes the exit decision of the bank,  $V^{x=0}(a, \ell, \delta, z)$  the value of the bank if it chooses to continue and  $V^{x=1}(a, \ell, \delta, z)$  the value in case of exit. The problem of the bank

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<sup>6</sup>To make this price consistent with the problem of the bank it is important to assume no bankruptcy costs. Bankruptcy costs which do not affect the value of loans, can easily be included. If bankruptcy costs which affect the value of the loan are included, then one needs to include the probability of bank failure into the pricing equations (12) and (13).

when it chooses to continue is

$$V^{x=0}(a, \ell, \delta, z) = E_{\delta'|\delta} \left\{ \max_{\{L_s^s, a'\}} \mathcal{D} + E_{z'|z} [M_{z',z} V(a', \ell', \delta', z')] \right\}$$

*s.t.*

$$\pi = \sum_s \{ [p_s(z)(1 - m_s)c_s - (1 - p_s(z))\lambda_s] \ell_s \} - r^d \delta + r^a a - \kappa, \quad (15)$$

$$n = \pi + \sum_s \{ [p_s(z)m_s + (1 - p_s(z))] \ell_s \} + a - \delta, \quad (16)$$

$$\mathcal{F} = n + \delta' - a' - \sum_s \{ [I_+ q_s^n(z, L_s^s) + I_- q_s^o(z)] L_s^s - I_+ \phi(L_s^s) \}, \quad (17)$$

$$\mathcal{D} = \begin{cases} \mathcal{F} & \text{if } \mathcal{F} \geq 0 \\ \mathcal{F} - \nu(\mathcal{F}, z) & \text{if } \mathcal{F} < 0, \end{cases} \quad (18)$$

$$e = \sum_s \{ p_s(z)(1 - m_s)\ell_s + [I_{[L_s^s \geq 0]} q_s^n(L_s^s) + I_{[L_s^s < 0]} q_s^o(L_s^s)] L_s^s \} + a' - \delta' \quad (19)$$

$$e \geq \varphi \left( \sum_s \{ w_{\ell_s} (p_s(z)(1 - m_s)\ell_s + [I_{[L_s^s \geq 0]} q_s^n + I_{[L_s^s < 0]} q_s^o] L_s^s) \} + w_a a' \right), \quad (20)$$

$$\ell'_s = p_s(z)(1 - m_s)\ell_s + L_s^s. \quad (21)$$

The value in case of bank exit is given by

$$V^{x=1}(a, \ell, \delta, z) = \max \left\{ n + \sum_s \{ p_s(z) q_s^o (1 - m_s) \ell_s \}, 0 \right\}$$

From the solution to this problem, we obtain the exit decision rule  $x(a, \ell, \delta, z)$ , a loan decision rule  $L_s(a, \ell, \delta, z, \delta')$ , a security decision rule  $a'(a, \ell, \delta, z, \delta')$ , and a dividend policy  $\mathcal{D}(a, \ell, \delta, z, \delta')$ .

## 4 Calibration

One period corresponds to a quarter. We calibrate the model to bank holding companies with \$ 50 billion or more in assets, we refer to these as the “big” banks. These are the banks that are included in the stress test conducted by the Federal Reserve. We calibrate the model to allow for two types of loans: real estate (*re*) and commercial & industrial (*ci*). These are the loans with the largest share of total loans among large bank holding companies. The data is taken from the Consolidated Financial Statements for Bank Holding Companies (FR Y-9C), which provides detailed information about large U.S. commercial banks’ balance sheets and income statements. Our sample period for the calibration is from 1986 to 2007.<sup>7</sup> All parameters are in real terms. We deflate using total CPI index.

<sup>7</sup>Data limitations prevents us from using data prior to 1986 and we decide to exclude the period since last financial crisis from the calibration exercise.



First, we describe the calibration of the loan demand and non-performing loans. In both cases, the relevant elasticities are pinned down directly from the data. Second, we present the calibration of the parameters that require solving the bank problem to then match a set of moments generated by the model with those from the data.

## 4.1 Loan Demand Estimation

To estimate bank  $b$  loan demand curve  $L_{bst}^d(z_t, \mathbf{q}_{bst}^n)$ , defined in Equation (3), we proceed as follows: first, we estimate market shares for the big banks in the U.S. as predicted by the discrete choice model (Equation (2)). Second, we estimate the evolution of aggregate loan demand (Equation (1)) by aggregating the bank level data.

### 4.1.1 Market Share Estimation

We estimate Equation (2), for each sector, using interest income (from which we derive the implicit interest rate and the discounted price of the loan) and loan volume data. We compute each bank’s market share,  $\sigma_{bst}$ , as loans by bank  $b$  relative to total credit in sector  $s$  where total credit is the sum over all loans of all incumbent banks in the sample. Following [Egan, Hortacsu, and Matvos \(2015\)](#), we allow the quality of the bank to vary over time. Let  $\zeta_{bst}$  denote the time-varying quality component. Then total bank quality is given by  $\nu_{bs} + \zeta_{bst}$ . We treat credit from those banks outside the big banks as an unobservable outside good, which we index by 0. We normalize non-interest utility of the outside good to zero,  $\nu_{0s} + \zeta_{0st} = 0$ . Dividing  $\sigma_{bst}$  in Equation (2) by  $\sigma_{0st}$ , taking logs and plugging in empirical counterparts, we get

$$\log \sigma_{bst} = \nu_{bs} + \varpi_t + \alpha_s q_{bst}^n + \zeta_{bst}, \quad (22)$$

where  $q_{bst}^n$  denotes the inverse of the loan credit rate,  $\nu_{bs}$  is a firm-fixed effect,  $\varpi_t \equiv \log(\sigma_{0st}) - \alpha^s q_{0st}^n$  is a time-fixed effect. The time fixed-effects absorb any aggregate variation (including the outside good/credit by other banks) in market shares and ensures we capture the price elasticity correctly. This equation is identical to the equation estimated in [Egan, Hortacsu, and Matvos \(2015\)](#). To identify the demand curve and circumvent simultaneity bias, we use data on the cost of federal funds at the bank level as a supply shifter (i.e. we follow a standard instrumental variables approach). Table 1(a) shows the estimation results. The estimates parameters are used to calibrate Equations (1) and (2). With the estimate of  $\alpha_s$  at hand, we set the average year and bank fixed (that we denote by  $\mu_s^\sigma$ ) to match the average loan market share of the big banks.

**Table 1:** Estimation Results: share and aggregate loan regression

<b>(a) Loan Market Share <math>\sigma_{bst}</math></b>		
	$s = re$	$s = ci$
$\alpha_s$	3.1879	8.5955
$p$ -value	0.00	0.00
obs.	693	684
Period	1986 - 2007	1986 - 2007
$R^2$	0.11	0.09
<b>(b) Aggregate Credit <math>M_s(z_t)</math></b>		
	$s = re$	$s = ci$
$\eta_{s1}$	2.6218	8.1276
$p$ -value	0.001	0.000
obs.	22	22
Period	1986 - 2007	1986 - 2007
$R^2$	0.022945	0.12934
<b>(c) Default Prob. <math>(1 - p_{bst})</math></b>		
	$s = re$	$s = ci$
$\xi_{s1}$	-0.14398	-0.1955
$p$ -value	0.000	0.000
obs.	712	712
Period	1986 - 2007	1986 - 2007
$R^2$	0.04	0.17

**Aggregate Level Estimation.** Unlike [Egan, Hortacsu, and Matvos \(2015\)](#), we do not take the mass of borrowers to be constant, but let aggregate loan demand respond to changes in the aggregate conditions. We estimate (1) by

$$\log(M(z_t)) = \eta_{s0} + \eta_{s1} \log(z_t) + \epsilon_t^z,$$

where  $\log(M(z_t))$  represents aggregate HP-filtered log-loan demand and  $\log(z_t)$  denotes log, HP-filtered log-real GDP. Since we will work with a normalization in our model (average  $z = 1$ ), the estimated constant  $\eta_{s0}$  will be calibrated match average credit by sector over GDP. Table 1(b) shows estimation results.

## 4.2 Non-Performing Loans Estimation

We estimate the elasticity of non-performing loans share,  $[1 - p_{bst}]$ , to changes in aggregate conditions by running the following panel regression:

$$(1 - p_{bst}) = \xi_{bs0} + \xi_{s1} \log z_t + \epsilon_{bt}^p, \quad (23)$$

where  $(1 - p_{bst})$  is measured as non-performing loans as a fraction of total loans of bank  $b$  and quarter  $t$  and  $\log(z_t)$  is HP-filtered log real GDP. We account for time-invariant heterogeneity between banking groups by adding bank fixed effects,  $\xi_{b0}$ . Table 1(c) shows the estimation

results. Using the estimate of  $\xi_{s1}$  and the average of  $\xi_{bs0}$ , denoted by  $\xi_{s0}$ , we derive  $p_s(z_t)$ . We calibrate  $\xi_{s0}$  to match the average non-performing loans in each sector. This elasticity is estimated using data prior to the crisis so it does not capture accurately the observed increase in the default probability. For that reason we set the value of  $p_{bst}(z_t)$  for crisis periods to those observed in 2008 and for stress periods the maximum in 2009 and 2010. This results in a value for the default probability in the real estate sector during crisis equal to 4.53% and during stress equal to 9.65%. The corresponding values for C&I loans are 2.09% and 4.31%.

### 4.3 Aggregate Shock Calibration

We relate the aggregate shock with the evolution of real GDP in the U.S. We detrend real log-GDP using the H-P filter and estimate the following equation:

$$\log(z_t) = \rho_z \log(z_{t-1}) + u_t^z,$$

with  $u_t \sim N(0, \sigma_{u^z})$ . Once parameters  $\rho_z$  and  $\sigma_{u^z}$  are estimated, we discretized the process using the [Tauchen and Hussey \(1991\)](#) method. We set the number of grid points to five, that is  $z_t \in Z = \{z_1, z_2, z_3, z_4, z_5\}$ . We choose the grid in order to capture the infrequent crisis states we observe in the data and the stress scenario we aim to capture in our main experiment. In particular, we choose  $z_4$  to match the mean of the process (i.e.  $z_4 = 1$ ), select  $z_3$  and  $z_5$  so they are at 1.5 standard deviations from  $z_5$ , set the value of  $z_2$  to be at 2.89 standard deviations from the mean to be consistent with the GDP levels observed during the 1982 crisis and the last financial crisis (years 2008/2009) and set  $z_1$  to be at 5 standard deviations from the mean to be consistent with the severe stress scenario proposed by U.S. regulators. This large negative event has a very low probability of occurrence and the probability of transitioning into  $z_1$  from  $z_4$  or  $z_5$  is zero (as determined by the Tauchen procedure).

### 4.4 Deposit Process Calibration

The idiosyncratic external funding shock process  $\delta_{bt}$  is calibrated using our panel of commercial banks in the U.S. In particular, after controlling for firm and year fixed effects as well as a time trend, we estimate the following autoregressive process for log-short term funds (deposits plus short term liabilities) for bank  $i$  in period  $t$ :

$$\log(\delta_{bt}) = (1 - \rho_d)k_0 + \rho_d \log(\delta_{bt-1}) + k_1 t + k_2 t^2 + k_{3,t} + \gamma_b + u_{bt}^\delta, \quad (24)$$

where  $t$  denotes a time trend,  $k_{3,t}$  are year fixed effects,  $\gamma_b$  are bank fixed effects, and  $u_{bt}^\delta$  is iid and distributed  $N(0, \sigma_u^2)$ . Since this is a dynamic model we use the method proposed by [Arellano and Bond \(1991\)](#). To keep the state space workable, we apply the method proposed by [Tauchen and Hussey \(1991\)](#) to obtain a finite state Markov representation  $G^f(\delta', \delta)$  to the autoregressive process in (24). We work with a normalization in the model, the mean  $k_0$  in (24) is not directly relevant. Instead, we leave the mean of the finite state Markov process, denoted  $\mu_d$ , as one of the parameters to be calibrated to match a target from the data (the most informative moment for this parameter is the average loan to deposit ratio).

## 4.5 Remaining Parameter Calibration

The parameters  $\{\lambda_s, r^a, r^d, m_s\}$  are chosen to match the average charge off rate, the return on securities (net of costs), the cost of deposits, and the average maturity of loans in each sector, respectively. We can pin down these parameters without the need to solve the model. We let the equity issuance cost function be  $\nu(\mathcal{F}, z) = (\nu_1 \mathcal{F})(\bar{z}/z)^{\nu_3}$  (a linear function increasing in  $z$ ). We assume that the discount factor is  $M_{t,t+1} = \beta$ , and that loan issuance cost is proportional to new lending, with parameter  $\phi_s$ . In summary, we are left with 15 parameters to calibrate  $\{c_s, \xi_{s0}, \mu_s^\sigma, \eta_{s0}, \beta, \kappa, \phi_s, \nu_1, \nu_3, \mu_d\}$ . These parameters are calibrated by minimizing the distance between a set of model simulated moments and their data counterpart. Table 2 presents the parameters and the targets.

**Table 2:** Model Parameters and Targets

Parameter		Value	Target
z-process	$\rho_z$	0.869	Real GDP
z-process	$\sigma_{e,z}$	0.007	Real GDP
Deposit process	$\rho_d$	0.973	Evolution Short Term Liabilities
Deposit process	$\sigma_{e,d}$	0.081	Evolution Short Term Liabilities
Return securities	$r^a$	0.022	Return on securities
Deposit interest rate	$r^d$	0.004	Cost of Funds
Non-performing loans	$\xi_{re1}$	-0.144	elasticity non-performing loans to gdp
Market Share elasticity	$\alpha_{re}$	3.188	elasticity market share to loan price
Aggregate Loan Demand	$\eta_{re1}$	2.622	Elasticity Aggregate Loan Demand to GDP
Loss given default	$\lambda_{re}$	0.157	Avg. LGD
Average maturity	$m_{re}$	0.0357	Avg. Maturity Real Estate Loans
Non-performing loans	$\xi_{ci1}$	-0.196	elasticity non-performing loans to gdp
Market Share elasticity	$\alpha_{ci}$	8.596	elasticity market share to loan price
Aggregate Loan Demand	$\eta_{ci1}$	8.128	Elasticity Aggregate Loan Demand to GDP
Loss given default	$\lambda_{ci}$	0.433	Avg. LGD
average maturity	$m_{ci}$	0.195	Avg. Maturity C&I Loans
Coupon	$c_{re}$	0.018	Avg. Interest Margin
Non-performing loans	$\xi_{re0}$	0.018	Avg. non-performing loans
Market Share constant	$\mu_{re}^\sigma$	683	Avg. Loan Market Share BHC
Aggregate Loan Demand	$\eta_{re0}$	-10.1	Loan to GDP Ratio
Cost new loans	$\phi_{re}$	0.149	Avg. net cost Sector 1
Coupon	$c_{ci}$	0.028	Avg. Interest Margin
Non-performing loans	$\xi_{ci0}$	0.021	Avg. non-performing loans
Market Share constant	$\mu_{ci}^\sigma$	100000	Avg. Loan Market Share BHC
Aggregate Loan Deman	$\eta_{ci0}$	-22.09	Loan to GDP Ratio
Cost new loans	$\phi_{ci}$	0.019	Avg. net cost Sector 2
Discount factor	$\beta$	0.99	capital ratio (risk-weighted)
Fixed cost	$\kappa$	0.0001	fixed cost to loans ratio
Equity issuance cost	$\nu_1$	0.005	Loans to Asset Ratio
Deposit Process	$\mu_d$	0.022	Equity Issuance over assets
Equity issuance cost	$\nu_3$	100	Frequency of Equity Issuance
			Loan to Deposit ratio
			Dividends over assets
			Loans $re$ / Total Loans

Note: Parameters above the line are set “off-line” (i.e., without the need to solve the model). Parameters below the line are chosen by minimizing the distance between the simulated model moments and the corresponding data moments.

Table 3 presents the data and model moments.<sup>8</sup>

<sup>8</sup>In this environment with discounted bonds, the interest margin is derived using the internal rate of return

**Table 3:** Targets and Model Moments

Moment (%)	Data	Model
Avg Interest Margin Sector $re$	4.07	1.45
Avg. Non-Performing Loans Sector $re$	1.79	1.79
Avg. Market share BHC Sector $re$	2.81	2.64
Loans $re$ to GDP Ratio	3.66	3.31
Avg net cost Sector $re$	2.63	3.19
Avg Interest Margin $ci$	4.66	0.67
Avg. Non-Performing Loans $ci$	2.04	2.04
Avg. Market share BHC $ci$	4.10	4.87
Loans $ci$ to GDP Ratio	1.82	1.63
Avg net cost $ci$	1.00	1.60
Capital Ratio (risk-weighted)	8.39	6.45
Fixed cost to loans ratio	0.88	2.92
Loans to Asset Ratio	62.00	73.56
Loans to Deposit ratio	76.86	78.78
Frequency of Equity Issuance (cond. > 0.1%)	24.55	9.99
Equity Issuance over assets (cond. > 0.1%)	1.62	2.84
Frequency of Dividends Payments (cond. > 0.1%)	62.28	76.81
Dividends over assets (cond. > 0.1%)	0.44	1.84
Loans $re$ to Loans	62.55	52.36

## 4.6 Non-targeted Moments

In this subsection, consider how the model does on non-targeted moments. In particular, we focus on how loans vary with the business cycle in Table 4. The model does well in matching the cyclicalities of C&I loans, but less so for real estate loans.

$r_s^n(L_s^s)$  on loans which makes the present discounted value of the promised sequence of future payments on a unit loan equal to the unit price. For a given unit price  $q_s^n(L_s^s)$  this can be obtained from:

$$r_s^n(L_s^s) = \frac{1}{q_s^n(L_s^s)} [E_{z'|z} \{p(z')[m + (1 - m)(c + q_o^L(z'))]\}] - 1,$$

Similarly, we can obtain

$$r_s^o = \frac{1}{q_s^o} [E_{z'|z} \{p(z')[m + (1 - m)(c + q_o^L(z'))]\}] - 1.$$

Then, the interest margin is computed as the weighted average of the loan returns net of the interest rate on deposits.

**Table 4:** Business Cycle Correlations

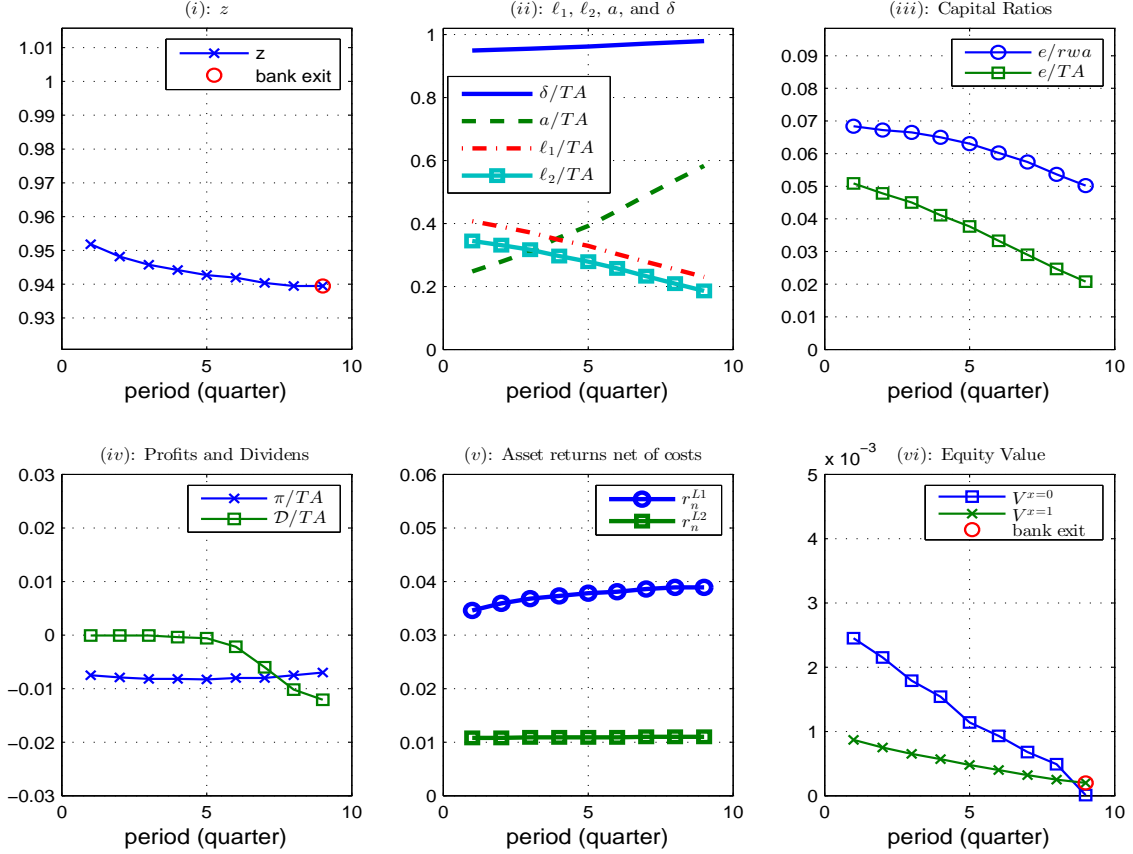
Correlation with GDP	Data	Model
$l_{re,t}$	0.420	0.124
$\Delta l_{re,t}$	0.102	0.723
$l_{ci,t}$	0.448	0.523
$\Delta l_{ci,t}$	0.519	0.592

Note: Data moments are computed using HP-filtered real variables at the bank level. We report the asset-weighted average of the individual correlations.

## 5 Exit Event Analysis

The paper aims to provide a tool to analyze stress scenarios. But before we present our main experiment, it is instructive to analyze what a typical bank failure looks like in our model. For that reason, we study the model’s dynamics using a simulated panel of banks, large enough to capture the long-run properties of the model. This sample contains failures in 0.1 percent of the simulated banks. Thus, the model produces endogenous exit with a low failure probability in equilibrium (consistent with the low failure probability of large banks in the U.S.).

**Figure 2: Exit Event Analysis**



Notes: Exit event in last period.

Figure 2 shows an event analysis based on the simulated panel. The plots show 2-year event windows (8 quarters) where the last period corresponds to the quarter of bank failure.<sup>9</sup> Each panel shows the average of the corresponding variable across banks that fail at the end of the sample. Panel (i) shows the evolution of the aggregate state ( $z$ ). Panel (ii) shows  $\ell_s$ ,  $a$ , and  $\delta$  as a fraction of total assets. Panel (iii) shows the evolution of the capital ratio as a fraction of risk-weighted assets ( $e/rwa$ ) and as a fraction of total assets ( $e/TA$ ). Panel (iv) shows the evolution of dividends and profits as a fraction of total assets ( $\mathcal{D}/TA$  and  $\pi/TA$ , respectively). Panel (v) compares new loan returns net of issuing cost, across the different types of loans.<sup>10</sup> Finally, Panel (vi) presents the equity value of the bank when it continues  $v^{x=0}(\ell, a, \delta, z)$  and when it chooses to exit  $v^{x=1}(\ell, a, \delta, z)$ .

Panel (i) of Figure 2 shows that bank failure following a sequence of low aggregate shocks (already below average  $z$ ). What is not evident in panel (ii) is that the bad aggregate shocks are actually combined with a reduction in the flow of deposits (below average  $\delta$ ) since panel (ii) shows deposits as a fraction of total assets (which are falling faster than deposits in an exit event). The bad state of the economy induces banks to cut loans (the stock of loans

<sup>9</sup>Our sample is constructed using the last 8 quarters of all banks that fail in our simulated panel.

<sup>10</sup>the return is computed as  $\frac{1}{q_s^n(L_s^n)} [E_{z'|z} \{p(z')[m + (1-m)(c + q_o^L(z')) + (1-p(z'))(1-\lambda_s)\}] - \phi_s - 1$



decreases by 45.0 percent) and shift to safe securities (the ratio of securities to total assets increases from 22.2 percent to 58.3 percent during the event window) in order to prevent losses. Panel (*iv*) shows that profits over assets  $\pi/TA$ , while negative from start to end, do not decline significantly due the change in the composition of assets as well as the slight increase in the return on new loans (Panel (*v*)) shows that the return on a new loan is higher than the expected return on securities. However, loans become sufficiently risky so that the bank chooses to build a buffer stock against future losses. To further protect it’s charter value, the bank also stops dividend payouts, and towards the end issue new equity. Absent profitable investment in loans, the bank also chooses to maintain a positive distribution of dividends as the failure period approaches. Even though security holdings increase, negative operating profits result in a notable decline in capital ratios. Panel (*iii*) shows that the risk-weighted capital ratio  $e/rwa$  declines by 26 percent and the leverage ratio (i.e., equity to total assets  $e/TA$ ) declines by more than 61 percent to end at 2.1 percent. The decline in capital ratios is accompanied by a decrease in the continuation value of the bank  $V^{x=0}$  that slowly approaches the exit value  $V^{x=1}$  until it crosses it and the bank exits. Total assets decline 8.33 percent from the start of the exit event to the period when the bank chooses to exit.

## 6 Structural vs. Reduced Form Stress Tests

In this section, we perform a stress test using our quantitative model and contrast the results with a reduced form model that is similar to the CLASS model presented in [Hirtle, Kovner, Vickery, and Bhanot \(2014\)](#).

To set the stress scenario, we follow the guidelines presented in the Supervisory Stress Test Methodology and Results by the Federal Reserve Board in June 2016. We focus on the “Severely Adverse” scenario.<sup>11</sup> According to the guidelines, in this stress scenario, the level of U.S. real GDP begins to decline in the first quarter of 2016 (the start of the stress window) and reaches a trough that is 6.25 percent below the pre-recession peak (similar to  $z = z_1$ ). The crisis continues for about two years until output slowly goes back to trend. We feed our panel of simulated banks the path for  $z$  presented in Panel (*i*) of Figure 3.<sup>12,13</sup> We let the value of  $\delta$  evolve according to its stochastic process. We present the results from the average behavior (i.e., in every stress period, we take the average across banks of each variable). To the extent there is failure along the the stress scenario, the averages displayed are influenced by selection bias.

We compare the evolution of variables as predicted by the structural model with those derived from the reduced form model. In short, the reduced form model consist of three key components. (*i*) First, the model estimates a set of linear equations that determine the evolution of the ratio of interest income and expenses and net operating cost to total assets,

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<sup>11</sup>All documentation can be found in <https://www.federalreserve.gov/bankinfo/stress-tests/2016-Preface.htm>

<sup>12</sup>More specifically, we simulate 400 banks over the stress horizon of 16 quarters for this experiment. The initial state of each bank (i.e. loans, securities, and deposits) is drawn from an average distribution of banks along our simulation, conditional on that the value of  $z$  corresponds to the initial value of  $z$  shown in Panel (*i*) of Figure 3 ( $z = z_4$ ).

<sup>13</sup>Note that fixing a path for  $z$  does not imply that the bank has perfect foresight about this path.

and the net charge-off rate as a function of macroeconomic conditions ( $z$ ). (ii) With the estimates of these equations at hand, the reduced form model imposes a set of assumptions on the evolution of assets to pin down profits. (iii) Finally, to predict the evolution of equity, the model imposes a rule for dividend payments. Note that the reduced form model is silent about the charter value of the bank, the main determinant of bank failure. We do not impose any failure in the baseline reduced form model.

## 6.1 The reduced Form Model

Recall that in the structural model profits realized at the beginning of the first subperiod (Equation 5) are equal to

$$\pi = \sum_s \{ [p_s(z)(1 - m_s)c_s - (1 - p_s(z))\lambda_s] \ell_s \} - r^d \delta + r^A a - \kappa. \quad (25)$$

Total profits in the period, after accounting for loan adjustments in the second subperiod, are

$$\pi^* = \pi + \sum_s \left\{ \underbrace{[1 - I_{[L_s^s \geq 0]} q_s^n - I_{[L_s^s < 0]} q_s^o] L_s^s}_{x_s} - \phi(L_s^s) \right\}, \quad (26)$$

where  $x_s$  captures capital gains associated with creating or liquidating loans. We can now define the key income and cost ratios to be estimated as in the CLASS model approach, where we map each component of  $\pi^*$  into either net-interest margin, net operating cost, or net charge off rates:

$$\text{nim} = \frac{\sum_s \{ p_s(z)(1 - m_s)c_s \ell_s + x_s \} - r^d \delta + r^A a}{\sum_s \ell_s + a} \quad (27)$$

$$\text{cost} = \frac{\sum_s \{ \phi(L_s^s) \} + \kappa}{\sum_s \ell_s + a} \quad (28)$$

$$\text{nco} = \frac{\sum_s \{ (1 - p_s(z))\lambda_s \ell_s \}}{\sum_s \ell_s} \quad (29)$$

The reduced form model assumes that these ratios follow an AR(1) process and estimate their evolution controlling for bank-fixed effects and aggregate conditions using the following specification

$$y_t = \beta_0^y + \beta_1^y y_{t-1} + \beta_2^y z_t + \epsilon_t, \quad y_t \in \{nim_t, nco_t, cost_t\}. \quad (30)$$

We use our simulated panel of banks and estimate equation (30) for  $y_t \in \{nim_t, nco_t, cost_t\}$ . The simulated data are generated from a simulation of the model that uses the historical path of  $z_t$ . Importantly, this path does not include stress events. We then use the estimated coefficients  $\{\hat{\beta}_i^y\}_{i=0}^2$  to generate the reduced form model stress projections  $\hat{y}_t = \hat{\beta}_0^y + \hat{\beta}_1^y y_{t-1} + \hat{\beta}_2^y z_t$  for  $y_t \in \{nim_t, nco_t, cost_t\}$  as in [Hirtle, Kovner, Vickery, and Bhanot \(2014\)](#). Table 5 presents the estimated coefficients.

**Table 5:** Reduced Form Model AR1 Estimation

	Dep. Variable $y_{it}$		
	$nim$	$cost$	$nco$
constant	-0.0039	-0.0042	0.0562
$y_{it-1}$	0.9899	0.9705	-0.0753
$z_t$	0.0004	0.0004	-0.0050

Given an allocation of loans and securities  $(\ell_{st}, a_t)$  we can use these estimates to derive profits, that is

$$\pi_t^{RF} = (\hat{n}im_t - \hat{c}ost_t) \left( \sum_s \ell_{st} + a_t \right) - \hat{n}co_t \left( \sum_s \ell_{st} \right) \quad (31)$$

Given the level of profits, the reduced form model follows the CLASS model and assumes that dividends follow:

$$\mathcal{D}_t^{RF} = \max\{0, 0.9\mathcal{D}_{t-1}^{RF} + (1 - 0.9)(\mathcal{D}_t^*)\}. \quad (32)$$

where  $\mathcal{D}_t^* = 0.45\pi_t^{RF}$  is the target level of dividends. Note that dividends are restricted to be non-negative. With the computed level of profits and dividends, the next period capital is completely pinned down from the law-of-motion for equity<sup>14</sup>

$$e_{t+1}^{RF} = e_t^{RF} + \pi_t^{RF} - \mathcal{D}_t^{RF}. \quad (33)$$

Equation (31) shows that this capital prediction depends on both the level and composition of assets. Thus, a key issue in the typical reduced form model is how to determine the balance sheet size and composition. Regarding the composition, we follow [Hirtle, Kovner, Vickery, and Bhanot \(2014\)](#), and we impose that in the reduced form model each bank's composition of assets stays fixed at its pre-stress values. This is not consistent with the data (and our structural model) that shows that balance sheet composition varies significantly with economic conditions. Formally, the ratio of loans and securities to total assets is given by:

$$\hat{\ell}_{st}^{RF} = \hat{\ell}_{s1}^{RF}, \forall s \quad (34)$$

$$\hat{a}_t^{RF} = 1 - \sum_s \hat{\ell}_{st}^{RF} \quad (35)$$

To close the reduced form model, one must take a stance on the evolution of either total assets or liabilities (only one is needed, as the other follows from the balance sheet identity). In the CLASS model, the assumption is that total assets grow at a constant rate during stress. In order to be consistent with our structural model, we instead assume that liabilities ( $\delta_t^{RF}$ ) is determined by its stochastic process (as in the structural model). Let  $TA_t^{RF}$  denote

<sup>14</sup>note that this measure of equity corresponds to the book value of equity:  $\sum_s \ell_{st} + a_t - \delta_s$

total assets. We then have that

$$TA_t^{RF} = e_t^{RF} + \delta_t^{RF} \quad (36)$$

which implies that the level of loans and securities is given by

$$\ell_{st}^{RF} = TA_t^{RF} \hat{\ell}_{st}^{RF}, \forall s \quad (37)$$

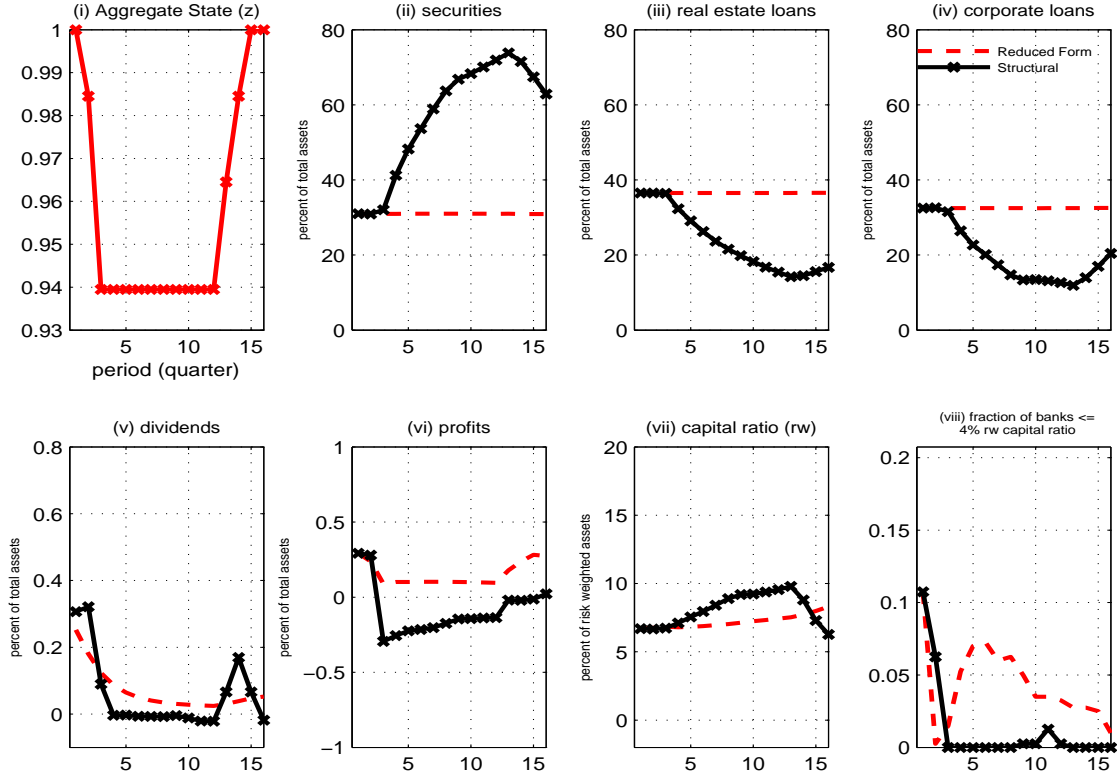
$$a_t^{RF} = TA_t^{RF} \left( 1 - \sum_s \hat{\ell}_{st}^{RF} \right) \quad (38)$$

As in the structural model, we simulate 400 banks over the fixed stress path. In order to generate reduced form projections, we need to (i) seed the model with initial values for income and cost ratios, total assets, and the asset composition and (ii) draw the paths for  $\delta_t^{RF}$ . We derive the seed for each of the reduced form banks from the initial conditions for the 400 structural banks, and set the path of  $\delta_t^{RF}$  identical to the shock realization in the structural model.

## 6.2 Stress Tests

We apply the stress test scenario to both models (i.e., structural and the reduced form) and obtain the results presented in Figure 3. Panel (i) presents the stress scenario (i.e., the evolution of  $z$  that we impose to represent the stress scenario). All other panels present the evolution of key variables. More specifically, Panel (ii) to (iv) (top row) present the evolution of securities and the loan portfolio, while panels (v) to (viii) (bottom row) present the ratio of dividends to assets, ratio of profits to assets, equity to risk weighted assets, and the fraction of banks below the minimum, respectively. In each panel, “Structural” refers to the average across active banks in the structural model, “reduced form” the average across active banks in the reduced form model.

**Figure 3:** Stress Test: Structural Model Vs. Linear Reduced Form



Notes:

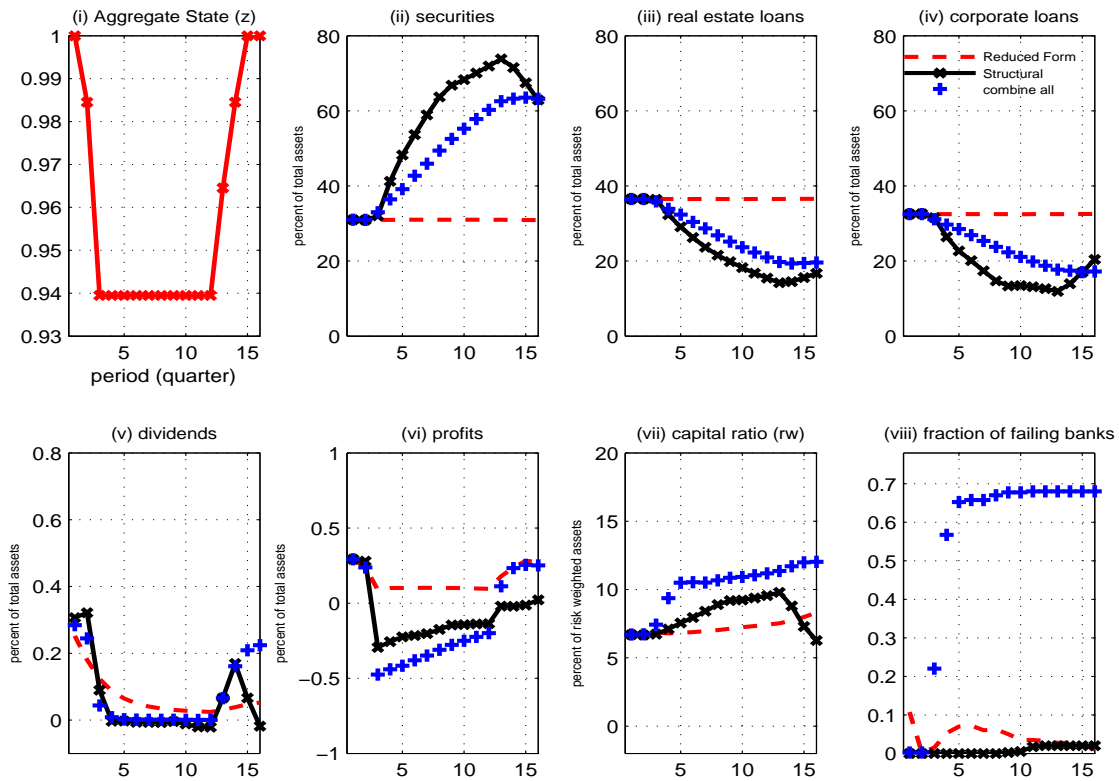
We observe that on impact, profits decline in both models (Panel *(vi)*). However, the decline is much deeper and more persistent in the structural model than in the reduced form model. The reason is that while banks in the structural model adjust their portfolio composition (Panels *(ii)* through *(iv)*), banks operating according to the reduced form model keep the same asset composition and underestimate the actual losses the bank will face. In addition, the reduced form model does not capture the non-linear dynamics that this change in the portfolio composition introduces. It is evident from Figure 3 Panels *(iii)* and *(iv)* that banks in the structural model lower C&I loans and real estate loans at a fast pace. Risk-weighted capital ratios (Panel *(vii)*) increase for most of the stress scenario in the structural model and in the reduced form model for different reasons. In the case of the structural model, the increase is driven mostly by the decline in risk-weighted assets (i.e., loans) and the increase in safe assets. This increase in safe assets not only increases capital ratios by construction but also mitigates the reduction in profitability. In the case of the reduced form model, the underestimation of bank losses during stress keep profits on the positive side and since banks distribute only a portion and there is no change in risk-weighted assets the risk-weighted capital ratio also increases but a slower pace than in the structural model.

With the baseline reduced form model at hand, we now perform a set of experiments in order to disentangle its differences with structural model. More specifically, we relax one by one the following assumptions: (1) linear equations for interest margins, charge-off rates and marginal costs, (2) Dividend Rule, (3) Constant asset composition, (4) No exit/failure

rule. We relax each assumption as follows: (1) using the data from the structural model, we estimate AR(1) equations with parameters that depend on the state of the economy ( $z$ ).<sup>15</sup> (2) We estimate a dividend rule that allows for negative dividends and does not impose that a particular fraction of profits go into dividends.<sup>16</sup> (3) We estimate asset shares assuming the same structure as in Equation (30). (4) We impose an exit rule based on the minimum capital requirement (i.e., banks that go below the minimum are forced to exit).

Figure 4 shows the results when all assumptions are jointly relaxed. Note that when all assumptions are jointly relaxed, the AR(1) for dividends and asset share are also estimated non-linearly. The appendix presents the figures with the individual decomposition.

**Figure 4:** Stress Test: Reduced Form - all extensions



Notes: *Reduced Form* refers to the baseline reduced form specification. *combine all* refers to the reduced form model with all baseline assumptions jointly relaxed.

While each assumption plays a role in bringing the reduced form model closer to the structural model, the main driver of the reduction in the observed differences between models is the introduction of changes in the evolution of asset shares along the stress scenario. When this assumption is relaxed, banks operating under the reduced form model reduce their holdings of loans following closely what the banks operating under the structural model do. This reduction in the behavior of the asset side of the bank allows the reduced form model to

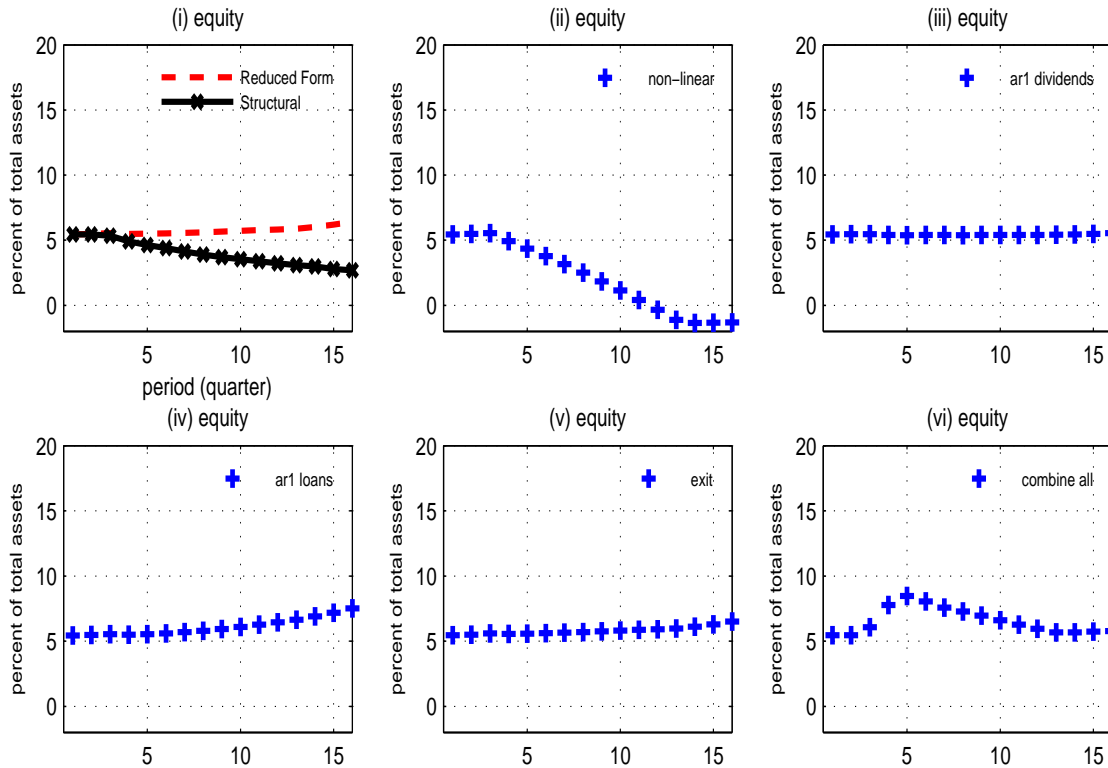
<sup>15</sup>We estimate an AR(1) using our simulated panel of banks from the structural model, separately on normal times observations:  $z_t \in (z_5, z_4, z_3)$ , crisis observations:  $z_t = z_2$  and stress observations  $z_t = z_1$

<sup>16</sup>We estimate a rule in which dividends follow an AR(1) as in Equation (30) with  $z_t$  replaced by  $\pi_t^*$ .

capture much better the evolution of profits (that are now negative even in the reduced form model), dividends and risk-weighted capital ratios. One feature of the extended reduced form model that does not capture correctly bank behavior is the exit rule based on the minimum capital ratio. The reduced form model that incorporates an exit rule overpredicts bank exit since it does not capture accurately the dynamics of the charter value of the bank.

Of particular importance is the evolution of capital ratios across models. Figure 5 shows how equity over assets evolve for the structural model, the baseline reduced form model, each of its extensions and all extensions combined.

**Figure 5:** Stress Test: Equity over total assets



Notes: Panel (i) displays the evolution of equity over total assets, in the structural and the baseline reduced form model. Panels (ii) - (v) display the reduced form model when relaxing assumption (1)-(4) one at a time. Panel (vi) refers to the reduced form model when all assumptions are jointly relaxed

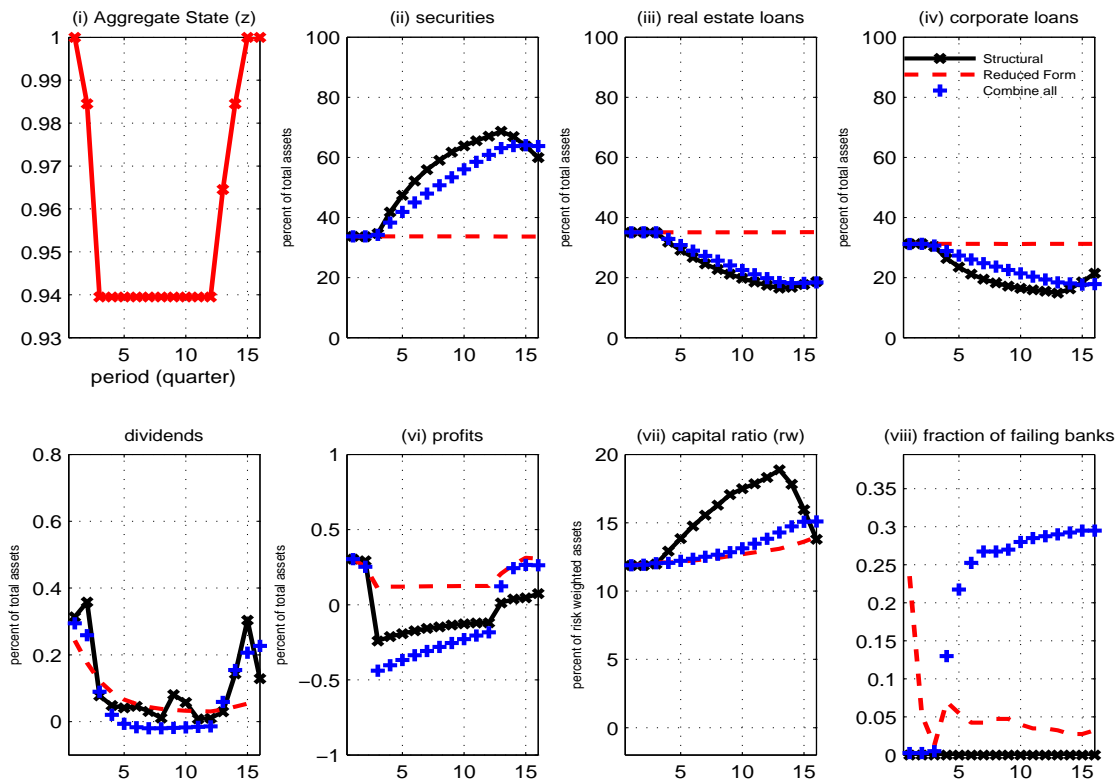
As we discussed in Figure 3, the reduced form model underpredicts the drop in profits and this has a direct effect on the evolution of the equity to asset ratio. While the structural form predicts a sharp decline (from above 5% to approximately 3%), the reduced form model predicts a slight increase. Incorporating non-linear functions for income ratios and the cost function (Panel (ii)) generates a decline in equity to assets. However, once the reduced form model incorporate all extensions, this decline vanishes. First, while allowing for adjustments in the asset composition captures the evolution much better than the baseline reduced form, it increases the difference in profits between the reduced form model and the structural model. Second, a selection effect that arises from the introduction of the exit rule generates

an increase in the observed capital ratio for incumbent banks operating under the reduced form model.

### 6.3 Higher Capital Requirements

In this section, we study the effects of increasing capital ratios to 10.5% (as moving from Basel II to Basel III, taking into account that banks in the stress test exercise are systemically important banks so they are required an additional 2% buffer over the minimum of 8.5%). Figure 6 presents the results.

**Figure 6:** Stress Test: Higher capital requirement



*Notes:*

As before, the baseline reduced form model underpredicts the drop in profitability generated by the structural model. The increase in the minimum capital ratio reduced the fraction of banks that fail in both cases, the structural model and the reduced form model. Again, the exit rule based on a particular level of capital does not capture accurately banks' exit decision during stress scenario. However, since banks in this regime are required to hold a larger level of capital, the selection effect under the reduced form model with all its extensions is reduced and this brings the two models closer to each other.



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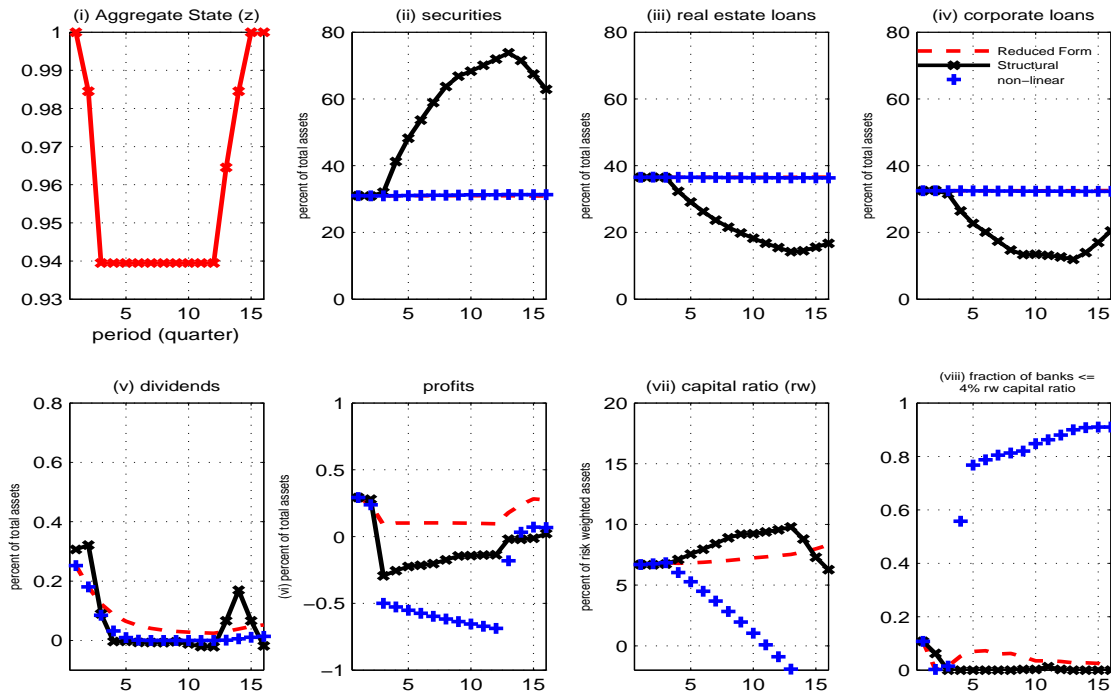
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# Appendix

## A-1 Reduced Form Model Extensions

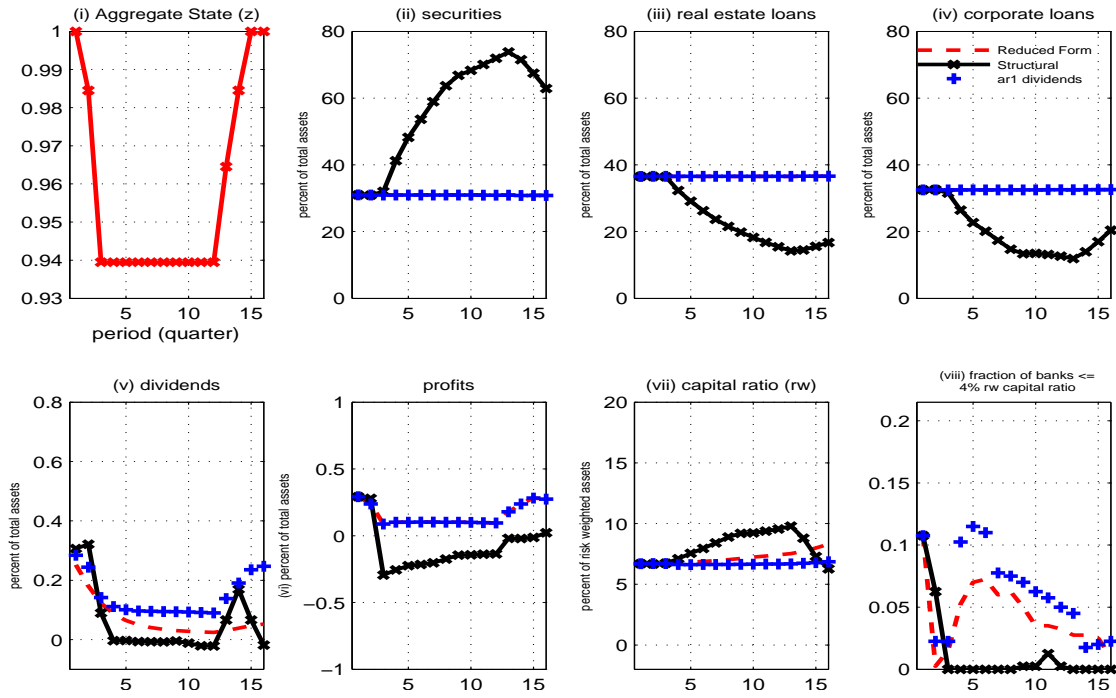
This appendix presents the results from each of the extensions to the reduced form model that we study as part of the analysis of the decomposition of the difference between the structural model and the reduced form model.

**Figure 7:** Stress Test: Non-linear Reduced Form



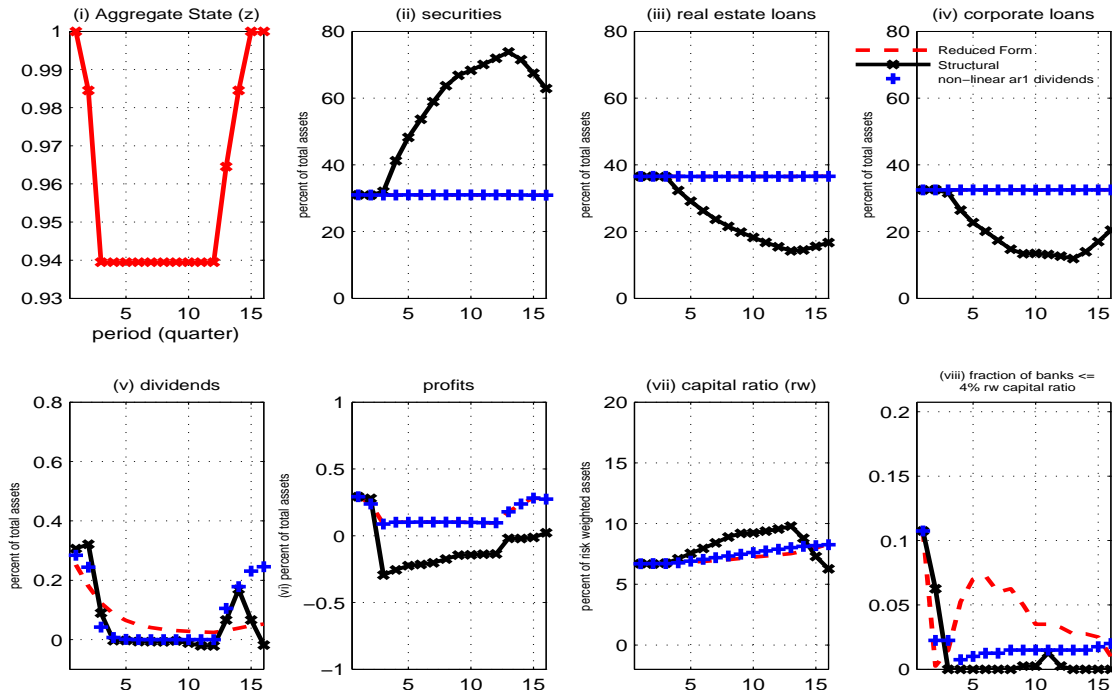
Notes: *Reduced Form* refers to the baseline reduced form specification. *non-linear* refers to the reduced form model in which the AR(1) coefficients for income and cost ratios depend on the aggregate state.

**Figure 8:** Stress Test: Reduced Form - dividend decomposition



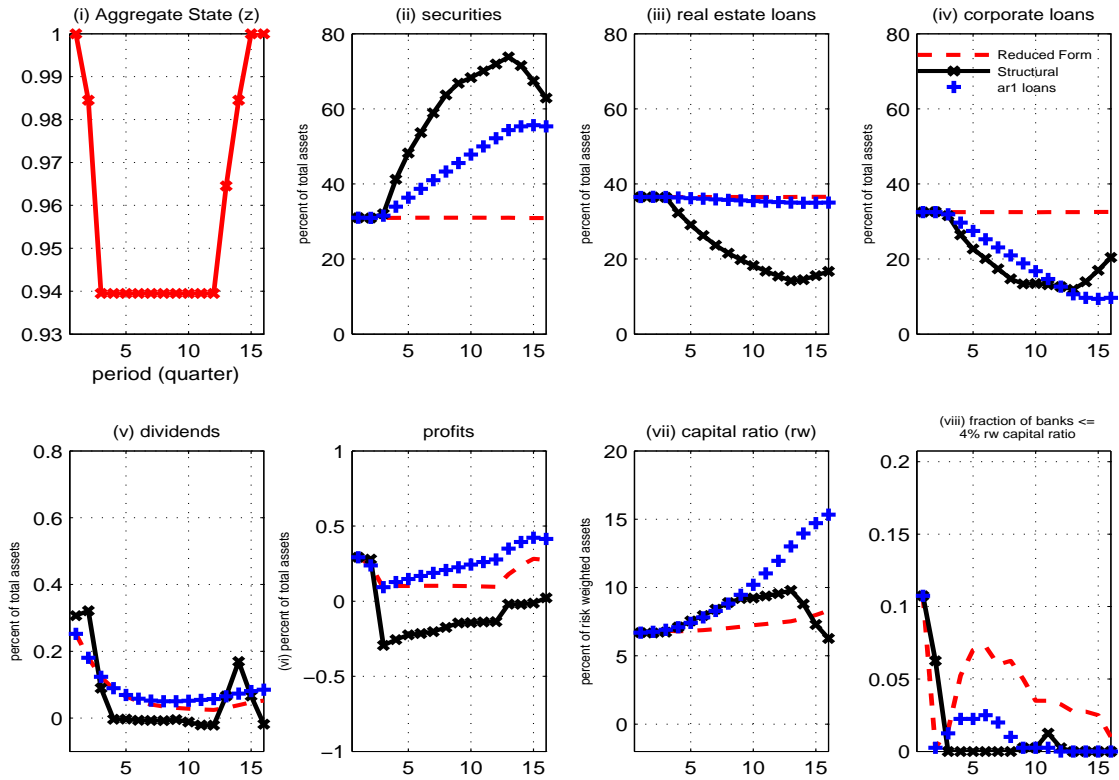
Notes: *Reduced Form* refers to the baseline reduced form specification. *ar1 dividends* refers to the reduced form model with an AR(1) dividend rule.

**Figure 9:** Stress Test: Reduced Form - non-linear dividend decomposition



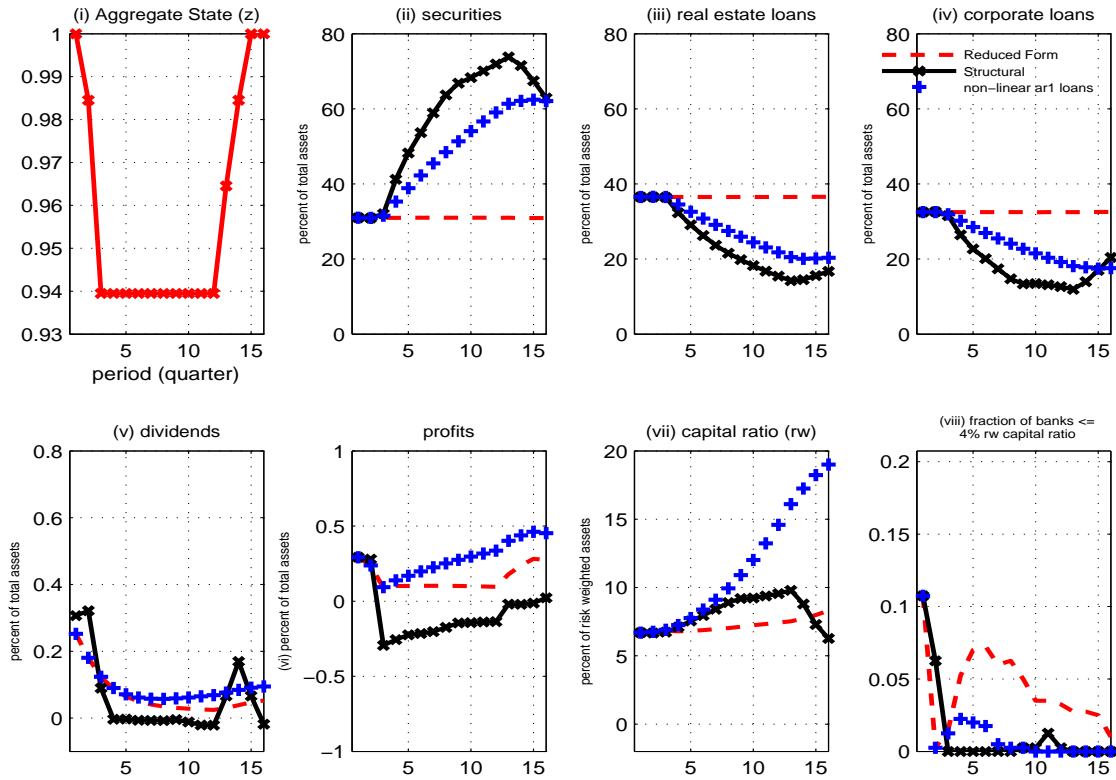
Notes: *Reduced Form* refers to the baseline reduced form specification. *non-linear ar1 dividends* refers to the reduced form model with an AR(1) dividend rule, where AR(1) coefficients that depend on the aggregate state.

Figure 10: Stress Test: Reduced Form - asset composition



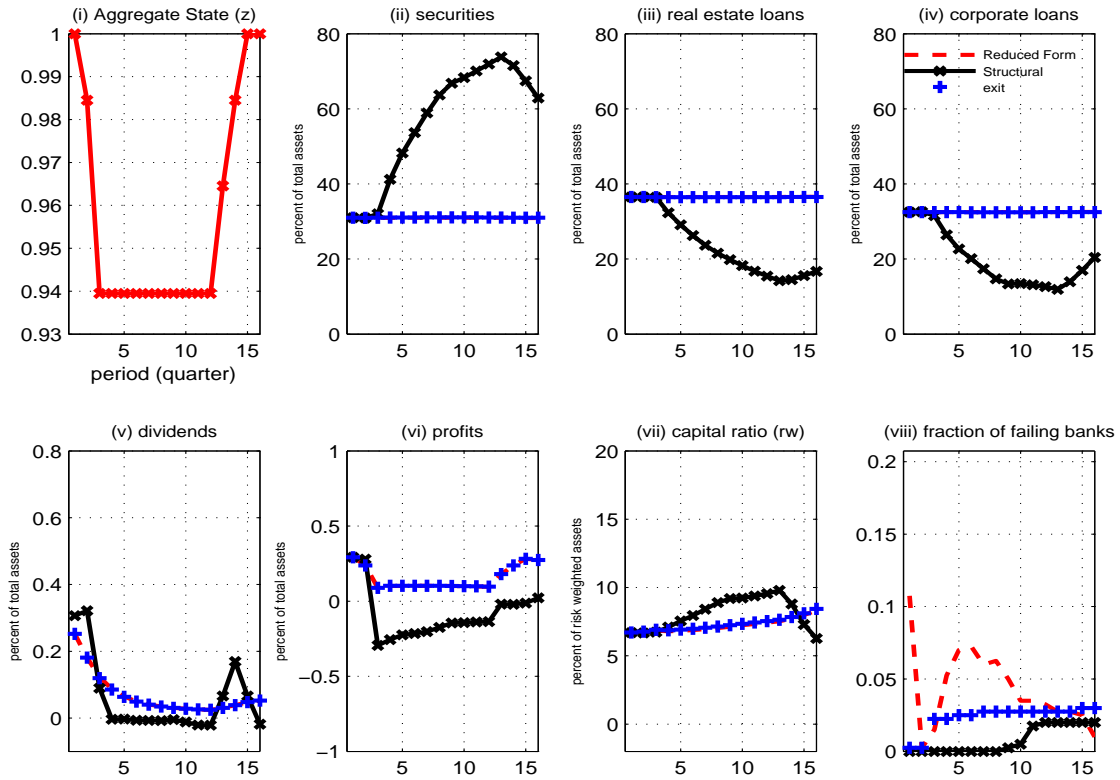
Notes: *Reduced Form* refers to the baseline reduced form specification. *ar1 loans* refers to the reduced form model with AR(1) projections for loans as a share of total assets.

**Figure 11:** Stress Test: Reduced Form - non-linear asset composition



Notes: *Reduced Form* refers to the baseline reduced form specification. *non-linear ar1 loans* refers to the reduced form model with AR(1) projections for loans as a share of total assets, where the AR(1) coefficients depend on the aggregate state.

Figure 12: Stress Test: Reduced Form - exit



*Reduced Form* refers to the baseline reduced form specification. *exit* refers to the reduced form model where we force banks that go below the minimum capital requirement to exit.