The Allocation of Talent to Financial Trading versus Entrepreneurship: Welfare Effects of Trading in General Equilibrium*

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Abstract

This paper investigates the implications of the Grossman-Stiglitz (1980) model on the informational efficiency of financial markets for the optimality of the allocation of talent to financial trading versus production. Informed traders make the financial market more informationally efficient, entrepreneurs create output and jobs. The model indicates that financial trading attracts too much, rather than too little, talent.

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1 Introduction

Is the allocation of talent to financial trading excessive or deficient? To investigate this question we incorporate occupational choice between financial trading and entrepreneurship and a labor market into the seminal Grossman-Stiglitz (1980) model on the informational efficiency of financial markets. Informed traders make the financial market more informationally efficient, entrepreneurs create output and jobs. In the model financial trading attracts too much, rather than too little, talent. For one thing, measures which raise entrepreneurial activity at the margin starting at the free markets equilibrium increase social welfare, in particular if there are labor market frictions and equilibrium unemployment. Second, social welfare is higher when agents are precluded from becoming informed traders altogether under fairly intuitive conditions. This is because informational efficiency is not generally conducive to social welfare: the fact that the asset price is more closely tied to stochastic fundamentals governing firms' profitability leads to a clustering of risks at entrepreneurs, thereby reducing the incentives to engage in entrepreneurship. Thus, in a context where the sole benefit of trading is to increase informational efficiency à la Grossman and Stiglitz (1980) the allocation of talent to trading tends to be excessive rather than deficient.[1] Put differently, other arguments than informational efficiency (mentioned in the literature review below) have to be invoked in order to argue for implicit or explicit support for financial trading in competition for the best and brightest.

Finance has attracted an increasing amount of talent over the past decades. Goldin and Katz (2008) observe that the proportion of male Harvard graduates from selected classes who work in the finance sector 15 years after graduation rose from 5 percent for early-1970s cohorts to 15 percent for early-1990s cohorts. According to the *Harvard Magazine*, the figure peaked at more than 20 percent in 2007, before labor demand collapsed with the onset of the subprime crisis.[2] Phillippon and Reshef (2012) report increasing average education of workers in the financial industry compared to the real sector since the 1980s. Despite rising relative employment, relative pay rose in "other finance" (i.e., mainly asset management and trading; cf. Greenwood and Scharfstein, 2013) compared to traditional banking and insurance. Insofar as wages reflect capabilities (and not compensating differ-

[1] Contrary to Bolton et al.'s (2016, p. 711) conjecture that "the standard framework of trading in financial markets first developed by Grossman and Stiglitz (1980) ... seems to suggest that the financial sector could be too small."

[2] Elizabeth Gudrais, "Flocking to Finance", *Harvard Magazine*, May-June 2008, `http://harvardmagazine.com/2008/05/flocking-to-finance.html`.

entials), this reflects an increasing flow of talent to trading related activities. (Contrary evidence is presented by Böhm et al., 2018, for Sweden and by Lindley and Mcintosh, 2017, for the U.K., who show that test scores and performance measures for finance workers have not increased since the 1990s.) Competition for talent does not stop when students have decided to specialize in science or engineering. Shortly before the financial crisis, serial entrepreneur and writer Vivek Wadhwa observed in his testimony to the the U.S. House of Representatives that "[T]hirty to forty percent of Duke Masters of Engineering Management students were accepting jobs outside of the engineering profession. They chose to become investment bankers or management consultants rather than engineers".[3] Similarly, *The Economist* reports that "[M]ost of the world's top hedge funds prefer seasoned traders, engineers and mathematicians, people with insight and programming skills, to MBAs".[4] Célérier and Vallée (2017) remark in their empirical study of French graduate engineers that a sizable portion of the post-2000 graduates worked in the City of London or on Wall street. Oyer (2008, p. 2622) finds "mixed evidence that initial jobs on Wall Street lead Stanford MBAs to start fewer businesses". He adds that there is path dependence in occupational choice: workers drawn into the financial sector by random events tend to stay there. As Cecchetti and Kharroubi (2012, p. 1) put it: "Finance literally bids rocket scientists away from the satellite industry."

While fierce competition for talent between finance and the real sector is undisputed, opinions diverge on whether this is a good thing. Esther Duflo replied to concerns that regulations would constrain the financial sector in the aftermath of the recent financial crisis: "Is there a risk of discouraging the most talented to work hard and innovate in finance? Probably. But it would almost certainly be a good thing."[5] At *The Economist's* 2013 Buttonwood Gathering, Robert Shiller ("When you study finance you are studying how to make things happen") and Wadhwa ("Google – not Goldman Sachs – deserves our best minds")[6] exchanged opinions. Long before the recent financial crisis Tobin (1984) bewailed that "we are throwing more and more of our resources, including the cream of our youth, into financial activities remote from the production of goods and services, into activities that generate high private rewards disproportionate to their social productivity." Similarly, Baumol (1990, p. 915) holds that "arbitrageurs" at least occasionally

[3]Quoted from Philippon (2010, p. 159).

[4]Philip Delves Broughton, "Think twice", *The Economist*, January 2011, `http://www.economist.com/whichmba/think-twice`.

[5]*Vox*, October 8, 2008, `http://www.voxeu.org/article/too-many-bankers`.

[6]Washington Post, November 1, 2013, `http://www.washingtonpost.com/blogs/innovations/wp/2013/11/01/google-not-goldman-sachs-deserves-our-best-minds/`.

engage in what he calls "unproductive entrepreneurship" as opposed to "productive entrepreneurship", which is essential for long-term development. Murphy et al. (1991, p. 506) state in their classic paper on the allocation of talent: "Trading probably raises efficiency since it brings security prices closer to their fundamental values ... But the main gains from trading come from the transfer of wealth to the smart traders ... Even though efficiency improves, transfers are the main source of returns in trading."[7] Empirical studies of the impact of finance on real economic activity usually focus on economic growth as the dependent variable. Early studies, such as King and Levine (1993), found a positive impact of the financial sector on economic growth. Subsequent research points to an inverse U-shaped relation between finance and growth. For instance, in Cecchetti and Kharroubi (2012) the marginal effect of finance on growth turns negative when private credit exceeds 100 percent of GDP or financial sector employment exceeds 3.9 percent of total employment (see also Rousseau and Wachtel, 2011, Gruendler and Weitzel, 2013, Law and Singh, 2014, Arcand et al., 2015, Cornède et al., 2015, and Ductor and Grechyna, 2015). Consistent with the view that trading is more likely to be unproductive than intermediation, Beck and Degryse (2014) find that in a broad cross section of countries the size of the financial sector (measured by its value added share in GDP) is insignificant if intermediation is controlled for. Kneer (2013) and Boustanifar et al. (2017) relate this to brain drain from skill-intensive manufacturing industries and from foreign countries, respectively.

This paper investigates the welfare effects of financial trading by incorporating occupational choice (OC) and a labor market with or without frictions into the seminal Grossman-Stiglitz (1980, henceforth: "GS") noisy rational expectations equilibrium (REE) model. We conduct a second-best welfare analysis which takes agents' investment decisions as given. Our measure of social welfare (SW) is the sum of *all* agents' expected utilities, suitably transformed so that pure redistribution does not affect SW, assuming that the noise traders present in the GS model have the same utility function as the other agents. Equivalently, SW is the sum of all agents' certainty equivalents. For the case of small noise trader shocks we show analytically that at equilibrium a marginal increase in the amount of resources devoted to entrepreneurship has a positive or zero first-order effect on SW, depending on whether there is equilibrium unemployment or not. Numerical

[7] Bai et al. (2016) confirm that the impact of current asset prices on future earnings and the portion of the standard deviation of the latter explained by cross-sectional variation in asset prices have risen in parallel to the expansion of the financial sector over the period 1960–2014 for nonfinancial S&P 500 firms.

analysis shows that for reasonably large noise trader shocks the marginal impact of entrepreneurship on SW remains positive in the vast majority of model specifications with unemployment and turns positive more often than not in the absence of labor market imperfections. That is, while the equilibrium allocation is constrained optimal without noise and labor market frictions, *the marginal benefits of job creation outweigh the marginal cost in terms of lower informational efficiency of financial markets in the presence of unemployment.* We also compare equilibrium with OC to the equilibrium that occurs when agents are precluded from becoming dealers, so that there is no informed trading at all. We analytically derive simple sets of weak sufficient conditions under which SW is higher in the absence of informed trading for small values of the variance of noise traders' asset demand. For instance, one set of sufficient conditions is that, first, neither noise traders nor rational agents short the asset on average and, second, individuals are small relative to corporations, in that noise traders' per capita demand for assets is less than the asset supply generated by a single entrepreneur. The reason why *agents are better-off with no informed trading at all* is that informational efficiency makes the asset price more responsive to stochastic fundamentals, which leads to an inefficient clustering of risks at entrepreneurs. Numerical analysis shows that equilibrium SW remains higher without OC for the vast majority of model parameterizations with reasonably large noise trader shocks. Our overall conclusion is that it is hard to argue that professional trading is socially beneficial on net if the only benefit it brings about is increased informational efficiency in the asset market as in GS.

Our model contributes to a growing literature which studies the efficiency of the allocation of resources to financial trading versus other activities. The GS model is regularly cited in the contributions but has not yet been used as the setting of the analysis. In Bolton et al. (2016) a class of agents have the option to become "dealers" and thus acquire the ability to assess the quality of newly issued stocks and buy the most profitable ones over-the-counter from originators. In the simplest version of their model (Bolton et al., 2016, Section II) such "cream skimming" is pure rent seeking, so if agents specialize in dealing, this is socially wasteful. In an earlier version of the paper, Bolton et al. (2012) derive similar results in a variant of the model with OC between becoming a dealer or an originator. In Glode and Lowery (2016) financial firms hire "experts" either as "bankers", who identify investment opportunities, or as "traders", who identify valuable investments in other firms (hit by a negative liquidity shock). As in Bolton et al.'s (2016) simplest model, trading has no social benefit, so any employment it attracts is excessive (see also

Glode et al., 2012).[8] In Biais et al. (2015) firms invest in fast trading capabilities. Since fast traders buy more when asset payoffs are high, and vice versa, a positive bid-ask spread is needed for market makers to break even. The bid-ask spread causes a welfare loss, because it prevents beneficial trades. There is over-investment in fast trading capabilities, as firms do not internalize their influence on the bid-ask spread.

Other models concerned with the allocation of resources to financial trading comprise a socially beneficial role for trading. For instance, Bolton et al. (2016, Section III) consider a version of their model with moral hazard in firms, in which cream skimming by dealers provides incentives for originators to supply high-quality assets.[9] Another branch of the literature (surveyed by Bond et al., 2012) emphasizes two further positive effects of trading in secondary financial markets on resource allocation via firm decisions. First, financial markets reveal information to producers they would otherwise not have (the "learning channel"). Second, asset prices can help improve efficiency when used as a determinant of managerial compensation (the "incentives channel"). Like our model, these models highlight potential tradeoffs between informational efficiency of asset prices and economic efficiency (see also Dow and Gorton, 1997, and Goldstein and Yang, 2014). The positive effects of trading may outweigh the negative effects in these models. Our model indicates that in order to argue for implicit or explicit support for financial trading in competition for talent, one should rely arguments related to the impact of trading in production decisions and not on informational efficiency.

Bond and García (2018) analyze indexing in an extended version of the Diamond-Verrecchia (1981) model, in which active trading entails a fixed cost compared to indexing. As in our model, better information due to more active trading reduces the scope for risk sharing and equilibrium welfare. Compared to their setup, the novelty of our model is the OC decision, which creates opportunity costs of trading in the form of less entrepreneurial activity and lower wages or fewer jobs.

The paper is organized as follows. Section 2 introduces the model. Section 3 derives the price function and agents' expected utilities. Sections 4 and 5 characterize equilibrium without and with noise trader shocks, respectively. Section 6 incorporates a labor market.

[8] Both models also provide explanations for the finance wage premium (see, e.g., Oyer, 2008, Philippon and Reshef, 2012, Axelson and Bond, 2015, Boustanifar et al., 2017, and Lindley and Mcintosh, 2017). Our model is silent on this issue, as agents are indifferent between entrepreneurship and trading at an equilibrium with a positive mass of agents active in each occupation.

[9] This relates to models with OC between finance and production in which agents who specialize in finance act as financial intermediaries (see, e.g., Phillipon, 2010, Cahuc and Challe, 2012, and Shakhnov, 2017).

Sections 7–9 provide a welfare analysis and investigate the implementation of the second-best welfare maximizing allocation as an equilibrium. Section 10 concludes. Proofs are delegated to an Appendix.

2 Model

The model we consider is the GS model augmented to include OC between entrepreneurship and finance and a labor market.

Consider a CARA-Gaussian economy with three dates, "early", "intermediate", and "late". There are three types of agents: a continuum of "high potential agents (hipos)" indexed by the interval $[0,\, L]$ $(L > 0)$, who choose between becoming a "dealer" or an "entrepreneur", a continuum of passive investors indexed by the interval $[0,\, M]$ $(M > 0)$, and noise traders. There is a single homogeneous consumption good. Prices are quoted in terms of this consumption good. Hipos are endowed with e_L (≥ 0) units of the good early, passive investors with e_M units. Hipos and passive investors are characterized by the CARA utility function $U(\pi) = -\exp(-\rho\pi)$, where π is late consumption and ρ (> 0) is the coefficient of absolute risk aversion. All agents have access to a storage technology that transforms endowments one-for-one into late consumption.

Hipos who become entrepreneurs run firms, create real wealth, and employ workers. Hipos who decide to become GS-type dealers collect information about macroeconomic fundamentals and contribute to informational efficiency by trading in the financial market. There is no physical cost of becoming an entrepreneur or a dealer. Hipos choose the occupation which maximizes their expected utility. They also have the option not to become an entrepreneur or a dealer, in which case they act like the passive investors (since there is no physical cost of becoming a dealer, this can only be beneficial if private information is worthless).

To assess the welfare effects of professional trading, we also consider the variant of the model without OC, in which hipos do not have the opportunity to become dealers, so that there is no informed trading.

Each entrepreneur sets up a continuum of firms indexed by the interval $[0, 1/a]$ $(a > 0)$. Denote the mass of hipos who decide to become entrepreneurs as L_E. Then a firm is an element of $[0, L_E] \times [0, 1/a]$. The mass of firms is L_E/a, and for each entrepreneur, the subset of firms he owns has measure zero, so entrepreneurs have no market power. We start with the version of the model without a labor market, in which firm output is exogenous: each firm produces θ units of output late. θ is a macroeconomic shock, which

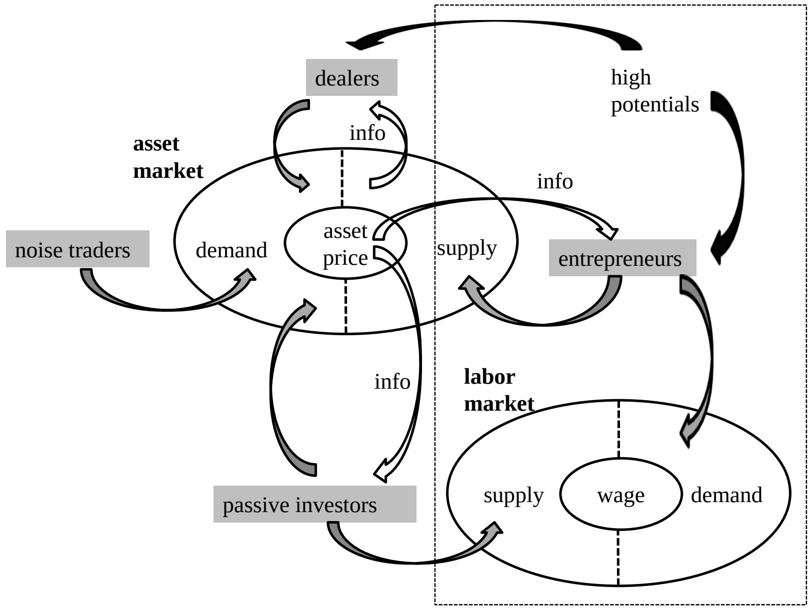

Figure 1: Structure of the model

is uniform across firms. It is the sum of two independent jointly normal random variables: $\theta = s + \varepsilon$, where $s \sim \mathrm{N}(\bar{s}, \sigma_s^2)$ and $\varepsilon \sim \mathrm{N}(0, \sigma_\varepsilon^2)$. Later we introduce a labor market, in which output θ depends on the input of unskilled labor (supplied by passive investors). The financial market is modeled as in GS (see also Grossman, 1976, and Hellwig, 1980). At the intermediate date shares in the firms are traded in a competitive stock market. Noise traders inelastically demand $\nu \sim \mathrm{N}(\bar{\nu}, \sigma_\nu^2)$ units of the risky asset. Dealers observe s and face residual uncertainty ε about firms' payoff. Entrepreneurs and passive investors observe neither s nor ε, so dealers' private information about s contributes to informational efficiency in the stock market via its impact on the asset price (since there are no firm-specific shocks, this information structure does not entail that dealers have information about individual firms that entrepreneurs do not have. It also implies that there are no benefits of going private in terms of hiding information (see Ferreira et al., 2014). Entrepreneurs and passive investors do not observe the other agents' trades. So they cannot tell exactly if a high stock market value of the firms is due to high demand by noise traders or by dealers having obtained favorable private information about profitability. Figure 1 depicts the structure of the model (with OC and for the case in which hipos do not decide to act as passive investors). Agents interact in the asset and labor markets. The black, grey, and white arrows indicate hipos' OC decision, agents' market supplies and demands, and information flows, respectively. The part of the model outside the dashed box are the GS model. The novel parts are inside the dashed box: hipos' OC decision, endogenous supply in the asset market, and the labor market.

The limitations of the model are similar as in Bolton et al. (2016). There is no moral hazard due to implicit or explicit state guarantees. There is no leverage, dealers trade only on their own account. The only input required to set up a firm is entrepreneurial labor, so there is no financial intermediation. Entrepreneurs set up and run firms, no distinction is made between engineering and management tasks.

3 Price function and expected utilities

This section defines equilibrium and derives agents' asset demands, the price function that relates the asset price to macroeconomic shocks, and agents' expected utilities.

Model variants

As is well known, there is a fundamental discontinuity in the GS model in the absence of noise: the asset price is fully informative about s for any positive mass of dealers, whereas it does not reveal any information about s if the mass of dealers is zero. This discontinuity does not show up in the presence of noise. Accordingly, we treat the cases $\sigma_\nu^2 = 0$ and $\sigma_\nu^2 > 0$ separately in Sections 4 and 5, respectively. The model variants with or without OC also have to be treated separately. We denote the economies with $\sigma_\nu^2 = 0$ and with or without OC, respectively, as $\mathsf{M}_0^{\mathbf{1}}$ and $\mathsf{M}_0^{\mathbf{0}}$. For $\sigma_\nu^2 > 0$, the models with or without OC are denoted $\mathsf{M}_\sigma^{\mathbf{1}}$ and $\mathsf{M}_\sigma^{\mathbf{0}}$, respectively. The analogous convention will be applied to other variables as well: superscripts $\mathbf{1}$ and $\mathbf{0}$ indicate whether there is OC or not, respectively, and subscripts 0 and σ indicate if the variance of noise trader demand is zero or positive, respectively.

Equilibrium

The mass of firms and, hence, the supply of stocks is L_E/a. Let P denote the stock market value of each firm and I_E, I_D, and I_M entrepreneurs', dealers', and passive investors stock holdings, respectively. Hipos make their OC and investment decisions so as to maximize expected utility conditional on available information. Consumption is $\pi_E = e_L + P/a + (\theta - P)I_E$ for entrepreneurs and $\pi_D = e_L + (\theta - P)I_D$ for dealers. Passive investors' consumption is $\pi_M = e_M + (\theta - P)I_M$. Since investment is independent of initial wealth, hipos who become neither entrepreneurs nor dealers invest the same amount I_M

and their final wealth differs only by the constant $e_L - e_M$. While dealers know s when they make their investment decision, entrepreneurs and passive investors can only use the price level P they observe to infer information about s. Throughout the paper, we focus on equilibria at which the mass of entrepreneurs L_E is positive, since otherwise asset supply is zero. Moreover, in the model variants with OC we focus on equilibria at which hipos become either entrepreneurs or dealers (and not passive investors). We argue below that this entails no loss of generality.

(L_E, I_E, I_D, I_M, P) is an *equilibrium* (an REE) of $\mathsf{M}^{\mathbf{1}}_{\varsigma}$ ($\varsigma = 0, \sigma$) if I_E maximizes $\mathrm{E}[U(\pi_E)|\, P]$, I_D maximizes $\mathrm{E}[U(\pi_D)|\, s, P]$, I_M maximizes $\mathrm{E}[U(\pi_M)|\, P]$, the market for the risky asset clears (i.e., $L_E/a = L_E I_E + (L - L_E)I_D + M I_M + \nu$), and OC is optimal (i.e., $\mathrm{E}[U(\pi_E)] = \mathrm{E}[U(\pi_D)]$ and $0 < L_E \le L$ or $\mathrm{E}[U(\pi_E)] \ge \mathrm{E}[U(\pi_D)]$ and $L_E = L$). An equilibrium of $\mathsf{M}^{\mathbf{0}}_{\varsigma}$ is defined analogously, except that I_D and the condition that it is chosen optimally drop out of the definition and the asset market clearing condition becomes $L_E/a = L_E I_E + (L - L_E + M)I_M + \nu$.

Price function

The optimal investment levels are

$$I_E = I_M = \frac{\mathrm{E}(\theta|\, P) - P}{\rho\,\mathrm{var}(\theta|\, P)}, \;\; I_D = \frac{s - P}{\rho\,\sigma_\varepsilon^2} \tag{1}$$

(see the Appendix). Substitution into the market clearing condition for the risky asset yields the price function

$$P = \frac{\frac{L-L_E}{\rho\sigma_\varepsilon^2} s + \frac{L_E+M}{\rho\,\mathrm{var}(\theta|\, P)} \mathrm{E}(\theta|\, P) - \left(\frac{L_E}{a} - \nu\right)}{\frac{L-L_E}{\rho\sigma_\varepsilon^2} + \frac{L_E+M}{\rho\,\mathrm{var}(\theta|\, P)}} = \frac{w + \frac{L_E+M}{\rho\,\mathrm{var}(\theta|\, w)} \mathrm{E}(\theta|\, w) - \frac{L_E}{a}}{\frac{L-L_E}{\rho\sigma_\varepsilon^2} + \frac{L_E+M}{\rho\,\mathrm{var}(\theta|\, w)}}, \tag{2}$$

where

$$w \equiv \frac{L - L_E}{\rho\sigma_\varepsilon^2} s + \nu. \tag{3}$$

From the updating rule for the mean of a normal random variable,

$$\mathrm{E}(\theta|\, w) = \bar{s} + \frac{\mathrm{cov}(\theta, w)}{\mathrm{var}(w)}[w - \mathrm{E}(w)]. \tag{4}$$

From $\mathrm{var}(\theta|\, w) = \mathrm{var}(s|\, w) + \sigma_\varepsilon^2$ and the updating rule $\mathrm{var}(s|\, w) = \sigma_s^2 - [\mathrm{cov}(s, w)]^2/\mathrm{var}(w)$, it follows that

$$\mathrm{var}(\theta|\, w) = \sigma_s^2 - \frac{[\mathrm{cov}(s, w)]^2}{\mathrm{var}(w)} + \sigma_\varepsilon^2 \tag{5}$$

is non-random. From (3), a high value of (w and) P can be due to good fundamentals (high s) or high noise trader demand (high ν). This gives rise to the GS signal extraction problem. The conditional payoff variance $\mathrm{var}(\theta|\, w)$ in (5) is a measure of the informational efficiency of the asset market: a lower value of $\mathrm{var}(\theta|\, w)$ means that the asset price helps make a more accurate prediction of macroeconomic fundamentals. The fact that $\mathrm{var}(\theta|\, w)$ is lower than the unconditional variance $\sigma_s^2 + \sigma_\varepsilon^2$ whenever $L_E < L$ means that the information contained in P (or w) is valuable to agents who do not know s.

Expected utilities

Let

$$z \equiv \frac{\mathrm{E}(\theta|\, w) - P}{[2\,\mathrm{var}(\theta|\, w)]^{\frac{1}{2}}}. \tag{6}$$

z measures expected payoff relative to risk for financial investments conditional on w ($z\sqrt{2}/P$ is the Sharpe ratio). An entrepreneur's expected utility conditional on P is

$$\mathrm{E}[U(\pi_E)|\, P] = -\exp(-\rho e_L)\exp\left(-\rho\frac{P}{a} - z^2\right).$$

(see the Appendix). From (2)–(5), z is a linear function of w. It can be shown that the linear dependence is negative (see the Appendix), so that $\mathrm{cov}(P, z) = -[\mathrm{var}(P)\,\mathrm{var}(z)]^{1/2}$. Using the law of iterated expectations and Lemma 1 in Demange and Laroque (1995, p. 252), we obtain the following expression for an entrepreneur's unconditional expected utility $\mathrm{E}[U(\pi_E)]$:

$$\begin{aligned} -\log\{-\mathrm{E}[U(\pi_E)]\} &= \rho e_L + \overbrace{\frac{\rho}{a}\left[\mathrm{E}(P) - \frac{\rho}{2a}\,\mathrm{var}(P)\right]}^{=\mathsf{GE}} \\ &\quad \underbrace{+\frac{\left[\mathrm{E}(z) - \frac{\rho}{a}\,\mathrm{cov}(P,z)\right]^2}{1 + 2\,\mathrm{var}(z)} + \frac{1}{2}\log\left[1 + 2\,\mathrm{var}(z)\right]}_{=\mathsf{GT}_E} \end{aligned} \tag{7}$$

(see the Appendix). Notice that $-\log\{-\mathrm{E}[U(\pi_E)]\}/\rho$ is equal to the certainty equivalent (CE) of an entrepreneurs' risky income π_E. For the sake of convenience, we often call this transformed value "expected utility" in what follows (and analogously for the other agents). If hipos merely stored and consumed their endowment, their expected utility would be given by ρe_L. If they become entrepreneurs and sell the $1/a$ firms they set up and carry out no further financial transactions, they get extra expected utility $-\log\{-\mathrm{E}[U(e_L + P/a)]\} - \rho e_L = (\rho/a)[\mathrm{E}(P) - \rho/(2a)\,\mathrm{var}(P)] \equiv \mathsf{GE}$. These "gains from entrepreneurship" are determined by the first two moments of the random asset price P. Define the additional terms in (7) as the "gains from trading" for entrepreneurs GT_E. GT_E reflects the marginal impact of an entrepreneur's trade in the stock market on his expected utility, after having sold his firm. GT_E depends on the first two moments of z and on the covariance of z and P. This covariance matters because changes in w (linearly) affect both the price P at which entrepreneurs sell their firms and the expected payoff-risk ratio z. (This effect is not present in GS, where agents are not engaged in entrepreneurial activity, and makes the application of the Demange and Laroque, 1995, lemma necessary.)

A passive investor's unconditional expected utility is obtained analogously:

$$-\log\{-\mathrm{E}[U(\pi_M)]\} = \rho e_M + \underbrace{\frac{[\,\mathrm{E}(z)]^2}{1+2\,\mathrm{var}(z)} + \frac{1}{2}\log\left[1+2\,\mathrm{var}(z)\right]}_{=\mathsf{GT}_M}. \tag{8}$$

As passive investors do not own shares in firms early, this is (7) with $P = 0$. The final two terms in the sum on the right-hand, GT_M say, give the passive investor's gains from trading. As $\mathrm{cov}(P, z)$ is negative, $\mathsf{GT}_E > \mathsf{GT}_M$ whenever $\mathrm{E}(z) > 0$ (which, as will be seen below, holds true whenever rational agents do not short the asset in the aggregate at equilibrium). Under this condition, even though entrepreneurs trade on the same information as passive investors, they derive greater benefits from their trades, since fluctuations in z provide a hedge against the entrepreneurial risk they carry. The expected utility of hipos who act like passive investors is $-\log\{-\mathrm{E}[U(\pi_M)]\} + \rho(e_L - e_M)$.

A dealer's expected utility conditional on P is

$$\mathrm{E}[U(\pi_D)|\,P] = -\exp(-\rho e_L)\left[\frac{\sigma_\varepsilon^2}{\mathrm{var}(\theta|\,w)}\right]^{\frac{1}{2}}\exp\left(-z^2\right).$$

Using the law of iterated expectations, it follows that

$$\begin{aligned}-\log\{-\mathrm{E}[U(\pi_D)]\} &= \rho e_L + \overbrace{\frac{1}{2}\log\left[\frac{\mathrm{var}(\theta|\,w)}{\sigma_\varepsilon^2}\right]}^{=\mathsf{GI}} \\ &\quad + \frac{[\,\mathrm{E}(z)]^2}{1+2\,\mathrm{var}(z)} + \frac{1}{2}\log\left[1+2\,\mathrm{var}(z)\right]\end{aligned} \tag{9}$$

(see the Appendix). The sum on the right-hand side can be rewritten as $\rho e_L + \mathsf{GI} + \mathsf{GT}_M$, where $\mathsf{GI} \equiv (1/2)\log[\mathrm{var}(\theta|\,w)/\sigma_\varepsilon^2]$ represents the "gains from being informed", i.e., from knowing s rather than having to infer information about it from the asset price. GI is positive or zero, depending on whether P reveals s or not, respectively.

4 Equilibrium without noise

This section analyzes the model for non-random noise trader demand (i.e., $\sigma_\nu^2 = 0$). We start with the version of the model with OC, i.e., with model $\mathsf{M}_0^{\mathbf{1}}$. We first consider equilibria with a positive mass of dealers and then equilibria at which all hipos decide to become entrepreneurs. Finally, we consider model $\mathsf{M}_0^{\mathbf{0}}$.

Occupational choice

We focus on equilibria with a positive mass of entrepreneurs (i.e., $L_E > 0$). To begin with, suppose that the mass of dealers is also positive (i.e., $L_E < L$). Since noise trader demand is non-random, w defined in (3) reveals s to entrepreneurs and passive investors. From (3) and (4), $\mathrm{E}(\theta|\, w) = s$. From (5), $\mathrm{var}(\theta|\, w) = \sigma_\varepsilon^2$. So I_E equals I_D, as given by (1). The price function (2) simplifies to

$$P = s - \frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right). \tag{10}$$

From (6), since $\mathrm{E}(\theta|\, w) - P = s - P$ is non-random, z is non-random, even though both s and P are risky. In fact, from (6) and (10),

$$z = \left(\frac{\sigma_\varepsilon^2}{2}\right)^{\frac{1}{2}} \frac{\rho}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right). \tag{11}$$

The equilibrium entails no rents for either entrepreneurs or dealers. The condition for optimal OC is that entrepreneurs are as well-off as dealers: $\mathsf{GE} + \mathsf{GT}_E - \mathsf{GT}_M = \mathsf{GI}$. As $\mathrm{cov}(P, z) = 0$ here, the gains from trading are identical for entrepreneurs and for passive investors (i.e., $\mathsf{GT}_E = \mathsf{GT}_M$). As $\mathrm{var}(\theta|\, w) = \sigma_\varepsilon^2$, dealers do not benefit from their private information about market fundamentals (i.e., $\mathsf{GI} = 0$). So $\mathsf{GE} = 0$ or, using (10),

$$\underbrace{\frac{\rho}{a}\left[\bar{s} - \frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right) - \frac{\rho\sigma_s^2}{2a}\right]}_{=\Delta_0(L_E)} = 0 \tag{12}$$

at an equilibrium with $0 < L_E < L$. The left-hand side of (12) maps the half-open interval $[0, L)$ to the reals. (The mass of entrepreneurs L_E must not take on the value L, because otherwise there would be no-one left to reveal the information about s as an informed trader.) Denote this mapping as $\Delta_0(L_E)$. Then we have:

PROPOSITION 4.1. *If there is $L_E^{\mathbf{1}}$ $(0 < L_E^{\mathbf{1}} < L)$ such that $\Delta_0(L_E^{\mathbf{1}}) = 0$, then $L_E = L_E^{\mathbf{1}}$, $I_E = I_D$, $I_M = I_D$, I_D given by (1), and P given by (10) are an equilibrium of $\mathsf{M}_0^{\mathbf{1}}$.*

This is illustrated in the left panel of Figure 2. The downward-sloping line $\Delta_0(L_E)$ gives the expected utility of an entrepreneur over and above a passive investor's, GE. The expected utility differential for a dealer compared to a passive investor GI is zero. So equilibrium occurs at the point of intersection of $\Delta_0(L_E)$ and the horizontal axis (see the filled circle).

Remark 4.1.1. There is an equilibrium with $0 < L_E^{\mathbf{1}} < L$ if $\Delta_0(0) > 0$ and $\Delta_0(L_E) < 0$ for L_E large enough. As $\Delta_0(L_E)$ is monotonically decreasing, this type of equilibrium is unique. (However, an equilibrium without dealers may coexist, as will be seen below.) Clearly, the condition of the proposition is also necessary: if it is not satisfied, then an equilibrium of $\mathsf{M}_0^{\mathbf{1}}$ with $L_E < L$ does not exist.

Remark 4.1.2. The fact that, other than in GS, a fully revealing REE possibly exists in the absence of noise is due to the fact that the only cost of becoming a dealer is the opportunity cost of not becoming an entrepreneur, which is zero at equilibrium. This type of equilibrium would vanish if there was a physical cost of becoming a dealer.

Remark 4.1.3. The allocation is indeterminate in that dealers would be equally well-off as passive investors. However, since investments and the asset price are the same if only a subset of non-zero measure of the $L - L_E$ non-entrepreneurs become dealers, L_E and equilibrium (L_E, I_E, I_D, I_M, P) are uniquely determined. In this sense the focus on equilibria at which no hipo becomes a passive investor is without loss of generality.

Remark 4.1.4. An equilibrium with $L_E^{\mathbf{1}}$ entrepreneurs has the expected comparative statics properties: whatever raises GE $(= \Delta_0(L_E))$ raises the equilibrium mass of entrepreneurs.

Remark 4.1.5. That neither dealers nor entrepreneurs earn any rents at equilibrium (as $\mathsf{GI} = \mathsf{GE} = 0$) is due to the fact that the asset price is fully revealing. Positive and sizable rents at equilibrium will occur in the model with stochastic noise trader demand (see Remarks 5.1.5 and 8.1.3 below).

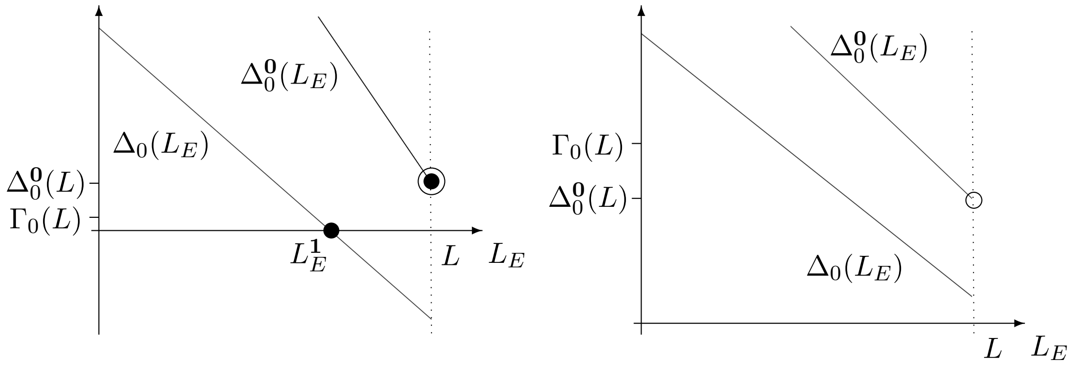

Figure 2: Equilibrium with no noise

No dealers

If there are no dealers, no market participant observes s and the asset price is uninformative: $\mathrm{E}(\theta|\, w) = \bar{s}$ and $\mathrm{var}(\theta|\, w) = \sigma_s^2 + \sigma_\varepsilon^2$. Agents' asset demand is

$$I_E = I_M = \frac{\bar{s} - P}{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}. \tag{13}$$

The asset price equates asset supply L_E/a and asset demand:

$$\frac{\rho}{a} P = \underbrace{\frac{\rho}{a}\left[\bar{s} - \frac{\rho\,(\sigma_s^2 + \sigma_\varepsilon^2)}{L + M}\left(\frac{L_E}{a} - \bar{\nu}\right)\right]}_{=\Delta_0^{\mathbf{0}}(L_E)}. \tag{14}$$

It is non-random. Denote the function on the right-hand side of (14) as $\Delta_0^{\mathbf{0}}(L_E)$. $\Delta_0^{\mathbf{0}}(L_E)$ maps $[0, L]$ to the reals. From (6), z is also non-random:

$$z = \frac{\bar{s} - P}{[2(\sigma_s^2 + \sigma_\varepsilon^2)]^{\frac{1}{2}}}. \tag{15}$$

An entrepreneur's expected utility (7) is

$$-\log\{-\mathrm{E}[U(\pi_E)]\} = \rho e_L + \frac{\rho}{a} P + z^2. \tag{16}$$

The second and third terms on the right-hand side represent the gains from entrepreneurship GE and the gains from trading GT_E, respectively. A passive investor's expected utility is $-\log\{-\mathrm{E}[U(\pi_M)]\} = \rho e_M + z^2$. The gains from trading GT_M are identical as for en-

trepreneurs.
Since being a dealer is preferred to being a passive investor whenever the mass of dealers is zero, equilibrium without dealers implies that all L hipos become entrepreneurs, so that the asset supply is L/a and the asset price is given by $(\rho/a)P = \Delta_0^{\mathbf{0}}(L)$. Optimal OC implies that a single agent must not have an incentive to become a dealer, given that the others are entrepreneurs. A single hipo who decides to become a dealer observes s and invests $I_D = (s - P)/(\rho\sigma_\varepsilon^2)$. His unconditional expected utility is

$$-\log\{-\mathrm{E}[U(\pi_D)]\} = \rho e_L + \underbrace{\frac{1}{2}\log\left(\frac{\sigma_s^2 + \sigma_\varepsilon^2}{\sigma_\varepsilon^2}\right)}_{=\Gamma_0(L)} + z^2$$

(see the Appendix). Denote the second term on the right hand side as $\Gamma_0(L)$. It does not pay to become a dealer if $\Delta_0^{\mathbf{0}}(L)$ is no less than $\Gamma_0(L)$.

PROPOSITION 4.2. *If $\Delta_0^{\mathbf{0}}(L) \geq \Gamma_0(L)$, then $L_E = L$, I_E given by (13), I_D arbitrary, $I_M = I_E$, and P given by (14) are an equilibrium of $\mathsf{M}_0^{\mathbf{1}}$.*

Remark 4.2.1. The condition of the proposition is also necessary: if it is not satisfied, then an equilibrium of $\mathsf{M}_0^{\mathbf{1}}$ with $L_E = L$ does not exist.

Remark 4.2.2. The assertion of Proposition 4.2 is similar as in GS: an equilibrium without dealers exists if the gains from trading GI $(= \Gamma_0(L))$ are sufficiently small (see the upper right filled circle in the left panel of Figure 2). In GS the gains from trading are compared to the physical cost of information gathering. Here they are compared to the benefits of being an entrepreneur GE $(= \Delta_0^{\mathbf{0}}(L))$.

Remark 4.2.3. Since the gains from entrepreneurship are positive, this type of equilibrium would survive the introduction of a sufficiently small positive physical cost (no greater than $\Delta_0^{\mathbf{0}}(L)/\rho$) of becoming an entrepreneur (cf. Remark 4.1.2).

Remark 4.2.4. There is at most one equilibrium with $L_E < L$ (cf. Remark 4.1.1), and there is at most one equilibrium with $L_E = L$. But the two types of equilibria can coexist (as illustrated in the left panel of Figure 2; in Section 7 we explain that the source of this multiplicity result is strategic complementarity in information acquisition).

Remark 4.2.5. It can also happen that an equilibrium fails to exist. This is illustrated in the right panel of Figure 2. $\Delta_0(L_E) > 0$ for all $L_E < L$ implies that an equilibrium with $L_E < L$ entrepreneurs and a positive mass of dealers does not exist (if the price if fully informative, it does not pay to be a dealer). $\Gamma_0(L) > \Delta_0^{\mathbf{0}}(L)$ implies that an equilibrium without dealers does not exist either (if the price is uninformative, a single agent has an incentive to incur a low cost of becoming informed). This is the GS non-existence result adapted to our setup.

No occupational choice

Finally, suppose hipos do not have the opportunity to become dealers: they act either as entrepreneurs or as passive investors. Equations (14) and (15) determine P and z, respectively.

PROPOSITION 4.3. *(i) If there is $L_E^{\mathbf{0}}$ $(0 < L_E^{\mathbf{0}} < L)$ such that $\Delta_0^{\mathbf{0}}(L_E^{\mathbf{0}}) = 0$, then $L_E = L_E^{\mathbf{0}}$, I_E given by (13), $I_M = I_E$, and P given by (14) are an equilibrium of $\mathsf{M}_0^{\mathbf{0}}$. (ii) If $\Delta_0^{\mathbf{0}}(L) \geq 0$, then $L_E = L$, I_E given by (13), $I_M = I_E$, and P given by (14) are an equilibrium of $\mathsf{M}_0^{\mathbf{0}}$.*

Remark 4.3.1. $\Delta_0^{\mathbf{0}}(0) > 0$ ensures existence of equilibrium. As $\Delta_0^{\mathbf{0}}(L_E)$ is monotonically decreasing, equilibrium is unique. The equilibrium mass of entrepreneurs $L_E^{\mathbf{0}}$ has the expected comparative statics properties. There are no positive rents for entrepreneurs at an equilibrium with $L_E^{\mathbf{0}} < L$.

Remark 4.3.2. Suppose there are equilibria with $L_E^{\mathbf{1}}$ and $L_E^{\mathbf{0}}$ (both less than L) entrepreneurs with and without OC, respectively. One might expect that $L_E^{\mathbf{0}} > L_E^{\mathbf{1}}$, since hipos do not have the opportunity to become dealers in the absence of OC. While this is not generally true, a simple sufficient condition is

$$L - a\bar{\nu} < \frac{L+M}{2} \tag{17}$$

(see the Appendix). Inequality (17) is in turn valid if “talent is scarce” in that there are fewer hipos than passive investors ($L \leq M$) and noise traders do not short the asset ($\bar{\nu} > 0$).

5 Equilibrium with noise

This section analyzes the model with positive noise trader shocks, again treating the cases with and without OC successively.

Occupational choice

At an equilibrium of $\mathsf{M}_\sigma^{\mathbf{1}}$, if a positive mass of hipos become entrepreneurs, this must be

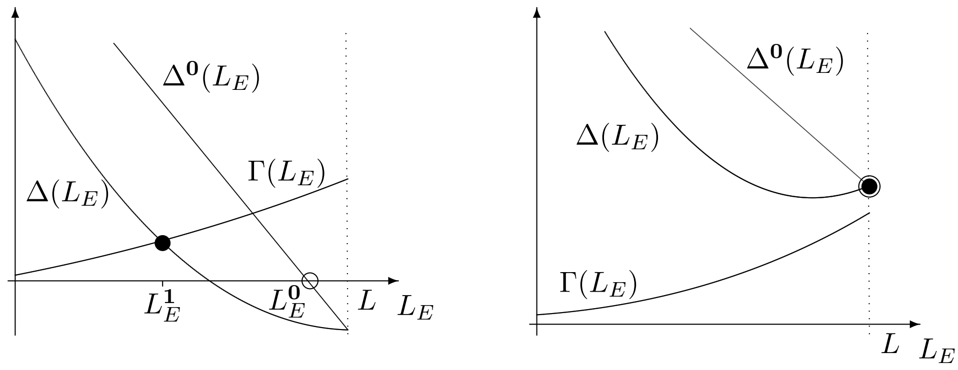

Figure 3: Equilibrium with noise

no less attractive than becoming a dealer: $\mathsf{GE} + \mathsf{GT}_E - \mathsf{GT}_M \geq \mathsf{GI}$ or, using (7) and (9),

$$\underbrace{\frac{\frac{\rho}{a}\operatorname{cov}(P,z)\left[\frac{\rho}{a}\operatorname{cov}(P,z) - 2\operatorname{E}(z)\right]}{1 + 2\operatorname{var}(z)} + \frac{\rho}{a}\left[\operatorname{E}(P) - \frac{\rho}{2a}\operatorname{var}(P)\right]}_{=\Delta(L_E)} \geq \underbrace{\frac{1}{2}\log\left[\frac{\operatorname{var}(\theta|\, w)}{\sigma_\varepsilon^2}\right]}_{=\Gamma(L_E)}. \quad (18)$$

Equations (2)–(5) determine the moments and the covariance of P and z as continuous functions of L_E alone (closed-form solutions are in the Appendix). Denote the composite function obtained from substituting these functions into the left-hand side of (18) as $\Delta(L_E)$. From (3) and (5), $\operatorname{var}(\theta|\, w)$ is also a continuous function of L_E alone (closed-form solution in the Appendix). Denote the function resulting from substituting this function into the right-hand side of (18) as $\Gamma(L_E)$. Both $\Delta(L_E)$ and $\Gamma(L_E)$ map $[0, L]$ to the reals. Since L_E also uniquely determines I_E, I_D, and P via (1) and (2), we have:

PROPOSITION 5.1. *(i) If there is $L_E^{\mathbf{1}}$ $(0 < L_E^{\mathbf{1}} < L)$ such that $\Delta(L_E^{\mathbf{1}}) = \Gamma(L_E^{\mathbf{1}})$, then $L_E^{\mathbf{1}}$, I_E, I_D, and I_M given by (1), and P given by (2) are an equilibrium of $\mathsf{M}_\sigma^{\mathbf{1}}$. (ii) If $\Delta(L) \geq \Gamma(L)$, then $L_E = L$, I_E and I_M given by (1), I_D arbitrary, and P given by (2) are an equilibrium of $\mathsf{M}_\sigma^{\mathbf{1}}$.*

The two types of equilibria are illustrated in Figure 3. The left and right panels refer to cases (i) and (ii), respectively. The filled circles represent the equilibrium mass of entrepreneurs L_E and the equilibrium difference in the expected utilities of entrepreneurs and passive investors $\mathsf{GE} + \mathsf{GT}_E - \mathsf{GT}_M$.

Remark 5.1.1. $\Delta(0) > \Gamma(0)$ is sufficient to ensure existence of equilibrium. Together with continuity of $\Delta(L_E)$ and $\Gamma(L_E)$, this condition implies that either there is $L_E^{\mathbf{1}} < L$ such that $\Delta(L_E^{\mathbf{1}}) = \Gamma(L_E^{\mathbf{1}})$ or else $\Delta(L) \geq \Gamma(L)$. This is in line with the result that the GS non-existence result (cf. Remark 4.2.5) vanishes in the presence of noise.

Remark 5.1.2. There exist parametrizations of the model such that there are multiple intersections of the functions $\Delta(L_E)$ and $\Gamma(L_E)$, so multiplicity of equilibria cannot be ruled out (cf. Remark 4.2.4 above and Remark 5.3.2 below).

Remark 5.1.3. An equilibrium $(L_E^{\mathbf{1}}, I_E, I_D, I_M, P)$ with a positive mass of dealers would also be an equilibrium in the presence of a physical cost of not being a passive investor no greater than $\Delta(L_E^{\mathbf{1}})/\rho$.

Remark 5.1.4. If $\Delta(L_E)$ intersects $\Gamma(L_E)$ from above, then the corresponding equilibrium of $\mathsf{M}_\sigma^{\mathbf{1}}$ has the expected comparative statics properties. For instance, a decrease in firms' expected profitability $\bar{s}$ reduces entrepreneurial activity, raises the number of dealers, and fosters informational efficiency (i.e., reduces $\mathrm{var}(\theta|\, w)$; see the Appendix).

Remark 5.1.5. Hipos earn positive rents $\mathsf{GE} + \mathsf{GT}_E - \mathsf{GT}_M$ $(= \Delta(L_E) > 0)$ compared to passive investors at equilibrium either as entrepreneurs or, if $L_E < L$, as dealers. The numerical analysis below shows that these rents can be sizable (see Remark 8.1.3).

No occupational choice

In $\mathsf{M}_\sigma^{\mathbf{0}}$, since no-one gathers information about s, the price is uninformative: $\mathrm{E}(\theta|\, P) = \bar{s}$ and $\mathrm{var}(\theta|\, P) = \sigma_s^2 + \sigma_\varepsilon^2$. Entrepreneurs' optimal investment level is given by (13) and the price function is given by (14) with ν instead of $\bar{\nu}$. As in $\mathsf{M}_\sigma^{\mathbf{1}}$, the left-hand side of (18) gives the expected utility differential for entrepreneurs compared to passive investors $\mathsf{GE} + \mathsf{GT}_E - \mathsf{GT}_M$. However, since the price function as well as z differ from their counterparts in the case with OC, the moments of P and z that appear on the left-hand side of (18) are different. The Appendix derives closed-forms solutions for the moments as functions of L_E alone. Denote the composite function that results from substituting these moments into the left-hand side of (18) as $\Delta^{\mathbf{0}}(L_E)$ (with domain $[0, L]$).

PROPOSITION 5.2. *(i) If $\Delta^{\mathbf{0}}(L_E^{\mathbf{0}}) = 0$ for some $L_E^{\mathbf{0}}$ $(0 < L_E^{\mathbf{0}} < L)$, then $L_E = L_E^{\mathbf{0}}$, I_E given by (13), $I_M = I_E$, and P given by (14) with ν instead of $\bar{\nu}$ are an equilibrium of $\mathsf{M}_\sigma^{\mathbf{0}}$. (ii) If $\Delta^{\mathbf{0}}(L) \geq 0$, then $L_E = L$, I_E given by (13), $I_M = I_E$, and P given by (14) with ν instead of $\bar{\nu}$ are an equilibrium of $\mathsf{M}_\sigma^{\mathbf{0}}$.*

Such equilibria are illustrated by the open circles in Figure 3.

Remark 5.2.1. In the Appendix, we show that $\Delta^{\mathbf{0}}(L_E)$ is a linear, decreasing function. So $\Delta^{\mathbf{0}}(0) > 0$ is sufficient for existence of equilibrium, and equilibrium is unique. $L_E^{\mathbf{0}}$ has the expected comparative statics properties.

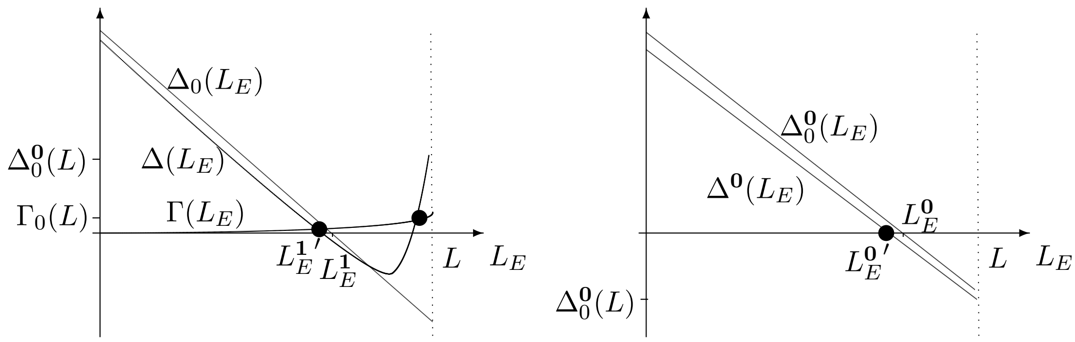

Figure 4: Equilibrium with small noise trader shocks

Remark 5.2.2. The Appendix also shows that $\Delta^{\mathbf{0}}(L) = \Delta(L)$ (see Figure 3): whether the non-entrepreneurs are passive investors or dealers does not make a difference for their expected utility compared to a passive investor's as their mass goes to zero.

Small noise trader shocks

While the subsequent welfare analysis requires numerical analysis in the case of large noise trader shocks, the cases of no noise and of small noise trader shocks can be treated analytically. To pave the way for the welfare analysis of the model with small noise trader shocks, consider the limiting case $\sigma_\nu^2 \to 0$ of $\mathsf{M}_\sigma^{\mathbf{j}}$. The following result states that, with or without OC, the equilibrium mass of entrepreneurs is a continuous function of σ_ν^2 at $\sigma_\nu^2 = 0$:

PROPOSITION 5.3. *Given an equilibrium of $\mathsf{M}_0^{\mathbf{j}}$ with mass of entrepreneurs $L_E^{\mathbf{j}}$, there is an equilibrium of $\mathsf{M}_\sigma^{\mathbf{j}}$ with mass of entrepreneurs $L_E^{\mathbf{j}\prime}$ arbitrarily close to $L_E^{\mathbf{j}}$ for σ_ν^2 sufficiently small ($\mathbf{j} = \mathbf{0}, \mathbf{1}$).*

The proof is in the Appendix. It relies on the fact that the functions which determine an equilibrium of $\mathsf{M}_\sigma^{\mathbf{j}}$ (i.e., $\Delta(L_E)$ and $\Gamma(L_E)$ for $\mathbf{j} = \mathbf{1}$) are close to the functions that determine an equilibrium of $\mathsf{M}_0^{\mathbf{j}}$ (i.e., $\Delta_0(L_E)$ and the horizontal line at height zero for $\mathbf{j} = \mathbf{0}$) for σ_ν^2 small (see Figure 4).

Remark 5.3.1. There is a qualitative difference between the models with and without noise for L_E-values in the vicinity of L. For σ_ν^2 small, $\Gamma(L_E)$ is close to zero for $L_E < L$ and $\Gamma(L)$ is close to $\Gamma_0(L)$ (see the Appendix). That is, the graph of $\Gamma(L_E)$ is almost kinked at L_E close to L.

Remark 5.3.2. This sheds light on the multiplicity result in Remark 5.1.2. If there is $L_E^{\mathbf{1}}$ ($< L$) such that $\Delta_0(L_E^{\mathbf{1}}) = 0$ and $\Delta_0^{\mathbf{0}}(L) > \Gamma_0(L)$, then two equilibria with $L_E^{\mathbf{1}}$ and L entrepreneurs, respectively, coexist in $\mathsf{M}_0^{\mathbf{1}}$ (see Remark 4.2.4). These conditions also imply that for σ_ν^2 small enough, $\Delta(L_E)$ and $\Gamma(L_E)$ intersect twice, viz., close to $L_E^{\mathbf{1}}$ and close to L, so that there are two equilibria of $\mathsf{M}_\sigma^{\mathbf{1}}$ with $0 < L_E < L$ (see Figure 4).

Remark 5.3.3. Since $L_E^{\mathbf{1}}$ and $L_E^{\mathbf{0}}$ are continuous functions of σ_ν^2, condition (17) in Remark 4.3.2 ensures that $L_E^{\mathbf{0}} > L_E^{\mathbf{1}}$ for σ_ν^2 positive but sufficiently small.

6 Wages and employment

This section incorporates a labor market into the model (cf. Figure 1). We consider both the full employment version of the model and a specification with real wage rigidity and equilibrium unemployment. Workers benefit from entrepreneurship in both models: a higher mass of entrepreneurs raises the real wage rate at a full employment equilibrium and lowers unemployment in the presence of labor market frictions.

Model

We maintain all assumptions made in Section 2 unless stated otherwise. Passive investors are now endowed with one unit of unskilled labor per capita and also called "workers". Hipos do not require unskilled labor to set up firms or to gather information. Irrespective of their OC decision (i.e., either as an entrepreneur or as a dealer or acting like a passive investor), they do not supply unskilled labor.

As before, an entrepreneur sets up firms indexed $[0, 1/a]$ early. Firm output and profit are $Y = \tilde{\theta} + F(m)$ and $\theta \equiv Y - Wm$, respectively, where m is firm-level employment and W is the wage rate. The production function F is twice continuously differentiable, strictly increasing, and strictly concave, with $\lim_{m\to 0} F'(m) = \infty$ and $\lim_{m\to\infty} F'(m) = 0$, so that profit maximization yields an interior solution. $\tilde{\theta}$ is the sum of two independent jointly normal random variables $\tilde{s} \sim \mathrm{N}(\hat{s}, \sigma_s^2)$ and $\varepsilon \sim \mathrm{N}(0, \sigma_\varepsilon^2)$.

Wages and employment are determined early, so W and m are non-random, and an employed worker's initial wealth is $e_M + W$. The disutility of working is equivalent to D (≥ 0) safe units of consumption. So the aggregate supply of labor is M for $W \geq D$ and zero otherwise. An entrepreneur's expected utility is an increasing function of $F(m) - Wm$, so he chooses employment m so as to maximize this objective function (see the Appendix).[10]

[10]This would not be true if the impact of $\tilde{\theta}$ on firm profit θ were non-additive, so that setting up a firm creates an asset with different return characteristics.

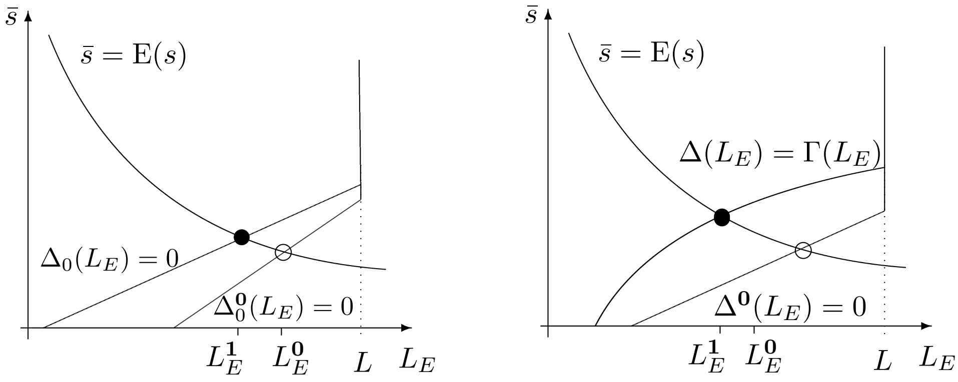

Figure 5: Full employment equilibrium

Dealers observe $\tilde{s}$ at the intermediate date, while entrepreneurs and workers have to infer information about $\tilde{s}$ from the equilibrium stock price.

Full employment

Denote the full employment version of the model as $\mathsf{F}_{\varsigma}^{\mathbf{j}}$, with $\mathbf{j}$ equal to $\mathbf{1}$ or $\mathbf{0}$ depending on whether there is OC or not, respectively, and ς equal to 0 or σ depending on whether σ_ν^2 is zero or positive. For simplicity, let $D = 0$ here, so that workers supply labor for any positive wage rate. Let $\hat{M} \equiv M/(L_E/a)$ denote the number of workers per firm. $(L_E, I_E, I_D, I_M, P, m, W)$ is an *equilibrium* of $\mathsf{F}_{\varsigma}^{\mathbf{j}}$ if, in addition to the conditions stated in Section 3, employment m maximizes $F(m) - Wm$ and the labor market clears (i.e., $m = \hat{M}$).

Denote the wage rate given full employment as $\hat{W} = F'(\hat{M})$. Define

$$s \equiv F\left(\frac{aM}{L_E}\right) - F'\left(\frac{aM}{L_E}\right)\frac{aM}{L_E} + \tilde{s}, \quad \bar{s} \equiv \mathrm{E}(s). \tag{19}$$

$s + \varepsilon$ is firm profit given full employment. s is normal with mean $\mathrm{E}(s) = \bar{s}$ and variance σ_s^2. Given the definitions of s and $\bar{s}$ in (19), the equilibrium analysis of $\mathsf{M}_{\varsigma}^{\mathbf{j}}$ in Sections 4 and 5 goes through without modification, and we have:

PROPOSITION 6.1. *Let s and $\bar{s}$ be given by (19). If (L_E, I_E, I_D, I_M, P) is an equilibrium of $\mathsf{M}_{\varsigma}^{\mathbf{j}}$, then $(L_E, I_E, I_D, I_M, P, \hat{M}, \hat{W})$ is an equilibrium of $\mathsf{F}_{\varsigma}^{\mathbf{j}}$ ($\mathbf{j} = \mathbf{0}, \mathbf{1}$, $\varsigma = 0, \sigma$).*

(It is understood that I_D drops out for $\mathbf{j} = \mathbf{0}$.) Figure 5 illustrates the determination of the equilibrium values of L_E and $\bar{s}$. The left panel applies to the model without noise trader shocks $\mathsf{F}_0^{\mathbf{j}}$. The upward-sloping lines depict the relations between $\bar{s}$ and L_E at an equilibrium of $\mathsf{M}_0^{\mathbf{j}}$ with $L_E < L$. The left line applies to the model with OC (i.e., $\mathbf{j} = \mathbf{1}$). It depicts the $(L_E, \bar{s})$ combinations which satisfy $\Delta_0(L_E) = 0$ (see (12)). The right line applies to the case of no OC (i.e., $\mathbf{j} = \mathbf{0}$). It is determined by $\Delta_0^{\mathbf{0}}(L_E) = 0$ (see (14)). The positive slopes of the lines reflect the fact that a higher value of $\bar{s}$ makes entrepreneurship more attractive. For sufficiently large values of $\bar{s}$, equilibria of $\mathsf{M}_\varsigma^{\mathbf{j}}$ are characterized by $L_E = L$. This is illustrated by the vertical line segment. The downward-sloping curve depicts the relation between $\bar{s}$ and L_E implied by (19). The negative slope reflects the fact that an increase in L_E decreases firm-level employment $\hat{M}$ and, therefore, $\bar{s}$: $d\bar{s}/dL_E = F''(\hat{M})\hat{M}^2/L_E < 0$. The filled circle and the open circle represent equilibria of $\mathsf{F}_0^{\mathbf{1}}$ and $\mathsf{F}_0^{\mathbf{0}}$, respectively. The right panel of 5 applies to the model with stochastic noise trader demand $\mathsf{F}_\sigma^{\mathbf{j}}$. The two upward sloping curves represent $(L_E, \bar{s})$ combinations such that $\Delta(L_E) = \Gamma(L_E)$ and $\Delta^{\mathbf{0}}(L_E) = 0$, respectively. Intersections with the downward sloping curve that represents (19) correspond to equilibria of $\mathsf{F}_\sigma^{\mathbf{1}}$ and $\mathsf{F}_\sigma^{\mathbf{0}}$, respectively.

Remark 6.1.1. With $\bar{s}$ given by (19) the conditions for existence of equilibrium in $\mathsf{F}_\varsigma^{\mathbf{j}}$ are given by the respective remarks to Propositions 4.1, 4.2, 4.3, 5.1, and 5.2. Since equilibrium in $\mathsf{M}_\sigma^{\mathbf{1}}$ is not generally unique (see Remarks 4.2.4 and 5.1.2), the same holds true for $\mathsf{F}_\sigma^{\mathbf{1}}$. Uniqueness of equilibrium in $\mathsf{M}_\sigma^{\mathbf{0}}$ (see Remarks 4.3.1 and 5.2.1) implies uniqueness in $\mathsf{F}_\sigma^{\mathbf{0}}$.

Remark 6.1.2. Equilibria of $\mathsf{F}_0^{\mathbf{j}}$ with $L_E < L$ have the expected comparative statics properties. Parameter changes which raise GE in (12) or (14), respectively, shift the upward-sloping lines to the right. Parameter changes which raise expected firm profit in (19) shift the downward-sloping curve upward. In either case the equilibrium mass of entrepreneurs L_E goes up. For $\sigma_\nu^2 > 0$, the equilibrium mass of entrepreneurs in $\mathsf{F}_\sigma^{\mathbf{1}}$ has the expected comparative statics properties if $\Delta(L_E)$ intersects $\Gamma(L_E)$ from above. $L_E^{\mathbf{0}}$ in $\mathsf{F}_\sigma^{\mathbf{0}}$ has the expected comparative statics properties. Because of diminishing marginal productivity, wages rise when the number of firms increases: $d\hat{W}/dL_E = -F''(\hat{M})\hat{M}/L_E > 0$. Therefore, any parameter change that raises L_E (and leaves F and $\hat{M}$ unaffected) increases workers' wages.

Remark 6.1.3. If condition (17) is satisfied, then the $\Delta_0(L_E) = 0$ line in the left panel of Figure 5 is located to the left of the $\Delta_0^{\mathbf{0}}(L_E) = 0$ line and the equilibrium mass of entrepreneurs is smaller in $\mathsf{F}_0^{\mathbf{1}}$ than in $\mathsf{F}_0^{\mathbf{0}}$, i.e., $L_E^{\mathbf{1}} < L_E^{\mathbf{0}}$ (see the Appendix).

Remark 6.1.4. If $(L_E, I_E, I_D, I_M, P, \hat{M}, \hat{W})$ is an equilibrium of $\mathsf{F}_0^{\mathbf{j}}$, then there is an equilibrium of $\mathsf{F}_\sigma^{\mathbf{j}}$ with a mass of entrepreneurs close to L_E for σ_ν^2 positive but sufficiently small. This follows from Proposition 5.3 together with the fact that σ_ν^2 does not show up in (19).

Unemployment

Next, we turn to the model with unemployment. The model is denoted $\mathsf{U}_\varsigma^{\mathbf{j}}$, where, as before, $\mathbf{j}$ is equal to $\mathbf{1}$ or $\mathbf{0}$ depending on whether there is OC or not, respectively, and ς is equal to 0 or σ depending on whether σ_ν^2 is zero or positive. There is equilibrium unemployment due to union wage setting (cf. McDonald and Solow, 1981). Workers are organized in decentralized firm-level unions. They are spread evenly across firms, so there are $\hat{M}$ workers per firm. Unions monopolistically set the wage rate. Firms have the "right to manage" and choose the profit maximizing level of employment. The production function F is Cobb-Douglas: $F(m) = Am^{1-b}$, where $A > 0$ and $0 < b < 1$. If there is unemployment, the probability of being employed is $m/\hat{M}$ for each worker. Unions maximize workers' expected utility, taking their asset demands as given. In the Appendix we show that the gains from trading are separable from the gains of having a job, so that unions maximize the following expression which gives the expected gain from having a job:

$$-\log\left(1 - \frac{m}{\hat{M}}\left\{1 - \exp\left[-\rho(W - D)\right]\right\}\right). \tag{20}$$

$(L_E, I_E, I_D, I_M, P, m, W)$ is an *equilibrium* of $\mathsf{U}_\varsigma^{\mathbf{j}}$ if in addition to the conditions stated in Section 3, employment m maximizes $F(m) - Wm$, W maximizes (20) given firms' optimal choice of m, and there is unemployment (i.e., $m < \hat{M}$).

As in the standard right-to-manage model without asset demands, the conditions for optimum union wage setting and profit maximization jointly determine the wage rate, $\tilde{W}$ say, and employment per firm $\tilde{M}$, independently of the other variables which make up an equilibrium $(L_E, I_E, I_D, I_M, P, m, W)$ (see the Appendix). In particular, the real wage rate is rigid in that it does not depend on the mass of firms L_E/a. So while an increase in the mass of entrepreneurs does not affect employment at the firm level (the intensive margin), it increases aggregate employment by raising the mass of firms (the extensive margin).[11]

The model has a block-recursive structure, which makes equilibrium even easier to characterize than in the full employment model. Analogously as in $\mathsf{F}_\varsigma^{\mathbf{j}}$ (cf. (19)), define

[11] We have solved the model with alternative sources of wage rigidity: union wage setting aimed at maximizing the wage bill (cf. Dunlop, 1944) and efficiency wages due to moral hazard (cf. Shapiro and Stiglitz, 1984) or fairness considerations (cf. Solow, 1979). The subsequent results carry over one-to-one to these models (see the Supplementary appendix).

$s \equiv F(\tilde{M}) - \tilde{W}\tilde{M} + \tilde{s}$. Since $\tilde{M}$ and $\tilde{W}$ are constants, s is normal with mean

$$\bar{s} \equiv F(\tilde{M}) - \tilde{W}\tilde{M} + \hat{s} \tag{21}$$

and variance σ_s^2 and independent of L_E.

PROPOSITION 6.2. *Let $\bar{s}$ be given by (21). If (L_E, I_E, I_D, I_M, P) is an equilibrium of $\mathsf{M}_\varsigma^{\mathbf{j}}$ and $\tilde{M} < \hat{M}$, then $(L_E, I_E, I_D, I_M, P, \tilde{M}, \tilde{W})$ is an equilibrium of $\mathsf{U}_\varsigma^{\mathbf{j}}$ ($\mathbf{j} = \mathbf{0}, \mathbf{1}$, $\varsigma = 0, \sigma$).*

Remark 6.2.1. With $\bar{s}$ given by (21), the existence and uniqueness properties of $\mathsf{M}_\varsigma^{\mathbf{j}}$ carry over to $\mathsf{U}_\varsigma^{\mathbf{j}}$.

Remark 6.2.2. More entrepreneurship means more jobs: parameter changes that increase L_E at an equilibrium of $\mathsf{M}_\varsigma^{\mathbf{j}}$ increase L_E at an equilibrium of $\mathsf{U}_\varsigma^{\mathbf{j}}$. This follows from the fact that $\bar{s}$ does not change and the increase in the mass of entrepreneurs raises aggregate employment $\tilde{M}L_E/a$. In terms of Figure 5, the $\bar{s} = \mathrm{E}(s)$ curve is horizontal, and the upward-sloping curves shift to the right.

Remark 6.2.3. Changes in labor market parameters which reduce the equilibrium wage rate (and leave the production function unaffected) lead to increases in employment at the intensive margin (as $\tilde{M}$ rises) and at the extensive margin (whenever $\bar{s}$ raises L_E in $\mathsf{M}_\varsigma^{\mathbf{j}}$). This is due to the fact that expected firm profitability rises: $d\bar{s}/d\tilde{W} = [F'(\tilde{M}) - \tilde{W}]d\tilde{M}/d\tilde{W} - \tilde{M}$ or, using the condition for profit maximization, $d\bar{s} = -\tilde{M}d\tilde{W} > 0$. In terms of Figure 5, the horizontal $\bar{s} = \mathrm{E}(s)$-curve shifts upwards.

Remark 6.2.4. From the fact that $\bar{s}$ is independent of L_E, it follows that condition (17) in Remark 4.3.2 is sufficient for $L_E^{\mathbf{1}} < L_E^{\mathbf{0}}$.

Remark 6.2.5. From Proposition 5.3 and the fact that $\bar{s}$ is independent of L_E, it follows that if $(L_E, I_E, I_D, I_M, P, \tilde{M}, \tilde{W})$ is an equilibrium of $\mathsf{U}_0^{\mathbf{j}}$, then there is an equilibrium of $\mathsf{U}_\sigma^{\mathbf{j}}$ with a mass of entrepreneurs close to L_E for σ_ν^2 positive but sufficiently small.

7 Welfare

Social welfare SW is defined as the unweighted sum of *all* agents' *transformed* expected utilities $-\log\{-\mathrm{E}[U(\pi_i)]\}$. In particular, noise traders' well-being is evaluated using the same CARA utility function that also characterizes rational agents.[12] In model $\mathsf{M}_\varsigma^{\mathbf{j}}$ (with-

[12]Similarly, Albagli et al. (2017, p. 7) use expected dividends as the measure of welfare in their analysis of corporate risk taking in a model with risk neutral rational agents and noise traders.

out labor supply) this is equivalent to assuming that a fraction $N/(M+N)$ of the $M+N$ non-hipos experience a shock at the intermediate date that keeps them from carrying out their utility maximization problem and randomize their asset demands.[13] Given the presence of noise traders in the GS setup, the alternative to including them in the welfare analysis is to ignore them. This leads to similar conclusions (see the remarks to Proposition 7.2 below). Since the transformed expected utilities are equal to ρ times the respective CEs, SW is ρ times the unweighted sum of CEs and SW maximization is equivalent to maximization of the sum of the CEs. This is a standard welfare criterion in risky environments (see, e.g., Chambers and Echenique, 2012). It implies that SW rises one-for-one with endowments, which rules out that redistribution of safe income affects SW.

We conduct a second-best welfare analysis, which considers SW given agents' investment and labor market decisions. We investigate the impact of marginal changes in entrepreneurial activity on SW at an equilibrium with OC as well as the question of whether SW is higher with or without OC (Section 9 shows how to implement welfare-enhancing changes in the equilibrium mass of entrepreneurs). This section treats the cases of zero and small noise trader shocks analytically, the next section investigates large noise trader shocks numerically. Our overall conclusion is that the allocation of talent to trading tends to be excessive from a welfare point of view in our model. This conclusion holds true in the absence of labor market imperfections and is strongly reinforced by the inclusion of labor market frictions.

Constrained efficiency of equilibrium without noise trader shocks

SW is denoted $S^{\mathbf{1}}(L_E, \bar{s})$ or $S^{\mathbf{0}}(L_E, \bar{s})$, depending on whether the non-entrepreneurs act as dealers or as passive investors, respectively.

To begin with, consider the case of no noise trader shocks. Consider model $\mathsf{M}_0^{\mathbf{1}}$ with L_E ($< L$) entrepreneurs and $L - L_E$ (> 0) dealers. Entrepreneurs' expected utility is $-\log\{-\mathrm{E}[U(\pi_E)]\} = \rho e_L + \mathsf{GE} + \mathsf{GT}_E$. Substituting $\mathsf{GE} = \Delta_0(L_E)$ (see (12)), $\mathsf{GT}_E = z^2$, and the expression for z in (11) yields

$$-\log\{-\mathrm{E}[U(\pi_E)]\} = \rho e_L + \Delta_0(L_E) + \frac{\sigma_\varepsilon^2}{2}\left[\frac{\rho}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right)\right]^2. \tag{22}$$

[13]See also Allen (1984, p. 4), who attributes a risk neutral utility function to the noise traders in his welfare analysis of the GS model. We do not specify a boundedly rational decision rule that relates noise traders' asset demands to their utility function. Dow and Gorton (2008) survey fully rational models in which noise trader demand is derived from stochastic liquidity needs or portfolio churning by asset managers.

Passive investors' expected utility is $\rho e_M + z^2$. Similarly, as $\mathsf{GI} = 0$, dealers' expected utility is $\rho e_L + z^2$. Let N denote the mass of noise traders and e_N their endowment per capita. In the Appendix, we show that a noise trader's expected utility is

$$-\log\{-\mathrm{E}[U(\pi_N)]\} = \rho e_N + \rho\frac{\bar{\nu}}{N}\frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right) - \frac{1}{2}\left(\rho\frac{\bar{\nu}}{N}\right)^2 \sigma_\varepsilon^2. \tag{23}$$

Let $e = Le_L + Me_M + Ne_N$ denote aggregate endowments. Summing the expected utilities over all agents gives SW with OC:

$$S^{\mathbf{1}}(L_E, \bar{s}) = \rho e + L_E\Delta_0(L_E) + \frac{\rho^2\sigma_\varepsilon^2}{2(L+M)}\left[\left(\frac{L_E}{a}\right)^2 - \left(1 + \frac{L+M}{N}\right)\bar{\nu}^2\right] \tag{24}$$

for $L_E < L$.

Next, consider model $\mathsf{M}_0^{\mathbf{0}}$. Hipos who are not entrepreneurs act as passive investors. Following the same steps as above, entrepreneurs' expected utility can be written as

$$-\log\{-\mathrm{E}[U(\pi_E)]\} = \rho e_L + \Delta_0^{\mathbf{0}}(L_E) + \frac{\sigma_s^2 + \sigma_\varepsilon^2}{2}\left[\frac{\rho}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right)\right]^2. \tag{25}$$

Noise trader expected utility is

$$-\log\{-\mathrm{E}[U(\pi_N)]\} = \rho e_N + \rho\frac{\bar{\nu}}{N}\frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right) - \frac{1}{2}\left(\rho\frac{\bar{\nu}}{N}\right)^2 (\sigma_s^2 + \sigma_\varepsilon^2) \tag{26}$$

(see the Appendix). So SW is

$$S^{\mathbf{0}}(L_E, \bar{s}) = \rho e + L_E\Delta_0^{\mathbf{0}}(L_E) + \frac{\rho^2(\sigma_s^2 + \sigma_\varepsilon^2)}{2(L+M)}\left[\left(\frac{L_E}{a}\right)^2 - \left(1 + \frac{L+M}{N}\right)\bar{\nu}^2\right]. \tag{27}$$

In the full employment economy $\mathsf{F}_0^{\mathbf{j}}$, the expressions for SW, $S^{\mathbf{1}}$ and $S^{\mathbf{0}}$, contain the additional term $\rho\hat{W}M$, which represents workers' extra expected utility drawn from their safe labor income. In the economy with unemployment $\mathsf{U}_0^{\mathbf{j}}$, SW contains M times the expression in (20) as an additional term representing workers' aggregate gains from employment (see the Appendix).

PROPOSITION 7.1. *(i) Suppose a solution $L_E^{\mathbf{1}} < L$ to $\Delta_0(L_E^{\mathbf{1}}) = 0$ exists, a solution $(L_E^{\mathbf{1}}, \bar{s})$ with $L_E^{\mathbf{1}} < L$ to $\Delta_0(L_E^{\mathbf{1}}) = 0$ and (19) exists, and a solution $L_E^{\mathbf{1}} < L$ to $\Delta_0(L_E^{\mathbf{1}}) = 0$ with $\bar{s}$ given by (21) exists. Then $L_E^{\mathbf{1}}$ maximizes $S^{\mathbf{1}}$ on $[0, L)$ in $\mathsf{M}_0^{\mathbf{1}}$ and $\mathsf{F}_0^{\mathbf{1}}$, and $L_E^{\mathbf{1}}$ falls short of the value that maximizes $S^{\mathbf{1}}$ in $\mathsf{U}_0^{\mathbf{1}}$.*

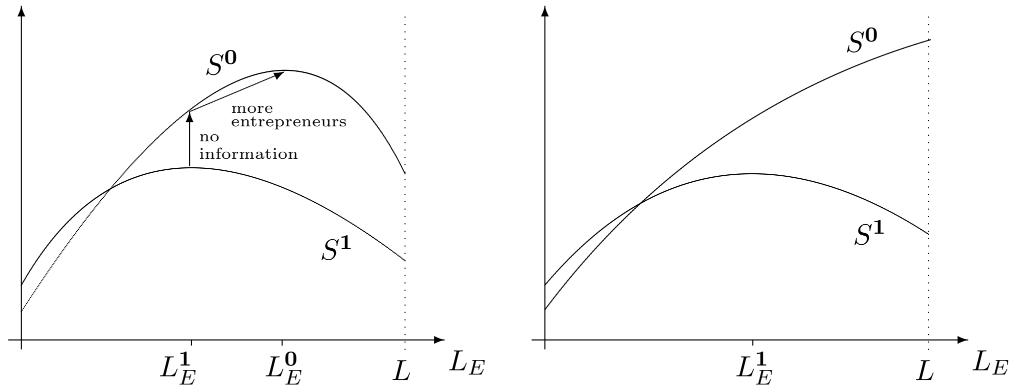

Figure 6: Social welfare

(ii) Suppose a solution $L_E^{\mathbf{0}} < L$ to $\Delta_0^{\mathbf{0}}(L_E^{\mathbf{0}}) = 0$ exists, a solution $(L_E^{\mathbf{0}}, \bar{s})$ with $\bar{s}$ given by (19) and $L_E^{\mathbf{0}} < L$ exists, and a solution $L_E^{\mathbf{0}} < L$ with $\bar{s}$ given by (21) exists. Then $L_E^{\mathbf{0}}$ maximizes $S^{\mathbf{0}}$ on $[0, L]$ in $\mathsf{M}_0^{\mathbf{0}}$ and $\mathsf{F}_0^{\mathbf{0}}$, and $L_E^{\mathbf{0}}$ falls short of the value that maximizes $S^{\mathbf{0}}$ in $\mathsf{U}_0^{\mathbf{0}}$.

This is the first of our two main results on the welfare implications of OC. The proof is in the Appendix. The result can be easily inferred from Figure 6. Without noise trader shocks and without labor market frictions is constrained efficient in the following sense: (i) given that the hipos who do not become entrepreneurs observe macro fundamentals and reveal them with their asset trades, the equilibrium mass of entrepreneurs is at its second-best optimum level in $\mathsf{M}_0^{\mathbf{1}}$ and $\mathsf{F}_0^{\mathbf{1}}$. (ii) The analogous result holds under the premise that the non-entrepreneurs act as passive investors, so that the asset price is uninformative (see the left panel of Figure 6). This is not surprising, since, conditional on the distribution of information, there are no market imperfections. The mere fact that entrepreneurs create jobs in $\mathsf{F}_0^{\mathbf{j}}$ does not imply that there is too little entrepreneurship. Viewed the other way round, the assertion of the proposition lends support to the welfare criterion chosen (with noise traders included): conditional on the distribution of information, it entails welfare maximization at the equilibria of the frictionless economies.[14]

Equilibrium is not constrained efficient, given the distribution of information, in the presence of labor market rigidities. An increase in the mass of entrepreneurs, starting at equilibrium, raises SW in $\mathsf{U}_0^{\mathbf{j}}$. This reflects the fact that job creation by entrepreneurs is

[14] Another implicit condition for second-best optimality of equilibrium in $\mathsf{M}_0^{\mathbf{1}}$ and $\mathsf{F}_0^{\mathbf{1}}$ is that the allocation of roles to agents is given. SW would be higher if the N noise traders acted as passive investors (see the Supplementary appendix).

inefficiently low in the presence of labor market frictions. As a result, measures which induce a shift of resources from professional trading to production are beneficial. These are third-best measures, however, as entrepreneurial activity is at its constrained optimum level once labor market frictions are removed, and SW is lower in $\mathsf{U}_0^{\mathbf{j}}$ than in $\mathsf{F}_0^{\mathbf{j}}$ for each L_E (see the Appendix).

Remark 7.1.1. If there is no $L_E^{\mathbf{0}} < L$ such that $\Delta_0^{\mathbf{0}}(L_E^{\mathbf{0}}) = 0$, then $S^{\mathbf{0}}$ increases monotonically with L_E in $\mathsf{M}_0^{\mathbf{0}}$, $\mathsf{F}_0^{\mathbf{0}}$ and $\mathsf{U}_0^{\mathbf{0}}$ (see the right panel of Figure 6). From part (ii) of Proposition 4.3, the equilibrium mass of entrepreneurs is L in this case and maximizes $S^{\mathbf{0}}$.

Remark 7.1.2. From (23) and (26), noise trader expected utility increases with L_E for $\bar{\nu} > 0$. Hence, if one deletes noise traders' expected utilities from the SW function, $S^{\mathbf{j}}$ decreases with L_E at $L_E^{\mathbf{j}}$, and there is too much entrepreneurship, conditional on information, at the margin in the absence of labor market frictions. This alleviates our overall conclusion that there is too little entrepreneurship in our model. However, the marginal impact of more entrepreneurship on SW is most often positive in the model with unemployment (see the numerical analysis below).

Remark 7.1.3. According to the proposition, SW attains a local maximum at the equilibrium mass of employment $L_E^{\mathbf{j}}$ in the absence of noise and labor market frictions. For σ_ν^2 small, there is a local maximum of $S^{\mathbf{j}}$ at a mass of entrepreneurs close to $L_E^{\mathbf{j}}$. This follows from the fact that all agents' expected utilities and, hence, SW $S^{\mathbf{j}}$ are continuous in σ_ν^2 at $\sigma_\nu^2 = 0$. Equations (7)–(9) and the formulas for the moments of P and z and $\mathrm{var}(\theta|\, w)$ in the Appendix imply continuity of $-\log\{-\mathrm{E}[U(\pi_i)]\}$ for $i = E, D, M$. In the Appendix, we derive a closed-form solution for noise trader expected utility $-\log\{-\mathrm{E}[U(\pi_N)]\}$ (which requires the use of a modified version of the Demange-Laroque, 1995, lemma) and show that it is also continuous in σ_ν^2 at $\sigma_\nu^2 = 0$.

Remark 7.1.4. From Proposition 5.3 and Remark 6.1.4, for σ_ν^2 small, there is an equilibrium of the economy without labor market frictions with a mass of entrepreneurs close to the noiseless equilibrium value. From Remark 7.1.3, SW is close to its local maximum then.

Remark 7.1.5. Recall that the model with OC behaves very differently with and without noise for L_E-values in the vicinity of L (see Remark 5.3.1). This also holds true for SW. $S^{\mathbf{1}}$ is continuous in σ_ν^2 at $\sigma_\nu^2 = 0$ for $L_E < L$ (see Remark 7.1.3). However, price informativeness converges to zero and, hence, $S^{\mathbf{1}}(L_E, \bar{s})$ converges to $S^{\mathbf{0}}(L, \bar{s})$ as $L_E \to L$ when σ_ν^2 is positive (whereas the price remains perfectly informative when $\sigma_\nu^2 = 0$).

Remark 7.1.6. Suppose that for $\sigma_\nu^2 = 0$ two equilibria exist, one with $L_E^{\mathbf{1}} < L$ and one with $L_E^{\mathbf{1}} = L$ (cf. Remark 4.2.4). Even though the former equilibrium yields SW at its *local* maximum, SW is higher at the latter equilibrium if $S^{\mathbf{0}}(L, \bar{s})$ exceeds $S^{\mathbf{1}}(L_E^{\mathbf{1}}, \bar{s})$ (a set of simple sufficient conditions is given in Remark 7.2.3). With σ_ν^2 positive but small, numerical analysis shows that SW at the equilibrium where almost all hipos become entrepreneurs is higher than SW at the local maximum (cf. Remark 5.3.2; see Figure 7 and the numerical analysis below).

Informational efficiency and real efficiency

According to Proposition 7.1, the equilibrium mass of entrepreneurs is at its constrained-efficient level in the absence of noise and labor market frictions conditional on whether prices are informative or not. We now proceed to compare the welfare levels at the two constrained optima. The welfare effect of switching from $L_E^{\mathbf{1}}$ entrepreneurs and $L - L_E^{\mathbf{1}}$ dealers to $L_E^{\mathbf{0}}$ entrepreneurs and no dealers can be decomposed into two effects (see Figure 6): the difference in $S^{\mathbf{0}}$ and $S^{\mathbf{1}}$ at $L_E^{\mathbf{1}}$ is the welfare effect of having no information about s, and the difference in $S^{\mathbf{0}}$ evaluated at $L_E^{\mathbf{0}}$ and at $L_E^{\mathbf{1}}$ is the impact of additional entrepreneurial activity.

The welfare effect of having no information about macro fundamentals s can be positive in our model (cf. Allen, 1984, Gorton, 1997, Goldstein and Yang, 2014, and Bond and García, 2018). The reason is that price informativeness leads to a concentration of risk at entrepreneurs. This is easy to see in the case without noise trader shocks. At an equilibrium with no short selling by noise traders (i.e., with $\bar{\nu} > 0$), each entrepreneur supplies a positive net amount of assets equal to $(1/a - I_E =)\ (1/a)[1-(L_E - a\bar{\nu})/(L+M)]$. While the selling price is safe without dealers (see (14)), it fluctuates with the observable part of the macro fundamentals s when there is a positive mass of dealers (see (10)). That is, the revelation of macro information makes it harder for entrepreneurs to get rid of the risks inherent in their production activity. This also sheds light on the multiplicity results in Remarks 4.2.4 and 5.3.2, which can be interpreted as the outcome of strategic complementarity. An increase in the mass of entrepreneurs reduces the mass of dealers and, hence, price informativeness and variability, making it more attractive to become an entrepreneur. If this effect dominates the two effects that reduced price informativeness also strengthens the incentives to become a dealer and that higher asset supply reduces the expected asset price, then multiplicity occurs.

From (22) with $\Delta_0(L_E^{\mathbf{1}}) = 0$ and (25) with $\Delta_0^{\mathbf{0}}(L_E^{\mathbf{0}}) \geq 0$, it is evident that entrepreneurs' expected utility is lower at an equilibrium with OC than at an equilibrium without OC if

$$\bar{\nu} < \frac{L_E^{\mathbf{1}}}{a} < \frac{L_E^{\mathbf{0}}}{a}. \tag{28}$$

The first inequality requires that rational agents do not short the asset at equilibrium with OC (their aggregate equilibrium assets holdings are $L_E^{\mathbf{1}}/a - \bar{\nu}$). The second inequality states that the mass of hipos who become entrepreneurs rises when they are precluded from becoming dealers (cf. Remark 4.3.2). Since the other rational agents obtain the same level of expected utility as entrepreneurs at equilibrium, they are also worse-off with OC. Thus, no OC is Pareto-preferred by the set of rational agents. From the continuity property of the model, the same holds true for small noise trader shocks.
The following result compares equilibrium SW, including noise traders' expected utilities, with and without OC in the absence of noise trader shocks:

PROPOSITION 7.2. *Let $L_E^{\mathbf{1}} < L$. Then the difference $S^{\mathbf{1}} - S^{\mathbf{0}}$ in the equilibrium values of SW in economies $\mathsf{M}_0^{\mathbf{1}}$ and $\mathsf{M}_0^{\mathbf{0}}$ is negative if*

$$0 < \bar{\nu} < \frac{L_E^{\mathbf{1}}}{a}, \ \frac{\bar{\nu}}{N} < \frac{1}{a}. \tag{29}$$

The same holds true in economies $\mathsf{F}_0^{\mathbf{1}}$ versus $\mathsf{F}_0^{\mathbf{0}}$ and, if $L_E^{\mathbf{1}} < L_E^{\mathbf{0}}$, in economies $\mathsf{U}_0^{\mathbf{1}}$ versus $\mathsf{U}_0^{\mathbf{0}}$.

This is our second main results on the welfare implications of OC. It is illustrated by Figure 6. The proposition states a set of very simple sufficient conditions which ensure that equilibrium SW is lower with than without OC in the absence of noise trader shocks. The second inequality in (29) also appears in (28). Jointly with the first one it says that neither noise traders nor rational agents short the asset. The third inequality states that individuals are small relative to corporations, in that noise traders' per capita demand for assets is less than the asset supply generated by a single entrepreneur. This inequality ensures that noise traders benefit, or at least are not too strongly negatively affected, by the absence of OC. To see why, note that with OC both the mean and the variance of noise traders' final wealth $\pi_N = e_N + (\theta - P)\bar{\nu}/N$ are lower than without OC (as the asset price P is more closely tied to fundamentals θ). The impact of lower expected wealth on expected utility is linear in $\bar{\nu}/N$. It dominates the quadratic effect of lower risk for $\bar{\nu}/N$ small enough. In models $\mathsf{M}_0^{\mathbf{j}}$ and $\mathsf{U}_0^{\mathbf{j}}$ with $L_E^{\mathbf{0}} < L$, $\bar{\nu}/N < 1/a$ ensures that noise trader demand is small enough in this sense. In model $\mathsf{F}_0^{\mathbf{j}}$, one has to make the stronger assumption $\rho(\bar{s}^{\mathbf{1}} - \bar{s}^{\mathbf{0}}) + \bar{\nu}/N < 1/a$ (see the Appendix). If one includes the second inequality in (28), then the SW differential $S^{\mathbf{1}} - S^{\mathbf{0}}$ is also negative in the model with unemployment.

Price informativeness has traditionally been considered conducive to real efficiency: “the ideal is a market in which prices provide accurate signals for resource allocation” (Fama, 1970, p. 383). Our model contributes to the strand of the literature that challenges this view. This literature dates back to Hirshleifer (1971), who emphasizes that the revelation of information precludes ex ante efficient mutual insurance (see also Allen, 1984, Hu and Qin, 2013, and Bond and García, 2018). Recent papers, cited in the introduction, argue that in certain contexts trading is pure rent seeking (e.g., Bolton et al., 2016, Section II, Glode and Lowery, 2016), while in others there is a genuine tradeoff between informational efficiency and real efficiency (Dow and Gorton, 1997, Bond et al., 2012, Goldstein and Yang, 2014, Bolton at al., 2016). In a nutshell, our model says that the gains in terms of more informed portfolio decisions do not outweigh the impact of higher asset price volatility on entrepreneurial activity and the “brain drain” from the real sector that go along with more informative asset prices in a GS environment with little noise.

Remark 7.2.1. Jointly (28) and (29) imply that no OC is Pareto-preferred to OC in $\mathsf{M}_0^{\mathbf{j}}$ and $\mathsf{U}_0^{\mathbf{j}}$: (28) ensures that rational agents are better-off and the final inequality in (29) ensures that the same holds true for noise traders.

Remark 7.2.2. An alternative set of sufficient conditions for $S^{\mathbf{1}} - S^{\mathbf{0}} < 0$ (in $\mathsf{M}_0^{\mathbf{j}}$ as well as in the models with a labor market) is

$$0 < \left(1 + \frac{L+M}{N}\right)^{\frac{1}{2}} \bar{\nu} < \frac{L_E^{\mathbf{0}}}{a},\ L_E^{\mathbf{1}} < L_E^{\mathbf{0}}.$$

These conditions strengthen the requirement that the aggregate supply of assets is large relative to noise traders' aggregate demand but make the condition that a single noise trader's demand falls short of the supply generated by a single entrepreneur obsolete.[15] In the numerical analysis in Section 8, we focus on the conditions in Proposition 7.2, so the proof of the sufficiency of the alternative conditions is delegated to the Supplementary appendix.

Remark 7.2.3. The conditions in (29) plus $L < M$ and $\sigma_\varepsilon^2 < \sigma_s^2$ jointly imply that in $\mathsf{M}_0^{\mathbf{j}}$ we have $(S^{\mathbf{0}}(L,\bar{s}) =)\, S^{\mathbf{1}}(L,\bar{s}) > S^{\mathbf{1}}(L_E^{\mathbf{1}},\bar{s})$ (see Remark 7.1.6; proof in the Supplementary appendix).

[15]In the economy with no noise traders and $M+N$ passive investors (see footnote 14) equilibrium SW is generally higher without than with OC (see the Supplementary appendix).

Remark 7.2.4. From Proposition 7.1, the assertion of Proposition 7.2 is also valid for the *maximum* (rather than *equilibrium*) welfare levels in $\mathsf{M}_0^{\mathbf{j}}$ and $\mathsf{F}_0^{\mathbf{j}}$. From the proof of Proposition 7.2 in the Appendix, it can be seen that the same holds true for $\mathsf{U}_0^{\mathbf{j}}$ (irrespective of whether $L_E^{\mathbf{1}} < L_E^{\mathbf{0}}$ or not).

Remark 7.2.5. Propositions 7.1 and 7.2 jointly imply that in the absence of noise trader shocks and labor market imperfections the only market intervention needed to achieve the second-best welfare-maximizing allocation is to keep hipos from becoming dealers. In the presence of labor market frictions, increases in entrepreneurship further raises SW (implementation is discussed in Section 9).

Remark 7.2.6. Together with Remark 7.1.3, it follows that for σ_ν^2 positive but small enough equilibrium SW is higher without than with OC if the conditions of Proposition 7.2 are satisfied. As $\bar{\nu}$ is the *mean* of noise traders' asset demand ν ($\sim \mathrm{N}(\bar{\nu}, \sigma_\nu^2)$), (29) then says that neither noise traders nor rational traders short the asset *on average*, and noise traders' *mean* per capita demand is less than the supply generated by a single entrepreneur.

8 Large noise trader shocks

For the case of small noise trader shocks the preceding section has established two results on the benefits of more entrepreneurship. First, at equilibrium a marginal increase in the mass of entrepreneurs has a positive effect on SW when there is unemployment due to labor market frictions (Proposition 7.1). Second, given simple sufficient conditions, such as (28), SW is higher if hipos who do not become entrepreneurs act as passive investors and not as dealers (Proposition 7.2). This section investigates the case of large noise trader shocks numerically. We show that the two results carry over to the vast majority of model specifications with reasonably large volatility of noise trader demand and that the impact of a marginal increase in entrepreneurship also tends to be positive in the absence of labor market imperfections.

Example and strategy

The closed-form solutions for the moments of P and z as functions of L_E in the Appendix allow the computation of the equilibrium mass of entrepreneurs $L_E^{\mathbf{j}}$ via (18), (19), and (21). The expressions corresponding to equations (24) and (27) in the models with noise trader shocks and with a labor market in the Appendix can then be used to compute equilibrium SW.

Figure 7 illustrates an example of model $\mathsf{M}^{\mathbf{j}}_{\sigma}$. The model parameters listed in the first column of Table 1 are set as multiples of the terms in the third column. The values in the second column are 0.05 or 0.2 for σ_ν and 0.2, 1, 1, 100, 0.5, 0.75, 0.1, 1, 10, in that order, for the other variables. Initial endowments are zero. The solid curves depict the levels of SW $S^{\mathbf{1}}$ and $S^{\mathbf{0}}$ with and without noise, respectively. The dashed curves give the corresponding welfare levels without noise. SW without OC $S^{\mathbf{0}}$ is hump-shaped.[16] By contrast, while $S^{\mathbf{1}}$ is also hump-shaped in the absence of noise, it converges to the level without OC as $L_E \to L$ when $\sigma_\nu > 0$ (see Remark 7.1.5). Consider the case with σ_ν equal to 5 percent of L/a (see the left panel of Figure 7). SW is very close to the noiseless case (for even smaller values of σ_ν, the solid and dashed curves become indistinguishable) except at the right end. $L^{\mathbf{1}}_E$ and $L^{\mathbf{0}}_E$ fall short of the constrained welfare-maximizing levels by 6.36 and 0.24 percent, respectively. The resulting deviations from maximum SW are less than 0.25 percent. So an increase in L_E starting at equilibrium has a weak positive impact SW in both cases. Equilibrium SW without OC is eleven times as high as equilibrium SW with OC (47.170 compared to 4.271). This confirms the conclusions of Propositions 7.1 and 7.2 for the noiseless case. In the example with $\sigma_\nu = 0.2L/a$ (see the right panel of Figure 7), while $L^{\mathbf{0}}_E$ is 6.43 percent lower than its constrained optimal value, $L^{\mathbf{1}}_E$ is 26.94 percent higher. So a marginal increase in L_E has a positive effect on SW starting at the equilibrium without OC but a negative effect on SW starting at the equilibrium with OC (observing negative marginal effects becomes more common as σ^2_ν gets large, but sill remains highly unlikely for σ_ν up to 20% of L/a, cf. Table 2). While the differences between equilibrium and optimum SW are relatively small (-13.680 versus -12.987 with OC, 19.473 versus 19.954 without OC), equilibrium SW is much larger without than with OC. As is also clear from Figure 7, while the presence of information collected by dealers reduces SW at equilibrium (i.e., at $L^{\mathbf{1}}_E$), information is socially beneficial at lower values of L_E in both cases.

We simulate the model for a large set of parameter values using Matlab. The strategy is as follows. We specify parameters such that for zero and small noise trader shocks, equilibrium exists and is unique and condition (29) holds, so that the analytical results on under-investment in entrepreneurship in Section 7 apply. We then consider increasing values of the volatility of noise trader demand σ_ν and check whether the under-investment results remain valid. In doing so, we restrict attention to the subset of the original parameter combinations for which equilibrium exists and is unique and (29) holds (i.e.,

[16]It can be shown that this is generally true for all $\sigma^2_\nu \geq 0$.

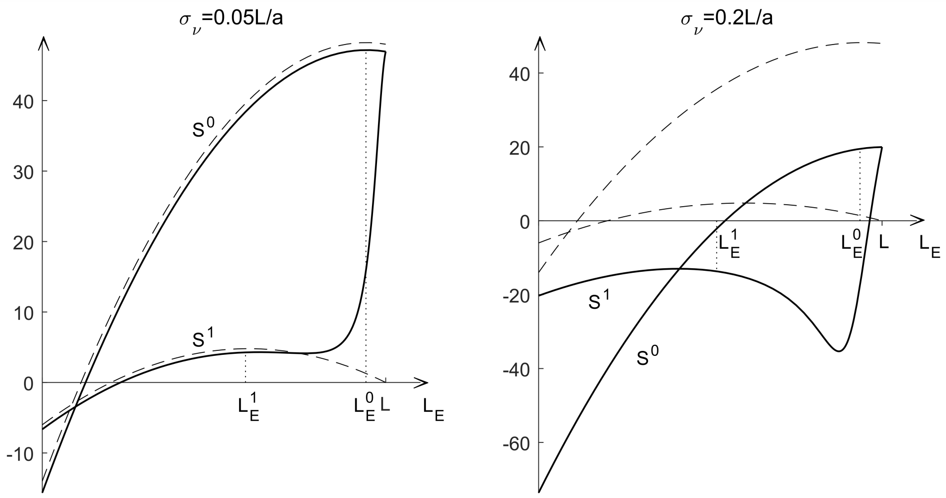

Figure 7: Example with $\sigma_\nu = 0.05L/a$ (left panel) or $\sigma_\nu = 0.2L/a$ (right panel)

no-one shorts the asset on average and corporations are large relative to individual noise traders).

Basic model

Consider first the model without a labor market $\mathsf{M}^{\mathbf{j}}_{\sigma}$. Agents' initial endowments e_i ($i = L, M, N$) play no role in the determination of equilibrium and SW, so they are set equal to zero. Table 1 summarizes the values chosen for the other parameters. The parameters in the first column are specified as multiples of the magnitudes in the third column.

Table 1: Parameter values in the simulation of $\mathsf{M}^{\mathbf{j}}_{\sigma}$

parameter	values	multiple of
σ_ν	0.001, 0.01, 0.05, 0.1, 0.2, 0.5	$\frac{L}{a}$
$\bar{\nu}$	0.001, 0.01, 0.05, 0.1, 0.2, 0.5	$\frac{L}{a}$
M	1, 2, 3, 5, 10, 100	L
N	1, 2, 3, 5, 10, 100	$0.25(L+M)$
L	100	1
$\bar{s} - \frac{\rho\sigma_s^2}{2a}$	0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99	$\frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L}{a} - \bar{\nu}\right)$
σ_ε^2	0.1, 0.25, 0.5, 0.75, 1	σ_s^2
σ_s^2	0.0001, 0.0005, 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, 1	$16\left(\frac{a}{\rho}\right)^2$
ρ	1	1
a	10	1

The maximum feasible supply of assets, which would materialize if all hipos became entrepreneurs, is L/a. For the standard deviation of noise traders' asset demand σ_ν, we consider different proportions of this maximum asset supply. While 10 percent or 20 percent appears to be a reasonable upper bound, we also consider $\sigma_\nu = 0.5L/a$ to allow for very large noise trader shocks. Similarly, the average noise trader demand for assets $\bar{\nu}$ rises from 0.1 percent up to 50 percent of L/a (so the first inequality in (29) is satisfied). The mass of passive investors M is a multiple of the mass of hipos L, so that talent is scarce and, from Remark 4.3.2, $L_E^{\mathbf{0}} > L_E^{\mathbf{1}}$ in the absence of noise. The mass of noise traders N is at least 25 percent of the mass of rational agents $L + M$, so that $N/a \geq 0.5L/a$. Jointly with the fact that $\bar{\nu} \leq 0.5L/a$, this implies the validity of the third inequality in (29).

The moments which appear in rational agents' expected utility functions $\mathrm{E}[U(\pi_i)]$ and in (18) are homogeneous of degree zero in σ_ν, $\bar{\nu}$, L, M, N, and L_E jointly (see the formulas in the Appendix). Since the former five variables vary proportionately with L in the simulations, so does the equilibrium mass of entrepreneurs L_E. Like rational agents' expected utilities, $\mathrm{E}[U(\pi_N)]$ is also unaffected by proportionate changes in σ_ν, $\bar{\nu}$, L, M, N, and L_E (see the Appendix). So whether there is under-investment in entrepreneurship or not is independent of the choice of L, and we confine attention to a single value, 100 say.

The mean of the macro fundamentals $\bar{s}$ is such that $\bar{s} - \rho\sigma_s^2/(2a)$ is in between zero and $\rho\sigma_\varepsilon^2(L/a - \bar{\nu})/(L + M)$. From (12), this ensures that in $\mathsf{M}_0^{\mathbf{1}}$ the equilibrium mass of entrepreneurs $L_E^{\mathbf{1}}$ is in between $a\bar{\nu}$ (> 0) and L, so that the second inequality, and hence all inequalities, in (29) are satisfied.

The variance of the unobservable productivity parameter σ_ε^2 is between 10 and 100 percent of the variance of the observable macro productivity parameter σ_s^2. It is well known that large gamble sizes potentially lead to very low CEs with CARA utility (see, e.g., Babcock et al., 1993, p. 19). In our model, when σ_s^2 and, hence, σ_ε^2 grow large, the variance of entrepreneurs' payoff π_E becomes so high that the CE of π_E becomes very small. To rule out excessive risk aversion on behalf of entrepreneurs, we restrict the admissible values for σ_s^2 appropriately. The CE of π_E is below the 95 percent confidence interval if $\mathrm{var}(\pi_E) < 16/\rho^2$ (see the Supplementary appendix). In $\mathsf{M}_0^{\mathbf{1}}$, the variance of $\mathrm{var}(\pi_E)$ is in $[\sigma_s^2/a^2, 1.25\sigma_s^2/a^2]$ (see the Supplementary appendix). So the CE is in the confidence interval at least in the case of minimum payoff variance if $\sigma_s^2 < 16(a/\rho)^2$. The admissible values of σ_s^2 are specified as proportions of this threshold. For all proportions below unity, we have $1.25\sigma_s^2/a^2 < 16/\rho^2$, so that the CE of π_E is inside the 95 percent confidence interval in the noiseless case.

Given the specification of $\bar{s} - \rho\sigma_s^2/(2a)$, σ_ε^2, and σ_s^2 as multiples of the respective expressions in the third column of Table 1 and the homogeneity properties of the model, the equilibrium levels of $L_E^{\mathbf{j}}$ and the resulting welfare levels $S_\varsigma^{\mathbf{j}}$ are independent of ρ and of a. So we can fix these two parameters arbitrarily. We set the coefficient of relative risk aversion ρ equal to unity, which is common in numerical analysis (cf. Biais et al., 2010), and $a = 10$.

The parameters in Table 1 yield a total of 87,480 combinations for each given value of σ_ν. By construction, an equilibrium exists and (29) is satisfied in the noiseless case. To rule out multiplicity in the noiseless case, we impose the condition (U_0, say, for "uniqueness") that $\Delta_0^{\mathbf{0}}(L) < \Gamma_0(L)$ (see Proposition 4.2), which leaves us with 68,112 combinations. (Almost all cases ruled out entail high values of σ_s^2. Multiplicity does not occur at all for the first five values of σ_s^2 in Table 1 but for more than 80 percent of the parameters with $\sigma_s^2 = 16(a/\rho)^2$.) From the analysis in Section 7, we know that the impact of a marginal change in the mass of entrepreneurs on SW is zero (Proposition 7.1) and SW is higher without OC (Proposition 7.2) without noise for each parameter combination.

We add two regularity conditions, which apply to the noisy version of the model. First, to make sure that a unique equilibrium with $L_E^{\mathbf{1}}/a > \bar{\nu}$ exists for each parameterization and rational agents do not go short on average, we focus on cases with $\Delta(a\bar{\nu}) > \Gamma(a\bar{\nu})$ and $\Delta(L) < \Gamma(L)$ and rule out cases with multiple solutions to $\Delta(L_E) = \Gamma(L_E)$ (see Propositions 5.1 and 5.2). This condition is called EU_σ (for "existence and uniqueness"). Second, while noise trader expected utility $\mathrm{E}[U(\pi_N)]$ is well defined in the case without noise (see (23) and (26)), it does not generally exist when $\sigma_\nu^2 > 0$ (as the square root of a potentially negative term, which depends on L_E, appears in the denominator of $\mathrm{E}[U(\pi_N)]$, see the Appendix). We confine attention to parameters for which noise trader expected utility is well defined at the equilibria with and without OC (condition BNU_σ, for "bounded noise trader utility"). Applying these two regularity conditions reduces the number of admissible parameter combinations further to between 61,290 for $\sigma_\nu = 0.05L/a$ and 17,529 for maximum noise trader demand volatility (where condition BNU_σ, applied after condition EU_σ, is responsible for less than 1,707 additional cases lost).

Table 2 summarizes the main results of the Matlab simulation of model $\mathsf{M}_\sigma^{\mathbf{j}}$. For given values of σ_ν relative to L/a, columns 2–4 report the following figures: the number of parameter combinations which satisfy conditions U_0–BNU_σ, the proportion of cases in which the marginal impact of an increase in L_E on SW $S^{\mathbf{1}}$ at an equilibrium with OC is positive, and the proportion of cases such that equilibrium SW is higher without than with OC. The overall conclusions are that noise turns the marginal impact of L_E on SW at equilibrium positive and the assertion of Proposition 7.2 carries over to model specifications with sizable noise trader shocks.

RESULT 8.1: *Consider the parameters in Table 1, with a standard deviation of noise trader demand σ_ν up to 50 percent of the maximum feasible asset supply L/a, and let conditions U_0, EU_σ, and BNU_σ hold. (i) The marginal impact of an increase in L_E at equilibrium is most often positive in $\mathsf{M}_\sigma^{\mathbf{1}}$ for σ_ν up to 20 percent of L/a. (ii) Equilibrium SW is most often higher in $\mathsf{M}_\sigma^{\mathbf{0}}$ than in $\mathsf{M}_\sigma^{\mathbf{1}}$.*

Table 2: Matlab simulation of $\mathsf{M}_\sigma^{\mathbf{j}}$

$\frac{\sigma_\nu}{L/a}$	# cases	$\frac{dS^{\mathbf{1}}(L_E^{\mathbf{1}})}{dL_E} > 0$	$S^{\mathbf{0}}(L_E^{\mathbf{0}}) > S^{\mathbf{1}}(L_E^{\mathbf{1}})$
0.001	68,112	99.63%	100.00%
0.01	67,926	99.63%	100.00%
0.05	61,290	98.89%	100.00%
0.1	52,287	97.72%	99.95%
0.2	39,180	89.04%	99.54%
0.5	17,529	17.75%	95.29%

Remark 8.1.1. SW is higher without than with OC not only in the majority of cases but also by a large amount. For σ_ν up to $0.2L/a$, the ration of the two SW levels is greater than ten on average. This huge difference is due to the fact that equilibrium SW without OC is very close to its maximum, which is far greater than equilibrium SW with OC (see the Supplementary appendix).

Remark 8.1.2. The overall conclusion that the allocation of talent to finance tends to be excessive does not hinge on the use of the *transformed* expected utilities (i.e., CEs) in the calculation of SW. With SW defined as the sum of all agents' *untransformed* expected utilities, the figures in columns 3 and 4 of Table 2 are similar.

Remark 8.1.3. Hipos earn sizable rents compared to passive investors in the model specifications with positive noise trader shocks (contrary to the noiseless case; cf. Remarks 4.1.5 and 4.3.1). To see this, consider the ratio of the CEs for entrepreneurs and passive investors $(-\log\{-\mathrm{E}[U(\pi_E)]\})/(-\log\{-\mathrm{E}[U(\pi_M)]\})$. This ratio equals 8.96 on average for $\sigma_\nu = 0.001L/a$ and rises strongly with increases in the volatility of noise trader demand (the differences are much less pronounced in terms of untransformed expected utility, however). There are two further interesting outcomes of the simulations. First, equilibrium asset price volatility tends to be lower rather than higher in the presence of noise trader shocks. The direct positive impact of volatility of noise trader demand ν on the asset price P is more than offset by the effect that noise makes P less sensitive to the macro fundamentals s. Second, the equilibrium mass of entrepreneurs is always higher in the absence of dealers.

Table 3: Parameter values in the simulations of $\mathsf{F}^{\mathbf{j}}_{\sigma}$ and $\mathsf{U}^{\mathbf{j}}_{\sigma}$

	parameter	values	multiple of
$\mathsf{F}^{\mathbf{j}}_{\sigma}$	$\bar{s}' - \frac{\rho\sigma_s^2}{2a}$	0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99	$\frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L}{a} - \bar{\nu}\right) + \bar{s}' - \bar{s}''$
	b	0.10, 0.25, 0.40, 0.55	1
	A	0.25, 0.5, 0.75, 1	$\frac{\rho\sigma_s^2}{2ab}\left(\frac{\bar{\nu}}{M}\right)^{1-b}$
$\mathsf{U}^{\mathbf{j}}_{\sigma}$	$\bar{s} - \frac{\rho\sigma_s^2}{2a}$	0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99	$\frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L}{a} - \bar{\nu}\right)$
	A	0.25, 0.5, 0.75, 1	$\frac{\rho\sigma_s^2}{2ab}\left(\frac{L}{aM}\right)^{1-b}$
	D	$\tilde{W} - \frac{1}{\rho}\ln\left(1 + \rho b\tilde{W}\right)$	1

Wages and employment

Next, consider the model with full employment $\mathsf{F}^{\mathbf{j}}_{\sigma}$. The model parameters which also appear in $\mathsf{M}^{\mathbf{j}}_{\sigma}$ take on the values in Table 1. New parameters are summarized in the upper part of Table 3.

Consider $\mathsf{F}^{\mathbf{1}}_{0}$. Let $\bar{s}'$ $(= bA(M/\bar{\nu})^{1-b} + \hat{s})$ and $\bar{s}''$ $(= bA(aM/L)^{1-b} + \hat{s})$ denote the values of $\bar{s}$ given by (19) for $L_E = a\bar{\nu}$ and $L_E = L$, respectively. To make sure that an equilibrium at which no-one goes short exists for $\sigma_\nu = 0$, $\Delta_0(L_E)$ has to be positive for $L_E = a\bar{\nu}$ and $\bar{s} = \bar{s}'$ and negative for $L_E = L$ and $\bar{s} = \bar{s}''$ (see Proposition 6.1). The choice of $\hat{s}$ and, hence, $\bar{s}'$ and $\bar{s}''$ as in the first row of the $\mathsf{F}^{\mathbf{j}}_{\sigma}$ part of Table 3 makes sure that this is the case: $\Delta_0(a\bar{\nu}) > 0 > \Delta_0(L)$ (from (12)).

The production function is $F(m) = Am^{1-b}$. To have a labor elasticity of about 0.75, we consider b around 0.25. A is set such that the two terms which add up to expected firm profit $\bar{s} = Abm^{1-b} + \hat{s}$ are of comparable size. From (12) and (19), the two terms are equally large at an equilibrium with $\sigma_\nu^2 = 0$ and $L^{\mathbf{1}}_E/a = \bar{\nu}$ for A equal to 0.5 times the term in the final column in Table 3. Accordingly we consider values of A scattered around this value.[17]

The total number of parameter combinations is 1,399,680 for each given σ_ν. As before, we require $\Gamma_0(L) > \Delta^{\mathbf{0}}_0(L)$ to rule out multiplicity in the noiseless case (condition U_0), which leaves us with 1,159,134 combinations. In order to have a unique equilibrium and no short sales on average in the noisy case too, we focus on parameters such that $\Delta(L_E)$ with $\bar{s}$ given by (19) is larger than $\Gamma(L_E)$ for $L_E = a\bar{\nu}$ and vice versa for $L_E = L$ and rule

[17]In the noisy simulations, the two terms account for 37 percent and 63 percent, respectively, of $\bar{s}$ on average for $\sigma_\nu = 0.05L/a$, for instance.

out cases with multiple intersections of $\Delta(L_E)$ with $\bar{s}$ given by (19) and $\Gamma(L_E)$ (condition EU_σ). As in the basic model, we omit parameter combinations which yield unbounded noise trader expected utility at equilibrium (condition BNU_σ). Applying EU_σ and BNU_σ leaves us with no less than 971,974 cases for σ_ν up to $0.2L/a$ and 661,087 cases for $\sigma_\nu = 0.5L/a$.

Columns 2 and 3 of Table 4 report the implications of the simulations for under- versus over-investment in entrepreneurship. The results are similar to the basic model without a labor market: the marginal impact of an increase in L_E at equilibrium is most often positive except for the maximum admissible value for σ_ν, and equilibrium SW is almost always higher without than with OC.

Table 4: Matlab simulation of $\mathsf{F}^{\mathbf{j}}_\sigma$ and $\mathsf{U}^{\mathbf{j}}_\sigma$

	$\mathsf{F}^{\mathbf{j}}_\sigma$		$\mathsf{U}^{\mathbf{j}}_\sigma$	
$\frac{\sigma_\nu}{L/a}$	$\frac{dS^{\mathbf{1}}(L_E^{\mathbf{1}})}{dL_E} > 0$	$S^{\mathbf{0}}(L_E^{\mathbf{0}}) > S^{\mathbf{1}}(L_E^{\mathbf{1}})$	$\frac{dS^{\mathbf{1}}(L_E^{\mathbf{1}})}{dL_E} > 0$	$S^{\mathbf{0}}(L_E^{\mathbf{0}}) > S^{\mathbf{1}}(L_E^{\mathbf{1}})$
0.001	99.62%	100.00%	100.00%	100.00%
0.01	99.62%	100.00%	100.00%	100.00%
0.05	91.85%	99.99%	99.99%	100.00%
0.1	84.92%	99.78%	99.93%	99.97%
0.2	74.26%	98.80%	99.63%	99.67%
0.5	23.00%	93.51%	97.00%	97.89%

Finally, consider model $\mathsf{U}^{\mathbf{j}}_\sigma$. The parameters are as in the model with full employment unless stated otherwise in the bottom part of Table 3. $\bar{s}$ is given by (21) and independent of L_E. $\hat{s}$ is chosen such that, similarly as in model $\mathsf{F}^{\mathbf{j}}_\sigma$, $\bar{s}$ is as in the table, so that $L_E^{\mathbf{1}}$ is in between $a\bar{\nu}$ and L for $\sigma_\nu = 0$. Similarly as before, A is chosen around the value (0.5 times the expression in the final column) for which $Ab(aM/L)^{1-b}$ and the expected macro shock $\hat{s}$ contribute equal amounts to expected firm profit $\bar{s}$ for $\sigma_\nu^2 = 0$ and $L_E^{\mathbf{1}}/a = \bar{\nu}$.

Workers' disutility of effort D is chosen such that labor demand equals labor supply exactly if all hipos become entrepreneurs. This condition, JC_σ say (for "job creation"), ensures that the job creation effect of entrepreneurship is operative at equilibrium: employment rises when L_E rises. The condition is implemented as follows. Profit maximization and JC_σ jointly imply $A(1-b)(aM/L)^{-b} = \tilde{W}$. For each single case (i.e., for given values of A, b, a, M, and L), we set the disutility of work D such that the wage rate which maximizes workers' expected utility (which also depends on ρ and b) coincides with this value of $\tilde{W}$. We maintain conditions U_0, EU_σ, and BNU_σ.

Out of a total of 1,399,680 parameter combinations for each value of σ_ν, 1,089,792 satisfy condition U_0. Applying conditions EU_σ and BNU_σ reduces the number of cases further, to no less than 836,500 for σ_ν up to $0.1L/a$, to 626,880 for $\sigma_\nu = 0.2L/a$, and to 280,464 for $\sigma_\nu = 0.5L/a$. As is clear from the analytical treatment of the noiseless case, the marginal impact of entrepreneurship is positive for a bigger proportion of cases than in the frictionless case. This holds true even for σ_ν equal to 50 percent of L/a. No OC is almost always better than OC. To sum up:

RESULT 8.2: *Consider the parameters in Tables 1 and 3, with a standard deviation of noise trader demand σ_ν up to 50 percent of the maximum feasible asset supply L/a, and let conditions U_0, EU_σ, BNU_σ, and, in $\mathsf{U}^{\mathbf{j}}_\sigma$, JC_σ hold. (i) The marginal impact of an increase in L_E at equilibrium is most often positive in $\mathsf{F}^{\mathbf{1}}_\sigma$ and almost universally positive in $\mathsf{U}^{\mathbf{1}}_\sigma$ for σ_ν up to 20 percent of L/a. (ii) Equilibrium SW is most often higher in $\mathsf{F}^{\mathbf{0}}_\sigma$ than in $\mathsf{F}^{\mathbf{1}}_\sigma$ and in $\mathsf{U}^{\mathbf{0}}_\sigma$ than in $\mathsf{U}^{\mathbf{1}}_\sigma$.*

9 Implementation

This section shows how to implement second-best allocations via subsidies and taxes. If there are too few entrepreneurs and too many dealers, what is required is a subsidy to the former and/or a tax on the latter. Note that a tax on deal*ers* is not a tax on trad*ing*, as it is not levied on other agents' asset trades. While this distinction is clear in theory, a tax on informed but not on uninformed trading would he hard to implement in practice. We think of it as a proxy for fiscal and regulatory measures aimed at constraining agents and institutions specialized in trading securities in secondary markets.

The implementation of the SW maximum is easy in the model without noise trader shocks. Suppose there are (possibly negative) lumpsum taxes t_i on type-i agents ($i = E, D, M, N$). (For simplicity, the tax on hipos who act as passive investors is zero.) Consider first models $\mathsf{M}^{\mathbf{j}}_0$ and $\mathsf{F}^{\mathbf{j}}_0$. For the sake of brevity, assume that without taxation there are unique equilibria with and without OC and $L^{\mathbf{1}}_E < L$. With OC any $t_D > \Gamma_0(L)/\rho$ (> 0) is a prohibitive tax on dealers, given that all other tax rates are zero, since it exceeds the gains from being informed GI, irrespective of whether $L_E < L$ (so that $\mathsf{GI} = 0$) or $L_E = L$ (so that $\mathsf{GI} = \Gamma_0(L)$). So the model with $t_D > \Gamma_0(L)$ behaves exactly like the model without OC. From Propositions 7.1 and 7.2, this is sufficient to implement the constrained-efficient allocation. As the tax base is zero, the budget is balanced. In model $\mathsf{U}^{\mathbf{j}}_0$, the second-best optimal mass of entrepreneurs, $\hat{L}^{\mathbf{0}}_E$ say, exceeds the free markets equilibrium value $L^{\mathbf{0}}_E$ (see Proposition 7.1). The equilibrium mass of entrepreneurs is determined by $\Delta_0^{\mathbf{0}}(L_E) - \rho t_E = 0$ (or it is equal to L, if $\Delta_0^{\mathbf{0}}(L) - \rho t_E > 0$), so the optimum value is achieved with $t_E = \Delta_0^{\mathbf{0}}(\hat{L}_E^{\mathbf{0}})/\rho$ (< 0). That is, a tax on dealers and a subsidy to entrepreneurs are required in order to implement the second-best optimum with labor market frictions. The budget can be balanced by taxing workers and/or noise traders: $t_M M + t_N N = -t_E \hat{L}_E^{\mathbf{0}}$ (> 0).

The following result states how to implement an allocation with L'_E entrepreneurs in the presence of noise:

PROPOSITION 9.1. *Let the mass of dealers be zero and I_D arbitrary. Given L'_E ($0 < L'_E \leq L$), let $t_D > \Gamma(L)/\rho$, $t_E = \Delta^{\mathbf{0}}(L'_E)/\rho$, $t_M \geq 0$, and $t_M M + t_N N = -t_E L'_E$. Then: (i) L'_E, I_E given by (13), I_D, $I_M = I_E$, and P given by (14) with ν instead of $\bar{\nu}$ are an equilibrium of $\mathsf{M}_\sigma^{\mathbf{1}}$. (ii) With s and $\bar{s}$ given by (19), $(L'_E, I_D, I_E, I_M, P, \hat{M}, \hat{W})$ is an equilibrium of $\mathsf{F}_\sigma^{\mathbf{1}}$. (iii) With $\bar{s}$ given by (21), $(L'_E, I_D, I_E, I_M, P, \tilde{M}, \tilde{W})$ is an equilibrium of $\mathsf{U}_\sigma^{\mathbf{1}}$.*

The reasoning is the same as in the noiseless case. As $\Gamma(L_E)$ is strictly increasing, the tax t_D ensures that no-one chooses to be a dealer and makes the models with OC and taxes behave like the ones without OC and taxes. The tax t_E (or subsidy $-t_E$) implies that L'_E hipos decide to become entrepreneurs. And t_M and t_N balance the budget, where non-negativity of t_M ensures that hipos are not better-off as passive investors than as entrepreneurs.

Remark 9.1.1. Suppose the maximum of $S^{\mathbf{0}}(L_E)$ exceeds the maximum of $S^{\mathbf{1}}(L_E)$, for instance because the conditions of Proposition 7.2 are satisfied and σ_ν^2 is small enough. Setting L'_E equal to the value $\hat{L}_E^{\mathbf{0}}$ that maximizes $S^{\mathbf{0}}(L_E)$, the taxes in the proposition implement the constrained-optimal solution. The tax on dealers t_D is positive. For σ_ν^2 small, the tax on entrepreneurs t_E is small in the absence of labor market frictions and negative in $\mathsf{U}_\sigma^{\mathbf{1}}$ (viz., close to the noiseless case).

Remark 9.1.2. The tax on dealers in the proposition is prohibitive. In practice, taxes and regulations are likely not to be aimed at shutting down professional trading completely. Given non-prohibitive taxes, the equilibrium mass of entrepreneurs L_E is determined by $\Delta(L_E) - \rho t_E = \Gamma(L_E) - \rho t_D$. Given a target level of entrepreneurship L'_E, this equality gives the tax differential needed to implement L'_E: $t_D - t_E = [\Gamma(L'_E) - \Delta(L'_E)]/\rho$. Suppose, as usual, that $\Delta(L_E)$ intersects $\Gamma(L_E)$ from above and the intersection is unique (as in Figure 3). Then, in order to achieve L'_E higher than the equilibrium level without taxes, a preferential tax treatment of entrepreneurship compared to trading is required: $t_D - t_E > 0$. This can be achieved by taxing dealers and/or by subsidizing entrepreneurship.

10 Conclusion

Judging from the viewpoint of the seminal GS model, it appears hard to argue that financial trading attracts too little talent. The increased price informativeness brought about by professional trading is not necessarily beneficial on net. And even if it is, the benefits tend to be outweighed by the opportunity cost in terms of foregone output and lower wages or lower employment in manufacturing. This lends some support to popular concerns (several of them cited in the Introduction) that financial trading attracts too much, rather than too little, talent. The main policy conclusion is thus that, weighing informational efficiency of asset prices against output, wages, and jobs, competition for talent should at least not be distorted in favor of the former, for instance by explicit or implicit subsidies and guarantees to institutions specialized in trading. In order to make a case for support for financial trading, one would have to invoke other channels, which imply a positive impact of financial trading on firm decisions, e.g., by providing incentives to issue valuable assets or by providing the opportunity to link pay to performance.

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Appendix: Proofs

The derivations below make use of Lemma 1 in Demange and Laroque (1995, p. 252), which says that for jointly normal random variables x and y,

$$\mathrm{E}\left[\exp\left(x-y^2\right)\right] = \frac{\exp\left\{\mathrm{E}(x)+\frac{1}{2}\operatorname{var}(x)-\frac{[\mathrm{E}(y)+\operatorname{cov}(x,y)]^2}{1+2\operatorname{var}(y)}\right\}}{[1+2\operatorname{var}(y)]^{\frac{1}{2}}}. \tag{A.1}$$

Equation (1):
Making use of (A.1) with y identically equal to zero, we have

$$\mathrm{E}[U(\pi_E)|\,P] = -\exp\left(-\rho\left\{e_L+\frac{P}{a}+[\mathrm{E}(\theta|\,P)-P]\,I_E\right\}+\frac{\rho^2}{2}\operatorname{var}(\theta|\,P)I_E^2\right). \tag{A.2}$$

Maximizing with respect to I_E yields the first equation in (1). Given that passive investors trade on the same information as entrepreneurs, $I_M = I_E$ follows from the fact that optimum investment does not depend on endowments. Similarly, using $\mathrm{E}(\theta|\, s) = s$ and $\mathrm{var}(\theta|\, s) = \sigma_\varepsilon^2$,

$$\mathrm{E}[U(\pi_D)|\, s, P] = -\exp\left\{-\rho\,[e_L + (s-P)I_D] + \frac{\rho^2}{2}\sigma_\varepsilon^2 I_D^2\right\}, \tag{A.3}$$

and maximization with respect to I_D yields the expression for I_D in (1).

Equations (7)–(9):
Substituting for I_E from (1) into (A.2) yields

$$\mathrm{E}[U(\pi_E)|\, P] = -\exp\left\{-\rho e_L - \rho\frac{P}{a} - \frac{[\mathrm{E}(\theta|\, P) - P]^2}{\mathrm{var}(\theta|\, P)} + \frac{1}{2}\frac{[\mathrm{E}(\theta|\, P) - P]^2}{\mathrm{var}(\theta|\, P)}\right\}.$$

The expression in the main text follows from collecting terms and the definition of z. Taking expectations, using the law of iterated expectations, we obtain

$$\mathrm{E}[U(\pi_E)] = -\exp(-\rho e_L)\,\mathrm{E}\left[\exp\left(-\rho\frac{P}{a} - z^2\right)\right]. \tag{A.4}$$

Since P and z are normal, we can apply (A.1) to get

$$\mathrm{E}\left[\exp\left(-\rho\frac{P}{a} - z^2\right)\right] = \frac{\exp\left\{\mathrm{E}\left(-\rho\frac{P}{a}\right) + \frac{1}{2}\,\mathrm{var}\left(-\rho\frac{P}{a}\right) - \frac{\left[\mathrm{E}(z) + \mathrm{cov}\left(-\rho\frac{P}{a}, z\right)\right]^2}{1+2\,\mathrm{var}(z)}\right\}}{[1+2\,\mathrm{var}(z)]^{\frac{1}{2}}}.$$

Substituting this into (A.4) and rearranging terms gives

$$\mathrm{E}[U(\pi_E)] = -\exp(-\rho e_L)\frac{\exp\left\{-\frac{\rho}{a}\,\mathrm{E}(P) + \frac{1}{2}\left(\frac{\rho}{a}\right)^2\,\mathrm{var}(P) - \frac{\left[\mathrm{E}(z) - \frac{\rho}{a}\,\mathrm{cov}(P,z)\right]^2}{1+2\,\mathrm{var}(z)}\right\}}{[1+2\,\mathrm{var}(z)]^{\frac{1}{2}}},$$

which can be rewritten as (7).
A passive investor's expected utility (8) is obtained analogously. The terms containing P/a drop out.
Substituting for I_D from (1) into (A.3) yields

$$\mathrm{E}[U(\pi_D)|\, s, P] = -\exp\left\{-\rho e_L - \left[\frac{s-P}{(2\sigma_\varepsilon^2)^{\frac{1}{2}}}\right]^2\right\}. \tag{A.5}$$

Let $y \equiv (s-P)/(2\sigma_\varepsilon^2)^{1/2}$. Notice that $\mathrm{E}(s|\,P) = \mathrm{E}(\theta|\,P)$ and $\mathrm{var}(s|\,P) = \mathrm{var}(\theta|\,P) - \sigma_\varepsilon^2$, so that $\mathrm{E}(y|\,P) = [\mathrm{E}(\theta|\,P) - P]/(2\sigma_\varepsilon^2)^{1/2}$ and $\mathrm{var}(y|\,P) = [\mathrm{var}(\theta|\,P) - \sigma_\varepsilon^2]/(2\sigma_\varepsilon^2)$. Applying the law of iterated expectations to (A.5) and using (A.1), we obtain

$$\mathrm{E}[U(\pi_D)|\,P] = -\exp(-\rho e_L)\frac{\exp\left\{-\frac{\frac{[\mathrm{E}(\theta|\,P)-P]^2}{2\sigma_\varepsilon^2}}{\frac{\mathrm{var}(\theta|\,P)}{\sigma_\varepsilon^2}}\right\}}{\left[\frac{\mathrm{var}(\theta|\,P)}{\sigma_\varepsilon^2}\right]^{\frac{1}{2}}}.$$

The expression in the main text follows upon rearranging terms and using the definition of z. Taking expectations, again making use of the law of iterated expectations and (A.1), yields

$$\mathrm{E}[U(\pi_D)] = -\exp(-\rho e_L)\left[\frac{\sigma_\varepsilon^2}{\mathrm{var}(\theta|\,w)}\right]^{\frac{1}{2}}\frac{\exp\left\{-\frac{[\mathrm{E}(z)]^2}{1+2\,\mathrm{var}(z)}\right\}}{[1+2\,\mathrm{var}(z)]^{\frac{1}{2}}},$$

which can be rewritten as (9).

Expected utility of a single dealer in $\mathbf{M_0^1}$:
A dealer's expected utility conditional on s is given by (A.5). Taking expectations, using (A.1) with x identically equal to zero, the fact that P is safe, and (15) yields the expression in the main text.

Proof of Remark 4.3.2:
$L_E^{\mathbf{0}} < L$ implies

$$\bar{s} - \frac{\rho(\sigma_s^2+\sigma_\varepsilon^2)}{L+M}\left(\frac{L}{a} - \bar{\nu}\right) < 0.$$

Together with condition (17), it follows that

$$\bar{s} - \frac{\rho(\sigma_s^2+\sigma_\varepsilon^2)}{2a} < 0. \tag{A.6}$$

From (12) and (14),

$$\Delta_0^{\mathbf{0}}(L_E^{\mathbf{1}}) = -\frac{\rho}{a}\frac{\sigma_s^2}{\sigma_\varepsilon^2}\left[\bar{s} - \frac{\rho(\sigma_s^2+\sigma_\varepsilon^2)}{2a}\right].$$

Suppose $L_E^{\mathbf{0}} \le L_E^{\mathbf{1}}$. Since $\Delta_0^{\mathbf{0}}(L_E)$ is a decreasing function, this implies $\Delta_0^{\mathbf{0}}(L_E^{\mathbf{1}}) \le 0$. This contradicts (A.6), so $L_E^{\mathbf{0}} > L_E^{\mathbf{1}}$.

The functions $\mathbf{\Delta(L_E)}$ and $\mathbf{\Gamma(L_E)}$:
Let

$$\alpha \equiv \frac{L-L_E}{\rho\sigma_\varepsilon^2},\ \beta \equiv \frac{L_E+M}{\rho\,\mathrm{var}(\theta|\,w)},\ \gamma \equiv \frac{1}{\alpha^2\sigma_s^2+\sigma_\nu^2}. \tag{A.7}$$

Then,

$$
\begin{aligned}
\operatorname{var}(\theta|\, w) &= \gamma\sigma_s^2\sigma_\nu^2 + \sigma_\varepsilon^2 && \text{(A.8)}\\
\mathrm{E}(P) &= \bar{s} - \frac{\frac{L_E}{a} - \bar{\nu}}{\alpha+\beta} && \text{(A.9)}\\
\operatorname{var}(P) &= \frac{1}{\gamma}\left(\frac{1+\alpha\beta\gamma\sigma_s^2}{\alpha+\beta}\right)^2 && \text{(A.10)}\\
\mathrm{E}(z) &= \frac{\frac{L_E}{a} - \bar{\nu}}{(\alpha+\beta)\left[2\left(\gamma\sigma_s^2\sigma_\nu^2+\sigma_\varepsilon^2\right)\right]^{\frac{1}{2}}} && \text{(A.11)}\\
\operatorname{var}(z) &= \frac{\gamma\left(\sigma_\nu^2\right)^2}{(\alpha+\beta)^2 2\left(\gamma\sigma_s^2\sigma_\nu^2+\sigma_\varepsilon^2\right)} && \text{(A.12)}\\
\operatorname{cov}(P,z) &= -\frac{(1+\alpha\beta\gamma\sigma_s^2)\sigma_\nu^2}{(\alpha+\beta)^2\left[2\left(\gamma\sigma_s^2\sigma_\nu^2+\sigma_\varepsilon^2\right)\right]^{\frac{1}{2}}}. && \text{(A.13)}
\end{aligned}
$$

Note that $\bar{s}$ affects only $\mathrm{E}(P)$. For future reference, also notice that the right-hand sides of (A.8)–(A.13) are homogeneous of degree zero in σ_ν, $\bar{\nu}$, L, M, N, and L_E jointly.
By definition, $w = \alpha s + \nu$, so $\operatorname{var}(w) = \alpha^2\sigma_s^2 + \sigma_\nu^2$ and $\operatorname{cov}(s,w) = \alpha\sigma_s^2$. Substituting this into (5) yields

$$\operatorname{var}(\theta|\, w) = \sigma_s^2\left(1 - \frac{\alpha^2\sigma_s^2}{\alpha^2\sigma_s^2+\sigma_\nu^2}\right) + \sigma_\varepsilon^2.$$

Equation (A.8) follows from the definition of γ in (A.7). From the definition of α in (A.7), it follows that $\operatorname{var}(\theta|\, w)$ decreases when L_E increases. That is, an increase in the mass of dealers increases informational efficiency. $\operatorname{var}(\theta|\, w)$ converges to σ_ε^2 as σ_ν^2 goes to zero.
According to the updating rule for the mean of a normal random variable, $\mathrm{E}(\theta|\, w) = \mathrm{E}(\theta) + [\operatorname{cov}(\theta,w)/\operatorname{var}(w)][w - \mathrm{E}(w)]$. Using $\mathrm{E}(\theta) = \bar{s}$, $\operatorname{var}(w) = \alpha^2\sigma_s^2 + \sigma_\nu^2$, $\operatorname{cov}(\theta,w) = \alpha\sigma_s^2$, and the definitions of w, α, and γ,

$$\mathrm{E}(\theta|\, w) = \bar{s} + \alpha\gamma\sigma_s^2\left[\alpha(s-\bar{s}) + \nu - \bar{\nu}\right]. \tag{A.14}$$

This can be used to rewrite (2) as

$$P = \frac{\alpha s + \nu + \beta\left\{\bar{s} + \alpha\gamma\sigma_s^2\left[\alpha(s-\bar{s}) + \nu - \bar{\nu}\right]\right\} - \frac{L_E}{a}}{\alpha+\beta}$$

or, rearranging terms,

$$P = \bar{s} + \frac{(1+\alpha\beta\gamma\sigma_s^2)\left[\alpha(s-\bar{s}) + \nu - \bar{\nu}\right] - \left(\frac{L_E}{a} - \bar{\nu}\right)}{\alpha+\beta}. \tag{A.15}$$

Equation (A.9) follows upon taking expectations.

The variance of P is

$$\text{var}(P) = \frac{\left(1 + \alpha\beta\gamma\sigma_s^2\right)^2 \left(\alpha^2\sigma_s^2 + \sigma_\nu^2\right)}{(\alpha+\beta)^2}.$$

Using the definition of γ, we obtain (A.10).

Substituting $\text{E}(\theta|\, w)$ from (A.14) and P from (A.15) into the definition of z yields

$$z = \frac{\bar{s} + \alpha\gamma\sigma_s^2\left[\alpha(s-\bar{s}) + \nu - \bar{\nu}\right] - \bar{s} - \frac{(1+\alpha\beta\gamma\sigma_s^2)[\alpha(s-\bar{s})+\nu-\bar{\nu}] - \left(\frac{L_E}{a} - \bar{\nu}\right)}{\alpha+\beta}}{\left[2\left(\gamma\sigma_s^2\sigma_\nu^2 + \sigma_\varepsilon^2\right)\right]^{\frac{1}{2}}}.$$

Simplifying terms, using $1 - \alpha^2\gamma\sigma_s^2 = \gamma\sigma_\nu^2$, we get

$$z = \frac{-\gamma\sigma_\nu^2\left[\alpha(s-\bar{s}) + \nu - \bar{\nu}\right] + \frac{L_E}{a} - \bar{\nu}}{(\alpha+\beta)\left[2\left(\gamma\sigma_s^2\sigma_\nu^2 + \sigma_\varepsilon^2\right)\right]^{\frac{1}{2}}}. \tag{A.16}$$

Taking expectations yields (A.11). Clearly, $\text{E}(z) \geq 0$ if $L_E/a \geq \bar{\nu}$.

The variance of z is

$$\text{var}(z) = \frac{\gamma^2\left(\sigma_\nu^2\right)^2\left(\alpha^2\sigma_s^2 + \sigma_\nu^2\right)}{(\alpha+\beta)^2 2\left(\gamma\sigma_s^2\sigma_\nu^2 + \sigma_\varepsilon^2\right)}.$$

Equation (A.12) follows from the definition of γ.

From (A.15) and (A.16),

$$\text{cov}(P,z) = \frac{1+\alpha\beta\gamma\sigma_s^2}{\alpha+\beta}\frac{-\gamma\sigma_\nu^2}{(\alpha+\beta)\left[2\left(\gamma\sigma_s^2\sigma_\nu^2 + \sigma_\varepsilon^2\right)\right]^{\frac{1}{2}}}\left(\alpha^2\sigma_s^2 + \sigma_\nu^2\right).$$

Equation (A.13) follows from the definition of γ. Using (A.10) and (A.12), (A.13) can be rewritten as $\text{cov}(P,z) = -[\text{var}(P)\,\text{var}(z)]^{1/2}$, which proves that P and z are perfectly negatively correlated.

Moments of P and z with no OC:

The first and second moments of P and z without OC are:

$$\begin{aligned}
\mathrm{E}(P) &= \bar{s} - \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right) && \text{(A.17)}\\
\mathrm{var}(P) &= \left(\frac{\rho}{L+M}\right)^2 (\sigma_s^2 + \sigma_\varepsilon^2)^2 \sigma_\nu^2 && \text{(A.18)}\\
\mathrm{E}(z) &= \frac{\rho}{L+M}\left(\frac{\sigma_s^2 + \sigma_\varepsilon^2}{2}\right)^{\frac{1}{2}}\left(\frac{L_E}{a} - \bar{\nu}\right) && \text{(A.19)}\\
\mathrm{var}(z) &= \left(\frac{\rho}{L+M}\right)^2 \frac{(\sigma_s^2 + \sigma_\varepsilon^2)}{2}\sigma_\nu^2 && \text{(A.20)}\\
\mathrm{cov}(P,z) &= -\left(\frac{\rho}{L+M}\right)^2 \frac{(\sigma_s^2 + \sigma_\varepsilon^2)^{\frac{3}{2}}}{2^{\frac{1}{2}}}\sigma_\nu^2. && \text{(A.21)}
\end{aligned}$$

Clearly, the right-hand sides of (A.17)–(A.21) are homogeneous of degree zero in σ_ν, $\bar{\nu}$, L, M, N, and L_E.
Equations (A.17) and (A.18) follow immediately from (14).
Inserting $\mathrm{E}(\theta|\, w) = \bar{s}$ and $\mathrm{var}(\theta|\, w) = \sigma_s^2 + \sigma_\varepsilon^2$ into the definition of z in (6) yields

$$z = \frac{\rho}{L+M}\left(\frac{\sigma_s^2 + \sigma_\varepsilon^2}{2}\right)^{\frac{1}{2}}\left(\frac{L_E}{a} - \nu\right). \qquad \text{(A.22)}$$

Equations (A.19) and (A.20) follow immediately.
Equations (14) and (A.22) yield (A.21).
Equations (A.17)-(A.21) hold true for all $L_E \leq L$. This is because in the absence of OC there is no jump in the informational efficiency of prices at $L_E = L$.
It is easily checked that the moments in (A.17)-(A.21) coincide with their counterparts (A.9)-(A.13) for $L_E = L$, so that $\Delta^{\mathbf{0}}(L) = \Delta(L)$.
Differentiating the composite function defined by (18) and (A.17)–(A.21) shows that $\Delta^{\mathbf{0}}(L_E)$ is a linear, decreasing function:

$$(\Delta^{\mathbf{0}})'(L_E) = \left(\frac{\rho}{a}\right)^2 \frac{(\sigma_s^2 + \sigma_\varepsilon^2)}{L+M}\left[\frac{\left(\frac{\rho}{L+M}\right)^2(\sigma_s^2 + \sigma_\varepsilon^2)\sigma_\nu^2}{1 + \left(\frac{\rho}{L+M}\right)^2(\sigma_s^2 + \sigma_\varepsilon^2)\sigma_\nu^2} - 1\right] < 0. \qquad \text{(A.23)}$$

Notice that, since $\mathrm{E}(P)$ is a linear function of $\bar{s}$ and the other moments are independent of $\bar{s}$, the equilibrium value of L_E is a linear function of $\bar{s}$.

Proof of Proposition 5.3:
For $L_E < L$, inserting the limits of the functions defined in (A.7)–(A.13) as $\sigma_\nu^2 \to 0$ into

(18) yields $\Delta(L_E) \to \Delta_0(L_E)$ pointwise, $\Gamma(L_E) \to 0$ pointwise, and $\Delta^{\mathbf{0}}(L_E) \to \Delta_0^{\mathbf{0}}(L_E)$ pointwise (see the Supplementary appendix).

For $L_E = L$, we get (A.17)-(A.21) evaluated at $L_E = L$. Inserting these expressions into the left-hand side of (18) and taking the limit as $\sigma_\nu^2 \to 0$ yields $\Delta^{\mathbf{0}}(L) \to \Delta_0^{\mathbf{0}}(L)$, and substitution into the right-hand side of (18) and taking the limit $\sigma_\nu^2 \to 0$ yields $\Gamma_0(L)$ (see the Supplementary appendix).

Consider first the case with OC. An equilibrium of $\mathsf{M}_0^{\mathbf{1}}$ with $L_E^{\mathbf{1}} < L$ entrepreneurs is determined by the requirement that $\Delta_0(L_E^{\mathbf{1}}) = 0$ (see Proposition 4.1). Since $\Delta(L_E)$ and $\Gamma(L_E)$ converge pointwise to $\Delta_0(L_E)$ and zero, respectively, for all $L_E < L$, there is $L_E^{\mathbf{1}\prime}$ arbitrarily close to $L_E^{\mathbf{1}}$ such that $\Delta(L_E^{\mathbf{1}\prime}) = \Gamma(L_E^{\mathbf{1}\prime})$ for σ_ν^2 small enough (see the left panel of Figure **??**). From part (i) of Proposition 5.1, there is an equilibrium of $\mathsf{M}_\sigma^{\mathbf{1}}$ with $L_E^{\mathbf{1}\prime}$ entrepreneurs.

An equilibrium with mass of entrepreneurs L exists if $\Delta_0^{\mathbf{0}}(L) > \Gamma_0(L)$ (see Proposition 4.2). Since $\Delta(L) \to \Delta_0^{\mathbf{0}}(L)$ and $\Gamma(L) \to \Gamma_0(L)$ for $\sigma_\nu^2 \to 0$, if $\Delta_0^{\mathbf{0}}(L) > \Gamma_0(L)$, then $\Delta(L) > \Gamma(L)$ for σ_ν^2 small enough, and, from part (ii) of Proposition 5.1, there is an equilibrium of $\mathsf{M}_\sigma^{\mathbf{1}}$ at which all hipos become entrepreneurs.

Next, consider the case of no OC. An equilibrium of $\mathsf{M}_0^{\mathbf{0}}$ with $L_E^{\mathbf{0}} < L$ entrepreneurs and $L - L_E^{\mathbf{0}}$ hipos acting as passive investors exists if there is $L_E^{\mathbf{0}}$ such that $\Delta_0^{\mathbf{0}}(L_E^{\mathbf{0}}) = 0$ (see part (i) of Proposition 4.3). Since $\Delta^{\mathbf{0}}(L_E)$ converges pointwise to $\Delta_0^{\mathbf{0}}(L_E)$ as $\sigma_\nu^2 \to 0$, for σ_ν^2 small enough, there is $L_E^{\mathbf{0}\prime}$ arbitrarily close to $L_E^{\mathbf{0}}$ such that $\Delta^{\mathbf{0}}(L_E^{\mathbf{0}\prime}) = 0$ (see the right panel of Figure **??**) and, from part (i) of Proposition 5.2, an equilibrium of $\mathsf{M}_\sigma^{\mathbf{0}}$.

An equilibrium with no OC at which all hipos become entrepreneurs exists if $\Delta_0^{\mathbf{0}}(L) > 0$ (see part (ii) of Proposition 4.3). Since $\Delta(L) \to \Delta_0^{\mathbf{0}}(L)$ as $\sigma_\nu^2 \to 0$, it follows from part (ii) of Proposition 5.1 that there is an equilibrium of $\mathsf{M}_\sigma^{\mathbf{0}}$ with L entrepreneurs for σ_ν^2 small enough.

Proof that entrepreneurs maximize profit:

Suppose all firms employ m workers and make profit $\theta = \tilde{\theta} + F(m) - Wm$. Consider a single firm which deviates with employment $m' \neq m$, thereby creating a new asset. Given the fact that the productivity shock is additive, an arbitrage argument is sufficient in order to prove that this is not beneficial to the firm. The deviating firm makes profit $\theta' = \theta + \delta$, where

$$\delta \equiv F(m') - F(m) - W(m' - m).$$

Since the firm's profit differs from the other firms' profit by the non-random amount δ, buying a fraction λ of the firm's shares at cost $\lambda P'$ generates the same cash flow as buying a fraction λ of one of the other firms at cost λP and storing $\lambda\delta$. Hence, arbitrage-freeness implies $P' = P + \delta$.

The final wealth of an entrepreneur who employs m' workers in each of his firms is $\pi'_E = e_L + P'/a + (\theta - P)I_E = \pi_E + \delta/a$. Since the price differential δ is non-random, we have

$$\mathrm{E}[U(\pi'_E)] = \exp\left(-\rho\frac{\delta}{a}\right)\mathrm{E}[U(\pi_E)].$$

So the entrepreneurs' objective is to maximize δ or, equivalently, $F(m') - Wm'$.

Proof of Remark 6.1.3:

Let $\bar{s}^{\mathbf{1}}$ and $\bar{s}^{\mathbf{0}}$ denote the equilibrium levels of $\bar{s}$ in $\mathsf{F}_0^{\mathbf{1}}$ and $\mathsf{F}_0^{\mathbf{0}}$, respectively. $L_E^{\mathbf{0}} < L$ and (17) jointly imply that (A.6) holds for $\bar{s} = \bar{s}^{\mathbf{0}}$:

$$\bar{s}^{\mathbf{0}} - \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{2a} < 0. \tag{A.24}$$

From (12) and (14),

$$\Delta_0^{\mathbf{0}}(L_E^{\mathbf{1}}) = -\frac{\rho}{a}\frac{\sigma_s^2}{\sigma_\varepsilon^2}\left[-\frac{\sigma_\varepsilon^2}{\sigma_s^2}\left(\bar{s}^{\mathbf{0}} - \bar{s}^{\mathbf{1}}\right) + \bar{s}^{\mathbf{1}} - \frac{\rho\left(\sigma_s^2 + \sigma_\varepsilon^2\right)}{2a}\right].$$

Suppose $L_E^{\mathbf{0}} \le L_E^{\mathbf{1}}$. This implies $\bar{s}^{\mathbf{0}} \ge \bar{s}^{\mathbf{1}}$ and $\Delta_0^{\mathbf{0}}(L_E^{\mathbf{1}}) \le 0$ (since $\bar{s}$ and $\Delta_0^{\mathbf{0}}(L_E)$ are decreasing functions of L_E). This contradicts (A.24), so $L_E^{\mathbf{0}} > L_E^{\mathbf{1}}$.

Proof that the real wage is rigid in $\mathsf{U}_\varsigma^{\mathbf{j}}$:

Whenever firms pay a uniform wage W and employment per firm m falls short of the mass of workers per firm $\hat{M}$, a worker's expected utility is

$$\mathrm{E}[U(\pi_M)] = \exp(-\rho e_M)\left(\frac{m}{\hat{M}}\left\{1 - \exp\left[-\rho(W - D)\right]\right\} - 1\right)\mathrm{E}\left\{\exp\left[-\rho(\theta - P)I_M\right]\right\}. \tag{A.25}$$

The same argument as in the full employment case proves that firms choose the profit maximizing level of employment $m = (F')^{-1}(W)$ if unions set a uniform wage W. If a union deviates with a wage rate $W' \ne W$, firm profit becomes $\theta' = \theta + \delta$, where $\delta = F(m') - F(m) - W'm' + Wm$, and arbitrage implies that the firm value is $P' = P + \delta$. By the same argument as above, firms choose $m' = (F')^{-1}(W')$. Hence, unions anticipate that firms react to the wage they set by choosing employment on the labor demand curve. Equation (A.25) is unions' objective function. The three factors on the right-hand side are

non-random. So, irrespective of workers' subsequent investment decision, unions' objective is to maximize the second factor or, equivalently, the expression in (20).
The firm's labor demand curve is $m = [A(1-b)/W]^{1/b}$. Maximization of (20) subject to this constraint is equivalent to maximization of $bW^{-1/b}\{1 - \exp[-\rho(W - D)]\}$. Setting the derivative equal to zero yields

$$W^{-\frac{1}{b}-1} \exp\left[-\rho(W-D)\right]\left\{1 + \rho b W - \exp\left[\rho(W-D)\right]\right\} = 0.$$

There is a unique positive $\tilde{W}$ ($> D$) such that the condition holds for $W = \tilde{W}$, and the derivative changes from positive to negative at $\tilde{W}$, so that $\tilde{W}$ maximizes expected utility. Employment is $\tilde{M} = [A(1-b)/\tilde{W}]^{1/b}$. There is unemployment if $\tilde{M} < \hat{M}$.

Noise trader utility for $\sigma_\nu^2 = 0$:
Each noise trader invests ν/N, so final wealth is $\pi_N = e_N + (\theta - P)\bar{\nu}/N$. For $\sigma_\nu^2 = 0$, from (10) and (14), respectively,

$$\theta - P = \frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L_E^{\mathbf{1}}}{a} - \bar{\nu}\right) + \varepsilon \tag{A.26}$$

at an equilibrium of $\mathsf{M}_0^{\mathbf{1}}$ and

$$\theta - P = s - \bar{s} + \frac{\rho\left(\sigma_s^2 + \sigma_\varepsilon^2\right)}{L+M}\left(\frac{L_E^{\mathbf{0}}}{a} - \bar{\nu}\right) + \varepsilon \tag{A.27}$$

at an equilibrium of $\mathsf{M}_0^{\mathbf{0}}$ with $L_E^{\mathbf{0}} < L$ entrepreneurs. Notice that OC raises the expectation of $\theta - P$ but at the same time makes it risky.
Using $\pi_N = e_N + (\theta - P)\bar{\nu}/N$ and (A.26), noise traders' expected utility in the case with OC can be written as

$$\mathrm{E}[U(\pi_N)] = -\exp\left(-\rho e_N\right)\mathrm{E}\left(\exp\left\{\left[\frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L_E^{\mathbf{1}}}{a} - \bar{\nu}\right) + \varepsilon\right]\frac{\bar{\nu}}{N}\right\}\right).$$

As final wealth is normal, we can apply (A.1) to get (23).
Following the same steps, using (A.27) instead of (A.26), we get noise traders' expected utility in the absence of OC (26).
With L instead of $L_E^{\mathbf{0}}$ the formulas also apply to the case in which all hipos become entrepreneurs with no OC.
Since (10) and (14) are also valid in the economies with full employment or unemployment, the formulas are likewise valid in these models.

Proof of Proposition 7.1:

Denote the functions defined in (12) and (14) as $\Delta_0(L_E, \bar{s})$ and $\Delta_0^{\mathbf{0}}(L_E, \bar{s})$ (instead of $\Delta_0(L_E)$ and $\Delta_0^{\mathbf{0}}(L_E)$, respectively, as in the main text). The dependence on $\bar{s}$ is essential when analyzing the impact of changes in L_E on SW in the economy with full employment, where $\bar{s}$ depends on L_E via (19).

Consider first $\mathsf{M}_0^{\mathbf{1}}$. (i) Differentiating (24) twice yields

$$\frac{\partial S^{\mathbf{1}}(L_E, \bar{s})}{\partial L_E} = \Delta_0(L_E, \bar{s}) \tag{A.28}$$

and $\partial^2 S^{\mathbf{1}}(L_E, \bar{s})/\partial L_E^2 < 0$. So if $\Delta_0(L_E^{\mathbf{1}}, \bar{s}) = 0$ for some $L_E^{\mathbf{1}}$ $(< L)$, then $S^{\mathbf{1}}$ us a hump-shaped function of L_E with its maximum at $L_E^{\mathbf{1}}$. Otherwise $S^{\mathbf{1}}$ is monotonically increasing.

In $\mathsf{F}_0^{\mathbf{1}}$, workers' expected utility is $-\log\{-\mathrm{E}[U(\pi_M)]\} = \rho(e_M + \hat{W}) + z^2$, and SW is

$$\tilde{S}^{\mathbf{1}}(L_E, \bar{s}) = S^{\mathbf{1}}(L_E, \bar{s}) + \rho \hat{W} M,$$

where $\bar{s}$ is given by (19) (in another slight abuse of notation, SW is denoted $S^{\mathbf{1}}$ in the main text and "includes the term $\rho\hat{W}M$"). Differentiating with respect to L_E yields

$$\frac{d\tilde{S}^{\mathbf{1}}(L_E, \bar{s})}{dL_E} = \frac{\partial \tilde{S}^{\mathbf{1}}(L_E, \bar{s})}{\partial L_E} + \frac{\partial \tilde{S}^{\mathbf{1}}(L_E, \bar{s})}{\partial \bar{s}} \frac{d\bar{s}}{dL_E} + \rho \frac{d\hat{W}}{dL_E} M.$$

Using $\partial \tilde{S}^{\mathbf{1}}/\partial L_E = \partial S^{\mathbf{1}}/\partial L_E$, $\partial \tilde{S}^{\mathbf{1}}/\partial \bar{s} = \rho L_E/a$,

$$\frac{d\bar{s}}{dL_E} = \left[F'(\hat{M}) - \hat{W}\right] \frac{d\hat{M}}{dL_E} - \frac{d\hat{W}}{dL_E}\hat{M},$$

$F'(\hat{M}) = \hat{W}$, and $\hat{M} = M/(L_E/a)$, it follows that

$$\frac{d\tilde{S}^{\mathbf{1}}(L_E, \bar{s})}{dL_E} = \frac{\partial S^{\mathbf{1}}(L_E, \bar{s})}{\partial L_E}.$$

From (A.28) and the fact that $\Delta_0(L_E^{\mathbf{1}}, \bar{s}) = 0$ at equilibrium, it follows that $d\tilde{S}^{\mathbf{1}}(L_E^{\mathbf{1}}, \bar{s})/dL_E = 0$.

In $\mathsf{U}_0^{\mathbf{1}}$, a worker is employed with probability $\tilde{M}/\hat{M}$, in which case he gets extra payoff $\tilde{W} - D$. From (A.25) with $m = \tilde{M}$ and $W = \tilde{W}$, SW is (with the same slight abuse of notation as above)

$$\tilde{S}^{\mathbf{1}}(L_E, \bar{s}) = S^{\mathbf{1}}(L_E, \bar{s}) - M \log \left(1 - \frac{\tilde{M}}{\hat{M}} \left\{ 1 - \exp[-\rho(\tilde{W} - D)] \right\} \right), \tag{A.29}$$

and $\bar{s}$ is given by (21), i.e., it does not depend on L_E. The log term on the right-hand side is decreasing in $\hat{M} = M/(L_E/a)$ (since the term in braces is negative). So $\partial \tilde{S}^{\mathbf{1}}(L_E, \bar{s})/\partial L_E > \partial S^{\mathbf{1}}(L_E, \bar{s})/\partial L_E$ for all L_E, and $\partial \tilde{S}^{\mathbf{1}}(L_E^{\mathbf{1}}, \bar{s})/\partial L_E > \partial S^{\mathbf{1}}(L_E^{\mathbf{1}}, \bar{s})/\partial L_E = 0$.
(ii) Next, suppose the $L - L_E$ non-entrepreneurs act as passive investors. SW $S^{\mathbf{0}}$ is given by (27). Taking the first two derivative yields

$$\frac{\partial S^{\mathbf{0}}(L_E, \bar{s})}{\partial L_E} = \Delta_0^{\mathbf{0}}(L_E, \bar{s})$$

and $\partial^2 S^{\mathbf{0}}(L_E, \bar{s})/\partial L_E^2 < 0$. That is, if there is $L_E^{\mathbf{0}} < L$ such that $\Delta_0^{\mathbf{0}}(L_E^{\mathbf{0}}, \bar{s}) = 0$, then it maximizes $S^{\mathbf{0}}(L_E, \bar{s})$ on $[0, L]$. Otherwise $S^{\mathbf{0}}$ is monotonically increasing on the interval $[0, L]$.
In $\mathsf{F}_0^{\mathbf{0}}$ SW $\tilde{S}^{\mathbf{0}}(L_E, \bar{s})$ encompasses the additional term $\rho \hat{W} M$ representing the contribution of wage income to passive investors' expected utility. Using the same results as in the previous case, it follows that $d\tilde{S}^{\mathbf{0}}(L_E, \bar{s})/dL_E = \partial S^{\mathbf{0}}(L_E, \bar{s})/\partial L_E$. In $\mathsf{U}_0^{\mathbf{0}}$ SW encompasses the log term on the right-hand side of (A.29), which is increasing in L_E, so that $d\tilde{S}^{\mathbf{0}}(L_E^{\mathbf{0}}, \bar{s})/dL_E > 0$.

SW in the models with or without labor market frictions:
Compare model $\mathsf{F}_\sigma^{\mathbf{1}}$ to $\mathsf{U}_\sigma^{\mathbf{1}}$. To do so, let $D = 0$, in the latter, since this is assumed in the former. The levels of SW differ for two reasons. First, L_E and, hence, $\bar{s}$ differ (see (19) and (21)). Second, workers' extra utility is given by $\rho \hat{W}$ in $\mathsf{F}_\sigma^{\mathbf{1}}$ and by M times (20) in $\mathsf{U}_\sigma^{\mathbf{1}}$. As noise traders' utility, GI, GT_E, GT_M, and var(P) are independent of $\bar{s}$, making use of (A.9), we get the difference in SW for given L_E:

$$\tilde{S}^{\mathbf{1}} - \tilde{S}^{\mathbf{1}} = \frac{L_E}{a} \left(\rho \left[F(\hat{M}) - F(\tilde{M}) + \tilde{W}\tilde{M} \right] + \hat{M} \log \left\{ 1 - \frac{\tilde{M}}{\hat{M}} \left[1 - \exp\left(-\rho \tilde{W} \right) \right] \right\} \right).$$

As $F(\hat{M}) - F(\tilde{M}) > 0$, the SW difference is positive if the sum composed of the remaining terms on the right-hand side is positive, which is equivalent to

$$\frac{\tilde{M}}{\hat{M}} \left[1 - \exp(-\rho \tilde{W}) \right] < 1 - \exp\left(-\rho \tilde{W} \frac{\tilde{M}}{\hat{M}} \right).$$

Both sides of this inequality take on the same value for $\tilde{M}/\hat{M} = 0$ and for $\tilde{M}/\hat{M} = 1$. The validity of the inequality for all $\tilde{M}/\hat{M}$ in between follows from the fact that the left-hand side is linear and the right-hand side is strictly concave.

The proof for $\mathsf{F}^{\mathbf{0}}_{\sigma}$ compared to $\mathsf{U}^{\mathbf{0}}_{\sigma}$ proceeds analogously, using (A.17) instead of (A.9). Comparing $\mathsf{F}^{\mathbf{0}}_{\varsigma}$ to $\mathsf{U}^{\mathbf{0}}_{\varsigma}$ gives the same results.

Noise trader utility for $\sigma^2_\nu > 0$:

For the calculation of noise traders' expected utility for $\sigma^2_\nu > 0$, we need a variant of the Demange-Laroque (1995) lemma (A.1): for normal random variables x and y,

$$\mathrm{E}\left[\exp\left(x + y^2\right)\right] = \frac{\exp\left\{\mathrm{E}(x) + \frac{1}{2}\,\mathrm{var}(x) + \frac{[\mathrm{E}(y)+\mathrm{cov}(x,y)]^2}{1-2\,\mathrm{var}(y)}\right\}}{[1-2\,\mathrm{var}(y)]^{\frac{1}{2}}} \tag{A.30}$$

for $\mathrm{var}(y) < 1/2$. $\mathrm{E}[\exp(x + y^2)]$ does not exist otherwise.

The proof is in the Supplementary appendix.

A noise trader's expected utility conditional on ν is

$$\mathrm{E}[U(\pi_N)\,|\nu] = -\exp(-\rho e_N)\exp\left[-\rho(\theta - P)\frac{\nu}{N}\,\middle|\,\nu\right].$$

Applying (A.1) or (A.30) with $x = (\theta - P)\nu/N$ (which is normal) and $y = 0$ yields

$$\mathrm{E}[U(\pi_N)\,|\nu] = -\exp(-\rho e_N)\exp\left[-\rho\frac{\nu}{N}\mathrm{E}(\theta - P|\,\nu) + \frac{1}{2}\left(\rho\frac{\nu}{N}\right)^2\mathrm{var}(\theta - P|\,\nu)\right]$$

or, using $\mathrm{E}(\theta|\,\nu) = \bar{s}$ and the standard updating rules,

$$\begin{aligned}\mathrm{E}[U(\pi_N)\,|\nu] \;=\; -\exp(-\rho e_N)\exp\Bigg(&\overbrace{-\rho\frac{\nu}{N}\left[\bar{s} - \mathrm{E}(P) + \frac{\mathrm{cov}(P,\nu)}{\sigma^2_\nu}\bar{\nu}\right]}^{=\Phi}\\ &\underbrace{+\rho\left(\frac{\nu}{N}\right)^2\left\{\frac{\mathrm{cov}(P,\nu)}{\sigma^2_\nu}N + \frac{\rho}{2}\left[\mathrm{var}(\theta - P) - \frac{(\mathrm{cov}(\theta - P,\nu))^2}{\sigma^2_\nu}\right]\right\}}_{=\Psi^2}\Bigg).\end{aligned}$$

Let Φ be defined as the first term in the sum in the second exponential and Ψ as the square root of the second term, so that

$$
\begin{aligned}
\mathrm{E}(\Phi) &= -\rho\frac{\bar{\nu}}{N}\left[\bar{s}-\mathrm{E}(P)+\frac{\mathrm{cov}(P,\nu)}{\sigma_\nu^2}\bar{\nu}\right] \\
\mathrm{var}(\Phi) &= \left[\frac{\mathrm{E}(\Phi)}{\bar{\nu}}\right]^2\sigma_\nu^2 \\
\mathrm{E}(\Psi) &= \rho^{\frac{1}{2}}\frac{\bar{\nu}}{N}\left\{\frac{\mathrm{cov}(P,\nu)}{\sigma_\nu^2}N+\frac{\rho}{2}\left[\mathrm{var}(\theta-P)-\frac{(\mathrm{cov}(\theta-P,\nu))^2}{\sigma_\nu^2}\right]\right\}^{\frac{1}{2}} \\
\mathrm{var}(\Psi) &= \left[\frac{\mathrm{E}(\Psi)}{\bar{\nu}}\right]^2\sigma_\nu^2 \\
\mathrm{cov}(\Phi,\Psi) &= \frac{\mathrm{E}(\Phi)\mathrm{E}(\Psi)}{\bar{\nu}^2}\sigma_\nu^2.
\end{aligned}
$$

Since both Φ and Ψ are normal, from the law of iterated expectations and (A.30),

$$
\mathrm{E}[U(\pi_N)] = -\exp(-\rho e_N)\frac{\exp\left\{\mathrm{E}(\Phi)+\frac{1}{2}\mathrm{var}(\Phi)+\frac{[\mathrm{E}(\Psi)+\mathrm{cov}(\Phi,\Psi)]^2}{1-2\,\mathrm{var}(\Psi)}\right\}}{[1-2\,\mathrm{var}(\Psi)]^{\frac{1}{2}}}. \tag{A.31}
$$

In the presence of dealers $\mathrm{E}(P)$ is given by (A.9). From (A.15), the other moments in the definitions of Φ and Ψ are

$$
\begin{aligned}
\mathrm{cov}(P,\nu) &= \frac{1+\alpha\beta\gamma\sigma_s^2}{\alpha+\beta}\sigma_\nu^2 \\
\mathrm{var}(\theta-P) &= \left(1-\alpha\frac{1+\alpha\beta\gamma\sigma_s^2}{\alpha+\beta}\right)^2\sigma_s^2+\left(\frac{1+\alpha\beta\gamma\sigma_s^2}{\alpha+\beta}\right)^2\sigma_\nu^2+\sigma_\varepsilon^2 \\
\mathrm{cov}(\theta-P,\nu) &= -\mathrm{cov}(P,\nu).
\end{aligned}
$$

Given (A.7) and these formulas, $\mathrm{E}[U(\pi_N)]$ can be expressed as a composite function of L_E alone.

In the absence of dealers $\mathrm{E}(P)$ is given by (A.17). From (14) with ν instead of $\bar{\nu}$, the other moments in the definitions of Φ and Ψ are

$$
\begin{aligned}
\mathrm{cov}(P,\nu) &= \frac{\rho\,(\sigma_s^2+\sigma_\epsilon^2)}{L+M}\sigma_\nu^2 \\
\mathrm{var}(\theta-P) &= \sigma_s^2+\sigma_\varepsilon^2+\left[\frac{\rho\,(\sigma_s^2+\sigma_\epsilon^2)}{L+M}\right]^2\sigma_\nu^2 \\
\mathrm{cov}(\theta-P,\nu) &= -\mathrm{cov}(P,\nu).
\end{aligned}
$$

Given these formulas, $\mathrm{E}[U(\pi_N)]$ can be expressed as a composite function of L_E alone.

$\mathrm{E}[U(\pi_N)]$ goes to minus infinity as $\mathrm{var}(\Psi)$ rises towards 1/2. For $\mathrm{var}(\Psi) \geq 1/2$, noise trader expected utility is undefined.

It is easily checked that $\mathrm{E}[U(\pi_N)]$ is homogeneous of degree zero in σ_ν, $\bar{\nu}$, L, M, N, and L_E.

Continuity of noise trader expected utility at $\sigma_\nu^2 = 0$:

For $L_E < L$, taking the limits as $\sigma_\nu^2 \to 0$ in the moments which appear in (A.31) yields (23) (see the Supplementary appendix). Substitution of the analogous expressions for the case $L_E = L$ into (A.31) yields (26) (see the Supplementary appendix).

Proof of Proposition 7.2:

Consider the difference in SW $S^{\mathbf{1}} - S^{\mathbf{0}}$ *for given* L_E. From (12), (14), (24), and (27),

$$S^{\mathbf{1}}(L_E) - S^{\mathbf{0}}(L_E) = \frac{\rho^2\sigma_s^2}{2(L+M)}\left[\left(\frac{L_E}{a}\right)^2 - 2\frac{L_E}{a}\left(\frac{L+M}{2a} + \bar{\nu}\right) + \left(1 + \frac{L+M}{N}\right)\bar{\nu}^2\right]$$

(where use is made of the fact that for given L_E, $\bar{s}$ is the same with and without OC; see (19) and (21)). $S^{\mathbf{1}}(L_E) - S^{\mathbf{0}}(L_E)$ is a convex function with a minimum at $L_E = (L+M)/2 + a\bar{\nu}$. Evaluating the difference at $a\bar{\nu}$ and at L yields

$$S^{\mathbf{1}}(a\bar{\nu}) - S^{\mathbf{0}}(a\bar{\nu}) = -\frac{\rho^2\sigma_s^2}{2N}\bar{\nu}\left(\frac{N}{a} - \bar{\nu}\right)$$

and

$$S^{\mathbf{1}}(L) - S^{\mathbf{0}}(L) = -\frac{\rho^2\sigma_s^2}{2(L+M)}\left[\frac{M}{N}\left(\frac{L}{a}\frac{N}{a} - \bar{\nu}^2\right) + \bar{\nu}\left(\frac{L}{a} - \bar{\nu}\right) + \bar{\nu}\frac{L}{N}\left(\frac{N}{a} - \bar{\nu}\right)\right],$$

respectively. From (29), both differences are negative. This implies $S^{\mathbf{1}}(L_E) - S^{\mathbf{0}}(L_E) < 0$ for $a\bar{\nu} < L_E < L$.

In economies $\mathsf{M}_0^{\mathbf{j}}$ and $\mathsf{F}_0^{\mathbf{j}}$, since $L_E^{\mathbf{0}}$ maximizes $S^{\mathbf{0}}(L_E)$, we thus have

$$S^{\mathbf{1}}(L_E^{\mathbf{1}}) < S^{\mathbf{0}}(L_E^{\mathbf{1}}) < S^{\mathbf{0}}(L_E^{\mathbf{0}}), \tag{A.32}$$

which proves that *equilibrium* SW is lower with OC. If $L_E^{\mathbf{1}} < L_E^{\mathbf{0}}$, then the inequalities in (A.32) also hold true in $\mathsf{U}_0^{\mathbf{j}}$. Here the second inequality follows from the fact that $L_E^{\mathbf{0}}$ falls short of the L_E-value that maximizes $S^{\mathbf{0}}(L_E)$, so that $S^{\mathbf{0}}(L_E)$ increases as L_E rises from $L_E^{\mathbf{1}}$ to $L_E^{\mathbf{0}}$.

Proof that noise trader expected utility is lower with OC:

Subtract (26) from (23) to obtain the difference in noise traders' expected utility with versus without OC:

$$\rho \left\{ -\frac{\rho}{L+M} \left[\sigma_s^2 \left(\frac{L_E^{\mathbf{0}}}{a} - \bar{\nu} \right) + \sigma_\varepsilon^2 \frac{L_E^{\mathbf{0}} - L_E^{\mathbf{1}}}{a} \right] \frac{\bar{\nu}}{N} + \frac{\rho}{2} \sigma_s^2 \left(\frac{\bar{\nu}}{N} \right)^2 \right\}.$$

Let $L_E^{\mathbf{0}} < L$. Substituting for $L_E^{\mathbf{1}}$ and $L_E^{\mathbf{0}}$ from (12) and (14), respectively, we find that in $\mathsf{M}_0^{\mathbf{j}}$ and $\mathsf{U}_0^{\mathbf{j}}$ this condition can be written as

$$\rho \frac{\bar{\nu}}{N} \left(\bar{s}^{\mathbf{1}} - \bar{s}^{\mathbf{0}} \right) + \left(\frac{\bar{\nu}}{N} \right)^2 < \frac{\bar{\nu}}{N} \frac{1}{a}.$$

In model $\mathsf{M}_0^{\mathbf{j}}$, $\bar{s}$ is exogenous. In $\mathsf{U}_0^{\mathbf{j}}$, $\bar{s}$ is pinned down by (27). So in both cases $\bar{s}^{\mathbf{1}} = \bar{s}^{\mathbf{0}}$, and the inequality is satisfied if the first and third inequalities in (29) are satisfied. In $\mathsf{F}_0^{\mathbf{j}}$, the condition is stronger, since $\bar{s}^{\mathbf{1}} > \bar{s}^{\mathbf{0}}$ when $L_E^{\mathbf{1}} < L_E^{\mathbf{0}}$ (see the proof of Remark 6.1.3).

Supplementary appendix

Details of the proof of Proposition 5.3:

For $L_E < L$, we obtain the following limits of the functions defined in (A.7)–(A.13) as $\sigma_\nu^2 \to 0$:

$$\begin{aligned}
\alpha\gamma\sigma_s^2 &\to \frac{1}{\alpha} \\
\mathrm{var}(\theta|\, w) &\to \sigma_\varepsilon^2 \\
\beta &\to \frac{L_E + M}{\rho\sigma_\varepsilon^2} \\
\alpha + \beta &\to \frac{L + M}{\rho\sigma_\varepsilon^2} \\
\mathrm{E}(P) &\to \bar{s} - \frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right) \\
\mathrm{var}(P) &\to \sigma_s^2 \\
\mathrm{E}(z) &\to \frac{\rho}{L+M}\left(\frac{\sigma_\varepsilon^2}{2}\right)^{\frac{1}{2}}\left(\frac{L_E}{a} - \bar{\nu}\right) \\
\mathrm{var}(z) &\to 0 \\
\mathrm{cov}(P, z) &\to 0.
\end{aligned} \tag{S.1}$$

Inserting these expressions into the left-hand side of (18) yields $\Delta_0(L_E)$. This proves that $\Delta(L_E) \to \Delta_0(L_E)$ pointwise. From (S.1), it follows that the right-hand side of (18) goes to zero, i.e., $\Gamma(L_E) \to 0$ pointwise.

Similarly, Substituting the limits of (A.17)-(A.21) as $\sigma_\nu^2 \to 0$ in to the right hand side of (18) proves $\Delta^{\mathbf{0}}(L_E) \to \Delta_0^{\mathbf{0}}(L_E)$ pointwise.

For $L_E = L$, we get (A.17)-(A.21) evaluated at $L_E = L$. Inserting these expressions into the left-hand side of (18) and taking the limit as $\sigma_\nu^2 \to 0$ yields $\Delta_0^{\mathbf{0}}(L)$. Since $\Delta^{\mathbf{0}}(L) = \Delta(L)$, this also proves $\Delta^{\mathbf{0}}(L) \to \Delta_0^{\mathbf{0}}(L)$. Substitution into the right-hand side of (18) and taking the limit $\sigma_\nu^2 \to 0$ yields $\Gamma_0(L)$.

Alternative sources of wage rigidity:

Denote model $\mathsf{U}^{\mathbf{j}}_{\varsigma}$ in the running text as ${}_1\mathsf{U}^{\mathbf{j}}_{\varsigma}$. Consider the following models ${}_2\mathsf{U}^{\mathbf{j}}_{\varsigma}$–${}_4\mathsf{U}^{\mathbf{j}}_{\varsigma}$.

${}_2\mathsf{U}^{\mathbf{j}}_{\varsigma}$: Employees can "work" or "shirk" at their workplace (cf. Shapiro and Stiglitz, 1984). A worker who works gets the wage rate W. Effort is not perfectly observable: a shirker is caught shirking with probability q $(0 < q < 1)$. So he gets the wage rate W with probability $1-q$ and no payment otherwise. If all workers work, firm output is $Y = F(m)$.

If everyone shirks, output is zero. So firms have to pay workers such that they choose not to shirk. $(L_E, I_E, I_D, I_M, P, m, W)$ is an *equilibrium* if, in addition to the conditions stated in Section 3, employment m maximizes $F(m) - Wm$, W is such that workers' expected utility is as high if they work as if they shirk, and there is unemployment (i.e., $m < \hat{M}$).

${}_3\mathsf{U}^{\mathsf{j}}_{\varsigma}$: Unions are organized as in $\mathsf{U}_1(\sigma_\nu^2, j)$. Firms have the right to manage. Rather than maximizing a utility function, firm-level unions set the wage rate W such that the wage bill Wm is maximal (cf. Dunlop, 1944). F is CES with low substitutability: $F(m) = A[b + (1-b)m^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$, where $A > 0$, $0 < b < 1$, and the elasticity of substitution η obeys $0 < \eta < 1$. $(L_E, I_E, I_D, I_M, P, m, W)$ is an *equilibrium* if, in addition to the conditions stated in Section 3, employment m maximizes $F(m) - Wm$, W maximizes Wm given firms' optimal choice of m, and $m < \hat{M}$.

${}_4\mathsf{U}^{\mathsf{j}}_{\varsigma}$: Firm output is $Y = F[E(W)m]$, where $E(W)$ is the effort provided by workers given the wage they receive (cf. Solow, 1979). Workers' provision of effort is determined by how fair they conceive the wage W they are paid. It is assumed that there is a unique "efficiency wage" $\tilde{W}$ that maximizes $E(W)/W$. Effort is normalized such that $E(\tilde{W}) = 1$. $(L_E, I_E, I_D, I_M, P, m, W)$ is an *equilibrium* if, in addition to the conditions stated in Section 3, employment m and the wage rate W jointly maximize $F(m) - Wm$, and $m < \hat{M}$.

These models display the same kind of wage rigidity as the right-to-manage model in the main text.

${}_2\mathsf{U}^{\mathsf{j}}_{\varsigma}$: Workers' asset demand I_M is independent of their employment status. If an employed worker doesn't shirk, his expected utility is

$$\mathrm{E}[U(\pi_M)] = \exp(-\rho e_M)\exp[-\rho(W - D)]\mathrm{E}\left\{-\exp\left[-\rho(\theta - P)I_M\right]\right\}.$$

If he shirks,

$$\begin{aligned}\mathrm{E}[U(\pi_M)] &= (1-q)\exp(-\rho e_M)\exp(-\rho W)\mathrm{E}\left\{-\exp\left[-\rho(\theta - P)I_M\right]\right\} \\ &\quad + q\exp(-\rho e_M)\mathrm{E}\left\{-\exp\left[-\rho(\theta - P)I_M\right]\right\}\end{aligned}$$

Equalizing these expected utilities yields the efficiency wage, necessary to prevent shirking:

$$\tilde{W} = \frac{1}{\rho}\log\left[\frac{\exp(\rho D) - (1-q)}{q}\right]. \tag{S.2}$$

Employment is $\tilde{M} = (F')^{-1}(\tilde{W})$. There is unemployment for M sufficiently large.

${}_3\mathsf{U}^{\mathsf{j}}_{\varsigma}$: Equating the marginal product of labor to the wage rate yields the following expression for each firm's wage bill:

$$Wm = A(1-b)\left[b + (1-b)m^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} m^{\frac{\eta-1}{\eta}}.$$

Setting the derivative equal to zero gives:

$$\frac{1-\eta}{\eta}W\left[\frac{1-b}{1-\eta}\frac{m^{\frac{\eta-1}{\eta}}}{b+(1-b)m^{\frac{\eta-1}{\eta}}} - 1\right] = 0.$$

Optimum employment is given by the value $m = \tilde{M}$ at which the derivative changes from positive to negative:

$$\tilde{M} = \left(\frac{\eta}{1-\eta}\frac{1-b}{b}\right)^{\frac{\eta}{1-\eta}}.$$

The corresponding wage rate is

$$\tilde{W} = A\left[\frac{1-\eta}{(1-b)^{\eta}}\right]^{\frac{1}{1-\eta}}.$$

${}_4\mathsf{U}^{\mathsf{j}}_{\varsigma}$: Firm profit can be expressed as

$$F\left[\frac{E(W)}{W}Wm\right] - Wm.$$

Maximization with respect to m and W is equivalent to maximizing $E(W)/W$ by an appropriate choice of W and then maximizing profit for given $E(W)/W$ by an appropriate choice of the wage bill Wm. The former step gives $W = \tilde{W}$, the latter $\tilde{M} = (F')^{-1}(\tilde{W})$. With ${}_k\mathsf{U}^{\mathsf{j}}_{\varsigma}$ $(k = 2, 3, 4)$ instead of $\mathsf{U}^{\mathsf{j}}_{\varsigma}$, the assertions of Proposition 6.2 and the subsequent remarks carry over one-to-one to these models.

Economy without noise traders:

Let the N noise traders act as passive investors, so that the mass of the latter is $M + N$ and there is no noise trading. From (12) and (24) with $M + N$ instead of M and $\bar{\nu} = 0$, SW with OC for given L_E is

$$S^{\mathbf{1}'} = \rho e + L_E\frac{\rho}{a}\left(\bar{s} - \frac{\rho\sigma_\varepsilon^2}{L+M+N}\frac{L_E}{a} - \frac{\rho\sigma_s^2}{2a}\right) + \frac{\rho^2\sigma_\varepsilon^2}{2(L+M+N)}\left(\frac{L_E}{a}\right)^2.$$

Subtracting $S^{\mathbf{1}}$ yields

$$S^{\mathbf{1}\prime} - S^{\mathbf{1}} = \frac{\rho\sigma_\varepsilon^2}{2}\frac{N}{(L+M)(L+M+N)}\left[\frac{L_E}{a} - \left(1+\frac{L+M}{N}\right)\bar{\nu}\right]^2 > 0$$

for all L_E. Let $L_E^{\mathbf{1}\prime}$ denote the equilibrium mass of entrepreneurs in the economy without noise trading. $L_E^{\mathbf{1}\prime}$ maximizes $S^{\mathbf{1}\prime}$ (this follows from the fact that $L_E^{\mathbf{1}}$ maximizes $S^{\mathbf{1}}$, considering the special case with $\bar{\nu} = 0$). Together with the result that $S^{\mathbf{1}\prime}$ exceeds SW with noise trading $S^{\mathbf{1}}$ at $L_E^{\mathbf{1}}$, it follows that equilibrium SW is higher without than with noise trading.
Letting $S^{\mathbf{0}\prime}$ denote SW without noise traders and without OC, $S^{\mathbf{1}\prime} < S^{\mathbf{0}\prime}$ at equilibrium follows immediately from replacing M with $M+N$ and $\bar{\nu} = 0$ in expression for $S^{\mathbf{1}} - S^{\mathbf{0}}$ in the proof of Proposition 7.2.

Proof of Remark 7.2.2:
Consider $\mathsf{M}_0^{\mathbf{j}}$. Let $L_E^{\mathbf{1}} < L$, so that $\Delta_0(L_E^{\mathbf{1}}) = 0$. From (24), (27), and $\Delta_0(L_E^{\mathbf{0}}) \geq 0$, it follows that $S^{\mathbf{1}} - S^{\mathbf{0}}$ is negative if

$$\sigma_\varepsilon^2\left[\left(\frac{L_E^{\mathbf{1}}}{a}\right)^2 - \left(\frac{L_E^{\mathbf{0}}}{a}\right)^2\right] - \sigma_s^2\left[\left(\frac{L_E^{\mathbf{0}}}{a}\right)^2 - \left(1+\frac{L+M}{N}\right)\bar{\nu}^2\right] < 0.$$

The assertion of the remark follows immediately.
In $\mathsf{F}_0^{\mathbf{j}}$ and $\mathsf{U}_0^{\mathbf{j}}$ workers' aggregate welfare includes the extra terms $\rho\hat{W}M$ and M times (20), respectively. Since these terms are increasing in L_E, workers benefit from a greater mass of entrepreneurs. So the conditions in Remark 7.2.2 are sufficient for higher SW without OC.

Proof of the modified Demange-Laroque lemma:
By direct calculation

$$\mathrm{E}\left[\exp\left(x+y^2\right)|\, y\right] = \exp\left(y^2\right)\exp\left[\mathrm{E}(x|\, y) + \frac{1}{2}\mathrm{var}(x|\, y)\right]\int_{-\infty}^{\infty}\frac{\exp\left(-\frac{\{x-[\mathrm{E}(x|\, y)+\mathrm{var}(x|\, y)]\}^2}{2\mathrm{var}(x|\, y)}\right)}{[2\pi\,\mathrm{var}(x|\, y)]^{\frac{1}{2}}}dx.$$

The integral is unity, since the integrand is the density of $\mathrm{N}[\mathrm{E}(x|\, y)+\mathrm{var}(x|\, y),\ \mathrm{var}(x|\, y)]$.

Using the updating rules for normal random variables, it follows that

$$\begin{aligned}\mathrm{E}\left[\exp\left(x+y^2\right)\mid y\right] &= \exp\left[y^2+\frac{\mathrm{cov}(x,y)}{\mathrm{var}(y)}y\right]\\ &\quad\cdot\exp\left[\mathrm{E}(x)+\frac{1}{2}\mathrm{var}(x)-\frac{\mathrm{cov}(x,y)}{\mathrm{var}(y)}\mathrm{E}(y)-\frac{1}{2}\frac{\mathrm{cov}(x,y)^2}{\mathrm{var}(y)}\right].\end{aligned} \tag{S.3}$$

The unconditional expectation of the first exponential on the right-hand side can be rewritten as

$$\begin{aligned}\mathrm{E}\left\{\exp\left[y^2+\frac{\mathrm{cov}(x,y)}{\mathrm{var}(y)}y\right]\right\} &= \frac{\exp\left[\frac{2\,\mathrm{E}(y)\,\mathrm{cov}(x,y)+\mathrm{cov}(x,y)^2+2\,\mathrm{E}(y)^2\mathrm{var}(y)}{2[1-2\mathrm{var}(y)]\mathrm{var}(y)}\right]}{[1-2\,\mathrm{var}(y)]^{\frac{1}{2}}}\\ &\quad\cdot\int_{-\infty}^{\infty}\frac{\exp\left\{-\frac{\left[y-\frac{\mathrm{E}(y)+\mathrm{cov}(x,y)}{1-2\,\mathrm{var}(y)}\right]^2}{2\frac{\mathrm{var}(y)}{1-2\mathrm{var}(y)}}\right\}}{\left[2\pi\frac{\mathrm{var}(y)}{1-2\mathrm{var}(y)}\right]^{\frac{1}{2}}}dy.\end{aligned}$$

The integral is unity, since the integrand is the density of $\mathrm{N}\{[\mathrm{E}(y)+\mathrm{cov}(x,y)]/[1-2\,\mathrm{var}(y)],\ \mathrm{var}(y)/[1-2\mathrm{var}(y)]\}$. So applying the law of iterated expectations to (S.3) yields (A.30).

Noise trader expected utility at $\sigma_\nu^2=0$:
For $L_E<L$, we have the following limits as $\sigma_\nu^2\to 0$:

$$\begin{aligned}\mathrm{cov}(P,\nu) &\to 0\\ \mathrm{var}(\theta-P) &\to \sigma_\varepsilon^2\\ \mathrm{cov}(\theta-P,\nu) &\to 0\\ \mathrm{E}(\Phi) &\to -\rho\frac{\bar{\nu}}{N}\left[\frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L_E}{a}-\bar{\nu}\right)+\frac{\rho\sigma_\varepsilon^2}{L-L_E}\bar{\nu}\right]\\ \mathrm{var}(\Phi) &\to 0\\ \mathrm{E}(\Psi) &\to \rho^{\frac{1}{2}}\frac{\bar{\nu}}{N}\left(\frac{\rho\sigma_\varepsilon^2}{L-L_E}N+\frac{\rho\sigma_\varepsilon^2}{2}\right)^{\frac{1}{2}}\\ \mathrm{var}(\Psi) &\to 0\\ \mathrm{cov}(\Phi,\Psi) &\to 0.\end{aligned}$$

Substitution into (A.31) yields (23).

For $L_E = L$,

$$
\begin{aligned}
\mathrm{cov}(P,\nu) &\to 0 \\
\mathrm{var}(\theta - P) &\to \sigma_s^2 + \sigma_\varepsilon^2 \\
\mathrm{cov}(\theta - P,\nu) &\to 0 \\
\mathrm{E}(\Phi) &\to -\frac{\rho^2\bar{\nu}(\sigma_s^2 + \sigma_\varepsilon^2)}{N(L+M)}\frac{L}{a} \\
\mathrm{var}(\Phi) &\to 0 \\
\mathrm{E}(\Psi) &\to \rho^{\frac{1}{2}}\frac{\bar{\nu}}{N}\left[\frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{L+M}N + \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{2}\right]^{\frac{1}{2}} \\
\mathrm{var}(\Psi) &\to 0 \\
\mathrm{cov}(\Phi,\Psi) &\to 0.
\end{aligned}
$$

Substitution into (A.31) yields (26).

Conditions for $\mathbf{S^0(L) > S^1(L_E^1)}$:
Consider $\mathsf{M}_0^{\mathbf{j}}$. Let the conditions in (29) be satisfied. The difference between $S^{\mathbf{0}}(L)$ and $S^{\mathbf{1}}(L_E^{\mathbf{1}})$ can be written as

$$
\begin{aligned}
S^{\mathbf{0}}(L) - S^{\mathbf{1}}(L_E^{\mathbf{1}}) \;=\; & \rho\bar{s}\left(\frac{L}{a} - \bar{\nu}\right) - \frac{L+M}{2\sigma_\varepsilon^2}\left(\bar{s} - \frac{\rho\sigma_s^2}{2a}\right)^2 + \frac{\rho^2\sigma_s^2}{2}\bar{\nu}\left(\frac{1}{a} - \frac{\bar{\nu}}{N}\right) \\
& -\frac{1}{2}\frac{\rho^2\left(\sigma_s^2 + \sigma_\varepsilon^2\right)}{L+M}\left(\frac{L}{a} - \bar{\nu}\right)^2 .
\end{aligned} \tag{S.4}
$$

We proceed to show that this expression is non-negative under the additional conditions of Remark 7.2.3. Consider the expression on the right-hand side as a function of $\bar{s}$. This function takes on a unique maximum for

$$
\bar{s} = \frac{\rho\sigma_s^2}{2a} + \frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L}{a} - \bar{\nu}\right).
$$

The assumption that there are dealers when there is OC (i.e., $L_E^{\mathbf{1}} < L$) implies that $\bar{s}$ is less than this maximizing value. So $S^{\mathbf{0}}(L) - S^{\mathbf{1}}(L_E^{\mathbf{1}})$ is an increasing function for the admissible values of $\bar{s}$. The condition that rational agents do not go short at equilibrium (i.e. $L_E^{\mathbf{1}}/a > \bar{\nu}$) puts a lower bound on the set of admissible values of $\bar{s}$, viz., $\bar{s} = \rho\sigma_s^2/(2a)$.

Evaluating (S.4) at this value gives

$$\frac{\rho^2\sigma_s^2}{2a}\left(\frac{L}{a}-\bar{\nu}\right)+\frac{\rho^2\sigma_s^2}{2}\bar{\nu}\left(\frac{1}{a}-\frac{\bar{\nu}}{N}\right)-\frac{1}{2}\frac{\rho^2\left(\sigma_s^2+\sigma_\varepsilon^2\right)}{L+M}\left(\frac{L}{a}-\bar{\nu}\right)^2.$$

If $L<M$ and $\sigma_\varepsilon^2<\sigma_s^2$, then a sufficient condition for this expression to be greater than zero is

$$\frac{\rho^2\sigma_s^2}{2a}\bar{\nu}\left[\left(\frac{1}{a}-\frac{\bar{\nu}}{N}\right)+\frac{1}{L}\left(\frac{L}{a}-\bar{\nu}\right)\right]>0.$$

This proves $S^{\mathbf{0}}(L)>S^{\mathbf{1}}(L_E^{\mathbf{1}})$ under the assumptions made.

Specification of $\sigma_{\mathbf{s}}^{\mathbf{2}}$ in the numerical analysis:
The CE of π_E is $\mathrm{E}(\pi_E)-(\rho/2)\mathrm{var}(\pi_E)$, and the lower bound of the 95-percent confidence interval for π_E is $\mathrm{E}(\pi_E)-2\mathrm{var}(\pi_E)^{1/2}$, so the CE is not below the lower bound if $\mathrm{var}(\pi_E)<16/\rho^2$.
Using (1) and (10), the variance of $\pi_E=e_L+P/a+(\theta-P)I_E$ in $\mathsf{M}_0^{\mathbf{1}}$ is

$$\mathrm{var}(\pi_E)=\frac{\sigma_s^2}{a^2}+\left(\frac{L_E-a\bar{\nu}}{L+M}\right)^2\frac{\sigma_\varepsilon^2}{a^2}.$$

It is bounded below by σ_s^2/a^2. As $L_E\le L$ and $M\ge L$, the term in parentheses is no greater than 0.5. So, given $\sigma_\varepsilon^2\le\sigma_s^2$, the variance is bounded above by $1.25\sigma_s^2/a^2$.

SW with and without OC in the numerical analysis:
Table S.1 explains by how much and why SW is higher without than with OC. $\hat{S}^{\mathbf{0}}$ denotes maximum SW without OC. $\Delta(x,y)=|(x-y)|/\max\{|x|,|y|\}$ denotes the absolute difference between x and y relative to the greater of the two.[18] Since the domain of Δ is usually confined to x and y with the same sign (see Törnqvist et al., 1980, p. 3), we calculate $\Delta(x,y)$ only for the subset of cases in which this condition is satisfied, thereby losing between 16.7% (for $\sigma_\nu=0.001L/a$) and 88.4% (for $\sigma_\nu=0.5L/a$) of the cases stated in the second column of Table 2 in the main text. For a given value of σ_ν relative to L/a, column 2 of Table S.1 reports the average difference between the equilibrium SW levels without and with OC (here and in what follows standard deviations in percentage points in parentheses). When $S^{\mathbf{0}}(L_E^{\mathbf{0}})$ is positive and greater than $S^{\mathbf{1}}(L_E^{\mathbf{1}})$, $\Delta(S^{\mathbf{1}}(L_E^{\mathbf{1}}),\,S^{\mathbf{0}}(L_E^{\mathbf{0}}))>0.9$ implies $S^{\mathbf{0}}(L_E^{\mathbf{0}})>10S^{\mathbf{1}}(L_E^{\mathbf{1}})$. Hence, for σ_ν up to $0.2L/a$, SW is more than ten times as large without than with OC on average. Columns 3 and 4 give

[18] This measure ranges between 0 and 100 percent. Symmetry and boundedness make it preferable to a simple percentage difference within our setting.

the average difference between equilibrium SW with OC and maximum SW without OC and the average difference between equilibrium and optimum SW without OC, respectively. Here we additionally restrict attention to cases where noise trader expected utility is bounded not only at equilibrium (condition BNU_σ) but for all L_E and where $S^{\mathbf{0}}(L_E^{\mathbf{0}})$ and $\hat{S}^{\mathbf{0}}$ have the same sign (this costs between 0 and 152 cases for given σ_ν). The large difference between the equilibrium levels of SW without and with OC for $\sigma_\nu \leq 0.2L/a$ is due to the fact that equilibrium SW without OC is very close to its maximum (column 4), which is far greater than equilibrium SW with OC (column 3).

Table S.1: Matlab simulation of $\mathsf{M}_\sigma^{\mathbf{j}}$

$\frac{\sigma_\nu}{L/a}$	$\Delta\left(S^{\mathbf{1}}(L_E^{\mathbf{1}}),\, S^{\mathbf{0}}(L_E^{\mathbf{0}})\right)$	$\Delta\left(S^{\mathbf{1}}(L_E^{\mathbf{1}}),\, \hat{S}^{\mathbf{0}}\right)$	$\Delta\left(S^{\mathbf{0}}(L_E^{\mathbf{0}}),\, \hat{S}^{\mathbf{0}}\right)$
0.001	93.82% (09.50%)	93.92% (09.50%)	0.00% (0.00%)
0.01	93.29% (10.14%)	93.29% (10.14%)	0.00% (0.00%)
0.05	91.57% (12.32%)	91.57% (12.31%)	0.00% (0.06%)
0.1	90.34% (13.35%)	90.39% (13.31%)	0.05% (0.80%)
0.2	90.29% (14.09%)	90.51% (13.44%)	0.39% (4.40%)
0.5	50.06% (28.06%)	47.94% (28.51%)	1.01% (5.92%)

Additional references

Törnqvist, L., Vartia, P., and Vartia, Y. (1980), "How should relative changes be measured?", *Keskusteluaiheita Discussion Paper, No. 68*, Elinkeinoelämän Tutkimuslaitos Research Institute of the Finnish Economy, Helsinki.