

# Sovereign Risk, Fiscal Policy, and Macroeconomic Stability

Giancarlo Corsetti, Keith Kuester, André Meier, and Gernot J. Müller\*

August 16, 2012

## Abstract

This paper analyzes the impact of strained government finances on macroeconomic stability and the transmission of fiscal policy. Using a variant of the model by Cúrdia and Woodford (2009), we study a “sovereign risk channel” through which sovereign default risk raises funding costs in the private sector. If monetary policy cannot offset increased credit spreads because it is constrained by the zero lower bound or otherwise, the sovereign risk channel exacerbates indeterminacy problems: private-sector beliefs of a weakening economy may become self-fulfilling. In addition, sovereign risk may amplify the effects of cyclical shocks. Under those conditions, fiscal retrenchment can help curtail the risk of macroeconomic instability and, in extreme cases, even bolster economic activity.

*Keywords:* Fiscal policy, monetary policy, zero lower bound,  
risk premium, sovereign risk

*JEL-Codes:* E32, E52, E62

---

\*Corsetti: Cambridge University and CEPR, Kuester: Federal Reserve Bank of Philadelphia, Meier: International Monetary Fund, Müller: University of Bonn and CEPR. An earlier draft of this paper was circulated under the title “Sovereign risk and the effects of fiscal retrenchment in deep recessions.” For very helpful comments, we thank Santiago Acosta-Ormaechea, John Bluedorn, Fabian Bornhorst, Hafedh Bouakez, Antonio Fatas, Philip Lane, Thomas Laubach, Daniel Leigh, Ludger Schuknecht, and the participants in seminars at the Board of Governors, Bundesbank-Banque de France, Goethe University, IMF, Midwest Macro Meetings, Society for Computational Economics, and Sveriges Riksbank. Corsetti’s work on this paper is part of PEGGED, Contract no. SSH7-CT-2008-217559 within the 7th Framework Programme for Research and Technological Development. Support from the Pierre Werner Chair Programme at the EUI is gratefully acknowledged. The views expressed herein are those of the authors and do not necessarily represent those of the IMF, the Federal Reserve Bank of Philadelphia, or the Federal Reserve System.

# 1 Introduction

In the wake of the global financial crisis, sovereign risk premia have risen sharply in several countries. This trend has been accompanied by a marked tightening of private credit markets in the same countries. The panels in Figure 1 contrast two sets of euro area countries: those with relatively low sovereign spreads (left panel) and those with relatively high sovereign spreads.<sup>1</sup> Each panel displays time series data on credit default swap (CDS) spreads for government debt (solid lines) and nonfinancial corporate debt (dashed lines). In each set of countries the two series display substantial comovement, particularly in the countries facing intense fiscal strain (right panel). In what follows, we start from the widely accepted view that at least part of this comovement is due to sovereign funding strains spilling over into private credit markets.<sup>2</sup> Specifically, we assume a “sovereign risk channel” through which higher public indebtedness adversely affects private-sector financing costs and explore its implications for macroeconomic stability and fiscal stabilization policies.

Our analysis builds on the model proposed by Cúrdia and Woodford (2009), in which heterogeneous households engage in borrowing and lending via financial intermediaries. Our variant of the model features two critical innovations. First, we allow for sovereign risk premia that respond to changes in the fiscal outlook of the country. Although the precise numerical relationship is uncertain and likely to vary over time, the basic premise that risk premia are affected by fundamentals should be uncontroversial. Second, private credit spreads rise with sovereign risk because strained public finances raise the cost of financial intermediation. This assumption reflects the observation that as sovereign default looms, domestic firms face a higher risk of financial difficulties due to the risk of tax hikes, increases in tariffs, disruptive strikes, social unrest, and general economic turmoil, all of which may raise the challenge of monitoring and enforcing loan contracts.<sup>3</sup> Our approach to modeling spillovers allows for a

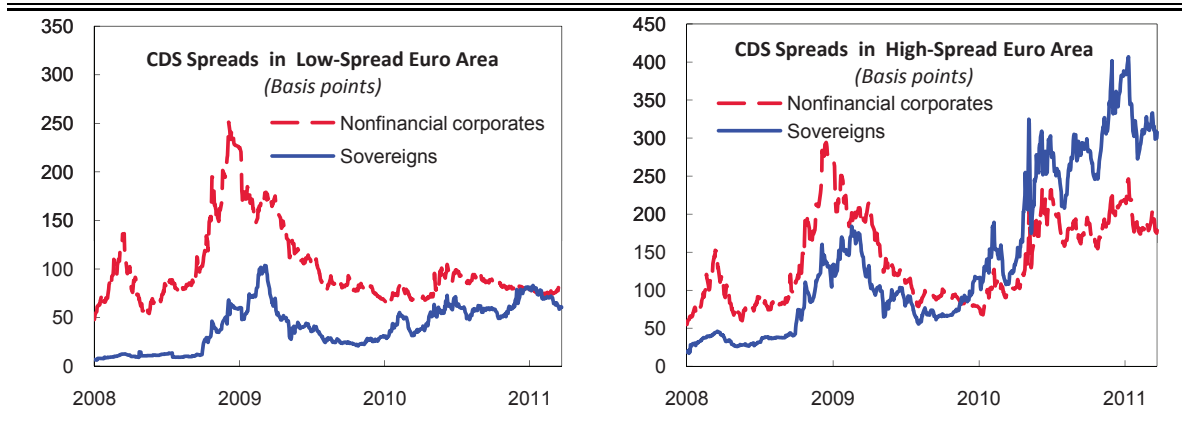
---

<sup>1</sup>We focus here on evidence for the euro area in order to control for the impact of monetary policy—a key factor in determining the strength of the sovereign risk channel.

<sup>2</sup>This view reflects the notion of a “sovereign ceiling.” In a strict sense, it posits that no debtor in a given country can have a better credit quality than the government, a primary reason being the state’s capacity to extract private-sector resources through taxation. In reality, several authors, including Durbin and Ng (2005), have documented exceptions to this rule, notably for firms with substantial export earnings or foreign operations. Even then, however, there is clear evidence that government bond yields strongly influence corporate bond yields; see International Monetary Fund (2010) and European Central Bank (2010).

<sup>3</sup>Alternatively, the assumption can be interpreted as a shortcut to capture the fact that banks are exposed to their sovereigns in many ways, including through often large government bond holdings. In times of fiscal strain, these exposures weigh on the banks’ own creditworthiness, raise their funding costs, and depress new lending to customers.

Figure 1: Sovereign and nonfinancial corporate CDS spreads



*Notes:* 5-year CDS spreads in low-spread and high-spread euro area countries, as well as for nonfinancial corporations headquartered there. Low-spread euro area includes Austria (number of firms in our sample: 1), Finland (1), France (24), Germany (18), and the Netherlands (8). High-spread euro area includes Belgium (number of firms: 1), Greece (1), Ireland (0), Italy (4), Portugal (2), and Spain (4). The corporations in our sample are the constituents of the Itraxx Europe index. The same relative weights are adopted for the sovereign and corporate index series. For example, of the 52 firms in the low-spread euro area sample, 24 are headquartered in France. As a result, in the sovereign low-spread euro area series, France has a weight of 24/52. Data sources: Bloomberg; Markit.

tractable representation of the sovereign risk channel within a simple variant of the canonical New Keynesian model. We can thus supplement our numerical results with analytical expressions for interesting special cases (by evaluating a linear approximation of the equilibrium conditions around different steady states).

The sovereign risk channel amplifies the transmission of shocks to aggregate demand, unless monetary policy is able to neutralize the spillover from sovereign default risk to private funding costs. Offsetting higher credit risk premia would typically require cuts in the policy rate. However, the central bank's capacity to enact such cuts may be hampered, most notably if the nominal interest rate is already at the zero lower bound (ZLB), as has been the case for several major economies in recent years. In what follows, we develop our model with an explicit reference to this ZLB problem as a prominent example of a constraint on central bank action. Yet, we emphasize that monetary policy would be similarly constrained under a currency peg or in other situations where political or institutional considerations prevent the central bank from counteracting a rise in sovereign risk premia.<sup>4</sup>

<sup>4</sup>The experience of the euro area since early 2010 is a case in a point. Although the European Central Bank has purchased significant amounts of government bonds under its Securities Markets Program, it has thus far failed to prevent a marked rise in risk premia, let alone accept an open-ended commitment to put a ceiling on bond yields. Moreover, even if such a commitment were forthcoming, it would not guarantee that central bank intervention can force *market* credit spreads down to any targeted level. Rather, the central bank

Our analysis generates two distinct sets of results. First, under these circumstances sovereign risk may give rise to indeterminacy, or belief-driven equilibria. Specifically, to the extent that a pessimistic shift in expectations (unrelated to fundamentals) implies an upward revision of the projected government deficit, the risk premium on public debt rises and, through the sovereign risk channel, spills over to private borrowing costs. Higher private funding costs, in turn, slow down activity, validating the initial adverse shift in expectations. Under normal circumstances, this scenario could be averted by the central bank’s commitment to appropriately lower the policy rate. To the extent that monetary policy is constrained, however, expectations may become self-fulfilling, especially when sovereign risk is very high. In this scenario, the anticipation of a procyclical spending response—that is, fiscal tightening in response to a cyclical fall in tax revenue—can help to ensure determinacy.

Second, the sign and the size of the government spending multiplier depend critically on the state of the economy. When the central bank is unconstrained, the sovereign risk channel is not operative in our model, as looser monetary policy can fully offset the impact of higher risk premia. By contrast, when the central bank is constrained, the sovereign risk channel tends to reduce the fiscal multiplier. While the effect is fairly modest as long as sovereign risk is contained, it becomes strong when public finances are very fragile *and* monetary policy is constrained for an extended period. For extreme cases the multiplier even changes its sign. In a concrete numerical example considered below, we find that the government spending multiplier turns negative if monetary policy is expected to be constrained by the ZLB for 10 quarters and the debt-to-GDP ratio is as high as 130 percent.

As a caveat, we emphasize that the present paper is not meant to add to the theory of sovereign default. Following Eaton and Gersovitz (1981), a number of authors, including Arellano (2008) and Mendoza and Yue (2011), have recently modeled default as a strategic decision of a sovereign that balances the gains from foregone debt service against the costs of exclusion from international credit markets and (exogenous) output losses. In equilibrium this implies that the probability of default increases in the level of debt. In order to maintain the tractability of our model for business cycle analysis, we impose such a relationship without explicitly modeling a strategic default decision. Specifically, we link the sovereign risk premium to the expected path of public debt (or, alternatively, future fiscal deficits). Implicit in our

---

might wind up buying the entire stock of bonds without sufficiently affecting private investor assessments of the appropriate risk premium.

approach is the assumption that there are limits to credible commitment on the part of fiscal policymakers; otherwise, there would be no risk premium in the first place, and policymakers seeking to protect growth would arguably prefer to delay retrenchment until the economy is on a firm recovery path.

The rest of the paper is structured as follows. Section 2 describes the model economy. Section 3 discusses our calibration. Sections 4 and 5 present, respectively, analytical results for a simplified version of the model and simulations based on the full nonlinear model. Section 6 concludes.

## 2 The model

The key motivation for our model is the observation that sovereign risk systematically affects private-sector borrowing conditions. The model, therefore, needs to account for the possibility that borrowing and lending take place in equilibrium. We rely on the framework developed by Cúrdia and Woodford (2009) (CW, henceforth), which gives rise to an interest rate spread in an otherwise standard New Keynesian model. The spread emerges as a result of heterogeneity among households and because of costly financial intermediation. CW maintain the tractability of the New Keynesian baseline model by assuming “asymptotic risk sharing:” Households trade a complete set of state-contingent assets but receive the associated payoffs only intermittently.<sup>5</sup> We add a slightly richer specification of fiscal policy to their model and allow the state of public finances to affect financial intermediation. In the following we briefly outline the model and stress where we depart from the original CW formulation.

### 2.1 Households

The economy is populated by a unit measure of households indexed by  $i \in [0, 1]$ . Household  $i$  is of one of two types, indexed by superscript  $\tau_t(i) \in \{b, s\}$ . In equilibrium, households of type  $\tau_t(i) = b$  will be “borrowers,” and households of type  $s$  will be “savers.” Households infrequently change their type. In each period, the probability of redrawing a type is  $1 - \delta$ , where  $\delta \in (0, 1)$ . Conditional on redrawing, the household will end up being a borrower with probability  $\pi^b$  and a saver with probability  $\pi^s = 1 - \pi^b$ . The objective of household  $i$  is given

---

<sup>5</sup>The term “asymptotic risk sharing” is used in Cúrdia and Woodford (2010).

by

$$E_0 \sum_{t=0}^{\infty} (e_t \beta^t) \left[ \frac{(\xi^\tau)^{(\sigma^\tau)^{-1}} [c_t(i)]^{1-(\sigma^\tau)^{-1}}}{1 - (\sigma^\tau)^{-1}} - \frac{\psi^\tau}{1 + \nu} h_t(i)^{1+\nu} \right],$$

where  $c_t(i)$  is an aggregate of household expenditures:

$$c_t(i) = \left[ \int_0^1 c_t(j, i)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}; \quad \theta > 1. \quad (1)$$

Here,  $c_t(j, i)$  is a differentiated output good produced by firm  $j \in [0, 1]$ .  $h_t(i)$  denotes hours worked by the household.  $e_t$  is a unit-mean shock to the time-discount factor,  $\beta \in (0, 1)$ , and  $\xi^\tau, \sigma^\tau, \psi^\tau$ , and  $\nu$  are positive parameters.

Households can insure against idiosyncratic risk through state-contingent contracts. Yet the resulting transfer payments are assumed to occur infrequently, that is, only in those periods in which a household is assigned a new type. Meanwhile, households may borrow or save through financial intermediaries. The beginning-of-period wealth of household  $i$  is given by

$$A_t(i) = [B_{t-1}(i)]^+ (1 + i_{t-1}^d) + [B_{t-1}(i)]^- (1 + i_{t-1}^b) + (1 - \vartheta_t) B_{t-1}^g(i) (1 + i_{t-1}^g) + D_t^{int} + T_t(i) + T_t^c. \quad (2)$$

$[B_{t-1}(i)]^+$  denotes deposits at financial intermediaries at the end of the previous period, which earn the deposit rate  $i_{t-1}^d$ . Conversely,  $[B_{t-1}(i)]^-$  denotes debt at financial intermediaries, which charge the borrowing rate  $i_{t-1}^b$ . In equilibrium, household  $i$  either borrows or saves. In the case in which it saves, the household may also hold government debt  $B_{t-1}^g(i) \geq 0$ .

We depart from CW by assuming that government debt is not riskless: In any period, the government may honor its debt obligations, in which case  $\vartheta_t = 0$ ; or it may partially default, in which case  $\vartheta_t = \vartheta_{\text{def}}$ , with  $\vartheta_{\text{def}} \in [0, 1]$  indicating the size of the haircut.  $i_{t-1}^g$  is the notional interest rate on government debt.  $D_t^{int}$  are profits from competitive financial intermediaries that are distributed across households in a lump-sum manner.  $T_t(i)$  denotes transfers resulting from state-contingent contracts (which are zero for those households that do not redraw their type).  $T_t^c$  is a lump-sum transfer that, in case of a sovereign default, compensates bondholders for losses associated with the default. Yet the payment is not proportional to the size of an individual's holdings of government debt (see Schabert and van Wijnbergen (2008) for a similar setup). This assumption, along with the risk of a haircut, drives a wedge between the risk-free rate,  $i_t^d$ , and the interest rate on government debt,  $i_t^g$ .

The end-of-period wealth of household  $i$  is given by

$$B_t(i) = A_t(i) - P_t c_t(i) + P_t w_t h_t(i) + D_t - T_t^g, \quad (3)$$

where  $P_t$  denotes the consumption price index and  $w_t$  is the economy-wide real wage;  $D_t$  are profits earned by goods-producing firms and  $-T_t^g$  are lump-sum transfers by the government. Note that savers' end-of-period wealth comprises deposits and government debt.

Assuming identical initial wealth for all households, state-contingent contracts ensure that post-transfer wealth is identical for all households that are selected to redraw their type. It is given by

$$A_t = [d_{t-1}(1 + i_{t-1}^d) + (1 - \vartheta_t)b_{t-1}^g(1 + i_{t-1}^g) - b_{t-1}(1 + i_{t-1}^b)]P_{t-1} + D_t^{int} + T_t^c, \quad (4)$$

where  $b_t^g$  denotes government debt in real terms,  $d_t$  denotes real aggregate savings deposited with intermediaries, and  $b_t$  denotes real aggregate private borrowing. The latter evolves according to

$$\begin{aligned} b_t = & \delta b_{t-1}(1 + \omega_{t-1})(1 + i_{t-1}^d)/\Pi_t - \pi^b \omega_t b_t + \pi^b [\delta b_{t-1}^g(1 + i_{t-1}^g)/\Pi_t - b_t^g] \\ & + \pi^b \pi^s [(c_t^b - c_t^s) - (w_t h_t^b - w_t h_t^s)]. \end{aligned} \quad (5)$$

Intuitively, the accumulation of private debt depends on four terms. The first term is the last period's private debt level plus interest (for those households that do not redraw their type). The second term,  $-\pi^b \omega_t b_t$ , is the gain accruing to borrowing households from fraudulent loans (discussed below). The third term captures the effect of sovereign indebtedness (suitably adjusted for the change in household types). In order to reduce sovereign indebtedness, current taxes need to be relatively high, which increases the need for borrowing by borrowers. Put differently, if sovereign indebtedness falls, so that  $[\delta b_{t-1}^g(1 + i_{t-1}^g)/\Pi_t - b_t^g] > 0$ , more resources are made available by savers for borrowers. The last term captures the difference in consumption levels relative to the difference in wage income across household types.

Turning to the intertemporal consumption decisions, note that, as a result of asymptotic risk sharing, all households of a specific type have a common marginal utility of real income,  $\lambda_t^i$ ,

and choose the same level of expenditure:

$$c_t^s = \xi^s (\lambda_t^s)^{-\sigma^s} \quad (6)$$

$$c_t^b = \xi^b (\lambda_t^b)^{-\sigma^b}. \quad (7)$$

The optimal choices regarding borrowing from and lending to intermediaries, as well as to the government, are then governed by the following Euler equations:

$$e_t \lambda_t^s = \beta E_t \left[ e_{t+1} \frac{1 + i_t^d}{\Pi_{t+1}} \left\{ (1 - \delta) \pi^b \lambda_{t+1}^b + [\delta + (1 - \delta) \pi^s] \lambda_{t+1}^s \right\} \right], \quad (8)$$

$$e_t \lambda_t^s = \beta E_t \left[ e_{t+1} \frac{(1 - \vartheta_{t+1})(1 + i_t^g)}{\Pi_{t+1}} \left\{ (1 - \delta) \pi^b \lambda_{t+1}^b + [\delta + (1 - \delta) \pi^s] \lambda_{t+1}^s \right\} \right], \quad (9)$$

$$e_t \lambda_t^b = \beta E_t \left[ e_{t+1} \frac{1 + i_t^b}{\Pi_{t+1}} \left\{ (1 - \delta) \pi^s \lambda_{t+1}^s + [\delta + (1 - \delta) \pi^b] \lambda_{t+1}^b \right\} \right]. \quad (10)$$

Optimal labor supply by households, in turn, is given by

$$h_t^s = \left( \frac{\lambda_t^s}{\psi^s} w_t \right)^{1/\nu}, \quad (11)$$

$$h_t^b = \left( \frac{\lambda_t^b}{\psi^b} w_t \right)^{1/\nu}. \quad (12)$$

Across household types, average labor supply,  $h_t = \pi^b h_t^b + (1 - \pi^b) h_t^s$ , is given by

$$h_t = \left( \frac{\Lambda_t}{\psi} w_t \right)^{1/\nu}, \quad (13)$$

where

$$\Lambda_t := \psi \left[ \pi^b \left( \frac{\lambda_t^b}{\psi^b} \right)^{1/\nu} + \pi^s \left( \frac{\lambda_t^s}{\psi^s} \right)^{1/\nu} \right]^\nu \quad (14)$$

and  $\psi^{-1/\nu} = \pi^b (\psi^b)^{-1/\nu} + \pi^s (\psi^s)^{-1/\nu}$ . Finally, for future reference we define

$$\lambda_t = \pi^b \lambda_t^b + (1 - \pi^b) \lambda_t^s \quad (15)$$

as the average marginal utility of real income across types.



## 2.2 Financial intermediaries

Saving and borrowing across households of different types takes place through perfectly competitive financial intermediaries. As in CW, we assume that in each period a fraction of loans,  $\chi_t$ , cannot be recovered, irrespective of the characteristics of borrowers (due to, say, fraud). Moreover, deposits,  $d_t$ , are assumed to be riskless, and intermediaries collect the largest quantity of deposits that can be repaid from the proceeds of the loans that they originate, that is,  $(1 + i_t^d)d_t = (1 + i_t^b)b_t$ . The cash flow in period  $t$  of a financial intermediary is thus given by  $d_t - b_t - \chi_t b_t$ . Using  $\omega_t$  to define the spread between lending and deposit rates, we have

$$1 + \omega_t = \frac{1 + i_t^b}{1 + i_t^d}. \quad (16)$$

Substituting  $d_t = (1 + \omega_t)b_t$  and choosing  $b_t$  to maximize the profits of the intermediary yields the first-order condition for loan origination

$$\omega_t = \chi_t. \quad (17)$$

In departing from CW, we assume that  $\chi_t$  depends on sovereign risk. This assumption captures the adverse effect of looming sovereign default risk on private-sector financial intermediation. Conceptually related is the notion that in case of a sovereign default, the government diverts funds from the repayments made by borrowers, see Mendoza and Yue (2011). Specifically, we assume that

$$\chi_t = \chi_\psi [(1 + i_t^g)/(1 + i_t^d)]^{\alpha_\psi} - 1, \quad (18)$$

where parameter  $\chi_\psi > 0$  is used to scale the private spread in the steady state, and  $\alpha_\psi$  measures the strength of the spillover from the (log) sovereign risk premium to the (log) private risk premium. Finally, transfers from intermediaries to households include loans that are not recovered by the intermediaries such that  $D_t^{int} = P_t \omega_t b_t$ .

## 2.3 Firms

There is a continuum of firms  $j \in [0, 1]$ , each of which produces a differentiated good on the basis of the following technology

$$y_t(j) = zh_t(j)^{1/\phi}, \quad (19)$$

where  $z$  is the aggregate productivity level. In each period only a fraction  $(1 - \alpha)$  of firms is able to reoptimize its prices. Firms that do not reoptimize adjust their price by the steady-state rate of inflation,  $\Pi$ . Prices are set in period  $t$  to maximize expected discounted future profits.<sup>6</sup> The resulting first-order condition for a generic firm that adjusts its price,  $P_t^*$ , is

$$\left(\frac{P_t^*}{P_t}\right)^{1+\theta(\phi-1)} = \frac{K_t}{F_t}, \quad (20)$$

with

$$K_t = \lambda_t e_t \mu^p \phi w_t \left(\frac{y_t}{z}\right)^\phi + \alpha \beta E_t \left[ \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta\phi} K_{t+1} \right], \quad (21)$$

$$F_t = \lambda_t e_t y_t + \alpha \beta E_t \left[ \left(\frac{\Pi_{t+1}}{\Pi}\right)^{(\theta-1)} F_{t+1} \right], \quad (22)$$

where  $\mu^p = \theta/(\theta - 1)$ . The law of motion for prices (inflation) is given by

$$1 - \alpha \left(\frac{\Pi_t}{\Pi}\right)^{\theta-1} = (1 - \alpha) \left(\frac{P_t^*}{P_t}\right)^{1-\theta}. \quad (23)$$

For future reference, it is also useful to define price dispersion  $\Delta_t := \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\theta\phi} dj$ , which evolves as follows

$$\Delta_t = \alpha \Delta_{t-1} \left(\frac{\Pi_t}{\Pi}\right)^{\theta\phi} + (1 - \alpha) \left(\frac{1 - \alpha (\Pi_t/\Pi)^{\theta-1}}{1 - \alpha}\right)^{\frac{\theta\phi}{\theta-1}}. \quad (24)$$

Finally, profits distributed to households are given by  $D_t = \int_0^1 P_t(j)y_t(j) - P_t w_t h_t(j) dj$ ; or, in equilibrium,  $D_t = P_t \left( y_t - w_t (y_t/z)^\phi \Delta_t \right)$ .

---

<sup>6</sup>Future nominal profits are discounted with the factor  $(\alpha\beta)^{T-t} \frac{\lambda_T}{\lambda_t} \frac{P_t}{P_T}$ , taking into account that demand for product  $j$  is given by the demand function  $y_t(j) = y_t (P_t(j)/P_t)^{-\theta}$ , where  $P_t(j)$  denotes the price of good  $j$ , and  $y_t$  is aggregate output.

## 2.4 Government

Real government debt evolves as follows:

$$b_t^g = (1 - \vartheta_t) \frac{b_{t-1}^g (1 + i_{t-1}^g)}{\Pi_t} + g_t + \frac{T_t^c}{P_t} - \frac{T_t^g}{P_t},$$

where  $g_t$  denotes government spending. Below we will consider different assumptions regarding the law of motion for government spending. As is customary, we assume throughout that the expenditure share of each particular differentiated good in government spending is the same as the share of that good in private consumption. Further, we assume that transfers  $T_t^c$  are set in such a way that a sovereign default does not alter the actual debt level. Therefore, *ex post*, whether sovereign default has actually occurred or not, will not affect the sovereign risk premium.<sup>7</sup> In particular, we set

$$T_t^c = P_t \vartheta_t \frac{b_{t-1}^g (1 + i_{t-1}^g)}{\Pi_t}.$$

The consolidated government flow budget constraint is thus given by

$$b_t^g = \frac{b_{t-1}^g (1 + i_{t-1}^g)}{\Pi_t} + g_t - \frac{T_t^g}{P_t}. \quad (25)$$

Letting  $tr_t = T_t^g/P_t$  denote the real tax revenue related to the business cycle and to debt stabilization, we assume that

$$(tr_t - tr) = \phi_{T,y}(y_t - y) + \phi_{T,b^g}(b_{t-1}^g - b^g), \quad \phi_{T,y} > 0, \quad \phi_{T,b^g} > 0. \quad (26)$$

Here and in the following, variables without time subscript refer to steady-state values. Tax revenue rises when economic activity improves, with parameter  $\phi_{T,y}$  denoting the semi-elasticity of revenue with respect to output. Similarly, taxes are increased whenever debt exceeds its target value. Throughout the paper, we assume that  $\phi_{T,b^g}$  is large enough so as to eventually stabilize public debt.

---

<sup>7</sup>Sovereign default is typically considered to cause some redistribution among households, that is, from savers to borrowers. This effect would be absent in our model even without the assumption of offsetting transfer payments, as asymptotic risk sharing allows households to insure themselves against the distributional consequences of sovereign default. Yet, the absence of transfers would imply lower risk premia *prior to* default, as the lower post-default debt stock would already be taken into account. Our assumption eliminates this counterintuitive effect.

While *actual* default *ex post* is neutral in the sense described above, the *ex ante probability* of default is crucial for the pricing of government debt ( $i_t^g$ ) and for real activity.<sup>8</sup> A fully specified model of sovereign default is beyond the scope of the present paper. Instead, we draw on earlier work in this area, which has pursued two distinct approaches. First, following Eaton and Gersovitz (1981), Arellano (2008) and others have modeled default as a strategic decision of the sovereign. Second, and more recently, Bi (2012) and Juessen et al. (2011) consider default as the consequence of the government's inability to raise the funds necessary to honor its debt obligations. Under both approaches, the probability of sovereign default is closely and nonlinearly linked to the level of public debt.

In the current paper we operationalize sovereign default by appealing to the notion of a fiscal limit in a manner similar to Bi (2012). Whenever the debt level rises above the fiscal limit, default will occur. The fiscal limit is determined stochastically, capturing the uncertainty that surrounds the political process in the context of sovereign default. Specifically, we assume that in each period the limit will be drawn from a generalized beta distribution with parameters  $\alpha_{bg}$ ,  $\beta_{bg}$ , and  $\bar{b}^{g,\max}$ . As a result, the *ex ante* probability of default,  $p_t$ , at a certain level of sovereign indebtedness,  $b_t^g$ , will be given by the cumulative distribution function of the beta distribution as follows:

$$p_t = F_{\text{beta}} \left( \frac{b_t^g}{4y \bar{b}^{g,\max}}; \alpha_{bg}, \beta_{bg} \right). \quad (27)$$

Note that  $\bar{b}^{g,\max}$  denotes the upper end of the support for the debt-to-GDP ratio. Regarding the haircut this implies

$$\vartheta_t = \begin{cases} \vartheta_{\text{def}} & \text{with probability } p_t, \\ 0 & \text{with probability } 1 - p_t. \end{cases} \quad (28)$$

Turning to monetary policy, we assume throughout that the central bank follows a Taylor-type interest rate rule that also seeks to insulate aggregate economic activity from fluctuations in risk spreads. In particular, we assume:

$$\log(1 + i_t^{d,*}) = \log(1 + i^d) + \phi_{\Pi} \log(\Pi_t/\Pi) - \phi_{\omega} \log((1 + \omega_t)/(1 + \omega)). \quad (29)$$

---

<sup>8</sup>This implication of our setup is in line with evidence reported by Yeyati and Panizza (2011). Investigating output growth across a large number of episodes of sovereign default, they find that the output costs of default materialize in the run-up to defaults rather than at the time when the default actually takes place.

Here,  $i_t^{d,*}$  marks the target level for the deposit rate,  $i_t^d$ , and  $\phi_\Pi > 1$ ,  $\phi_\omega > 0$ . CW show that optimal policy in the presence of credit frictions involves some adjustment of policy rates in response to interest rate spreads.<sup>9</sup> However, in deep recessions the target level and the actual interest rate can diverge. The reason is that in implementing rule (29), the central bank relies on steering the riskless nominal interest rate  $i_t^d$ , which cannot fall below zero. Therefore,  $i_t^d = i_t^{d,*}$  can only be ensured if  $i_t^d \geq 0$ . Otherwise,  $i_t^d = 0$ . As a result, an increase in the spread  $\omega_t$  cannot be offset if monetary policy is constrained by the ZLB.<sup>10</sup>

## 2.5 Market clearing

Goods-market clearing requires

$$y_t = \int_0^1 c_t(i) di + g_t = \pi^b c_t^b + \pi^s c_t^s + g_t. \quad (30)$$

The total supply of output is given by

$$y_t \Delta_t^{1/\phi} = z h_t^{1/\phi}. \quad (31)$$

## 3 Calibration

To solve the model numerically, we assign parameter values on the basis of observations for U.S. data. The relationship between sovereign risk, private-sector spreads, and debt levels is calibrated based on cross-country evidence. A time period in the model is one quarter.

With respect to monetary policy, we assume an average inflation rate of 2 percent per year.

The coefficient on inflation in the Taylor rule is set to a customary value of  $\phi_\Pi = 1.5$ . With regard to the response of the interest rate to the private spread,  $\phi_\omega$ , we choose a value such

---

<sup>9</sup>Taylor (2008) suggests that the Fed actually makes such an adjustment for “stress in the markets.” There is also evidence from the minutes of the Fed and the Bank of England that “credit tightening” and/or problems in the banking sector were considered in the determination of actual interest rate policies; see, for example, Board of Governors of the Federal Reserve System (2007) and Bank of England (2008).

<sup>10</sup>Although we focus here on a simple representation of monetary policy, the model would, in principle, allow for more complicated types of monetary policy. For example, a central bank faced with the ZLB could promise low future real rates to help the economy ease out of the lower-bound situation; see Eggertsson and Woodford (2003). This would not only increase output relative to the current interest rate rule (29), but it would also raise tax revenues and therefore alleviate some of the fiscal strain. The question to what extent central banks can credibly engage in such forward guidance is not settled, however. Similarly, the effects of other unconventional monetary policy operations, such as large-scale bond purchases in the secondary market, are uncertain and likely to be bounded in practice.

that, up to a first-order approximation, the central bank fully neutralizes the effect of the sovereign risk premium on aggregate economic activity in normal times; Section 4.1 shows that this is the case for  $\phi_\omega = 0.71$ .

The steady-state level of government spending (consumption and investment) relative to GDP is  $g/y = 0.2$ . The level of gross public debt in the steady state is set to 60 percent of annual GDP. These values are broadly in line with U.S. averages over the last 20 years. We assume that taxes react to debt sufficiently strongly ( $\phi_{T,b^g}$  large enough) so as to ensure that the debt level remains bounded throughout. We set  $\phi_{T,y} = 0.34$  in line with OECD evidence for the U.S.; see Girouard and André (2005).

With regard to the preference parameters, we set the curvature of the disutility of work to  $\nu = 1/1.9$ , in line with the arguments provided by Hall (2009). We set an elasticity of demand of  $\theta = 7.6$  to generate a gross price markup of  $\mu^p = 1.15$ , which is in the range of values typically used in the literature. Finally, we assume that the average intertemporal elasticity of substitution is given by  $\bar{\sigma} = c/y$ , where  $\bar{\sigma} := \pi^b \cdot (c^b/y) \cdot \sigma^b + \pi^s \cdot (c^s/y) \cdot \sigma^s$ . If the model had a representative household, this would correspond to the case of log-utility. Further, we assume that aggregate hours worked in the steady state are given by  $h = 1/3$ . We choose the relative values of the intertemporal elasticity of substitution for the two types of households ( $\sigma^b$  and  $\sigma^s$ ), and of the scaling parameters for the disutility of work ( $\psi^b$  and  $\psi^s$ ), such that the linearized model can be represented in the canonical three-equation New Keynesian format. This representation allows us to derive a number of analytical results for a linear approximation in the next section. Importantly, under this calibration only the current value of the interest rate spread enters the dynamic IS-relationship and the New Keynesian Phillips curve. In addition, the evolution of output and inflation is independent of the level of private debt. Appendix A spells out in detail the conditions under which this representation is valid. Specifically, given the other parameter values, we set  $\sigma^b/\sigma^s = 0.53$  and  $\psi^b/\psi^s = 0.82$ . We target a ratio of private debt to annual GDP,  $b/4y$ , of 80 percent, in line with Great Moderation averages for nonfinancial, nonmortgage, nongovernment credit market debt outstanding recorded in the U.S. flow of funds accounts; compare CW. Along with the market clearing condition, this determines scaling parameters  $\xi^b$  and  $\xi^s$ . Next, as in CW, we assume that households redraw their type on average every 40 quarters, meaning  $\delta = 0.975$ . This implies that the average time during which a specific type is without access to payoff streams

from asymptotic risk sharing is 10 years.

A central element in our calibration is the share of borrowers in the economy,  $\pi^b$ . It determines the share of economic activity that is affected by an increase in the credit spread and therefore deserves some discussion. One possible reference is the U.S. Survey of Consumer Finances. Averaging over the latest surveys (1998, 2001, 2004, and 2007), the share of U.S. families that hold some kind of debt is 76 percent; see Aizcorbe et al. (2003) and Bucks et al. (2009). This suggests a value of  $\pi^b = 0.76$ . However, loans secured by the primary residence make up a large share of that debt. Setting  $\pi^b$  at this level might, therefore, overstate the effect that an increase in credit spreads could have on economic activity. Another metric, also from the Survey of Consumer Finances, that is more directly related to the notion of “borrowers” and “savers” in our model is the 57 percent of families in the survey who report that, over the year preceding the survey date, they have spent less than their income, i.e., they have saved. This suggests a value for  $\pi^b = 0.43$ . That said, both of the aforementioned figures do not explicitly take into account corporate borrowing (other than by single-owner firms). To the extent that households in our model own firms and also take intertemporal decisions for these firms, any purely household-based measure of indebtedness is bound to underestimate the degree of indebtedness and thereby the importance of the interest rate spread. In particular, using the same measure of private borrowing as above (nonfinancial, nonmortgage, nongovernment credit market debt), a large share of private borrowing is accounted for by corporations rather than households. To capture this, we set  $\pi^b = (1 - 0.17) \cdot 0.43 + 0.17 \cdot 1 = 0.53$  in our baseline calibration. This formula hypothetically divides households into consumption entities, some of which are indebted, and investment entities, all of which have debt. In the calculations, 0.17 is the share of nonresidential private domestic investment in private domestic economic activity.

In regard to the normal spread between deposit and lending rates, we target a steady-state value of 2.1 percent (annualized), in line with commercial and industrial loan rate spreads in the Federal Reserve’s Survey of Terms of Business Lending. This pins down parameter  $\chi_\psi$ . The steady-state level for the central bank’s target interest rate,  $i^d$ , is set to 4.5 percent (annualized), which determines the time discount factor,  $\beta$ .

Turning to the production parameters, we set  $\phi = 1$ , implying a linear production function. We furthermore target a unit value for steady-state output; this pins down the steady-state

level of productivity,  $z$ . The price stickiness parameter is fixed at  $\alpha = 0.925$ . Judging by *microeconomic* evidence on price rigidities (for example, Bils and Klenow, 2004), the implied frequency of price adjustment may appear too low. However, our calibration implies an appropriately flat Phillips curve,  $\kappa = 0.0068$ , causing inflation to respond relatively little to a recessionary shock, in line with the actual behavior of inflation during the latest crisis.<sup>11</sup>

Finally, it remains to determine the parameters that govern the link between strained public finances and elevated private-sector spreads. Actual haircuts in case of a sovereign default show large variation; see Panizza et al. (2009) and Moody’s Investors Service (2011).  $\vartheta_{\text{def}} = 0.5$  appears to be a reasonable average value. With respect to the specification of the fiscal limit, we seek to replicate the relationship between the sovereign risk premium and public debt shown in Figure 2. The figure plots CDS spreads of industrialized economies against the level of projected gross debt of the general government (relative to GDP). The projections are taken from the IMF’s April 2011 World Economic Outlook. The blue dots show projections for the end of 2011. For comparison, the figure also plots IMF forecasts for the debt-to-GDP ratio by the end of 2015 (green triangles). For the countries shown in the figure, CDS spreads are systematically higher, the higher the level of projected gross public debt.<sup>12</sup> In fact, the risk premium appears to rise disproportionately as the debt level rises. We choose parameters  $\alpha_{bg} = 3.70$ ,  $\beta_{bg} = 0.54$ , and  $\bar{b}^{\text{g,max}} = 2.56$  to match this empirical relationship. The black solid line displays the implied steady-state relationship between debt levels and the sovereign risk premium.

Next, we need to calibrate the spillovers from sovereign to private-sector risk. Figure 1 is

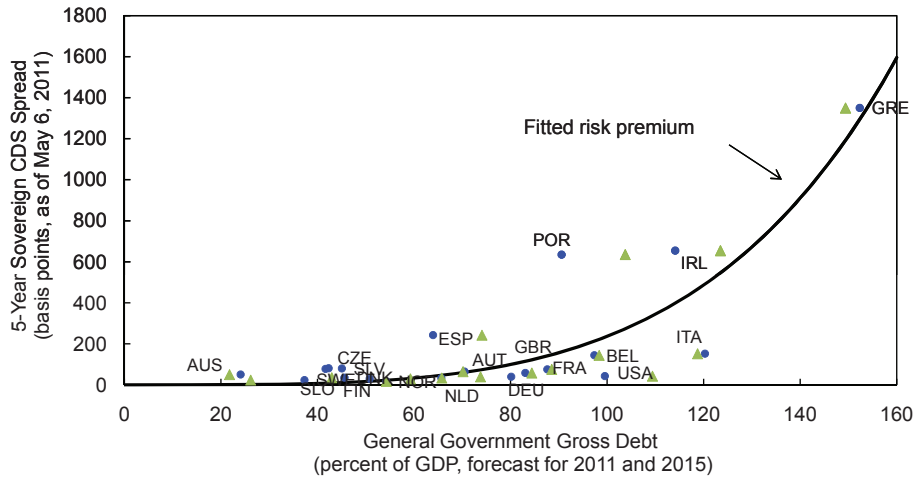
---

<sup>11</sup>In fact, our simulations below show that inflation initially falls to -4 percent (annualized) when output declines by 6.7 percent under the baseline scenario. For comparison, actual quarterly headline PCE inflation rates were negative for only two quarters in the last recession, dropping to -5.7 percent in 2008Q4 and -1.7 percent in 2009Q1 on the back of sharp energy price declines. A similar argument for the empirical realism of a flat Phillips curve can be found in Erceg and Lindé (2010), who choose a price stickiness parameter of  $\alpha = 0.9$  in a model with real rigidities at the firm level. Note that such real rigidities—which our model abstracts from—provide a well-known microeconomic argument as to why Phillips curves may be flatter than would otherwise be the case for any given  $\alpha$ . Finally, our own value of  $\kappa$  remains within the confidence intervals provided by many empirical studies. Galí and Gertler (1999), for example, report point estimates in the range between 0.007 and 0.047 with wide confidence bands.

<sup>12</sup>For a systematic empirical analysis of the relationship between fiscal variables and yields on government bonds, see, among others, Reinhart and Sack (2000), Ardagna et al. (2007), Baldacci et al. (2008), Haugh et al. (2009), Laubach (2009), Baldacci and Kumar (2010), and Borge et al. (2011). Ardagna et al. (2007) explicitly focus on possible nonlinearities in the relationship and find that bond rates rise disproportionately for very high levels of debt. Note, however, that sovereign risk premia are bound to be affected by more than just one fiscal variable, including such factors as the quality of fiscal institutions or the composition of the investor base. We abstract from these complications to keep our exercise tractable and focus on the fact that high current and/or projected debt is consistently found to be a key determinant of government financing costs.



Figure 2: Sovereign risk premia vs. debt



*Notes:* The figure plots 5-year sovereign CDS spreads as of May 6, 2011 against forecasts for end-2011 gross general government debt/GDP (blue circles) and end-2015 debt/GDP (green triangles). The countries shown are Australia, Austria, Belgium, the Czech Republic, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, the Slovak Republic, Slovenia, Spain, Sweden, the United Kingdom, and the United States. Excludes Japan. Forecasts are taken from the IMF World Economic Outlook April 2011.

suggestive of a sovereign risk channel that runs from sovereign spreads to spreads in private credit markets. Consistent with that notion, Harjes (2011) finds that for a sample of large, publicly traded euro area companies, a 100-basis-point increase in sovereign spreads raises these firms' credit spreads by about 50 to 60 basis points. We therefore set  $\alpha_\psi = 0.55$ . Although this value implies significant spillovers, it may still be somewhat conservative: First, it is based on credit spreads of companies that are large, with often sizeable export activities, and access to international credit markets. Spillover effects from sovereign risk are likely to be more pronounced for smaller and less international companies that rely on local bank-based financing. Second, Figure 1 suggests that the comovement between spreads is considerably stronger in countries that face intense fiscal strain.

#### 4 Sovereign risk, fiscal policy, and macroeconomic stability

We now analyze how sovereign risk affects the economy's dynamics and the effects of stabilization policy. Monetary policy plays a key conditioning role, notably through its capacity

to insulate private borrowing costs from fluctuations in sovereign risk premia. To highlight this aspect, we consider a scenario in which the central bank's capacity to act is constrained by the ZLB. The current section looks at a special case of our model that allows us to obtain analytical solutions for a linear approximation of the equilibrium conditions.<sup>13</sup> For this case, we assume that the probability of sovereign default depends on the expected primary deficit, rather than on the level of debt. To capture the nonlinearity that characterizes the relationship between the risk premium and the level of debt, we evaluate the linearized model for different steady-state values of debt. The higher the initial debt level, the stronger the response of default risk to changes in the expected deficit. We first examine the stability properties of the economy. In the presence of the sovereign risk channel, it turns out that the economy becomes more vulnerable to self-fulfilling equilibria. We investigate to what extent this observation may argue for procyclical spending policy when the ZLB is binding. Thereafter, and focusing on parameterizations that guarantee the absence of self-fulfilling equilibria, we study how the sovereign risk channel affects the size of the spending multiplier.

#### 4.1 A tractable special case of the model

We focus on a first-order approximation of the equilibrium conditions around the deterministic steady state. The aggregate equilibrium dynamics of the model can be represented by a variant of the New Keynesian Phillips curve and a dynamic IS-relationship. The Phillips curve relates inflation to expected inflation, output, and government purchases:

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} + \kappa_y \tilde{y}_t - \kappa_g \tilde{g}_t, \quad (32)$$

where  $\kappa_y = \kappa(\nu + \bar{\sigma}^{-1})$  and  $\kappa_g = \kappa\bar{\sigma}^{-1}$ , with  $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$ . In terms of notation,  $\tilde{y}_t = y_t - y$ ,  $\tilde{g}_t = g_t - g$ , and  $\widehat{\Pi}_t = \log(\Pi_t/\Pi)$ , where variables without a time subscript continue to refer to steady-state values. Next, the dynamic IS-relationship links output to real government spending and the effective real interest rate through

$$\tilde{y}_t - \tilde{g}_t = E_t \tilde{y}_{t+1} - E_t \tilde{g}_{t+1} - \bar{\sigma} \left[ \widehat{i}_t^d + (\pi_b + s_\Omega) \widehat{\omega}_t - E_t \widehat{\Pi}_{t+1} + \Gamma_t \right], \quad (33)$$

---

<sup>13</sup>The extent to which this approximation is numerically accurate continues to be the subject of debate; for opposing views, see Braun et al. (2012) and Christiano and Eichenbaum (2012).

where  $\widehat{\omega}_t := \log((1 + \omega_t)/(1 + \omega))$ ,  $\widehat{i}_t^d := \log((1 + i_t^d)/(1 + i^d))$ , and  $\Gamma_t := E_t \log(e_{t+1}) - \log(e_t)$ . From the IS-relationship, it is clear that fluctuations in the private-sector spread can influence economic activity unless they are neutralized by monetary policy. The degree to which the spread affects economic activity for a given policy rate is determined by parameters  $\pi_b + s_\Omega$ . As discussed in CW, parameter  $s_\Omega := \pi_b \pi_s (\sigma_b c_b / y - \sigma_s c_s / y) / \bar{\sigma}$  indicates whether an increase in the interest rate affects the aggregate demand of borrowers more adversely than that of savers. In our calibration this is the case ( $s_\Omega > 0$ ). As regards monetary policy, equation (29) implies that during normal times (in deviations from steady state):

$$\widehat{i}_t^d = \phi_\pi \widehat{\Pi}_t - \phi_\omega \widehat{\omega}_t. \quad (34)$$

We restrict ourselves to the case  $\phi_\omega = (\pi_b + s_\Omega) = 0.71$ . Under this assumption, up to a first-order approximation, in normal times the central bank fully neutralizes the effect of the sovereign risk premium on aggregate economic activity; see equation (33). However, the central bank will no longer be able to offset a rise in the credit spread when the ZLB binds. In our analysis we posit that this is the case in the initial period. We follow Christiano et al. (2011) and Woodford (2011) in assuming that monetary policy will return to Taylor rule (34) in the next period and be unconstrained thereafter with probability  $1 - \mu$ , where  $\mu \in (0, 1)$ . Otherwise, the zero interest rate persists into the next period. The same Markov structure applies to all subsequent periods. As a result, the expected length of the ZLB episode is given by  $1/(1 - \mu)$ . Shocks to the time discount factor,  $\Gamma_t$ , follow the same Markov structure.<sup>14</sup> Given that there are no endogenous state variables in the special case considered here, once the ZLB episode ends, the economy immediately reverts to the steady state.

As indicated above, we make one further simplifying assumption in this section that allows us to derive analytical results: We assume that the probability of sovereign default—and thus the sovereign risk premium—depends on the primary deficit rather than the level of public debt as in the full model. Consequently, the interest rate spread now depends on the expected deficit. We postulate a linear relationship of the form

$$\widehat{\omega}_t = \xi E_t (\tilde{g}_{t+1} - \phi_{T,y} \tilde{y}_{t+1}), \quad (35)$$

---

<sup>14</sup>Specifically, we assume a temporary increase in the effective discount factor, triggered by  $0 < e_t = e_L < 1$  while at the ZLB, so  $\Gamma_t = \mu \log(e_L) - \log(e_L) = -(1 - \mu) \log(e_L) \geq 0$ .

Table 1: Quantifying parameter  $\xi$ 

Debt/GDP	$\xi'$	$\xi$ by length of ZLB episode (qtrs)				
		6	7	8	9	10
90 percent	0.0016	0.014	0.015	0.017	0.018	0.020
110 percent	0.0030	0.025	0.028	0.031	0.034	0.037
130 percent	0.0051	0.042	0.047	0.052	0.057	0.062
140 percent	0.0065	0.054	0.060	0.066	0.073	0.079
150 percent	0.0083	0.068	0.076	0.084	0.092	0.100

*Notes:* The table presents estimates for the slope  $\xi$  of the private-sector interest rate spread (multiplied by  $\pi_b + s_\Omega$ ) with respect to the fiscal deficit for different average lengths (in quarters) of the ZLB episode and for different debt/GDP ratios. The entries in the columns “ $\xi$  by length of ZLB episode (qtrs)” are based on the formula  $\xi = \frac{1+\mu(1-\mu)}{\mu(1-\mu)} \xi'$  that is explained in the main text and in Appendix B.

where, in order to ease the burden on notation, we have defined the spread that enters the IS-relationship over and above the risk-free deposit rate as  $\widehat{\omega}_t := (\pi_b + s_\Omega)\widehat{\omega}_t$ . Parameter  $\xi \geq 0$  indicates the extent to which a weak fiscal position—as measured by primary deficits—adversely affects private-sector spreads. Parameter  $\phi_{T,y} \in [0, 1)$  measures the sensitivity of tax revenue with respect to economic activity. The assumption made in equation (35) is meant to capture the main implications of the full model. We will, therefore, refer to high values of parameter  $\xi$  as indicating economies with high public debt and correspondingly steep sovereign risk premia; compare Figure 2.

## 4.2 The size of the spillover from public to private risk premia

To appreciate our results below, it is useful to discuss the range of plausible values for  $\xi$  in equation (35). Let  $\xi'$  be the slope of the private-sector interest rate spread (multiplied by  $\pi_b + s_\Omega$ ) with respect to debt at a specific debt level; all of this evaluated in the steady state. Our assumptions in Section 2 imply that

$$\xi' = \alpha_\psi \frac{(\pi_b + s_\Omega) \vartheta_{\text{def}}}{1 - \vartheta_{\text{def}} F_{\text{beta}}\left(\frac{b^g}{4y} \frac{1}{\bar{b}^{g,\max}}; \alpha_{b^g}, \beta_{b^g}\right)} \frac{1}{4y} \frac{1}{\bar{b}^{g,\max}} f_{\text{beta}}\left(\frac{b}{4y} \frac{1}{\bar{b}^{g,\max}}; \alpha_b, \beta_b\right). \quad (36)$$

The first column of Table 1 reports the values of  $\xi'$  for different steady-state levels of sovereign debt. All other parameters take on the values introduced in Section 3. At first sight, the entries for  $\xi'$  appear to be fairly small. Recall, however, that the relationship in equation (35) links the interest rate spread to the expected deficit, whereas the full model implies

a link between the spread and the expected level of debt, that is, the result of a series of accumulated deficits. The values for  $\xi'$  are thus bound to understate the sensitivity of the interest rate spread to a persistent fiscal deficit. An appropriate mapping from the slope of the risk premium into the simplified model needs to take into account the horizon over which deficits accumulate. The following expression is meant to capture this fact for empirically reasonable values of  $\mu > 0.5$  (so the ZLB is expected to be binding for at least two periods):<sup>15</sup>

$$\xi = \frac{1 + \mu(1 - \mu)}{\mu(1 - \mu)} \xi'. \quad (37)$$

The columns of Table 1 labeled “ $\xi$  by length of ZLB period” report the corresponding values of  $\xi$  for different steady-state debt levels if the ZLB has an expected duration of six through 10 quarters. These calculations suggest that a value of  $\xi$  as large as 0.1 cannot be ruled out if initial debt is high and the recessionary shock persists for an extended period.

### 4.3 The sovereign risk channel and equilibrium determinacy

Assume, as a baseline scenario, that the level of government spending is exogenously given. For this case, we find that the sovereign risk channel significantly alters the determinacy properties of the model when the ZLB is binding. Specifically, the range of parameters that ensure determinacy shrinks in the presence of sovereign risk. The following proposition establishes parameter restrictions that yield a (locally) determinate equilibrium.<sup>16</sup>

**Proposition 1** *In the economy summarized by equations (32) – (35), let the interest rate be equal to zero in the initial period. In each subsequent period, let the interest rate remain at zero with probability  $\mu \in (0, 1)$ . Otherwise, let monetary policy be able to permanently return to Taylor rule (34). There is a locally unique bounded equilibrium if and only if*

$$a) \quad a < 1/(\beta\mu), \quad \text{and} \quad b) \quad (1 - \beta\mu)(1 - a) > \mu\bar{\sigma}\kappa_y,$$

where  $a := \mu + \mu\xi\phi_{T,y}\bar{\sigma}$  and  $\kappa_y := \kappa[\nu + 1/\bar{\sigma}]$ .

**Proof.** See Appendix C. ■

<sup>15</sup>Appendix B presents a more detailed motivation for formula (37).

<sup>16</sup>We focus on local determinacy once the economy has reached the ZLB. A separate strand of the literature examines global determinacy in the New Keynesian model. Benhabib et al. (2002), for example, propose a fiscal policy setup that rules out liquidity traps by making the low-inflation steady state fiscally unsustainable. Mertens and Ravn (2011) study the efficacy of fiscal policy in belief-driven equilibria.

In the absence of an endogenous risk premium (that is, for  $\xi = 0$ ) as in Christiano et al. (2011) and Woodford (2011), condition a) is always satisfied. Hence, there will be a unique bounded equilibrium if and only if condition b) holds. If  $\xi = 0$ , condition b) is given by  $(1 - \beta\mu)(1 - \mu) > \mu\bar{\sigma}\kappa_y$ . The previous literature has shown that the set of “fundamental” parameters for which this condition holds is larger (i) the less persistent the ZLB situation (in our parameterization, the smaller  $\mu$ ); (ii) the lower the interest sensitivity of demand (the smaller  $\bar{\sigma}$ ); and (iii) the flatter the Phillips curve (the smaller  $\kappa_y$ ). In addition to these findings, our analysis shows that the range of parameters for which the equilibrium is determinate shrinks in the presence of a sovereign risk channel. Specifically, with  $\xi > 0$ , condition a) is violated if either the interest rate spread is sufficiently responsive to the deficit or if tax revenue is sufficiently responsive to output ( $\phi_{T,y}$  is large enough). Note that the same parameters are also key determinants for whether condition b) is satisfied.<sup>17</sup>

It is instructive to contrast this baseline result with a situation in which government spending adjusts endogenously to output while the economy is at the ZLB. The following proposition summarizes the pertinent conditions for the existence of a unique bounded equilibrium.

**Proposition 2** *In the economy specified in Proposition 1, let government spending  $\tilde{g}_t$  take on a value of  $\tilde{g}_t = \varphi\tilde{y}_t$  when the economy is at the ZLB, and  $\tilde{g}_t = 0$  otherwise. Suppose further that  $\varphi < 1$ . Define  $a^* := \mu + \mu\xi\phi_{T,y}^*\bar{\sigma}^*$ ;  $\kappa_y^* = \kappa_y - \varphi\kappa_g$ ;  $\phi_{T,y}^* := \phi_{T,y} - \varphi$ ; and  $\bar{\sigma}^* = \bar{\sigma}/(1 - \varphi)$ . There exists a locally unique bounded equilibrium if and only if:*

1. with  $a^* > 0$

$$a) \quad a^* < 1/(\beta\mu), \quad \text{and} \quad b) \quad (1 - \beta\mu)(1 - a^*) > \mu\bar{\sigma}^*\kappa_y^*,$$

2. with  $a^* < 0$ :

$$a) \quad (1 + \beta\mu)(1 + a^*) > -\mu\bar{\sigma}^*\kappa_y^* \quad \text{and} \quad b) \quad (1 - \beta\mu)(1 - a^*) > \mu\bar{\sigma}^*\kappa_y^*.$$

**Proof.** See Appendix C. ■

---

<sup>17</sup>The analytical results in Proposition 1 do not depend on the strength of the central bank’s response to inflation once the economy has left the ZLB,  $\phi_\pi$  (apart from whether the parameter satisfies the Taylor principle). At first glance this seems to contradict the results in Davig and Leeper (2007), who study an economy with monetary regime changes in which the Taylor principle is satisfied in one regime but not the other. Their calculations, however, explicitly exclude the possibility of a ZLB scenario in which monetary policy does not react to inflation at all.

To appreciate the implications of this proposition, consider first the possibility that there is no sovereign risk channel ( $\xi = 0$ ). In this case the range of parameters for which the equilibrium is determinate is larger if spending is countercyclical ( $\varphi < 0$ ). With an endogenous risk premium, however, the opposite may hold. More precisely, if  $\xi > 0$  and if the conditions of Item 1 of Proposition 2 hold, then subject to some limits on the elasticity of taxes with respect to output, namely,  $\phi_{T,y} < 1 - \frac{\kappa\nu}{(1-\beta\mu)\xi}$ , the range of fundamental parameters for which the equilibrium is determinate is at least as large with a procyclical spending response,  $\varphi \in (0, 1)$ , as without any response, and can be strictly larger. Note that this case is more likely the lower the elasticity of tax revenue to economic activity (the smaller  $\phi_{T,y}$ ), and the more strongly the interest rate spread responds to the deficit (the larger  $\xi$ ). The main conclusion is straightforward, if unconventional: A procyclical fiscal stance may reduce the risk of equilibrium indeterminacy in the presence of sovereign risk.<sup>18</sup>

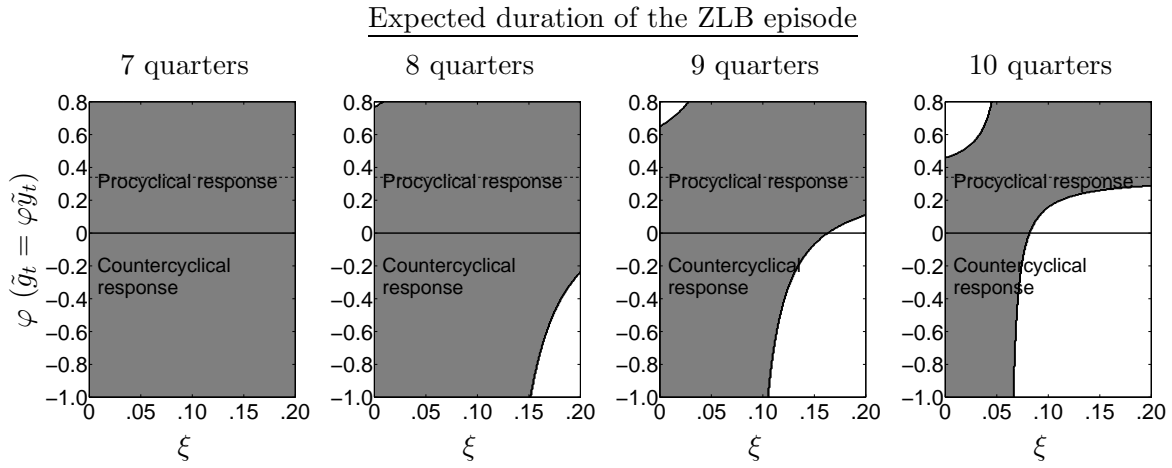
The two propositions above deserve further discussion. We assume throughout that fiscal policy is passive in the sense of Leeper (1991), that is, taxation will eventually reduce sovereign debt to reasonably low levels in the long run. Yet sovereign default can occur, and affect economic outcomes, along the way. Indeed, we find that an economy with an endogenous risk premium and a constrained central bank is prone to belief-driven equilibria.

Moreover, we find that systematic spending cuts during ZLB episodes may actually help to anchor expectations to a unique equilibrium. To see why, assume that during the ZLB period agents expect some nonfundamental drop in output. Lower output would mean less tax revenue and, in the absence of a fiscal response, higher deficits. In high-debt economies these deficits would imply a significantly higher interest rate spread. Since a widening of the interest rate spread cannot be offset by monetary policy at the ZLB, the real interest rate would rise. A sharp rise in real rates will weigh sufficiently on private demand to make nonfundamental expectations of adverse output developments self-fulfilling. By contrast, a procyclical fiscal stance can be sufficient in a high-debt economy to prevent an adverse expectational shock from confirming itself. The reason is that systematic cuts in public spending would offset the expected decline in tax revenue triggered by a fall in output, thereby dampening the increase in the credit spread and the real rate.

---

<sup>18</sup>See Appendix D, Corollary 4 for details. It bears stressing that we focus here on very simple fiscal and monetary rules to maintain analytical tractability. More complicated rules that would make future policy behavior depend on past developments might, in principle, help overcome problems of indeterminacy as well.

Figure 3: Determinacy regions (grey) – endogenous response of government spending



*Notes:* Determinacy regions for the case of an endogenous response of government spending to economic activity during a deep recession. Grey areas mark parameterizations that imply determinacy. y-axis: response of government spending to output,  $\varphi$  ( $\tilde{g}_t = \varphi \tilde{y}_t$ ). x-axis: response of the interest rate spread to the deficit,  $\xi$ . From left to right: ZLB is expected to bind for 7, 8, 9, or 10 quarters ( $\mu = 6/7, 7/8, 8/9, 9/10$ ). The dashed horizontal line pertains to a value of  $\varphi = 0.34$ .

Figure 3 illustrates Propositions 1 and 2 graphically, adopting the parameterization discussed in Section 3. The x-axis in each panel traces out different slopes,  $\xi$ , of the interest spread with respect to the deficit. The y-axis traces out different responses of government spending to output,  $\varphi$ . We plot a range from  $\varphi = -1$  (so that for each one-dollar drop in GDP, government spending countercyclically rises by one dollar) to a value of  $\varphi = 0.8$ , marking a very procyclical policy. Each panel of Figure 3 displays results for a different value of  $\mu$ , implying, from left to right, an expected duration of the ZLB episode of 7, 8, 9, and 10 quarters, respectively. For each of the different combinations of  $\varphi$ ,  $\xi$ , and the expected duration of the ZLB episode, the panels indicate whether a unique equilibrium exists (grey area) or not (white area).

As the panels show, the sovereign risk channel implies a bound on the admissible degree of countercyclicality of government spending for high-debt economies. If the ZLB is expected to bind for only seven quarters and the slope of the risk premium is within the range plotted in Figure 3, the equilibrium is determinate for all values of the response parameter  $\varphi$  shown. In other words, equilibrium determinacy is not affected by whether government spending is pro- or countercyclical. However, the longer the ZLB is expected to bind, the more the determinacy region shrinks for high debt levels (high values of parameter  $\xi$ ). In a ZLB episode lasting eight quarters, for example, a strongly countercyclical response of government spending ( $\varphi$



close to -1) would induce indeterminacy for values of  $\xi$  above about 0.15; see the lower-right corner of the second panel. A less countercyclical or even procyclical response, instead, would ensure that expectations remain anchored. As the expected length of the ZLB episode increases, the indeterminacy region grows in size. Thus, with a ZLB episode expected to last nine quarters and  $\xi$  on the order of 0.16 or larger, a determinate equilibrium is ensured only with procyclical spending policy; see third panel. The cutoff value for  $\xi$  moves even closer to the range of values reported in Table 1 when the ZLB is expected to bind for 10 quarters (rightmost panel). In sum, when monetary policy is constrained, the decision to cut spending during a downturn can help high-debt countries to lower the risk of belief-driven equilibria. In terms of the magnitude of the required spending cuts, the panels show a dashed horizontal line at the value of  $\varphi = 0.34$  as a point of reference. At that value, under our calibration, the government promises to cut spending so as to exactly offset the reduction in tax revenue caused by a drop in GDP, that is, the deficit would be unchanged.<sup>19</sup>

Last, we note that a procyclical spending response helps to anchor expectations under the specific conditions named above, that is, constrained monetary policy and high debt levels. At lower levels of debt in a ZLB episode, however, procyclical government spending can have the opposite, destabilizing effect. The reason is that at low levels of debt (or a low  $\xi$ ), spending cuts reduce the risk premium by very little. The direct negative effect on demand of lower spending therefore prevails. As a result, procyclical fiscal policy tends to validate, rather than invalidate, recessionary sunspot expectations. This risk rises as the length of the ZLB episode increases (since the spending multiplier increases). In particular, observe that in the three rightmost panels of Figure 3, there is a growing area of indeterminacy in parameter regions marked by a moderate slope of the risk premium and procyclical spending policies (the upper-left white corner). Thus, the effect of different spending responses hinges critically on the specific fiscal and monetary circumstances at hand.

---

<sup>19</sup>As a caveat, we note that the linearized environment considered here may miss an implementability constraint for (perhaps implausibly) extreme sunspot expectations. In particular, government spending in our calibration accounts for 20 percent of GDP in steady state. Suppose that agents expect output to fall by close to 100 percent. Since government spending cannot be negative, the government cannot commit to cutting spending by more than 20 percent of steady-state GDP. Such extreme sunspot expectations, therefore, can only be fended off if a value of  $\varphi$  no larger than 0.2 already ensures determinacy.

#### 4.4 Output effects of government spending cuts

Next, we turn to the crucial question of how exogenous cuts to government spending affect output, the deficit, and the interest spread. For that purpose, we limit our analysis to parameterizations that imply a stable and unique equilibrium. Similar to the findings in the previous subsection, sovereign risk and the expected duration of the ZLB episode turn out to be key determinants of both the size and the sign of the fiscal multiplier.

We continue to focus on an economy that is initially at the ZLB. Following Woodford (2011) and Christiano et al. (2011), we assume that government spending deviates from its steady-state level only during the ZLB episode, by taking on a value of  $\tilde{g}_t = g_L$ . The following proposition summarizes our results.

**Proposition 3** *Under the conditions spelled out by Proposition 1 (which ensure that a locally unique bounded equilibrium exists), let shock  $\Gamma_t$  take on a value of  $\Gamma$  for as long as monetary policy is constrained to a zero interest rate, and a value of 0 otherwise. Similarly, let government spending take on a value of  $g_L$  when the economy is at the ZLB, and 0 otherwise. As before, define  $a = \mu + \mu\xi\phi_{T,y}\bar{\sigma}$ , and  $b = \mu + \mu\bar{\sigma}\xi$ . Then, while monetary policy is constrained, output is given by*

$$y_L = \vartheta_r(\log(1 + i^d) - \Gamma) + \vartheta_g g_L,$$

where

$$\vartheta_r = \frac{\bar{\sigma}(1 - \beta\mu)}{(1 - \beta\mu)(1 - a) - \mu\bar{\sigma}\kappa_y} > 0 \quad (38)$$

and

$$\vartheta_g = \frac{(1 - \beta\mu)(1 - b) - \mu\bar{\sigma}\kappa_g}{(1 - \beta\mu)(1 - a) - \mu\bar{\sigma}\kappa_y}. \quad (39)$$

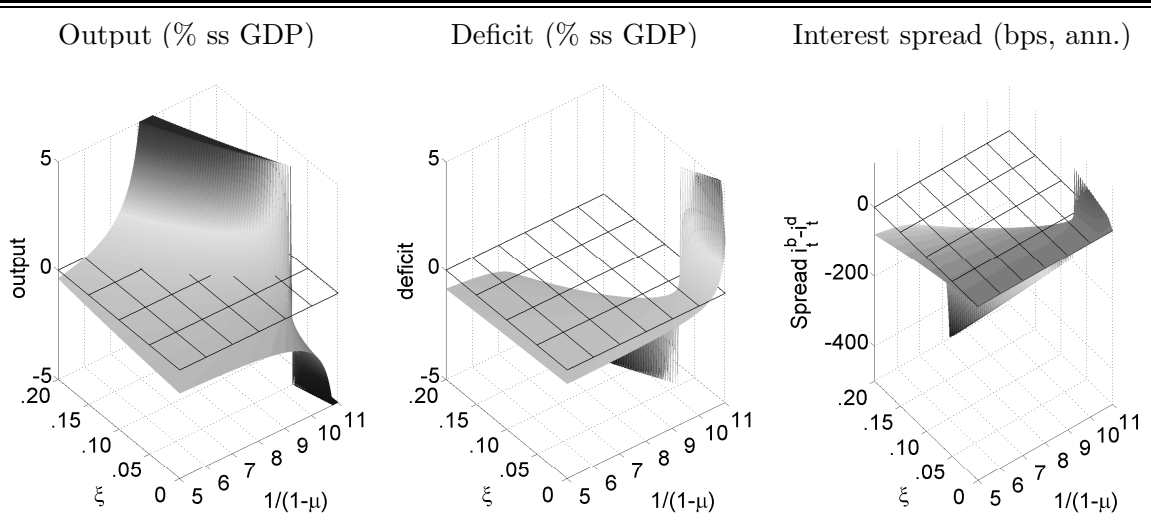
**Proof.** See Appendix C. ■

Note that  $\vartheta_g$  provides a measure for the government spending multiplier on output at the ZLB. It is characterized in more detail by Corollary 5 in Appendix D. Specifically, under the determinacy conditions established above, equation (39) implies that the multiplier is positive if and only if

$$(1 - \mu) - \frac{\mu\bar{\sigma}\kappa_g}{1 - \beta\mu} > \mu\xi\bar{\sigma}. \quad (40)$$

If this condition is satisfied, a spending cut at the ZLB will reduce output, consistent with conventional wisdom. Figure 4 illustrates the proposition graphically. The left panel displays the output effect of a government spending cut during the ZLB episode for different strengths

Figure 4: Effects of early fiscal retrenchment (relative to no retrenchment)



*Notes:* The figure shows the effects of a unit cut in government spending by 1 percent of steady-state GDP for the length of the ZLB episode. Effect on output (left panel), on the deficit (middle panel: negative means the deficit shrinks), and on the interest rate spread (right panel). On the axes: responsiveness of interest spread to expected deficit,  $\xi$ , and expected duration of ZLB episode,  $1/(1-\mu)$ . Only parameterizations that imply determinacy are shown. Parameters other than  $\xi$  and  $\mu$  take on values as described in Section 3. For better readability, multipliers and deficits are capped at the lowest and highest levels indicated in the charts.

of the sovereign risk channel (measured as before by alternative values for  $\xi$ ) and for different expected durations of the ZLB episode (measured by alternative values for  $1/(1-\mu)$ ). The fiscal multiplier depends crucially on both of these dimensions. Consider first the case in which  $\xi = 0$ , that is, a situation without a sovereign risk channel. In this case, a spending cut invariably causes a more than one-for-one decline in output when the ZLB binds. The effect is stronger the longer the expected duration of the ZLB episode, as stressed by Christiano et al. (2011) and Woodford (2011). The reason is that the deflationary effect of spending cuts cannot be counteracted by a reduction in the policy rate, thus causing an increase in the real interest rate during the entire period of the fiscal retrenchment. Private demand, in turn, is determined by the expected path of current and future real interest rates.

The effect on the primary budget deficit is twofold (middle panel). First, the spending cut directly reduces the deficit one for one. A second, indirect effect works through the tax revenue and thus depends on the behavior of aggregate demand. For  $\xi = 0$  and relatively short expected durations of the ZLB episode, the direct effect of spending cuts dominates, so that the primary deficit falls. As the expected duration of the ZLB increases, the spending multiplier rises, implying a larger hit to aggregate demand. At some point, the indirect effect

on tax revenue starts to dominate, and the fiscal consolidation becomes self-defeating: the deficit rises in response to a spending cut.

Consider now the case in which the sovereign risk channel is active,  $\xi > 0$ . Let us focus, first, on a scenario in which the ZLB episode is expected to be short, say, only five quarters ( $1/(1-\mu) = 5$ ). In that case, the fiscal multiplier will be smaller—and hence the output effect of a spending cut less negative—the higher the initial debt level (as captured by a larger value of  $\xi$ ); see the left panel. Still, the role of the sovereign risk channel is limited: even for very high values of  $\xi$ , the spending cut remains contractionary. That said, spending cuts succeed in lowering the deficit (middle panel) and, therefore, in reducing the interest rate spread (right panel) in the case of a short ZLB episode.

Much stronger effects through the sovereign risk channel emerge if monetary policy is expected to be constrained for an extended period. Specifically, for long ZLB episodes and high (but still empirically relevant) values of  $\xi$ , the sign of the output multiplier may actually turn negative: A spending cut becomes expansionary. As a practical example under our parameterization, in a ZLB episode expected to last for 10 quarters, the multiplier turns negative at a value of  $\xi = 0.061$ , which according to Table 1 corresponds to a debt level of about 130 percent of GDP. To understand this finding, it is useful to consider the responses of the deficit and the risk premium. For most of the parameterizations shown in Figure 4, a cut in government spending reduces the deficit, similar to the findings in Erceg and Lindé (2010). If fiscal strain is severe at the outset, the lower deficit leads to a considerable decline in the risk premium, which reduces the interest rate spread (right panel). The stimulating effect this has on private demand is particularly strong if the ZLB is expected to bind for some time. The higher tax revenue associated with this in turn leads to a virtuous cycle of an additional decline in credit spreads, increased economic activity, and a further improvement of the fiscal outlook.

As the left panel of Figure 4 makes clear, for an expected ZLB duration of up to 10.4 quarters or so these effects prevail for all values of  $\xi$ . Beyond that, for moderate values of  $\xi$ , we observe that the dynamics change again. A higher initial debt level now implies a bigger decline in output in response to the spending cut. To see why, consider an expected duration of the ZLB of 10.5 quarters. In that case, for  $\xi = 0$ , the output multiplier is as high as 3.2. Combined with a semi-elasticity of taxes of  $\phi_{T,y} = 0.34$ , the indirect effect of the spending cut on the deficit, through lower tax revenue, becomes so large as to outweigh the direct effect from

lower spending. Under those conditions, the higher  $\xi$ , the more the rising deficit pushes up the sovereign risk premium, compounding the negative impact on economic activity.

Note, last, that for the durations of the ZLB beyond about 10.4 quarters that we discuss currently, there is a large range of values of  $\xi$  for which Figure 4 does not report observations.

The reason is that they imply indeterminacy. Among these, for values of  $\xi$  close to, but above the cutoff systematic countercyclical spending policy of the form discussed in the previous section would guarantee determinacy (not shown). Such a policy would not anchor expectations for the large range of values of  $\xi$  farther above the cutoff, however. For these, instead, a policy of *systematic* spending cuts (rather than the exogenous cuts considered here) would ensure determinacy, in line with our discussion of Figure 3.

In sum, similar to our earlier results on equilibrium determinacy, we find that the effect of spending cuts on output and the deficit depends in subtle and non-monotonic ways on the initial fiscal position and the severity of the constraints on monetary policy.

## 5 Dynamic analysis

We now turn to a numerical analysis of the full model as introduced in Section 2. This allows us to account for the possibility that sovereign risk depends on the expected debt level, rather than the expected deficit. We therefore no longer have to work with a heuristic mapping of sovereign indebtedness into the parameters of the model. We solve the nonlinear model under perfect foresight. We also depart from the simplifying assumption that the expected duration of the ZLB episode is exogenously given. Instead, we envisage a scenario in which a) the initial debt level matters for the depth of the recession, and b) fiscal retrenchment may alter the length of the ZLB episode.

### 5.1 Deep recessions and sovereign risk

The scenario that we compute is as follows. We start the economy in the steady state. In the first period, a shock to the discount factor materializes. The process for this shock is persistent,  $\log(e_t) = \rho_e \log(e_{t-1}) + u_t$ . We calibrate both the initial innovation to the shock,  $u_0$ , (with  $u_t = 0$  for all  $t > 0$ ) and the persistence parameter,  $\rho_e$ , such that the baseline calibration (with a ratio of government debt to annual GDP at 60 percent) reproduces salient

features of the U.S. economy during the 2007–2009 recession.<sup>20</sup> We then take this process for the shock as given and ask how the economy would have evolved if the debt level, and therefore sovereign risk premia, had been higher at the outset.

In our model, the evolution of tax revenue,  $tr_t$ , matters for government debt and, thereby, aggregate activity. It is determined by equation (26), which specifies the response of revenue to the business cycle (via parameter  $\phi_{T,y}$ ) and to the level of sovereign debt (via parameter  $\phi_{T,b^g}$ ). We set  $\phi_{T,y} = 0.34$  as before. As regards  $\phi_{T,b^g}$ , we face a quandary, once we compare alternative scenarios for the initial level of debt. On the one hand, for higher debt levels, the interest costs of financing the debt will be higher and sensitive to the state of the economy. As a result, the government will need to raise disproportionately more revenue than for low debt levels. This suggests that we need a higher value of  $\phi_{T,b^g}$ . On the other hand, this would mean that for many initial debt levels, the debt stock would fall very rapidly toward the target, which appears unrealistic. In order to assess the role of spending-based consolidation at high debt levels, we resolve this trade-off in the following way. Throughout our simulations, we set  $\phi_{T,b^g} = 0.048$ . This value is large both from an empirical and a theoretical perspective.<sup>21</sup> At the same time, we augment the tax equation by an additional term that—in the absence of fiscal consolidation measures—adjusts so as to keep the debt level unchanged for 25 quarters.<sup>22</sup> Turning to the calibration of the shock, we note the Congressional Budget Office’s (2011) assessment that the U.S. output gap reached 6.7 percent in 2009 and will still be at 1.7 percent in 2014 and at 0.5 percent in 2015. Values of  $u_0 = -0.1475$  and  $\rho_e = 0.93$  imply a good fit with this path under the baseline (which has a 60 percent debt-to-GDP ratio and no spending cuts).<sup>23</sup> Figure 5 shows the evolution of output, the private-sector interest rate spread, and the policy rate in response to the recessionary shock. It displays the behavior of the economy in the absence of fiscal consolidation for four different initial levels of debt:

---

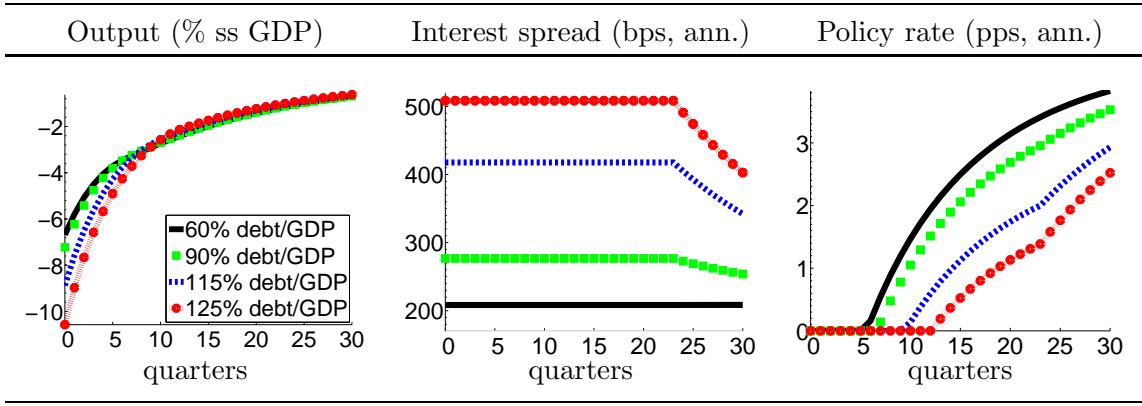
<sup>20</sup>The scenario for the underlying shock process differs from the Markov assumption entertained in Section 4. Nevertheless, the results that we obtain here are quite similar.

<sup>21</sup>It is seven times the strength of the response that would be needed to stabilize debt in an economy linearized around a 60 percent debt-to-GDP ratio.

<sup>22</sup>Specifically, we set  $T_t^g/P_t = tr_t + \text{exceptionaltax}_t$ . In the simulations, for the first 25 quarters, we find a sequence of values for the “exceptional tax” that keeps the debt level stable at a certain level, say 115 percent of GDP. For periods thereafter, the exceptional tax is set equal to zero. When considering spending cuts, the systematic part of taxes follows equation (26), and the exceptional tax takes on the sequence of values determined in the absence of fiscal retrenchment.

<sup>23</sup>This baseline scenario allows us to analyze the likely impact of fiscal measures over and above the actual (expansionary) fiscal measures taken in response to the crisis, as targeting the actual output gap implicitly controls for their effect.

Figure 5: Effect of recessionary shock at different levels of government debt



*Notes:* The figure shows responses to the recessionary shock for different initial debt levels. Black solid line: 60 percent debt-to-GDP, green squares: 90 percent, blue dashes: 115 percent, red dots: 125 percent. Output ( $y_t$ ) is expressed in terms of percent deviations from the steady state, the interest spread ( $\omega_t$ ) in annualized basis points, and the policy interest rate ( $i_t^d$ ) in annualized percentage rates. The timing of the exit from the ZLB is endogenously determined by implementing the nonnegativity constraint along with Taylor-rule equation (29).

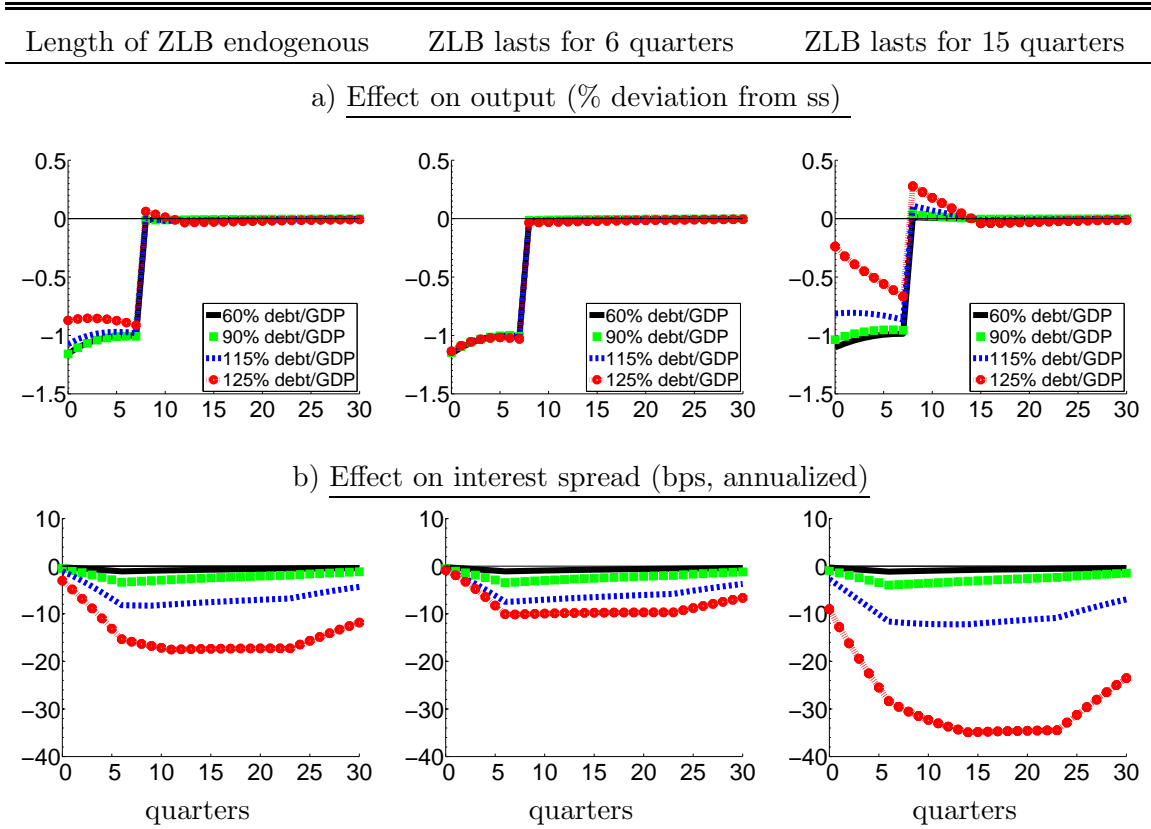
60 percent of GDP (that is, the debt level considered in calibrating the recessionary shock, marked here by a black solid line), 90 percent (green squares), 115 percent (blue dashed line), and 125 percent (red dots). Clearly, the adjustment dynamics differ substantially by debt level. In particular, the interest rate spread is much larger if debt is high (middle panel). Since the recessionary shock is so severe that the ZLB becomes binding, the higher spread translates into higher real interest rates. As a consequence, private expenditure and output fall more for high debt levels (left panel). A persistently high spread also extends the time span over which the ZLB remains binding (right panel): Moving from a 60 to a 125 percent debt level, the ZLB episode lengthens by seven quarters. Not only does the importance of the sovereign risk channel, therefore, depend on whether the central bank is constrained in steering interest rates, but sovereign risk may itself be an important determinant for how severely monetary policy is constrained in the face of certain shocks.

## 5.2 Fiscal retrenchment

We now turn to the effects of government spending cuts. To that end, we consider an episode of spending cuts that starts at the onset of the recession and lasts for two years, corresponding to an “immediate retrenchment” scenario similar to the setup in Section 4. During this episode, government spending falls below its steady-state level by 1 percent of (steady-state)

GDP. Figure 6 depicts the effect on output (top row) and the private-sector interest spread (bottom row) relative to the case of no retrenchment. In order to isolate the effect of the ZLB, the panels on the left show the case when the length of the ZLB episode is endogenously determined. The panels in the middle and on the right, instead, assume that the length of the ZLB is exogenously fixed at 6 and 15 quarters, respectively.

Figure 6: Impact—relative to the baseline—of immediate retrenchment



*Notes:* The figure shows the effects of an immediate retrenchment in government spending by 1 percent of steady-state output for 8 quarters. The panels display the effect on output,  $y_t$ , (top) and the private-sector interest spread,  $\omega_t$  (bottom). Black solid line: 60 percent debt-to-GDP, green squares: 90 percent, blue dashes: 115 percent, red dots: 125 percent. Left panels: effect of the retrenchment when the timing of the exit from the ZLB is endogenously determined by implementing the nonnegativity constraint along with equation (29). Other panels: policy rate exogenously kept at zero for six quarters (middle panels) and 15 quarters (right panels), respectively, whereafter the interest rate follows the Taylor without regard to the ZLB constraint. For all panels, the “extratax” revenue has been calibrated to stabilize debt, in the absence of fiscal retrenchment, at, respectively, 60, 90, 115, or 125 percent of GDP for the first 25 quarters.

We find, first, that for all initial debt levels, the retrenchment package is effective in reducing the deficit and the level of government debt. Consequently, the risk premium declines (bottom row). In some cases, the retrenchment also allows monetary policy to escape from the ZLB



constraint earlier—by 1 quarter, for example, in the case of debt at 125 percent of GDP (not shown). However, the spending cuts have quite different effects on output across the four scenarios. If initial debt stands at a moderate 60 percent of GDP, the spending cuts prompt an initial output decline of more than 1 percent, since private expenditure also falls. In contrast, if debt is as high as 125 percent of GDP, output initially falls by less than 1 percent, reflecting a rise in private expenditure.

The panels in the middle and right columns of Figure 6 show the effect of the austerity package on output for different debt levels, but fixing the length of the ZLB episode exogenously. If the lower bound is expected to be binding for only a short period of time (middle panels), the effect of the spending cuts on output hardly varies with the debt level. However, in line with our earlier results in Section 4, the picture changes quite dramatically if the lower bound is expected to bind for an extended period (right panels). In this case, we find that spending cuts are markedly more favorable for economic activity at high levels of debt. At 125 percent of debt to GDP, fixing the interest rate at zero for 15 quarters implies that the spending cuts turn out to be almost neutral for GDP on impact (dotted red line, top-right panel). The key factor is the benign response of the interest rate spread, which falls by nearly 40 basis points (dotted red line, bottom-right panel). In sum, our simulations of the full-fledged model confirm what we found for the simplified model earlier: Spending cuts can become far less detrimental to output at high levels of debt than they would be at low debt levels. The key conditioning factor, as before, is the extent of monetary accommodation, notably the constraints imposed on the central bank by the ZLB.

## 6 Conclusion

The present paper analyzes how the “sovereign risk channel” affects macroeconomic dynamics and stabilization policy. Through this channel, rising sovereign risk drives up private-sector borrowing costs, unless higher risk premia are offset by looser monetary policy. If the central bank is constrained in neutralizing elevated credit spreads, notably because policy rates may have fallen to the zero lower bound (ZLB), sovereign risk becomes a critical determinant of macroeconomic dynamics.

Building on the model proposed by Cúrdia and Woodford (2009), we show that the sovereign risk channel makes the economy (more) vulnerable to problems of indeterminacy. Specifically,

private-sector beliefs about a weakening economy can become self-fulfilling, driving up risk premia and choking off demand. In this environment, a procyclical fiscal stance—tighter spending policy during economic downturns—can help to ensure determinacy. Further, we find that sovereign risk tends to exacerbate the effects of cyclical shocks. In particular, the negative demand shock that we consider causes a deeper recession and a longer ZLB episode if the sovereign risk channel is stronger.

The sovereign risk channel also has a significant bearing on fiscal multipliers. Specifically, the effect of government spending on aggregate output hinges on (i) the responsiveness of risk premia to changes in public indebtedness; (ii) the length of time during which monetary policy is expected to be constrained; and (iii) the sensitivity of tax revenue to economic activity. Our analysis suggests that procyclical fiscal retrenchment is typically less detrimental to economic activity (that is, multipliers are smaller) in the presence of significant sovereign risk, as lower public deficits improve private-sector financing conditions. In extreme cases in which fiscal strains are very severe and monetary policy is expected to be constrained for an extended period, fiscal tightening in a downturn may even exert an expansionary effect. That said, we also find cases of self-defeating fiscal consolidation—characterized by intermediate degrees of fiscal strain and very long ZLB episodes—in which output contracts and the deficit rises.

As a caveat, we note that our analysis focuses on fiscal multipliers under a “go-it-alone” policy that does not involve external financial support at below-market rates. Availability of such support could allow countries to stretch out the fiscal adjustment as they benefit from lower funding costs and, possibly, positive credibility effects. Indeed, if and where announcements of future fiscal adjustment are credible, delaying some of the planned spending cuts remains a superior strategy in terms of protecting short-term growth (see Corsetti et al. 2010).

We also recognize that the ZLB is neither a necessary nor a sufficient condition for constraining the central bank’s room for maneuver. On the one hand, central banks may be unable to neutralize high spreads even when policy rates are positive, notably under pegged exchange rates. On the other hand, unconventional monetary policy tools, such as large-scale bond purchases, may provide scope to ease financial conditions above and beyond the determination of short-term money market rates. We find it plausible, however, that the additional impact from such policies is bounded in practice. Our results should thus remain qualitatively relevant even where a wider set of monetary policy tools is accounted for.

## References

- Aizcorbe, A., Kennickell, A., and Moore, K. (2003), ‘Recent Changes in U.S. Family Finances: Evidence from the 1998 and 2001 Survey of Consumer Finances,’ *Federal Reserve Bulletin*.
- Ardagna, S., Caselli, F., and Lane, T. (2007), ‘Fiscal Discipline and the Cost of Public Debt Service: Some Estimates for OECD Countries,’ *The B.E. Journal of Macroeconomics*, 7(1).
- Arellano, C. (2008), ‘Default Risk and Income Fluctuations in Emerging Economies,’ *American Economic Review*, 98, pp. 690–712.
- Baldacci, E., Gupta, S., and Mati, A. (2008), ‘Is It (Still) Mostly Fiscal? Determinants of Sovereign Spreads in Emerging Markets,’ *IMF Working Paper 08/259*.
- Baldacci, E. and Kumar, M. (2010), ‘Fiscal Deficits, Public Debt, and Sovereign Bond Yields,’ *IMF Working Paper 10/184*.
- Bank of England (2008), ‘Minutes of the Monetary Policy Committee Meeting 5 and 6 November 2008,’ [www.bankofengland.co.uk/publications/minutes/](http://www.bankofengland.co.uk/publications/minutes/).
- Benhabib, J., Schmitt-Grohé, S., and Uribe, M. (2002), ‘Avoiding Liquidity Traps,’ *Journal of Political Economy*, 110, pp. 535–563.
- Bi, H. (2012), ‘Sovereign Default Risk Premia, Fiscal Limits, and Fiscal Policy,’ *European Economic Review*, 56, pp. 389–410.
- Bils, M. and Klenow, P. J. (2004), ‘Some Evidence on the Importance of Sticky Prices,’ *Journal of Political Economy*, 112(5), pp. 947–985.
- Board of Governors of the Federal Reserve System (2007), ‘Minutes of the Federal Open Market Committee October 30-31 2007,’ [www.federalreserve.gov/newsevents/press/monetary/20071120a.htm](http://www.federalreserve.gov/newsevents/press/monetary/20071120a.htm).
- Borgy, V., Laubach, T., Mesonnier, J.-S., and Renne, J.-P. (2011), ‘Fiscal Policy, Default Risk and Euro Area Sovereign Bond Spreads,’ mimeo, Banque de France.
- Braun, A. R., Körber, L. M., and Waki, Y. (2012), ‘Some Unpleasant Properties of Log-Linearized Solutions When the Nominal Rate Is Zero,’ Federal Reserve Bank of Atlanta Working Paper 2012-5.
- Bucks, B., Kennickell, A., Mach, T., and Moore, K. (2009), ‘Changes in U.S. Family Finances from 2004 to 2007: Evidence from the Survey of Consumer Finances,’ *Federal Reserve Bulletin*.
- Christiano, L., Eichenbaum, M., and Rebelo, S. (2011), ‘When Is the Government Expenditure Multiplier Large?’ *Journal of Political Economy*, 119(1), pp. 78–121.

- Christiano, L. J. and Eichenbaum, M. (2012), ‘Notes on Linear Approximations, Equilibrium Multiplicity and E-Learnability in the Analysis of the Zero Lower Bound,’ mimeo, Northwestern University.
- Congressional Budget Office (2011), ‘The Budget and Economic Outlook: Fiscal Years 2011 to 2021,’ Pub. No. 4236.
- Corsetti, G., Kuester, K., Meier, A., and Müller, G. (2010), ‘Debt Consolidation and Fiscal Stabilization of Deep Recessions,’ *American Economic Review, Papers and Proceedings*, 100, pp. 41–45.
- Cúrdia, V. and Woodford, M. (2009), ‘Credit Frictions and Optimal Monetary Policy,’ Unpublished manuscript, Federal Reserve Bank of New York and Columbia University.
- Cúrdia, V. and Woodford, M. (2010), ‘Credit spreads and monetary policy,’ *Journal of Money, Credit, and Banking*, 42, pp. 3–35.
- Davig, T. and Leeper, E. (2007), ‘Generalizing the Taylor Principle,’ *American Economic Review*, 97, pp. 607–635.
- Durbin, E. and Ng, D. (2005), ‘The Sovereign Ceiling and Emerging Market Corporate Bond Spreads,’ *Journal of International Money and Finance*, 24, pp. 631–649.
- Eaton, J. and Gersovitz, M. (1981), ‘Debt with Potential Repudiation: Theoretical and Empirical Analysis,’ *Review of Economic Studies*, 48, pp. 289–309.
- Eggertsson, G. and Woodford, M. (2003), ‘The Zero Interest-Rate Bound and Optimal Monetary Policy,’ *Brookings Papers on Economic Activity*, 1, pp. 139–211.
- Erceg, C. and Lindé, J. (2010), ‘Is There a Fiscal Free Lunch in a Liquidity Trap?’ CEPR Discussion Paper No. 7624.
- European Central Bank (2010), ‘Financial Stability Review, June 2010,’ Frankfurt am Main, Germany.
- Galí, J. and Gertler, M. (1999), ‘Inflation Dynamics: A Structural Econometric Analysis,’ *Journal of Monetary Economics*, 44(2), pp. 195–222.
- Girouard, N. and André, C. (2005), ‘Measuring Cyclically-Adjusted Budget Balances for OECD Countries,’ *OECD Economics Department Working Paper No. 434*.
- Hall, R. (2009), ‘By How Much Does GDP Rise If the Government Buys More Output?’ *Brookings Papers on Economic Activity*, 2, pp. 183 – 231.
- Harjes, T. (2011), ‘Financial Integration and Corporate Funding Costs in Europe After the Financial and Sovereign Debt Crisis,’ in: ‘IMF Country Report No. 11/186,’ International Monetary Fund.

- Haugh, D., Ollivaud, P., and Turner, D. (2009), ‘What Drives Sovereign Risk Premiums? An Analysis of Recent Evidence From the Euro Area,’ *OECD Economic Department Working Paper, ECO/WKP 2009/59*.
- International Monetary Fund (2010), ‘Global Financial Stability Report, April 2010,’ Washington, D.C.
- Juessen, F., Linnemann, L., and Schabert, A. (2011), ‘Understanding Default Risk Premia on Public Debt,’ mimeo, University of Dortmund.
- Laubach, T. (2009), ‘New Evidence on the Interest Rate Effects of Budget Deficits and Debt,’ *Journal of the European Economic Association*, 7(4), pp. 858–885.
- Leeper, E. (1991), ‘Equilibria Under Active and Passive Monetary and Fiscal Policies,’ *Journal of Monetary Economics*, 27(1), pp. 129–147.
- Mendoza, E. and Yue, V. (2011), ‘A General Equilibrium Model of Sovereign Debt Default and Business Cycles,’ *Quarterly Journal of Economics*, forthcoming.
- Mertens, K. and Ravn, M. (2011), ‘Fiscal Policy in an Expectations Driven Liquidity Trap,’ mimeo, Cornell University.
- Moody’s Investors Service (2011), ‘Sovereign Default and Recovery Rates, 1983-2010,’ May 10<sup>th</sup>.
- Panizza, U., Sturzenegger, F., and Zettelmeyer, J. (2009), ‘The Economics and Law of Sovereign Debt and Default,’ *Journal of Economic Literature*, 47, pp. 651–698.
- Reinhart, V. and Sack, B. (2000), ‘The Economic Consequences of Disappearing Government Debt,’ *Brookings Papers on Economic Activity*, 2, pp. 163–220.
- Schabert, A. and van Wijnbergen, S., ‘Sovereign Default and the Stability of Inflation Targeting Regimes,’ (2008), mimeo, University of Amsterdam.
- Taylor, J., ‘Monetary Policy and the State of the Economy,’ (2008), Testimony before the Committee on Financial Services U.S. House of Representatives.
- Woodford, M. (2003), *Interest and Prices*, Princeton University Press, Princeton, NJ.
- Woodford, M. (2011), ‘Simple Analytics of the Government Expenditure Multiplier,’ *American Economic Journal: Macroeconomics*, 3, pp. 1 – 35.
- Yeyati, E. and Panizza, U. (2011), ‘The Elusive Costs of Sovereign Defaults,’ *Journal of Development Economics*, 94(1), pp. 95–105.

## A Three-equation representation of the aggregate economy

The following IS curve can be derived for the linearized model:

$$\begin{aligned}\widehat{y}_t &= \widehat{y}_{t+1} - (\tilde{g}_{t+1} - \tilde{g}_t) \\ &\quad - \bar{\sigma} \left[ \widehat{i}_t^d - \widehat{\Pi}_{t+1} + \widehat{\Gamma}_t + (\pi_b + s_\Omega)\widehat{\omega}_t - [\psi_\Omega + s_\Omega(1 - \bar{\delta})]\widehat{\Omega}_{t+1} \right],\end{aligned}$$

where

$$\widehat{\Omega}_t = \widehat{\omega}_t + \bar{\delta}\widehat{\Omega}_{t+1}.$$

The New Keynesian Phillips curve reads as follows:

$$\begin{aligned}\widehat{\Pi}_t &= \beta\widehat{\Pi}_{t+1} \\ &\quad + \kappa \left[ (\phi(1 + \nu) + 1/\bar{\sigma} - 1)\widehat{y}_t - \phi(1 + \nu)\widehat{z}_t - 1/\bar{\sigma}\tilde{g}_t + [s_\Omega + \pi_b - \gamma_b]\widehat{\Omega}_t \right];\end{aligned}$$

and the Taylor rule is given by:

$$\begin{aligned}\widehat{i}_t^{d,*} &= \phi_\Pi\widehat{\Pi}_t - \phi_\omega\widehat{\omega}_t. \\ \widehat{i}_t^d &= \max \left\{ \widehat{i}_t^{d,*}, -(1 + i^{d,*}) \right\}.\end{aligned}$$

This three-equation system, together with an assumption about the evolution of the interest spread  $\widehat{\omega}_t$ , describes the dynamics of output, inflation, and interest rates (independently of private debt) if

1.  $[\psi_\Omega + s_\Omega(1 - \bar{\delta})] = 0$ .
2.  $[s_\Omega + \pi_b - \gamma_b] = 0$ .

The parameters and laws of motion chosen in the paper satisfy these conditions.

## B Linking $\xi$ and $\xi'$

This appendix provides the foundation for equation (37) in the main text. The formula is motivated through a sequence of back-of-the-envelope calculations. Focus on the interest rate spread in the IS curve, neglecting other terms. Assume that initially, in period  $t$ , the economy is at the ZLB. This gives a relationship of

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \bar{\sigma}\widehat{\omega}_t + \dots$$

Iterating forward, we have

$$\tilde{y}_t = -\bar{\sigma} E_t \left\{ \sum_{j=0}^{\infty} \hat{\omega}_{t+j} I(\text{economy at ZLB in period } t+j) \right\},$$

where  $I()$  is the indicator function. If the spread depends on the future debt level, as in the full version of the model presented in Section 2, we have

$$\hat{\omega}_{t+j} = \xi' E_{t+j} \tilde{b}_{t+j+1}^g,$$

so

$$\tilde{y}_t = -\bar{\sigma} E_t \left\{ \sum_{j=0}^{\infty} I(\text{economy at ZLB in period } t+j) \xi' E_{t+j} \tilde{b}_{t+j+1}^g \right\}.$$

Abstracting from the effect of interest payments and the valuation of debt,  $\tilde{b}_{t+j+1}^g$  is roughly the sum of initial debt in  $t-1$  (assumed to be zero without loss of generality in the following exposition) and the deficits accumulated in period  $t$  through  $t+j+1$ . Let *deficit* be the primary deficit. Following the Markov structure, we assume that the primary deficit (before extraordinary debt-stabilization measures) is the same in every period at the ZLB. With the same abstraction as above, conditional on being at the ZLB in  $t+j$ ,

$$E_{t+j} \tilde{b}_{t+j+1}^g = \tilde{b}_{t+j}^g + \mu \cdot \text{deficit} = (j+1+\mu) \text{deficit}.$$

Then,

$$\tilde{y}_t = -\bar{\sigma} \xi' E_t \left\{ \sum_{j=0}^{\infty} I(\text{economy at ZLB in period } t+j) (j+1+\mu) \text{deficit} \right\}.$$

And so

$$\tilde{y}_t = -\bar{\sigma} \xi' \text{deficit} \sum_{j=0}^{\infty} \mu^j (j+1+\mu),$$

or, equivalently,

$$\tilde{y}_t = -\bar{\sigma} \xi' \frac{1+\mu(1-\mu)}{(1-\mu)^2} \text{deficit}. \quad (41)$$

In contrast, the analytical version of the model in Section 4 has

$$\tilde{y}_t = -\bar{\sigma} E_t \left\{ \sum_{j=0}^{\infty} I(\text{economy at ZLB in period } t+j) \xi \text{deficit} E_{t+j} [I(\text{at ZLB in } t+j+1)] \right\},$$

which boils down to

$$\tilde{y}_t = -\bar{\sigma} \xi \sum_{j=0}^{\infty} \mu^j \text{deficit } \mu.$$

Or, equivalently,

$$\tilde{y}_t = -\bar{\sigma} \xi \frac{\mu}{1-\mu} \text{deficit}. \quad (42)$$

Comparing (41) and (42) leads to equation (37) in the main text:

$$\xi = \frac{1 + \mu(1 - \mu)}{\mu(1 - \mu)} \xi'.$$

## C Proofs of propositions

### C.1 Proof of Proposition 1

The economy, stripped from exogenous variables, is given by

$$E_t z_{t+1} = A z_t,$$

where  $z_t = [\tilde{y}_t; \hat{\pi}_t]$  and

$$A = \frac{1}{a\mu\beta} \begin{bmatrix} \mu\beta + \bar{\sigma}\mu\kappa_y & -\bar{\sigma}\mu \\ -a\kappa_y & a \end{bmatrix},$$

where  $a = (\mu + \mu\bar{\sigma}\xi\phi_{T,y})$ . The Blanchard-Kahn conditions for determinacy require that matrix  $A$  has two roots outside the unit circle. Woodford (2003), pp. 670f., gives the following necessary and sufficient conditions for determinacy:

either (Case I): (i)  $\det(A) > 1$ , (ii)  $\det(A) - \text{tr}(A) > -1$ , and (iii)  $\det(A) + \text{tr}(A) > -1$ ,  
or (Case II): (i)  $\det(A) - \text{tr}(A) < -1$  and (ii)  $\det(A) + \text{tr}(A) < -1$ .

In the current case,  $\det(A) = \frac{1}{a\mu\beta}$  and  $\text{tr}(A) = \frac{1}{a\mu\beta}[\mu\beta + \bar{\sigma}\mu\kappa_y + a]$ . Since both  $\det(A) > 0$  and  $\text{tr}(A) > 0$  Case II cannot be satisfied. Checking Case I, condition (iii) holds since both terms are positive. Condition (i) of Case I is equivalent to condition a) in the proposition. Condition (ii) of Case I is equivalent to condition b) in the proposition. ■

### C.2 Proof of Proposition 2

In this case

$$A = \frac{1}{a^*\mu\beta} \begin{bmatrix} \mu\beta + \bar{\sigma}^*\mu\kappa_y^* & -\bar{\sigma}^*\mu \\ -a^*\kappa_y^* & a^* \end{bmatrix},$$

where  $a^*$ ,  $\bar{\sigma}^*$ , and  $\kappa_y^*$  are defined in the proposition.

1. Note that under the restriction that  $a^* > 0$ ,  $\det(A) > 0$ . Therefore it cannot be the



case that  $\det(A) - \text{tr}(A) < -1$  and  $\det(A) + \text{tr}(A) < -1$ . This means that determinacy can only obtain under the conditions of Case I spelled out in the proof of Proposition 1. Conditions (i) and (ii) of that case correspond to conditions a) and b) in Proposition 2. In addition, if  $\varphi < 1$  (as assumed), then  $\text{tr}(A) > 0$ , so  $\det(A) + \text{tr}(A) > -1$ . Condition (iii) of Case I is therefore obsolete.

2. For  $a^* < 0$ ,  $\det(A) < 0$ , so Case I cannot hold. The conditions given in the proposition are those pertaining to Case II.  $\square$

### C.3 Proof of Proposition 3

The assumed Markov structure means that output, inflation, and government spending (in deviations from the steady state) will take on the same respective values,  $y_L, \pi_L$ , and  $g_L$ , in every period in which monetary policy is constrained, and values of zero thereafter. The IS curve thus implies

$$y_L - g_L = \mu(y_L - g_L) - \bar{\sigma}[-\log(1 + i^d) + \Gamma + \mu\xi(g_L - \phi_{T,y}y_L) - \mu\pi_L],$$

while the Phillips curve gives

$$\pi_L = \mu\beta\pi_L + \kappa_y y_L - \kappa_g g_L.$$

Solving these equations for  $y_L$  and  $\pi_L$  yields:

$$y_L = \vartheta_r [\log(1 + i^d) - \Gamma] + \vartheta_g g_L,$$

where  $\vartheta_r$  and  $\vartheta_g$  take on the values given in the proposition. In addition,  $\vartheta_r > 0$ : the numerator is positive and so is the denominator, per condition b) for determinacy in Proposition 1.

■

## D Two corollaries

### D.1 Statement and proof of Corollary 4

**Corollary 4** *Under the conditions of Proposition 2, the following special cases obtain:*

1. *With no endogenous risk premium ( $\xi = 0$ ), the range of parameters for which the equilibrium is determinate is larger if government spending is countercyclical ( $\varphi < 0$ ), rather than acyclical. In addition, the range of fundamental parameters implying determinacy of the equilibrium is larger the more negative  $\varphi$ .*
2. *With an endogenous risk premium  $\xi > 0$ , instead, the range of parameters for which the equilibrium is determinate is often larger if government spending is procyclical, that is, if spending is cut systematically during a deep recession. More precisely: Under the conditions of Proposition 2, if  $a^* > 0$ ,  $\varphi \in (0, 1)$ , and  $\phi_{T,y} < 1 - \frac{\kappa_y}{(1-\beta\mu)\xi}$ , then the range of fundamental parameters for which the equilibrium is determinate is at least as large*

as in the absence of an endogenous spending response and can be larger. Note that this case is more likely the less tax revenue responds to the state of the economy (the smaller  $\phi_{T,y}$ ), and the more responsive the sovereign risk premium to the deficit (the larger  $\xi$ ).

**Proof.**

1. If  $\xi = 0$ ,  $a^* = \mu > 0$ . As a result, case 1 of Proposition 2 is the relevant one. First note that condition a) will always be satisfied. What remains to be checked is condition b). For the statement in the proposition, it suffices to observe that if  $\xi = 0$ , the left-hand side of condition b) is independent of  $\varphi$ , and the right-hand side is strictly increasing in  $\varphi$ .
2. In order for the range of fundamental parameters for which determinacy holds to be bigger with  $\varphi \in (0, 1)$  under the stated conditions than with  $\varphi = 0$ , we have to have  $a^* < a$ , and

$$(1 - \beta\mu)(a - a^*) > \mu\bar{\sigma}^* \kappa_y^* - \mu\bar{\sigma} \kappa_y.$$

$a^* < a$  boils down to  $\frac{\phi_{T,y} - \varphi}{1 - \varphi} < \phi_{T,y}$ , which is true for  $0 < \varphi < 1$ .

The second condition reduces to

$$(1 - \beta\mu)\xi(1 - \phi_{T,y})\frac{\varphi}{1 - \varphi} > \kappa\nu\frac{\varphi}{1 - \varphi}.$$

For  $\varphi \in (0, 1)$  this yields  $\phi_{T,y} < 1 - \frac{\kappa\nu}{(1 - \beta\mu)\xi}$ , the condition in the corollary.

■

## D.2 Statement and proof of Corollary 5

**Corollary 5** *Under the conditions of Propositions 1 and 3:*

1. *The government spending multiplier,  $\vartheta_g$ , is positive if and only if*

$$(1 - \mu) - \frac{\mu\bar{\sigma}\kappa_g}{1 - \beta\mu} > \mu\xi\bar{\sigma}. \tag{43}$$

*Note, conversely, that the spending multiplier can be negative if the risk premium has a sufficiently detrimental effect on the economy, that is, if  $\xi$  is large enough.*

2. *Government spending at the lower bound is self-financing if  $\vartheta_g > 1/\phi_{T,y}$ . This is the case if*

$$\phi_{T,y} > \frac{(1 - \beta\mu)(1 - \mu) - \mu\bar{\sigma}\kappa_y}{(1 - \beta\mu)(1 - \mu) - \mu\bar{\sigma}\kappa_g}.$$

*This cutoff is independent of  $\xi$ .*

3. *If  $\xi = 0$ , provided that the conditions for determinacy in Proposition 1 are satisfied, the government spending multiplier is strictly larger than one. This case corresponds to the analysis by Christiano et al. (2011) and Woodford (2011).*

4. If  $\xi > 0$ , the government spending multiplier is unambiguously larger than 1 if  $\phi_{T,y} > 1 - \frac{\kappa\nu}{\xi(1-\beta\mu)}$ , that is, if tax revenue is sufficiently responsive to output.

**Proof.**

1. Under the restrictions for determinacy provided by Proposition 1, the denominator of  $\vartheta_g$  is unambiguously positive.  $\vartheta_g > 0$  thus requires  $(1 - \beta\mu)(1 - b) - \mu\bar{\sigma}\kappa_g > 0$ , which solves to the expression in equation (43).
2. The deficit (in deviation from the steady state) is given by  $g_L - \phi_{T,y}y_L$ . Government spending will thus be self-financing if  $1 - \phi_{T,y}\vartheta_g < 0$ . Insert the multiplier from equation (39), and observe that the denominator is positive by the assumption of determinacy. Substitute for  $a$  and  $b$  using the expressions given in the proposition. Simplifying yields the desired inequality for  $\phi_{T,y}$ .
3. The conditions for determinacy require that  $(1 - \beta\mu)(1 - a) - \mu\bar{\sigma}\kappa_y > 0$ . The denominator of  $\vartheta_g$  is, therefore, positive.  $\vartheta_g > 1$  then boils down to

$$(1 - \beta\mu)(1 - b) - \mu\bar{\sigma}\kappa_g > (1 - \beta\mu)(1 - a) - \mu\bar{\sigma}\kappa_y.$$

Now, if  $\xi = 0$  then  $a = b$ .  $\vartheta_g > 1$  therefore requires  $-\mu\bar{\sigma}\kappa_g > -\mu\bar{\sigma}\kappa_y$ , or  $\kappa_g < \kappa_y$ , which is true.

4. For  $\xi > 0$ ,  $\vartheta_g > 1$  is equivalent, after substituting  $\kappa_y - \kappa_g = \kappa\nu$ , to  $\phi_{T,y} > 1 - \frac{\kappa\nu}{\xi(1-\beta\mu)}$ .

■