Putting $\frac{N}{a}$ = population density = d' we have

$$\frac{p}{d'} = R'i \qquad (9)$$

It will be seen that there is a certain analogy between the formulae (8), (9) as applied to the relation between rent (population pressure) and area occupied by a population, on the one hand, and the formulae (6), (7) as applied to the relation between the pressure p and the volume v of a gas. The analogy is particularly close if the income per head i, and the factor R' are constant as a changes. The former may be true, approximately, if the inhabitants of area a derive their income from some extraneous source quite independent of this area. It will not be true, even in rough approximation, if the inhabitants derive their income from the produce of the area a itself. It is not the author's intention to emphasize unduly mere analogies. Nevertheless, the one here presented seems worthy of passing notice. Compare also E. Woodruff, Expansion of Races.

The actual relation between population density and rent or land value is a subject of great economic interest. Research in this subject is in progress under the auspices of the Institute for Research in Land Economics, Madison, Wis., but has not, at this time of writing, matured to published results.

Law of Urban Concentration. An empirical law of urban concentration was pointed out some years ago by F. Auerbach (Petermanns Mittheilungen, 1923, p. 74). Arranging in order of magnitude the cities of a given country, he found that the product of population and ordinal number (rank) was approximately constant. Thus, plotting rank against population, he obtained a hyperbolic curve, or, plotting the product of these two quantities, he obtained, roughly speaking, a straight line. For some of his curves the reader must be referred to the original publication.

The illustration (fig. 60) shows the graph obtained by plotting the logarithm of the population of the cities of the United States (1920) against the logarithm of their respective rank. In the higher ranks only every fifth city has been plotted. It will be seen that, excepting cities of rank 4, 5 and 6, the plot does approximate quite closely to a straight line. The slope of this line, however, is not exactly unity, as demanded by Auerbach's law, but 0.93, so that the

actual law of urban concentration in the United States, in 1920, was, within the limits indicated, given by

$$PR^{0.93} = 5{,}000{,}000$$

where P denoted the population of the city of rank R.

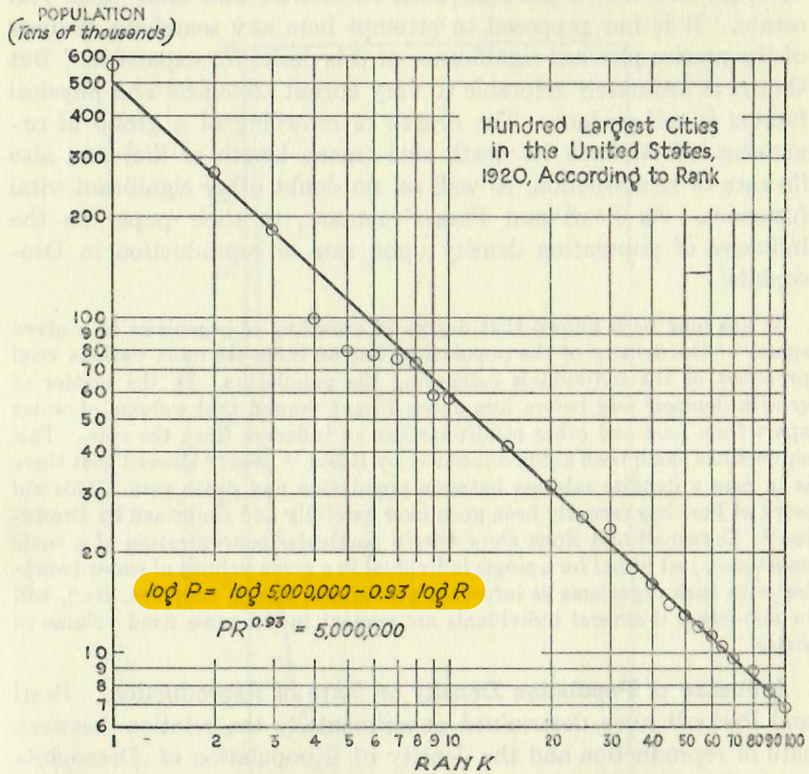

Fig. 60. Law of Urban Concentration

Graph obtained by plotting as ordinates, on logarithmic scale, Population of United States Cities, and as abscissae the corresponding Rank (in order of magnitude), also on logarithmic scale.

It may be left an open question how much significance is to be attached to this empirical formula. We shall meet with a similar relation in Chapter XXIII in dealing with Willis' theory of Age and Area.