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Start-up costs and pecuniary externalities as barriers to economic development

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Abstract

We use a dynamic monopolistic competition model to show that an economy that inherits a small range of specialized inputs can be trapped into a lower stage of development. The limited availability of specialized inputs forces the final goods producers to use a labor intensive technology, which in turn implies a small inducement to introduce new intermediate inputs. The start-up costs, which make the intermediate inputs producers subject to dynamic increasing returns, and pecuniary externalities that result from the factor substitution in the final goods sector, play essential roles in the model.

JEL classification: O11; O31

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1. Introduction

One critical aspect of economic development is that productivity growth is generally associated with an ever greater indirectness in the production process and an ever increasing degree of specialization. In developed economies, consumer goods industries make superior use of highly specialized capital goods,

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particularly in machinery, and enjoy access to a wide variety of producer services, such as equipment repair and maintenance, transportation and communication services, engineering and legal supports, accounting, advertising, and financial services, and so on (Greenfield (1966), Stanback (1979); see Rodríguez (1993) for more extensive references). Many underdeveloped economies, on the other hand, are characterized by relatively simple production methods, and a limited availability of specialized inputs. Attempts to transplant advanced technologies into these economies often meet disaster, as the vast network of auxiliary industries, taken for granted in industrialized economies, is not available in underdeveloped economies (Stigler, 1951; Jacobs, 1969).

We emphasize that there is a fundamental circularity between the choice of technologies by consumer goods producers and the variety of intermediate inputs available. With a wide range of specialized inputs and producer services, firms in the consumer goods sector adopt more indirect and roundabout ways of production and achieve high productivity. The growing demand by the consumer goods industry in turn creates a large market for intermediate goods and brings into being a host of specialized auxiliary industries to service its need. On the other hand, if the economy produces only a limited range of intermediate inputs and producer services, the consumer goods industry is forced to use more primitive modes of production. This in turn implies a limited incentive to start up firms and introduce new goods in the intermediate inputs sector.

The goal of this paper is to show that, under relatively weak and empirically plausible conditions, this circularity is strong enough that an economy that inherits a narrow range of intermediate inputs is trapped into a lower stage of economic development. Our model economy consists of two (final and intermediate goods) sectors and a single primary factor of production (labor). The final goods sector is perfectly competitive. It produces the homogeneous consumption good with constant returns to scale technologies, using labor and a variety of differentiated intermediate inputs. The second sector, which supplies intermediate inputs to final goods producers, is monopolistically competitive. Production of each intermediate good, carried out by a specialist firm, requires the use of labor, as well as possibly minor start-up operations upfront. The firm recovers the start-up costs by selling the good for a price higher than its marginal cost of production. Free entry into this process dissipates its profit in a present value sense. Our model differs from the model of Judd (1985), as reformulated by Grossman and Helpman (1991, Ch. 3.1), only in that the final goods sector may substitute between labor and intermediate inputs. We chose this specification because we believe that our argument can be made most transparent when presented in a familiar setting.

The logic behind the existence of a development trap is based on two factors. First, because of start-up costs, specialist firms that produce intermediate goods are subject to dynamic increasing returns. The inducement to start up operations thus depends on the anticipated market size. When high demand is expected, more firms enter and thus a wider range of specialized inputs will be available. Second, starting up a new firm and introducing a new variety of intermediate inputs generate benefits that are not completely appropriated by those who finance start-up costs. The main beneficiaries are, of course, the buyers of new products. But, an increasing availability of specialized inputs induces the final goods producer to adopt a more roundabout production method and to use intermediate inputs more intensively. In a range where the substitution of intermediate inputs for labor is large, then other producers of intermediate goods also see their demand and profits go up, because of the highly diverse need of the final goods producers. As a result, other firms in the intermediate goods sector also reap some of the benefits of an entry. The presence of such pecuniary externalities leads to an insufficient inducement to start up firms and to introduce new products. These two factors, start-up costs and pecuniary externalities, together imply the circularity between the degree of specialization and the market share of intermediate inputs and present barriers to economic development.

The circularity does not always imply a vicious circle of poverty, however. If the economy inherits a sufficiently broad range of specialized inputs and thus has more than the 'critical mass' of specialist firms, the very fact that the relation is circular generates a virtuous circle. Over time, the division of labor becomes far more elaborate, the production process more indirect, involving an increasing degree of specialized inputs. Through such a cumulative process, the economy experiences productivity growth and a rising standard of living. Our model thus suggests the existence of a threshold in economic development.

When the economy starts below the threshold level, one might wonder why a coordinated entry of specialist firms cannot push the economy above the threshold and make it possible to break away from the development trap. In the analysis below we indeed identify the situations in which entrepreneurial optimism leads specialist firms to start up and the economy escapes the trap due to a sort of self-fulfilling prophecy. In many cases, however, such a coordinated entry is impossible at a lower stage of economic development because start-up operations require reallocation of resources from production. This resource constraint makes a coordinated entry unprofitable when the productivity of the economy is low.

The rest of the paper is organized as follows. The next section reviews the related work in the literature. The basic model is described in Section 3. Section 4 characterizes the market equilibrium and shows that, with a limited substitution between differentiated inputs and labor, there is no development trap. Section 5 discusses why development traps do not exist with the limited substitution, using the notion of Hicks–Allen substitutes and complements. Section 6 then finally shows the existence of development traps with a large substitution between specialized inputs and labor. In Section 7, we extend the basic model to incorporate the technology spillovers associated with the introduction of new products, in the spirit of the recent literature on endogenous growth (Lucas, 1988; Romer, 1986, 1990; Grossman and Helpman, 1991). We provide some general discussions in Section 8.

2. The related work in the literature

The idea that productivity growth can be achieved through specialization goes back to Adam Smith's famous dictum that the division of labor is limited by the extent of the market. The mechanism of economic development presented in this paper is, however, related more directly to Allyn A. Young's (1928) classic article on 'Increasing Returns and Economic Progress'. In this article, he emphasized that the progressive division and specialization of industries, rather than the Smithian subdivision of labor within a firm, is an essential part of the process by which increasing returns are realized. Young also pointed out that there is a strong connection between the economies of specialization and the economies of the capitalistic methods of production, that is, use of labor in roundabout or indirect ways. And, although "the division of labour depends upon the extent of the market", ... the extent of the market also depends upon the division of labour" (Young, 1928, p. 539). It is this circularity that generates underdevelopment traps in our model.

Alfred Marshall (1920) introduced the notion of external economies, dependent on the general progress of the industrial environment, and emphasized the importance of the growth of correlated branches of industry, perhaps being concentrated in the same localities, that supply highly specialized intermediate goods as a source of external effects. Ever since, international, regional, and urban economists have stressed the importance of nontraded specialized producer services as the cause of agglomeration economies and geographical localization; Jacobs (1969), Richardson (1973), Stanback (1979), and very recently, Porter (1990) and Krugman (1991). Some writers have formalized this idea in models of monopolistic competition; see Matsuyama (1995a) for a survey. Among these studies, the work of Rodríguez (1993) is particularly related to ours, as he demonstrated that a switch between two final goods sectors in an open economy gives rise to the possibility of multiple equilibria with differing ranges of nontraded specialized inputs. [1]

Rosenstein-Rodan (1943), Nurkse (1953), and Scitovsky (1954), among other development economists, emphasized 'the complementarity of investment activities across industries'. The main idea, recently formalized by Murphy et al. (1989), is that the introduction of modern efficient methods of large-scale production in an industry, even itself unprofitable, can enhance profitability of investment activities in other industries. Due to such pecuniary externalities, simultaneous investment across a wide range of industries, by creating necessary demand for each other,

[1] External economies and multiple equilibria in the presence of nontraded inputs are also demonstrated in the model of Okuno-Fujiwara (1988), where the intermediate input sector consists of Cournot oligopolists producing the homogeneous good. The entry of new firms generate externalities in his model because of a lower mark-up rate, rather than an increasing variety.

can be profitable and thus should be an essential step for a successful industrial development. This so-called balanced growth doctrine, despite its apparent similarity, differs from our idea in many respects. First, as Fleming (1955) pointed out, the balanced growth doctrine typically stresses "the horizontal complementarity", that is, the interdependence of profitability across different consumer goods industries, while "the vertical complementarity", that is, the interdependence between intermediate inputs and final goods sectors plays an essential role in our model. Second, the balanced growth doctrine relies on the pecuniary externalities due to the income effects, that is the effect of investment on the increased purchasing power of workers. In our model, on the other hand, the pecuniary externalities are caused by the factor substitution, i.e., the shift to a more intermediate goods intensive technology by the final goods sector. Third, our general equilibrium formulation shows clearly how the resource constraint often makes coordinated investment unprofitable. As noted by Fleming (1955), this is the critical point often ignored by the advocates of the balanced growth doctrine. Fourth, the doctrine views the adoption of mass production techniques as an essence of economic development. On the other hand, the cumulative impact of small improvements caused by specialist firms, emphasized in Rosenberg (1982, Ch. 3), is critical in our model. Again, let us quote Young (1928, p. 539): "the mechanism of increasing returns is not to be discerned adequately by observing the effects of variations in the size of an individual firm or of a particular industry,...". "What is required is that industrial operations be seen as an interrelated whole".

Previously, one of us has studied the problem of a development trap and a take-off in dynamic general equilibrium settings. In Matsuyama (1991), a development trap is generated by the technological externalities in sectoral allocations of labor. [2] The model of Matsuyama (1992a), as a dynamic extension of a Murphy–Shleifer–Vishny model, should be viewed as a formulation of the balanced growth doctrine. In particular, the pecuniary externalities in that model are caused by income effects. Despite these differences, the equilibrium dynamics in the model presented below share many properties with the two earlier studies, such as the multiplicity of steady states, the existence of a threshold, and the possibility of a take-off due to a self-fulfilling prophecy.

Finally, Romer (1990), and Grossman and Helpman (1991, Ch. 3) extended the dynamic monopolistic competition model of Judd (1985) in the context of growth and development. Our model can be viewed as a generalization of their models. In this respect, our contribution is to demonstrate that, in the presence of a greater substitution between homogeneous and differentiated goods than they assumed,

[2] Other studies of development traps due to the technological externalities include Azariadis and Drazen (1990), Diamond (1982), and Diamond and Fudenberg (1989), although Diamond and Fudenberg are mainly concerned with business cycles.

the differentiated goods become Hicks–Allen complements to each other, which in turn implies strategic complementarities in the entry process, thereby generating a much richer set of equilibrium behaviors. [3]

3. The basic model

Our basic model differs from Grossman and Helpman (1991, Ch. 3.1) only in that we allow for the substitution between labor and differentiated goods. We preserve every other aspect of their model, as we believe that our idea can be made most transparent when presented in a familiar framework. Our companion paper, Ciccone and Matsuyama (1996), deals with a more general specification of this model.

Time is continuous and extends from zero to infinity. In the economy we consider, households supply L units of labor inelastically and consume the homogeneous final good (taken as the numeraire) over an infinite horizon. At any moment they choose consumption so as to maximize

$$U_t = \int_t^{\infty} \mathrm{e}^{-\rho(\tau - t)} \log(C_\tau) \mathrm{d}\tau,$$

$$\text{s.t.} \quad \int_t^{\infty} \mathrm{e}^{-(R_\tau - R_t)} C_\tau \mathrm{d}\tau \leq L \int_t^{\infty} \mathrm{e}^{-(R_\tau - R_t)} w_\tau \mathrm{d}\tau + W_t$$

where $\rho > 0$ is the subjective discount rate, R_t is the cumulative interest factor up to time t, w_t is the wage rate, and W_t is the value of asset holding, which consists of ownership shares of profit making firms. The solution to this maximization problem is characterized by the Euler condition,

$$\frac{\dot{C}_t}{C_t} = \dot{R}_t - \rho \tag{1}$$

(that is, consumption grows at the rate equal to the interest rate minus the subjective discount rate), as well as the binding budget constraint,

$$\int_t^{\infty} \mathrm{e}^{-(R_\tau - R_t)} (C_\tau - w_\tau L) \mathrm{d}\tau = W_t. \tag{2}$$

The final consumer good is produced by competitive firms. They share the identical constant returns to scale production function, $C_t = F(X_t, H_t)$, where H_t

[3] In a recent article, Young (1993) extended the endogenous growth model of Grossman and Helpman (1991, Ch. 3.2.) by introducing a *technological* complementarity between differentiated goods and showed the possibility of multiple balanced growth paths. In our model, the complementarity arises as a result of equilibrium interaction between the final goods and intermediate inputs sectors.

is the labor input, while X_t is the composite of perishable, differentiated intermediate inputs or 'producer services', which has a form of symmetric CES,

$$X_t = \left[\int_0^{n_t} [x_t(i)]^{1-1/\sigma} \mathrm{d}i \right]^{\sigma/(\sigma-1)}, \quad \sigma > 1, \tag{3}$$

where $x_t(i)$ is the amount of variety i used. We thus take the space of intermediate goods to be continuous and ignore integer constraints on the number of products. Each variety substitutes imperfectly with other varieties; the direct partial elasticity of substitution between every pair of products is equal to σ. At any moment only a subset of differentiated products, $[0, n_t]$, is available in the marketplace. The restriction $\sigma > 1$ implies that no intermediate input is essential; each intermediate good is useful independent of whether other intermediate goods are available. This is necessary as we are interested in the situation in which the range of differentiated inputs available vary over time. Despite that there is no 'left-shoe, right-shoe' problem in this model, the equilibrium interaction may lead to complementarities among differentiated inputs, as demonstrated below.

This specification of product differentiation, first developed by Spence (1976) and Dixit and Stiglitz (1977) and later extended in a dynamic setting by Judd (1985), has one property that is significant for the analysis of development; that is, total factor productivity increases with the range of differentiated inputs available. To see this, let M be the total quantity of intermediate inputs used. Because of symmetry, it is efficient to produce the same quantity of each variety, $x(i) = x$. Then, $nx = M$ and, from (3), $X/M = n^{1/(\sigma-1)}$. Since $\sigma > 1$, this shows the productivity of intermediate goods increases with n. Ethier (1982) and Romer (1987) ascribe this property of technology as increasing returns due to specialization in production. This interpretation has recently been given a formal treatment by Weitzman (1994).

Each intermediate input is supplied by a single, atomistic firm; n_t thus represents not only the range of available varieties but also the 'number' of specialist firms that operate in this economy as of time t. [4] Being a sole supplier, the firm has some monopoly power over its own product market, but it is negligible relative to the aggregate economy. Due to the CES specification, demand for each input exhibits a constant price elasticity, σ. Producing a unit of each input requires a_X units of labor, so that marginal cost is constant and equal to $w_t a_X$. For notational convenience, we choose the unit of measurement so as to

[4] It is necessary to assume that each specialized input is produced by one only firm. This is guaranteed in equilibrium. In our model, all inputs enter symmetrically in production, new firms never want to produce the inputs that are already available in the market, in the presence of start-up costs.

have $a_X = 1 - 1/\sigma$. Each intermediate goods producer hence sets the price equal to

$$p_t(i) = \left(1 - \frac{1}{\sigma}\right)^{-1} a_X w_t = w_t, \tag{4}$$

where the last equality is due to our choice of normalization. Because of the symmetry, all producers set the same price. Using (4), the price index of the intermediate goods composite thus becomes

$$P_t = \left[\int_0^{n_t} [p_t(i)]^{1-\sigma} \mathrm{d}i\right]^{1/(1-\sigma)} = n_t^{1/(1-\sigma)} w_t. \tag{5}$$

Note that the effective relative factor price, P/w, decreases with n, which is nothing but the mirror image of increasing returns due to specialization. As a broader range of differentiated inputs are available, it becomes advantageous to use them more intensively as a group, even though the price of each input remains the same.

Let α_t denote the factor share of intermediate inputs as of t. As the final goods sector is perfectly competitive, $\alpha = F_X(X, H)X/F(X, H)$. The linear homogeneity of $F(X, H)$ implies that this expression solely depends on the relative factor price, $P/w = F_X(X, H)/F_H(X, H)$. By denoting this relation by $\alpha = \alpha(P/w)$, we can express the factor share as a function of the product variety:

$$\alpha_t = \alpha\left(n_t^{1/(1-\sigma)}\right) \equiv A(n_t). \tag{6}$$

Here, $A(n)$ is a well-defined function of n almost everywhere (that is, except at the points where the elasticity of substitution between X and H is infinite). If $F(X, H)$ is a Cobb–Douglas, $A(n)$ is independent of n; it is increasing (decreasing) in n, whenever the elasticity of substitution between labor and the composite of intermediate inputs is greater (less) than one. We treat the case of an increasing $A(n)$ as the central case below, given the strong evidence the share of the producer services sector increases with the level of GNP, both in cross section and in time series. (See the reference given in the introduction).

Since all intermediate inputs enter symmetrically in the final goods production and their equilibrium prices are equal, the equilibrium output, $x_t(i)$, and the operating profit, $\pi_t(i)$, are also independent of variety. Therefore, by dropping i, we have $n_t p_t x_t = \alpha_t C_t$, and thus

$$\pi_t = (p_t - a_X w_t) x_t = \frac{p_t x_t}{\sigma} = \frac{\alpha_t C_t}{\sigma n_t},$$

or, from (6),

$$\pi_t = \frac{A(n_t)}{\sigma n_t} C_t. \tag{7}$$

Eq. (7) shows that an increase in the number of firms and available varieties has two effects on the profit of an incumbent firm. A larger set of competing varieties reduces the share of each variety, for a given factor share of intermediate inputs in the final goods production. However, it may also affect the factor share; with increasing degree of specialization, the final goods producers use the intermediate inputs more intensively. When the elasticity of substitution is greater than one, the share of intermediate inputs in the final goods production goes together with the degree of specialization, so that the two effects work in opposite directions.

The number of the specialist firms (and the range of producer services available) increases over time through the process of entry. Initially, the economy inherits a given number of firms, n_0. At any moment firms may enter freely into the intermediate goods sector, except that they need a start-up operation, which requires the use of a_n units of labor per variety. [5] They finance start-up costs by issuing ownership shares. Because of free entry, the value of an intermediate goods firm, v_t, never exceeds the start-up cost, $w_t a_n$, and whenever some entry occurs, they are equalized. Furthermore, the operating profit is always positive, so that no incumbent firm has an incentive to exit. That is, in equilibrium, we have

$$w_t a_n \geq v_t, \quad \dot{n}_t \geq 0, \quad (w_t a_n - v_t)\dot{n}_t = 0. \tag{8}$$

The market value of an intermediate goods producer is equal to the present discounted value of profits,

$$v_t \equiv \int_t^{\infty} \mathrm{e}^{-[R_\tau - R_t]} \pi_\tau \mathrm{d}\tau,$$

from which we obtain

$$\frac{\pi_t + \dot{v}_t}{v_t} = \dot{R}_t. \tag{9}$$

Eq. (9) states that the rate of return of holding ownership shares is equal to the interest rate.

Next, from $n_t p_t x_t = n_t w_t x_t = \alpha_t C_t$ and $w_t H_t = (1 - \alpha_t) C_t$, the labor market clears if

$$\begin{aligned} L &= a_n \dot{n}_t + H_t + n_t a_X x_t \\ &= a_n \dot{n}_t + (1 - \alpha_t)\left(\frac{C_t}{w_t}\right) + \left(1 - \frac{1}{\sigma}\right)\alpha_t\left(\frac{C_t}{w_t}\right) \\ &= a_n \dot{n}_t + \left(1 - \frac{A(n_t)}{\sigma}\right)\left(\frac{C_t}{w_t}\right). \end{aligned} \tag{10}$$

[5] It is worth pointing out that development traps could exist in our model *despite* that start-up costs are paid in labor. If the start-up operation instead used the output of the final goods sector, then development traps could be generated without any substitution between differentiated inputs and labor in the final goods production: see Ciccone (1993) and Matsuyama (1995a) for a demonstration. The assumption that start-up costs requires the use of labor thus helps to focus on the particular mechanism we are interested in.

By solving (10) for C_t/w_t and using $n_t w_t x_t = \alpha_t C_t$, $w_t H_t = (1-\alpha_t)C_t$ and $C_t = F(X_t, H_t)$, we obtain the expression of w_t. Inserting it back to (10) yields

$$a_n \dot{n}_t = L - \left(1 - \frac{A(n_t)}{\sigma}\right) \frac{C_t}{F\left(n_t^{1/(\sigma-1)} A(n_t), 1 - A(n_t)\right)}. \tag{11}$$

Eq. (11) shows the intertemporal trade-off the economy faces at any moment. Productivity growth and increasing specialization can be achieved only through reallocation of labor from manufacturing to start-up operations.

Finally, multiplying (10) by w_t and using (7) and (8), we obtain the national income account

$$w_t L + n_t \pi_t = C_t + v_t \dot{n}_t,$$

which, together with (9), can be integrated into the intertemporal budget constraint to yield

$$\lim_{T\to\infty} n_T v_T \mathrm{e}^{-R_T} = \int_t^\infty \mathrm{e}^{-(R_\tau - R_t)}(w_\tau L - C_\tau)\mathrm{d}\tau + n_t v_t = 0, \tag{12}$$

where use has been made of $n_t v_t = W_t$ and (2).

4. The market equilibrium: The case without an underdevelopment trap

To analyze the market equilibrium, it proves useful to describe the dynamic evolution of the economy in the two variables, n and $V = v/C$, where V represents the value of an intermediate inputs producing firm, measured in utility. From (1), (7), and (9),

$$\dot{V}_t = \rho V_t - \frac{A(n_t)}{\sigma n_t}, \tag{13a}$$

and, from (8) and (10),

$$\dot{n}_t = \max\left\{\frac{L}{a_n} - \left(1 - \frac{A(n_t)}{\sigma}\right)\frac{1}{V_t}, 0\right\} \tag{13b}$$

and, from (1) and (12),

$$\lim_{t\to\infty} V_t n_t \mathrm{e}^{-\rho t} = 0. \tag{13c}$$

For any initial number of firms the economy inherits, n_0, a market equilibrium of this economy is a path of $\{V_t, n_t\}$ that satisfies (13a)–(13c). Note that in the model of Grossman and Helpman (1991, Ch. 3.1), $A(n) = 1$ as they assume that $F(X, H) = X$. Setting $A(n) = 1$ in (13a)–(13b) leads to their equations (3.19)–(3.20). This seemingly minor extension, however, could lead to a drastically different equilibrium behavior of the economy, as shown below.

The qualitative property of the equilibrium dynamics crucially depends on the shapes of the two loci

$$V = \frac{A(n)}{\rho\sigma n}, \tag{VV}$$

and

$$V = \frac{a_n}{L}\left(1 - \frac{A(n)}{\sigma}\right). \tag{NN}$$

These two loci intersect at $n = n^*$ if and only if

$$\Phi(n^*) \equiv n^*\left(\frac{\sigma}{A(n^*)} - 1\right) = \frac{L}{\rho a_n}. \tag{14}$$

Eq. (14) states that, controlling for the factor share of intermediate inputs, the range of differentiated intermediate products increases with the size of the economy, measured by the total labor force. This expresses the Smith–Young notion that the division of labor depends on the extent of the market. At the same time, increasing availability of specialized inputs may induce the final goods producers to use a more roundabout method of production, which would increase the size of the market for intermediate inputs: that is, the extent of the market also depends on the division of labor. Because of this circularity, there may be multiple solutions to Eq. (14). Since the constant returns to scale property of the final goods production alone imposes few restrictions on the shape of $A(n)$, a wide range of dynamic behavior may be possible, unless we are willing to make further assumptions on the technology of final goods production. The following proposition provides a sufficient condition that guarantees a unique solution to (14).

Proposition 1. Let $\epsilon(P/w)$ be the elasticity of substitution between X and H in the final goods production when the relative factor price is P/w. If $\epsilon(P/w) \leq \sigma$ for all P/w, then VV is downward-sloping and intersects with locus NN at most once and from above. If $\epsilon(+\infty) < \sigma$, then (14) has a unique positive solution, $n^* > 0$. Furthermore, n^* is an increasing function of $L/\rho a_n$.

Proof. First, by integrating

$$\epsilon\left(\frac{P}{W}\right) = -\frac{\mathrm{d}\log(X/H)}{\mathrm{d}\log(P/W)},$$

the relative factor demand can be written as

$$\frac{H}{X} = \beta \exp\left[\int_1^{P/W} \frac{\epsilon(z)}{z}\mathrm{d}z\right] = \beta \exp\left[\int_1^{n} \frac{\epsilon(s^{1/(1-\sigma)})}{(1-\sigma)s}\mathrm{d}s\right],$$

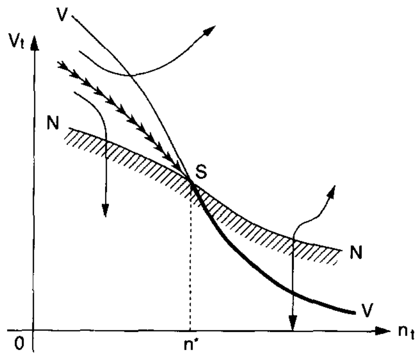

Fig. 1.

for a positive constant β, where use has been made of (5). From $1/A(n) = 1 + wH/PX$, we have

$$\frac{n}{A(n)} = n + \beta \exp\left[\int_1^n \frac{\epsilon(s^{1/(1-\sigma)}) - \sigma}{(1-\sigma)s} \mathrm{d}s\right]. \tag{15}$$

This shows that, if $\epsilon(\cdot) \leq \sigma$, $A(n)/n$ is a strictly decreasing in n, so that VV is downward-sloping. It also implies that the left-hand side of (14),

$$\Phi(n) = n\left[\frac{\sigma}{A(n)} - 1\right] = (\sigma - 1)n + \sigma\beta \exp\left[\int_1^n \frac{\epsilon(s^{1/(1-\sigma)}) - \sigma}{(1-\sigma)s} \mathrm{d}s\right],$$

is a strictly increasing in n, and $\lim_{n\to\infty}\Phi(n) = \infty$, and thus VV intersects with NN once and from above. If $\epsilon(+\infty) < \sigma$, $\lim_{n\to 0}\Phi(n) = 0$, and hence (14) has a unique positive solution n^*. Finally, n^* increases with $L/\rho a_n$ because $\Phi(n)$ is an increasing function of n. Q.E.D.

Fig. 1 depicts the situation given in Proposition 1. Locus VV shows the combinations of n and V for which V remains momentarily constant. It is downward sloping, because, with the limited substitution in the final goods production, a large number of competing varieties means lower profits. (In the next section, we provide a more extensive discussion on this point.) The lower profit makes investment in the shares of an intermediate goods producing firm less attractive and consumption more attractive. Above this locus, V needs to increase in order to make the representative consumer willing to hold the shares. Below the locus, V declines. Locus NN has a negative (positive) slope whenever $\epsilon(n^{1/(\sigma-1)})$ is greater (less) than one. Fig. 1 is drawn under the assumption that $A(n)$ is increasing in n. The number of differentiated products remains constant at points on or below locus NN. This is because starting up new firms requires a sufficiently high firm value to justify the cost of entry. Above this line, active entry leads to an expanding range of intermediate inputs.

The equilibrium dynamics of this economy are also depicted in Fig. 1. If n_0, the range of differentiated inputs the economy inherits, is less than n^*, then the economy follows the saddle path converging to the steady state, S. Along this path, the inducement to start up firms declines over time and the entry continues until the number of firms increases to n^*. If $n_0 \geq n^*$, on the other hand, the economy stays still on VV; the profit level is too low to justify any entry. Any points on VV to the right of S is thus a (trivial) steady state. The equilibrium path of this economy is thus unique and well-behaved for any initial condition. Entry of new firms and an expanding range of differentiated products would lead to a lower profit and firm value, without a sufficiently large increase in the factor share of producer services. Thus, the entry process, if it ever starts, will run into diminishing returns and eventually stop. An increase in the labor supply or a decline in start-up costs shifts NN down, while a small discount rate shifts up VV curve. Each of these changes therefore increases n^*, creating more room for new firms.

5. Digression on the Hicks–Allen substitutes and complements

It is worth stopping briefly at this point, to discuss why the limited substitution between specialized inputs and labor implies that the profit per firm declines with the number of firms. The key to understanding this result is the notion of complementary goods, defined by Hicks and Allen (1934) in the context of consumer demand. According to their definition, two goods are substitutes (complements) if the Allen partial elasticity of substitution between the two is positive (negative); that is, if the Hicksian demand for good 1 increases (decreases) with the price of good 2. To translate this notion in the present context, consider the following problem: for a fixed n, choose $x(i)$; $i \in [0, n]$ and H to minimize

$$\textstyle\int_0^n p(i)\,x(i)\mathrm{d}i + wL$$

subject to

$$F(X, H) = F\left(\left[\textstyle\int_0^n [x(i)]^{1-1/\sigma}\mathrm{d}i\right]^{\sigma/(\sigma-1)}, H\right) \geq C.$$

Then, the Hicksian demand for $x(i)$ is equal to

$$x(i) = \left[\frac{p(i)}{P}\right]^{-\sigma} X = \left[\frac{p(i)}{P}\right]^{-\sigma} \frac{\alpha(P/w)}{P} C.$$

Hence, specialized inputs are Hicks–Allen substitutes if the Allen partial elasticity of substitution between $x(i)$ and X,

$$\left.\frac{P}{x(i)}\frac{\mathrm{d}x(i)}{\mathrm{d}P}\right|_{C=\text{const.}} = (\sigma - 1) - \{1 - \alpha(P/w)\}\{\epsilon(P/w) - 1\}$$

is positive, while they are Hicks–Allen complements if the above expression is negative. Note that a high value of the direct partial elasticity of substitution among differentiated goods, σ, is not sufficient to make them Hicks–Allen substitutes. If the elasticity of substitution between X and H, $\epsilon(P/w)$, is sufficiently high, *and* the factor share of labor in the final goods production, $1-\alpha(P/w)$, is high, then demand for a specialized input increases when the prices of other specialized inputs are reduced, by shifting demand from labor to the composite of specialized inputs. The assumption, $\epsilon(P/w) \leq \sigma$ for all P/w, limits the magnitude of this indirect substitution, thereby ensuring that specialized inputs are always substitutes to each other in the sense of Hicks and Allen.

By differentiating (15), the condition under which the profit per firm measured in utility, $A(n)/n$, decreases with the number of firms, can be written as

$$\{1-A(n)\}\{\epsilon(n^{1/(1-\sigma)})-1\} < \sigma - 1$$

which holds if and only if the differentiated inputs are Hicks–Allen substitutes. This shows that, with the limited substitution between X and H, a small number of firms in the market leads to a large incentive to start up new firms, and hence development traps cannot exist in this case. [6]

This argument also explains why the equilibrium dynamics of this economy depicted in Fig. 1 resemble those of Grossman and Helpman (1991, Fig. 3.1). In their model, no labor is used directly in the final goods production and hence $A(n) = 1$. Without any possibility of substitution between X and H, differentiated inputs in their model necessarily become Hicks–Allen substitutes.

6. The market equilibrium: The case with underdevelopment traps

The argument in the previous section suggests that, with a large substitution between X and H, the differentiated inputs could become Hicks–Allen complements, and therefore, an increase in the number of incumbent firms gives a greater incentive to start up new firms, at least for a certain range. [7] As a result, the dynamic evolution of the economy would be much different from the case analyzed above. Depending on the shape of $\epsilon(P/w)$, one could generate a wide variety of equilibrium dynamics. In order to avoid a taxonomical exposition, however, we will focus on the following two examples, which illustrate how a large response by the final goods producers as they face an expanding range of differentiated intermediate products generates an underdevelopment trap.

[6] That the profit per firm declines with the number of firms means that, in the terminology of Bulow et al. (1985), entry activities are strategic complements. Although the recent literature, particularly after Cooper and John (1988), tends to stress the difference between the notion of strategic substitutes and complements and the Hicks–Allen notion of substitutes and complements, our model suggests that there may be a deep connection between the two notions.

[7] That is, entry activities are strategic complements in the sense of Bulow et al. (1985).

Example 1. To motivate this example, imagine that the final goods is food. There are two ways of producing food. The first is the primitive form of agriculture, which makes intensive use of horse, carts, and direct labor. The second relies on tractors, planes, and many supporting services, which, when used together, make

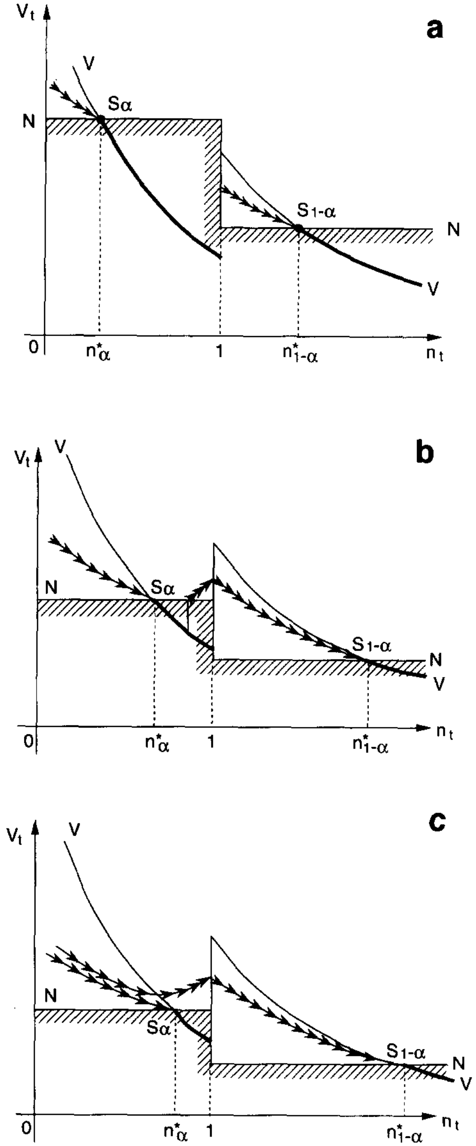

Fig. 2.

agribusiness operational. This situation can be modelled as the following form of the technology for the final goods production:

$$F(X,H) = \max_{X_1,X_2,H_1,H_2 \geq 0} \left\{ X_1^{\alpha} H_1^{1-\alpha} + X_2^{1-\alpha} H_2^{\alpha} \middle| X_1 + X_2 \leq X, H_1 + H_2 \leq H \right\}$$

where $0 < \alpha < 0.5$. The final goods producers have access to two Cobb–Douglas technologies. (The symmetry of the two technologies is not essential, but helps to simplify the algebra.) They select the more labor intensive one if $P/w > 1$ and the more intermediate goods intensive one if $P/w < 1$. If $P/w = 1$, they are indifferent between the two. The elasticity of substitution between X and H is hence

$$\epsilon(P/w) = \begin{cases} 1 & \text{if} \quad P/w \neq 1, \\ \infty & \text{if} \quad P/w = 1. \end{cases}$$

Note that $\epsilon(\cdot)$ satisfies the condition of Proposition 1 almost everywhere, but violates it at one point, at $P/w = 1$. The factor share becomes

$$A(n) \begin{cases} = & \alpha & \text{if} \quad n < 1, \\ \in & [\alpha, 1-\alpha] & \text{if} \quad n = 1, \\ = & 1-\alpha & \text{if} \quad n > 1. \end{cases}$$

Locus VV jumps up and locus NN jumps down at $n = 1$. If the parameters satisfy

$$\frac{\sigma}{1-\alpha} - 1 < \frac{L}{\rho a_n} < \frac{\sigma}{\alpha} - 1,$$

then the two loci intersect at $n < 1$ and at $n > 1$, generating two nontrivial steady states, S_{α}, and $S_{1-\alpha}$, to which the economy may approach, depending on the initial condition. Fig. 2a shows the phase diagram under the additional assumption

$$\frac{\sigma}{1-\sigma} - 1 < \frac{L}{\rho a_n} < \frac{\sigma - \alpha}{1-\alpha}. \tag{16}$$

In this case, the equilibrium path is unique for any initial condition. If $n_0 < n_{\alpha}^*$, entry occurs until the economy converges to S_{α}. If $1 < n_0 < n_{1-\alpha}^*$, then entry occurs until the economy converges to $S_{1-\alpha}$. If $n_{\alpha}^* \leq n_0 < 1$, or $n_{1-\alpha}^* \leq n_0$, on the other hand, no entry takes place and the economy stays still on VV. (When $n_0 = 1$, both no entry and convergence to $S_{1-\alpha}$ are market outcomes.) The model thus exhibits a kind of threshold effects of economic development. The economy needs a sufficiently large commercial and industrial base (in this case, $n > 1$) in order to reach the higher steady state, $S_{1-\alpha}$, which is characterized by high productivity, the wide range of specialized producer services that are more intensively used. The initial condition completely determines which stage of economic development the economy ends up in. [8]

[8] This example fits well with Durlauf and Johnson (1992), who showed that economies with similar initial conditions tend to converge to one another, but found little evidence of convergence across economies with substantially different initial conditions.

If Eq. (16) does not hold because of, say, smaller start-up costs, an equilibrium may not be unique for some initial conditions. Fig. 2b and 2c illustrate two possible situations. In the case depicted in Fig. 2b, two equilibria exist if n_0 is slightly smaller than the threshold level. In one of them, which may be called the pessimistic equilibrium, no entry is expected to occur, and thus the final goods sector is expected to use the less intermediate goods intensive technology. As a result, no entry takes place and the economy stays still on VV. In the other equilibrium, a rush of entries by new firms is expected, which leads to a widening range of specialized inputs, inducing the final goods sector to adopt the more intermediate goods intensive technology in the future. Such optimistic expectations indeed justify earlier entry to the intermediate goods sector. Along this equilibrium path, entrepreneurial optimism brings about a coordinated entry of specialist firms. The economy thus manages to break the vicious circle, take off, and converge to the high steady state, due to a sort of self-fulfilling prophecy. For even smaller start-up costs (Fig. 2c), we have the co-existence of optimistic and pessimistic equilibria for any initial condition below the threshold level ($n_0 < 1$).

Remark 1. Example 1 suggests the stringency of the sufficient condition for no development trap identified in Proposition 1. Development traps can exist, despite $\epsilon(P/w) = 1 < \sigma$ holds almost everywhere. If econometricians use the cross country data generated by this model and estimate the production function, F, they cannot reject that ϵ is equal to one. And if they simply collect data on the production technology from underdeveloped countries, they would end up estimating $F(X, H) = X^{\alpha}H^{1-\alpha}$, as, in equilibrium, this captures all the technologies actually used in these countries, both in and out of the steady state. This point should be kept in mind when interpreting that the empirical plausibility of the sufficient condition given in Proposition 1. The production function, $F(X, H)$, must include all the technologies available, including the technologies that may never be used in underdeveloped countries, as the central question in development economies is why certain advanced technologies, widely used in developed countries and available in blue-print, fail to be adopted in underdeveloped countries.

Remark 2. This example can be given an alternative interpretation (Rodríguez (1993), see also Rodrik (1994). Consider a small open economy with two tradeable consumer goods, 1 and 2, while intermediate inputs and labor are nontradeable. [9] The production function of good 1 is given by $X_1^{\alpha}H_1^{1-\alpha}$ and that of good 2 is $X_2^{1-\alpha}H_2^{\alpha}$. The relative price of the two consumer goods are exogenously determined in the world market and equal to one. Then, the aggregate production function of the consumer goods industry is given by

$$F(X, H) = \max_{X_1, X_2, H_1, H_2 \geq 0} \left\{ X_1^{\alpha}H_1^{1-\alpha} + X_2^{1-\alpha}H_2^{\alpha} \middle| X_1 + X_2 \leq X, H_1 + H_2 \leq H \right\}.$$

and this economy specializes in good 1 if $n_t < 1$, and in good 2 when $n_t > 1$. The equilibrium dynamics of this economy is thus depicted by Figs. (2a)–(2c). According to this interpretation, underdeveloped countries, lacking in an extensive network of support industries, specialize in primitive consumer goods. Now suppose that the intertemporal preferences of the representative consumer over the two consumer goods are given by the aggregator $C = U(C_1, C_2)$, where U is linearly homogeneous. Then, if this economy closed its trade in consumer goods, the aggregate production function would become

$$F(X, H) = \max_{X_1, X_2, H_1, H_2 \geq 0} \left\{ U\left(X_1^{\alpha} H_1^{1-\alpha}, X_2^{1-\alpha} H_2^{\alpha} \right) \mid X_1 + X_2 \leq X, H_1 + H_2 \leq H \right\}.$$

which may or may not satisfy the sufficient condition given in Proposition 1. This discussion should also offer a caution with which the empirical plausibility of the conditions for development traps must be judged. The properties of production function, $F(X, H)$, depend on the market structure, since it aggregates all the technologies available to the consumer goods industries, and the market structure affects the process of aggregation.

Example 2. Let $F(X, H)$ be a CES of the following form:

$$F(X, H) = \left[X^{1-1/\epsilon} + \beta^{1/\epsilon} H^{1-1/\epsilon} \right]^{\epsilon/(\epsilon-1)}, \quad \epsilon > \sigma.$$

(Recall that the case of $\epsilon \leq \sigma$ has been taken care of in Proposition 1.) Then,

$$\frac{1}{A(n)} = 1 + \beta n^{(1-\epsilon)/(\sigma-1)}, \quad \frac{n}{A(n)} = n + \beta n^{(\sigma-\epsilon)/(\sigma-1)}$$

so that NN is downward sloping, while VV has a single peak at

$$\check{n} = \left[\frac{\beta(\epsilon - \sigma)}{\sigma - 1} \right]^{(\sigma-1)/(\epsilon-1)}.$$

This implies that differentiated inputs are Hicks–Allen complements to each other if $n < \check{n}$ and Hicks–Allen substitutes to each other if $n > \check{n}$. Furthermore, $\Phi(\cdot)$ also has a single peak, and hence (14) has at most two solutions. There are three generic cases to be distinguished, depending upon the effective labor supply, L/a_n. First of all, when start-up costs are sufficiently high, NN lies above VV

[9] The nontradeablity of intermediate inputs, particularly producer services, while it is a reasonable assumption, is not an essential element of this model. For example, one can reinterpret a_n as the cost of setting up a distribution channel to each region and a_X as the unit price of an intermediate input abroad, plus its unit shipping cost, all measured in labor. As long as some start-up costs, say, for setting up a branch office, are required to service each intermediate input in a given region, then the analysis can be carried over even when all intermediate inputs are tradeable.

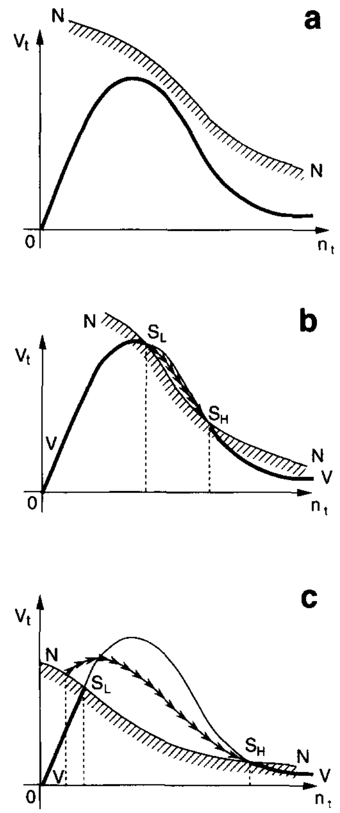

Fig. 3.

everywhere (Fig. 3a); in this case, any combination of n and V on loci VV is a (trivial) steady state. No entry takes place in this economy.

For somewhat smaller start-up costs, NN intersects with VV twice, at S_L and S_H, both at a downward sloping part of VV. This situation is depicted in Fig. 3b. The equilibrium path is unique for any initial condition. [10] If the economy starts below the lower steady state, S_L, the narrow industrial base forces the final goods producers to use the labor intensive technology. As a result, demand for intermediate inputs is too low to justify starting up new firms in the intermediate goods sector. The economy stays still on VV, and it is trapped in the lower stage of economic development. If the economy starts slightly above S_L, however, the

[10] If one linearizes (13a)–(13b), ignoring the nonnegativity constraint in (13b), then one can show the Jacobian matrix has two real, positive eigenvalues at S_L and one positive and one negative eigenvalues at S_H; there is a saddle path leaving S_L and converging to S_H monotonically.

range of differentiated products available is sufficiently large so as to induce final goods producers to make more intensive use of intermediate inputs. This generates a large market for intermediate products that lead new firms to enter. As a result, the economy experiences an expanding variety of differentiated inputs, productivity growth, and a rising share of intermediate goods sector in employment. This cumulative process continues until the economy reaches the high level steady state, S_H.

For an even smaller level of start-up costs (Fig. 3c), NN intersects with VV at its upward sloping part, at S_L, the lower steady state. This generates a possibility of multiple equilibria similar in many ways to the situation depicted in Fig. 2b. That is, there exist two equilibria if the economy starts just below the lower steady state. [11] In one of them, the pessimistic equilibrium, no entry is expected to occur, and the share of intermediate inputs remains small. As a result, no entry takes place. The economy stays still on VV. In the other equilibrium, optimistic expectations that an increasing range of specialized intermediate products will lead to a rising share of the intermediate inputs market in the future induces new firms to enter. Active entry in fact expands the range of intermediate goods and the economy converges to the higher steady state, S_H. A take-off becomes possible as a result of the self-fulfilling prophecy. The positive feedback between the entry and the rising share creates a virtuous circle along this equilibrium path. As shown in Fig. 3c, the initial number of specialist firms generally needs to be sufficiently large for such an optimistic equilibrium to exist. If n is very small, the only equilibrium is a trivial steady state, in which the economy stays still on VV. With a very narrow industrial base, even the optimistic expectations cannot generate the momentum necessary to generate the virtuous circle. The uniqueness of the equilibrium path for a sufficiently small n_0 is guaranteed if NN intersects with the vertical axis at the level higher than the peak of VV. It should be pointed out, however, that one can show that, for any initial condition, $n_0 > 0$, the optimistic equilibrium exists if the start-up costs are made sufficiently small. (It seems plausible that the optimistic equilibrium also exists for $n_0 = 0$ by taking the start-up costs sufficiently small, although we have not been able to demonstrate this.)

It should be clear that the properties of equilibrium dynamics demonstrated in Example 2 depend solely on the fact that the VV locus has a bell-shaped, and the NN locus is downward-sloping. (In particular, it does not depend on the constancy of the elasticity of substitution between labor and intermediate inputs in the final goods production.) We have thus proved the following proposition.

[11] If one linearizes (13a)–(13b), ignoring the nonnegativity constraint in (13b), then one can show that the Jacobian matrix at S_L has a pair of complex eigenvalues with positive real parts. Hence, the saddle path converging to S_H spirals around S_L if the nonnegativity constraint is ignored.

Proposition 2. Suppose that the NN locus has a negative slope, and the VV locus has a single peak, increasing in $n < \check{n}$, and decreasing in $n > \check{n}$.
(i) *If $L/a_n < \rho\Phi(\check{n})$, then NN does not intersect with VV before $\check{n}$. For any initial condition, the equilibrium is unique; the process of entry and increasing specialization takes place in the range where VV lies above NN and the economy stays still in the range where VV lies below NN.*
(ii) *If $L/a_n > \rho\Phi(\check{n})$, then NN intersects with VV once before $\check{n}$. The equilibrium is unique if the economy starts above this intersection. There is a range of initial conditions just below this intersection for which two equilibria, optimistic and pessimistic, exist.*

The equilibrium dynamics illustrated by the two examples above do not exhaust all the possibilities of our model. When the conditions in Propositions 1 or 2 are not met, one could have far more exotic dynamic behaviors. For example, the model may have an arbitrary number of steady states; such an example can be constructed by specifying that the final goods production function be characterized by a finite number of fixed coefficient (Leontief) technologies, so that the elasticity of substitution between intermediate inputs and labor alternates between zero and infinity as the number of specialist firms increases.

7. Knowledge spillover and barriers to modern economic growth

Along any equilibrium path described in the previous section, the process of entry, increasing specialization and productivity growth must eventually peter out. This can be understood by looking at the expression for the VV locus, $V = A(n)/\rho\sigma n$. Along VV, the value of a specialist firm goes to zero as the number of firms (and products) goes to infinity. The share of intermediate goods sector is bounded from above (by one), so that an expanding range of competing products eventually drives down the market share of each product, and therefore its profit, to zero. This means that locus VV eventually lies below locus NN, as the latter is bounded away from the horizontal axis. Once the economy reaches this region, there will be no incentive to start up firms and introduce new products. As a result, productivity growth ultimately stops.

In order to generate an ever increasing specialization and self-sustainable productivity growth, the start-up costs must go down over time, so that an incentive to introduce new products will not disappear in spite of a declining market share for each product. In this section, we modify our model to incorporate technology spillovers in the start-up operations. [12] More specifically, we assume that, due to technological externalities associated with learning-by-doing, the labor

[12] Alternatively, the self-sustainable growth can be generated by assuming, as in Rivera-Batiz and Romer (1991) and Barro and Sala-i-Martin (1992), that start-up operations require the use of the final output, instead of labor.

requirement necessary to introduce a new product as of time t is inversely related to the total number of products that has been introduced up to that time:

$$a_n = \frac{a_I}{n_t}, \tag{17}$$

where a_I is a positive constant.

This assumption of technology spillovers is most plausible if one interprets the start-up operations as research activities, which seek to invent a new product. It is useful to think that, when firms invest in research activities, they generate two different types of information. First, commercial research generates specific information, such as a blue-print, that allows a firm to supply new products. Second, it also produces general information with wide applicability, which facilitates further innovation. Following Grossman and Helpman (1991; Ch. 3.2) and Romer (1990), let us assume that the first type of information is completely proprietary and excludable, while the second is completely nonexcludable. That is, profit-seeking firms are engaged in the inventive activities to produce a new design, which enables the inventors to earn monopoly profits forever. At the same time, they inadvertently produce the general information, which enter the public domain. The inventive activities thus enhances the total stock of knowledge available in the economy, which can be exploited by any firm to develop even more products in the future. The specification given above can be considered to capture this sort of knowledge spillovers. In this formulation, the total stock of knowledge that researchers can rely on at any point in time, which can be defined as the labor productivity of the inventive activity, is proportional to the existing number of products. This linearity makes self-sustainable growth possible.

The dynamic behavior of the economy can be obtained simply by inserting (17) into (13a)–(13c). To characterize the equilibrium paths, it proves useful to define a new variable $Q_t = n_t v_t / C_t = W_t / C_t = n_t V_t$, which is the total value of the ownership shares of intermediate producing firms, measured in utility. Then, (13a)–(13c) can be rewritten as

$$\dot{Q}_t = \max\left\{\left(\rho + \frac{L}{a_I}\right)Q_t - 1,\ \rho Q_t - \frac{A(n_t)}{\sigma}\right\}, \tag{18a}$$

$$\frac{\dot{n}_t}{n_t} = \max\left\{\frac{L}{a_I} - \left(1 - \frac{A(n_t)}{\sigma}\right)\frac{1}{Q_t}, 0\right\}, \tag{18b}$$

$$\lim_{t\to\infty} Q_t \mathrm{e}^{-\rho t} = 0. \tag{18c}$$

For any number of specialist firms the economy inherits, n_0, a market equilibrium of this economy is a path of $\{Q_t, n_t\}$ that satisfies (18a)–(18c).

We focus on the case where $\epsilon(P/w) > 1$ for all P/w, so that $A(n)$ is strictly increasing and ranges from zero to one. Fig. 4 illustrates the equilibrium dynamics

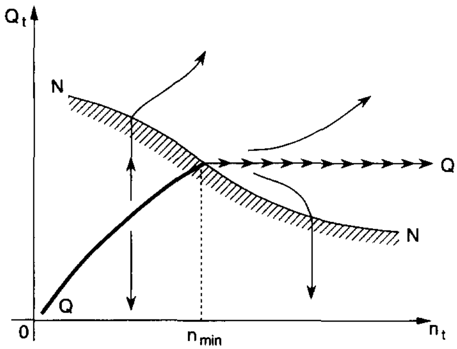

Fig. 4.

in this case under the additional assumption $L/\rho a_l > \sigma - 1$. Locus QQ, along which Q remains momentarily constant, is increasing from $n = 0$ to $n = n_{\min}$, and horizontal for $n > n_{\min}$, where $n_{\min}$ is defined by

$$A(n_{\min}) = \sigma\left(1 + \frac{L}{\rho a_l}\right)^{-1} < 1.$$

Locus NN, on the other hand, is downward sloping.

The equilibrium path of the model is now always unique. If the economy inherits the range of intermediate goods less than the critical mass, $n_{\min}$, the economy stays still on the QQ locus. The presence of both pecuniary and technological externalities make it impossible for this economy to grow; the vicious circle now becomes unbreakable. First of all, the narrow range of specialized inputs available forces the final goods producers to use the labor intensive technology, which limits the size of the intermediate goods market. This lack of demand spillovers from the existing products, or pecuniary externalities, means a lower inducement to start up firms and introduce new products. Second, the limited experiences of starting up firms, or a low level of knowledge capital that can be used to invent a new product, implies high start-up costs.

On the other hand, if the economy inherits the number of intermediate products more than the critical mass, the economy grows along the QQ locus. The presence of the two types of externalities now works positively and makes the cumulative advance possible. Along this growth path, Q remains constant, which implies that the value of the ownership shares, W_t, and consumption, C_t, grow at the same rate. Furthermore, the economy experiences an accelerating growth. This can be shown by looking at, for example, the growth rate in the number of specialist firms, which is given by, from (18b),

$$\frac{\dot{n}_t}{n_t} = \frac{L}{a_l} - \left(1 - \frac{A(n_t)}{\sigma}\right)\left(\rho + \frac{L}{a_l}\right) = \frac{A(n_t)}{\sigma}\frac{L}{a_l} - \left(1 - \frac{A(n_t)}{\sigma}\right)\rho.$$

Since the share of intermediate goods sector rises over time, the expression also increases. One can also show that consumption and productivity grows at an accelerating rate. Asymptotically, the growth rate converges to

$$\lim_{t\to\infty} \frac{\dot{n}_t}{n_t} = \frac{1}{\sigma}\frac{L}{a_I} - \left(1 - \frac{1}{\sigma}\right)\rho$$

which is identical to the growth rate of the balanced growth economy analyzed by Grossman and Helpman (1991; Eq. 3.28).

It should be pointed out, however, that the result of accelerating growth is entirely due to the assumption that $\epsilon(P/w) > 1$ for all P/w. More generally, the economy grow as long as locus QQ stays above locus NN, but the growth rate could decline over the range in which $\epsilon(n^{1/(1-\sigma)}) < 1$. What is crucial for a growth trap is that there is a range in which $A(n) < \sigma/(1 + L/\rho a_I)$ so that QQ stays below NN.

8. Discussion

The market equilibria discussed in the previous sections are inefficient. Characterizing the efficient allocations in these models needs some additional technicalities, which is far beyond the scope of this paper. Furthermore, any comparison between efficient and market allocations requires some simulation exercises. We therefore refer to our companion paper (Ciccone and Matsuyama, 1996), on these issues and instead provide some general discussions on policy issues.

First of all, there is the fundamental difficulty of correcting the distortions in these economies in a decentralized manner. In principle, one could compute the optimal allocations and design Pigouvian taxes and subsidies and lump-sum transfers in an attempt to implement them. Unfortunately, what one can best hope for by using such simple policy tools is to make the first-order conditions right. In a nonconvex economy such as ours, one also has to take care of some global conditions in order to implement efficient allocations. Another way of stating this difficulty is that the Euler and stock adjustment equations and the transversality condition of the central planner's problem are only necessary, but not sufficient, for the optimality in the presence of non-convexity. In general, there are multiple paths that satisfy these conditions (this is often so even when the market equilibrium is unique in the absence of any government intervention), and there is no simple way of ensuring that the private sector will select the optimal one.

Even if one does not need to worry about the problem of implementing the efficient allocations in a unique decentralized equilibrium (possibly because a sufficiently rich set of nonlinear policy tools is available), it should be noted that the task of computing the efficient allocations itself is quite formidable. In this paper, we emphasize the process of proliferation of intermediate inputs and producer services as the essential part of economic development and growth. No single input plays any decisive role in this process; productivity growth is realized through the cumulative impact of small improvements. This is precisely the situation where what Hayek (1945) called "the knowledge of the particular circumstance of time and place" matters, which presents the difficulty of computing efficient allocation of resources. In the model, this difficulty is artificially resolved by the form of the production function in the final goods sector. This functional form assumes that all specialized inputs enter symmetrically, so that the network of intermediate inputs producers can be summarized by a single number, n. Although it greatly simplifies the analysis, this is not a realistic feature of the model. In practice, some new intermediate inputs may be complementary to old ones, while others may be substitutes. The introduction of a new variety will generally alter the relation between any two existing varieties; it may even lead to complete obsolescence of some existing varieties. Since the start-up operations require the use of scarce resources, the selection problem is critical for the productivity performance. And yet, its solution necessitates highly detailed technical knowledge on the network of intermediate inputs, which is unlikely to be available to any social planner. [13]

As a general lesson, when discussing general economic issues related to the development of the economy, more attention should be paid to the specialized intermediate inputs and producer services. For example, as Carter (1970) pointed out, the common practice in the productivity growth analysis is to focus exclusively on the relation between the final output and the primary factors, such as labor, energy, and steel; a variety of specialized machine tools and business services that establishments furnish to each other are netted out. This practice, while useful for the purpose of measuring technological progress, hardly offers any insight on the causes of improvement. For many aspects of technological changes are visible only at the intermediate level. Neglecting supporting industries is often the major factor in the disappointing performances of technology transfers. Many Third World countries, often with the technical assistance of some industrialized countries or multinational institutions, have attempted to transplant advanced technologies. For the location of the factories, they often choose rural or otherwise economically backward areas, where "jobs are badly needed", in their attempt to curb migration into cities. Jacobs (1969, pp. 186–187) described how one of such projects failed: "No single problem seems to have been horrendous. Instead, endless small difficulties arose: the delays in getting the right tools, in repairing things that broke, in correcting work that had not been done to

[13] Matsuyama (1992b, Section 4) analyzes a model in which differentiated goods are at once substitutes for some and complements for others. Matsuyama (1995b) and Matsuyama (1996) deal with the inherent difficulty of figuring out which set of goods should be introduced, when the symmetry assumption is dropped, and offer some policy implications.

specifications, in sending off for a bit of missing material''. It is our hope that the models presented in this paper will help to direct more attention to the critical roles played by the availability of intermediate inputs and producer services in the problem of economic development and growth.

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