Linkages

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Abstract
Economic activities in different industries are linked to each other through aggregate income (horizontal linkages) and input–output relationships (vertical linkages). Could such linkages give rise to vicious circles of underdevelopment or virtuous circles of development when there are increasing returns to scale at the firm level?

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Cost linkages; Demand linkages; Horizontal and vertical linkages; Increasing returns to scale; Industrialization; Input chains; Linkages; Multiple equilibria; Pre-industrial production methods; Underdevelopment traps

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Introduction
Economic activities in different industries are linked to each other through aggregate income (horizontal linkages) and input–output relationships (vertical linkages). Could such linkages give rise to vicious circles of underdevelopment or virtuous circles of development when there are increasing returns to scale at the firm level? A standard account of a vicious circle goes as follows. Small-scale production methods in industry A lead to low output and income. This translates into low demand for industry B, which therefore also ends up using small-scale production methods and generating low output and income. The result is low demand for industry A, which justifies the small-scale production methods used in this industry. Low aggregate output and income are seen as the result of a vicious circle because the same economic environment is thought to be compatible with a high-income equilibrium where all industries use technologies that achieve high productivity at large scale. This high-income equilibrium is sustained by a virtuous circle. Large-scale production methods in industry A are profitable because of high income in industry B, and vice versa.

We will show that vicious or virtuous circles based on demand linkages are subject to a simple fallacy if increasing-returns-to-scale technologies differ from pre-industrial technologies only in that they are more productive at large scale. Still, vertical demand linkages will give rise to vicious
or virtuous circles if increasing-returns-to-scale technologies use intermediate inputs more intensively than the technologies they replace. And horizontal demand linkages will do so if firms adopting increasing-returns-to-scale technologies must pay a compensating wage differential. Moreover, when there are both vertical demand and cost linkages, underdevelopment traps can be consistent with economic principles even if increasing-returns-to-scale technologies differ from pre-industrial technologies only in that they are more productive at large scale. We first discuss the role of horizontal demand linkages, then that of vertical demand linkages, and finally turn to vertical cost linkages.

**Horizontal demand linkages.** Imagine an economy populated by households and by firms in different industries. Suppose that each industry sells only to households. Assume also that the amount households spend on each industry is independent of prices (industry demand functions are unit elastic). In this case, demand linkages among industries are said to be horizontal. This simply means that economic activity in one industry affects spending on other industries only through the aggregate income of households.

Could horizontal demand linkages lead to economies being trapped into a situation of low income due to a vicious circle of low income and output? Rosenstein-Rodan (1943) and Nurkse (1953) thought so. They imagined a situation where low aggregate income was an obstacle to the adoption of technologies that achieve high productivity at large scale. But large-scale production methods would be profitable if all industries adopted them, because incomes generated in one industry would create demand for other industries.

The elements necessary for underdevelopment traps to be consistent with economic principles have always been subject to debate. Increasing returns to scale appeared to be crucial. But Fleming (1955) made clear that this was not enough. He imagined a situation where, because of low aggregate income, industry A cannot make a profit from adopting the increasing-returns-to-scale technology and that the same is true for industry B. Is it possible that the increasing-returns-to-scale technology becomes profitable if both A and B adopt it? Consider forcing A to adopt. In this case, the loss made in industry A will lower aggregate income. As a result, industry B will now face even lower demand and therefore make an even greater loss if it adopts the increasing-returns-to-scale technology. This means that aggregate income will fall further if we also force industry B to adopt the increasing-returns-to-scale technology. Hence, if the adoption of increasing-returns-to-scale technologies is unprofitable for any single industry, adoption in all industries will not be profitable either. Increasing returns alone can therefore not explain why industrialization does not take place although it would ultimately be profitable.

All accounts of underdevelopment traps did in fact feature (several) additional elements. In particular, Rosenstein-Rodan maintained that firms using large-scale production methods had to pay a compensating wage differential (partly because of the higher costs of living in urban areas, where industrial firms were located). Section “A Model of Horizontal Demand Linkages” follows Murphy et al. (1989) in showing that underdevelopment traps may emerge when firms adopting the increasing-returns-to-scale technologies must pay a compensating wage premium.

**Vertical demand linkages.** Suppose now that industries sell goods to households and each other (to be used as intermediate inputs). Economic activity in one industry can then affect demand in another industry even if aggregate income remains unchanged. As a result, there are said to be vertical linkages. For example, consider the situation where industry B buys from A (industry A is upstream of B). In this case there is a vertical demand linkage as demand for the upstream industry A will depend on the economic activity in downstream industry B. There could also be a vertical cost linkage because the cost of production in downstream industry B is partly determined by the cost of goods produced in upstream industry A.

While the effects of horizontal demand linkages on economic development have always been subject to some controversy, there appears to be a consensus among early contributors that vertical demand linkages can lead to underdevelopment
traps when technologies are subject to increasing returns to scale (Fleming 1955; Scitovsky 1954; Hirschman 1958). It is simple to show however that this is not the case if increasing-returns-to-scale technologies differ from pre-industrial technologies only in that they are more productive at large scale. To see this, note that with vertical demand linkages the adoption of increasing-returns-to-scale technologies affects aggregate income directly and indirectly: directly through the profits made in the adopting industry, and indirectly through the profits made in supplying (upstream) industries. It would therefore seem that increasing-returns-to-scale technologies could be unprofitable in the adopting industry but still increase aggregate income. But this cannot happen when the increasing-returns-to-scale and the pre-industrial technologies use upstream inputs with the same intensity. In this case, the increase in the value of upstream goods demanded by a firm adopting increasing-returns-to-scale technologies is always a fraction of the (absolute value of the) loss that it makes. Moreover, as profits cannot exceed revenues, the increase in profits in supplying industries is necessarily smaller than the increase in the value of goods they sell. It therefore follows that the increase in profits in supplying industries (the positive indirect effect) can never compensate for the loss made in the industry adopting the increasing-returns-to-scale technology. The empirical evidence indicates that the intermediate-input intensity of production increases with a country’s level of industrialization. Increasing-returns-to-scale technologies may therefore be using intermediate inputs more intensively than the production methods they replace. Section “Vertical Demand Linkages in an Input Chain Model” draws on Ciccone’s (2002) model of input chains to show that vertical linkages can in this case explain why countries may be trapped into a vicious circle of underdevelopment, and why escaping this trap may be associated with large gains in aggregate income and productivity.

The interplay of vertical cost and demand linkages. The greater demand for intermediate inputs brought about by industrialization (vertical demand linkages) may partly be caused by falling intermediate input prices (vertical cost linkages). Falling intermediate input prices, on the other hand, are possible because of the higher productivity of large-scale production methods. Vertical cost and demand linkages therefore feed on each other (Young 1928; Okuno-Fujiwara 1988; Rodriguez-Clare 1996). For example, Rodriguez-Clare considers a small open economy framework where the entry of new intermediate input varieties lowers the cost of intermediate inputs relative to labour, which leads final-good producers to substitute towards intermediate inputs. When this substitution effect is strong enough, it translates into greater revenues and profits for intermediate-input producers, which may validate intermediate-input producers’ decision to start up new varieties in the first place. Rodriguez-Clare shows that this interplay of vertical demand and cost linkages may lead to two equilibria: a low-income equilibrium where final-good producers use labour-intensive production methods because of the limited range of intermediate inputs available, and a high-income equilibrium where a large variety of intermediate inputs leads final-good producers to use intermediate-input intensive production methods. Okuno-Fujiwara (1988) considers a situation where vertical demand and cost linkages interact because greater demand for intermediate inputs leads to lower prices due to competition among a larger number of Cournot oligopolists. The final section of this entry uses the model with input chains to show that the interplay between vertical demand and cost linkages interact because greater demand for intermediate inputs leads to underdevelopment traps even if increasing-returns-to-scale technologies differ from pre-industrial technologies only in that they are more productive at large scale.

A Model of Horizontal Demand Linkages

We will now examine the role of horizontal demand linkages for economic development using the model of Murphy et al. (1989) (for a historical and methodological perspective on the horizontal-linkages literature, see Krugman 1993, 1994). The first step is to describe the model set-
up—the household sector, the production sector, and market structure. The second step is to characterize equilibrium prices and equilibrium allocations.

*Households*. There are $L$ households, each of whom supplies one unit of labour in elastically (labour is the only production factor in this model and serves as the numeraire). Households spend an equal share of their incomes on each of the $N$ goods produced in the economy.

*Production*. Each of the $N$ goods demanded by households can be produced using two different production methods: a *pre-industrial* method requiring one unit of labour for each unit of output produced, and an *industrial* or increasing-returns-to-scale method, which is more efficient at the margin but subject to a fixed labour requirement ($f$). Formally, the increasing-returns-to-scale production method requires

$$l_i = f + cq_i$$

units of labour to produce $q_i$ units of good $i$, where $f > 0$ and $1 > c > 0$.

*Industry wage premium*. Working in the industrial sector generates a disutility $v \geq 0$ for households. Hence, relative to pre-industrial firms, industrial firms will have to pay a wage premium $v \geq 0$ as a compensating wage differential.

*Market structure*. Many firms are assumed to know the pre-industrial method to produce good $i$. As a result, the pre-industrial sector (also called competitive fringe) will be characterized by perfect competition. By contrast, only a single firm is taken to have the ability to produce each good in the industrial sector. These firms set prices optimally, taking the prices of all other firms as given. The labour market is taken to be perfectly competitive.

What keeps this model simple to analyse is that the equilibrium price of each good is unity whether the good is produced by the pre-industrial or the industrial sector. To see this, note that perfect competition and constant returns to scale in the pre-industrial sector imply that the price of goods produced in this sector must be equal to unity. A higher price would mean strictly positive profits and therefore further entry of pre-industrial producers, while a lower price would mean that no pre-industrial producer could break even. Now consider goods produced in the industrial sector. Clearly, the industrial producer will not set a price above unity, as she would lose the entire market to pre-industrial producers in this case. Moreover, industrial producers do not have an incentive to set a price below unity either, as households spend the same fraction of income on their good irrespectively of the price. Hence, industrial producers find it optimal to use a limit pricing strategy, setting prices exactly equal to the marginal cost of pre-industrial producers. As a result, the price of each of the $N$ goods is equal to unity independently of the production method.

*Pre-industrial equilibrium*. Under what conditions will there be an equilibrium where all goods are produced with the pre-industrial method? In such an equilibrium, firms just break even, and aggregate income $Y$ in the economy is therefore equal to aggregate labour income $L$. Because households spread income equally among all $N$ goods, the quantity of good $i$ demanded and supplied is $q_i = L/N$. The remaining question is whether firms in the industrial sector have an incentive to adopt the increasing-returns-to-scale method. The potential profit of such firms is

$$\pi_i = q^m_i - (f + c q^m_i)(1 + v),$$

where $q^m_i$ is the demand faced by the industrial producer of good $i$. As industrial and pre-industrial producers set the same price, the first industrial producer faces exactly the same demand as the pre-industrial producers she replaces, $q^m_i = L/N$. Her profits are therefore

$$\pi_i = L/N - (f + cL/N)(1 + v).$$

If $\pi_i < 0$, an industrial producer has no incentive to adopt the increasing-returns-to-scale method, and it will be an equilibrium for all goods to be produced with the pre-industrial method. Hence, (2) implies that there is an equilibrium where all goods are produced with the pre-industrial method if

$$L(1 - c(1 + v)) < F(1 + v),$$

where $F = fN$. 
Industrial equilibrium. What about equilibria where all goods are produced using the industrial method? We already know that prices of all goods will be equal to unity in this equilibrium also. Moreover, households will keep spending the same share of income on all goods. Hence, all industries will employ the same amount of labour, \( L/N \), in equilibrium. \( \text{(1)} \) therefore implies that the value of production in each industry is \((L/N - f)/c\). Summing across the \( N \) industries in the economy yields a value for gross domestic product, and hence aggregate household income, of \( Y = (L - F)/c \) (recall that \( F \equiv fN \)).

Do firms make the profit necessary to sustain the industrial production method when all production takes place in the industrial sector? Profits of firms in the industrial sector are \( \pi_i = q_i^m - (f + cq_i^m) \) \((1 + v) \geq 0\), where \( q_i^m \) is the demand faced by the industrial producer of good \( i \), \( q_i^m = Y/N(L - F)/cN \). Hence, there will be an equilibrium where firms using the increasing-returns-to-scale method make a profit if
\[
L(1 - c(1 + v)) \geq F. \tag{4}
\]

Efficient allocation. When is the adoption of increasing-returns-to-scale technologies efficient? The aggregate value of production is \( Y = (L - F)/c \) when industrial production methods are used and \( Y = L \) with pre-industrial methods. The amount of goods necessary to pay the compensating wage differential when all workers are employed in the industrial sector is \( vL \). Hence aggregate welfare will be higher with industrial production methods if and only if \((L - F)/c - vL \geq L \), or
\[
L(1 - c(1 + v)) \geq F. \tag{5}
\]

Note that \( 4 \) and \( 5 \) coincide. Hence, an industrial equilibrium exists if and only if it is efficient.

Multiple equilibria and underdevelopment traps. Only one of the two inequalities in \( 3 \) and \( 4 \) can hold if there is no industry wage premium \((v = 0)\). Hence, the equilibrium is unique in this case and, as a result, there cannot be development traps. Moreover, because an industrial equilibrium exists if and only if it is efficient, economies in a pre-industrial equilibrium actually do the best they can given the economic environment.

But when there is an industry wage premium \((v > 0)\) there may be multiple equilibria as the inequalities in \( 3 \) and \( 4 \) can both be satisfied. When this is the case, economies may be stuck in a pre-industrial equilibrium, although the same economic environment would be compatible with an (efficient) industrial equilibrium. To understand why, suppose the economy is in a pre-industrial equilibrium when we force an industry to adopt the increasing-returns-to-scale technology. If \( 3 \) holds, then the adopting firm will make a loss. Still, its contribution to aggregate income is strictly positive. To see this, note that demand for this industry is \( L/N \), and that this is also the amount of labour required to produce the amount of goods demanded using the pre-industrial production methods. Production with the increasing-returns-to-scale technology requires \( cL/N + f \) units of labour, which is strictly smaller than \( L/N \) if \( 4 \) holds. Hence, the adoption of the increasing-returns-to-scale technology saves labour in the adopting industry, and therefore increases aggregate output and income. This increases demand faced by other industries and therefore raises the profitability of further adoption of the increasing-returns-to-scale technology. Eventually, industrialization raises aggregate income enough for increasing-returns-to-scale industries to break even. Hence, the industrial equilibrium can be seen as the result of a virtuous circle. The adoption of increasing-returns-to-scale technologies raises aggregate income and therefore the profitability of adopting increasing-returns-to-scale technologies. At the same time, the economic environment also allows for a development trap where low aggregate income is both the cause and the consequence of the failure to adopt increasing-returns-to-scale technologies.
Vertical Demand Linkages in an Input Chain Model

The economic activity of different industries is linked to each other because the output of some industries is used as input in other industries. Can such vertical linkages give rise to vicious circles of underdevelopment or virtuous circles of development when there are increasing returns at the firm level? We will show that just as for horizontal linkages, this cannot happen if increasing-returns-to-scale technologies differ from pre-industrial technologies only in that they are more productive at large scale.

Chenery et al. (1986) comparative study of industrialization shows, however, that the industrialization of countries has typically been accompanied by an increase in the intermediate-input intensity of production. This suggests that industrial technologies may use intermediate inputs more intensively than the technologies they replace. We will therefore start by analysing a model of development where increasing-returns-to-scale technologies use intermediate inputs more intensively than pre-industrial technologies.

It will be useful to analyze the consequences of vertical linkages for industrialization in a framework that is as close as possible to the model of horizontal linkages of Murphy, Shleifer and Vishny. In particular, the aggregate amount of labour supplied by households continues to be $L$ and households spend an equal share of their incomes on each of the $N$ goods produced in the economy. On the production side, we continue to assume that each good can be produced using two different production methods, namely, a pre-industrial method and an industrial (increasing-returns-to-scale) method. The pre-industrial method requires one unit of labour for each unit of output. The increasing-returns-to-scale method will turn out to be cheaper at the margin but subject to a fixed labour requirement $f$. Many firms know the pre-industrial method, but for each good there is only a single firm with the ability to produce in the industrial sector.

Input chains and industrial production. The key difference with the horizontal linkages model is that now the increasing-returns-to-scale method is taken to be more intermediate-input intensive than the pre-industrial method. One way to model the intermediate-input structure of the economy is to think of goods being produced in $S$ different locations along a river. Each location produces $H$ different goods (the total number of goods is $N=HS$). Goods at location 1 are produced using labour only. Goods at any location $s > 1$, on the other hand, are produced using all goods at location $s-1$. This implies that all goods at locations $s < S$ may face intermediate-input demand from downstream industries in addition to consumption-goods demand from households (the exception are the $H$ goods furthest downstream, at location $S$, which face consumption-goods demand only). In particular, we assume that, after having incurred the overhead labour cost, one unit of any good $j$ located at $s > 1$ can be produced with $c$ units of an intermediate-input composite $z_{j,s}$ that combines all $H$ goods produced at location $s-1$,

$$z_{j,s} = \prod_{i=1}^{H} (Hq_{i,s-1})^{1/H}.$$  \hspace{1cm} (6)

where $q_{i,s-1}$ is the input of good $i$ at location $s-1$. This formulation implies that industrial firms spend the same amount on all upstream inputs. As a result, the marginal cost of the intermediate-input composite necessary for industrial production at location $s > 1$ is simply a geometrically weighted average of prices $p_{i,s-1}$ of the $H$ up stream goods,

$$MC_s = \prod_{i=1}^{H} p_{i,s-1}^H.$$  \hspace{1cm} (7)

Industrial production for goods at location $s = 1$ requires $f$ units of overhead labour and $c$ units of labour for each unit of output. (The assumption that the industrial overhead requires labour only while production at the margin requires intermediate inputs only simplifies the analysis considerably. Ciccone (2002) analyses the case where production of the overhead and at the margin use both labour and intermediate inputs.)
Just as in the horizontal linkages model, industrial firms find it optimal to use a limit pricing strategy for consumption goods vis-à-vis the competitive fringe. Their intermediate-input pricing strategy is potentially more complicated but also simplifies to a limit pricing strategy vis-à-vis the competitive fringe when $H$ is sufficiently large.

Pre-industrial equilibrium. When will there be an equilibrium where all goods are produced with the pre-industrial method? It turns out that if $H$ is sufficiently large the condition is

$$L(1 - c) < F,$$

which coincides with the condition for a pre-industrial equilibrium in the Murphy, Shleifer, and Vishny model of horizontal linkages. To see this, suppose that all goods are produced with the pre-industrial technology and their price is unity. When (8) holds, any single firm adopting the increasing-returns-to-scale method to produce consumption goods will make a loss. Moreover, when $H$ is sufficiently large, (7) also implies that single industrial firms are unable to generate intermediate-input demand for their good even if they lower their price to the marginal cost of production. To see this, suppose that one industrial firm at location $S - 1$ is considering selling its good at marginal cost to firms at location $S$ in order to generate intermediate-input demand. In this case, one of the $H$ inputs of potential industrial firms at $S$ would become available at price $c$ and (7) implies that the marginal cost of production would therefore fall from $c$ to $e^{(1 + H)yH}$ (recall that the remaining $H - 1$ inputs are available at price of unity). Goods at $S$ face demand $L/N$, which comes exclusively from households as there are no upstream industries. Hence, profits of the potential industrial firm at $S$ producing at marginal cost $e^{(1 + H)yH}$ would be $(1 - e^{(1 + H)yH})L/N - f$, which is strictly negative if (8) holds and $H$ is large enough. Potential industrial firms at location $S$ would therefore find it unprofitable to start production even after the price cut, which implies that potential industrial firms at location $S - 1$ must break even on consumption-goods demand only. Applying the same argument sequentially to potential industrial firms in locations $S - 2$, $S - 3$, ..., 1 yields that pre-industrial production of all goods is an equilibrium when (7) holds and $H$ is sufficiently large.

Industrial equilibrium. To determine the conditions for the existence of an industrial equilibrium, it is necessary to determine aggregate income when all goods at location $\sigma$ and upstream of location $\sigma$ are produced with the increasing-returns-to-scale technology. This turns out to be straightforward. If aggregate income is $Y$, the quantity of each good demanded by households is $Y/N$. The intermediate-input structure implies that industrial production of $Y/N$ units of each of the $H$ goods at location $\sigma$ requires $cY/N$ units of each of the $H$ goods at location $\sigma - 1$. Hence, as $Y/N$ units of good $\sigma - 1$ are demanded by households, production of each good at $\sigma - 1$ must be $Y/N + cY/N$. Production of this quantity of goods at $\sigma - 1$ requires $C(Y/N + cY/N)$ units of each good at $\sigma - 2$. Adding the $Y/N$ units of goods at $\sigma - 2$-demanded by households, yields that production at $\sigma - 2$ must be $Y/N + cY/N + c^2Y/N$. Continuing all the way up stream yields that the total production of each of the $H$ goods at location 1 must be

$$q_1 = Y/N + cY/N + c^2Y/N + ... + c^{\sigma-1}Y/N$$

$$= \frac{1 - c^\sigma}{1 - c} Y/N.$$

(9)

To turn to the labour market, $f$ units of labour must be used as overhead in the production of each good produced with the industrial technology. Moreover, $Y/N$ units of labour are required for the production of each good produced with the pre-industrial technology. Hence, the amount of labour available for production at the margin of the $H$ goods at $\sigma = 1$ is $L - \sigma Hf - (N - \sigma H)yY/N$. Labour market clearing requires

$$cHq_1 = L - \sigma Hf - (N - \sigma H)yY/N.$$ Substituting (9) yields aggregate income in an economy where the $\sigma$ industries furthest upstream have industrialized:
$Y(\sigma) = \frac{L - F(\sigma H/N)}{c\theta[\sigma](\sigma H/N) + (1 - (\sigma H/N))} = \frac{L - F(\sigma H/N)}{1 - (\sigma H/N)(1 - c\theta[\sigma])}, \quad (10)$

where

$\theta[\sigma] = \frac{1 - e^\sigma}{(1 - c)\sigma}$.

c$\theta[\sigma]$ has a simple interpretation. It is the amount of labour required to produce one additional unit of goods located at $\sigma$ if all industries upstream of (including) $\sigma$ have adopted the industrial technology. Note that the amount of labour required to produce one additional unit of goods at location $\sigma$ falls the longer the industrial input chain ($\theta[\sigma]$ is strictly decreasing in $\sigma$).

The intermediate-input structure implies that the demand for goods is greater the further upstream they are located. Hence, profits from adopting the increasing-returns-to-scale technology fall the further downstream industries are located. An equilibrium where all industrial firms make a profit will therefore exist if goods produced furthest downstream (at location $S$) can be produced using the increasing-returns-to-scale technology without a loss. Because firms furthest downstream sell to households only, their sales are equal to aggregate income divided by the number of goods, $\gamma[S]/N$ (recall that all firms set prices optimally at unity). As a result, their profits are positive if and only if $\pi_S = (1 - c)(\gamma[S]/N) - F \geq 0$ or, to make use of (10),

$$(1 - c)L \geq (c\theta[S] + (1 - c))F. \quad (11)$$

*Multiple equilibria and underdevelopment traps.* Comparison of (8) and (11) yields that, with input chains ($S > 1$), it is possible for the pre-industrial equilibrium and the industrial equilibrium to exist side by side. (When $S = 1$ then $\theta = 1$ and the model is that of Murphy, Shleifer, and Vishny without an industry wage premium.) This is because the adoption of increasing-returns-to-scale technologies now has a direct and indirect effect on income. The direct effect is given by the profit or loss in the adopting industry. The indirect effect is equal to the profits generated upstream of the adopting industry. When the indirect profits generated by the increased intermediate-input demand more than offset direct losses of industrial technologies, then industrialization increases aggregate income. As a result, further industrialization becomes more profitable. When (7) and (10) hold simultaneously, this effect is strong enough to ensure that all industrial firms make a profit once all goods are produced with increasing-returns-to-scale technologies.

The pre-industrial and industrial equilibrium can exist side by side even if aggregate income is much greater in the industrial equilibrium. Note that aggregate income in the industrial equilibrium is $Y[S] = (L - F)\theta[S]$, see (10). As intermediate-input chains become longer, $\theta[S]$ in (10) tends to zero, and aggregate income in the industrial equilibrium increases. Aggregate income in the pre-industrial equilibrium, on the other hand, is independent of $S$ as production does not rely on intermediate inputs. Moreover, the range of parameter values for which the industrial equilibrium exists increases. Hence, long input chains imply that equilibrium multiplicity is more likely and also that the aggregate income difference between industrial and pre-industrial equilibria may be very large.

*Vertical linkages and equilibrium uniqueness.* To see that the equilibrium is unique when increasing-returns-to-scale technologies use intermediate inputs as intensively as pre-industrial technologies, note that costs of production plus profit must add up to the value of firms’ sales, $COST + \pi = q$. Suppose that intermediate inputs are a share $\alpha$ of costs of production for both the pre-industrial and the industrial production method. In this case, the demand for goods produced at $s - 1$ is equal to $\alpha COSTS = \alpha(q_s - \pi_s)$. Now suppose that all goods upstream of $\sigma$ are produced with the increasing-returns-to-scale technology. Is it possible that aggregate income increases with the adoption of the increasing-returns-to-scale technology at $\sigma$ even if the adopting firm makes a loss? A switch to industrial production at $\sigma$ does not affect the value of goods
produced at this location (\(q_\sigma\) is unchanged). Hence, the adoption of the increasing-returns-to-scale technology at \(\sigma\) increases demand for each good produced at \(\sigma - 1\) by \(\pi_{\sigma}/H\). Loss-making industrialization at \(\sigma\) therefore leads to greater demand at \(\sigma - 1\). But the profits generated by this input demand cannot be greater than the initial loss \(\pi_{\sigma}\). To see this, notice that total profits at location \(\sigma - 1\) increase by 

\[-(1 - c)\pi_{\sigma}, \text{ where } -(1 - c)\pi_{\sigma}/H \text{ is the increase in demand for each good produced at } \sigma - 1.\]

The general formula is that total profits at location \(\sigma - i\) increase by 

\[-(1 - c)\pi_{\sigma} + \cdots + (\pi_{\sigma}) < 1\], which is smaller than 

\[-(1 - c)\pi_{\sigma}(1 + \pi_{\sigma})^\alpha = -\pi_{\sigma}(\alpha - \pi_{\sigma})/(1 - \pi_{\sigma}).\]

Hence, \(\alpha \leq 1\) implies that the sum of profits generated upstream of \(s\) by loss-making industrialization at \(s\) is always smaller than the initial loss (\(\pi_{\sigma}\)). Loss-making industrialization necessarily lowers aggregate income. The aggregate demand externality necessary for multiple equilibria is therefore absent when increasing-returns-to-scale technologies are no more intermediate-input intensive than pre-industrial technologies.

**Vertical Demand and Cost Linkages with Input Chains**

So far firms adopting increasing returns to scale technologies did not have an incentive to cut prices. This eliminated virtuous circles of development where lower intermediate-input prices (vertical cost linkages) and greater intermediate-input demand (vertical demand linkages) feed on each other. A simple way to capture the interplay between vertical demand and cost linkages is to suppose that firms in the competitive fringe can produce one unit of goods at location \(s > 1\) with \(1 + \varepsilon > 1\) units of the intermediate-input composite in (6) or one unit of labour. That is, firms have access to two modes of production, a labour-intensive mode and an intermediate-input intensive mode. The exception continues to be goods at location 1, for which there is a labour-intensive mode of production only. Industrial firms at locations \(s > 1\) also have access to a labour-intensive and an intermediate-input intensive mode of production, but are more efficient than pre-industrial firms at the margin. Once they have incurred the overhead labour requirement \(f\), industrial firms can produce one unit of output with \(c(1 + \varepsilon) < 1\) of the intermediate-input composite in (6) or \(c < 1\) units of labour. Industrial firms producing goods at location 1 have access to the labour-intensive mode of production only. The assumption that the overhead is produced using labour only continues to simplify the analysis considerably. A new by-product of this assumption is that industrial firms now actually use intermediate inputs less intensively than pre-industrial firms at the same factor prices – the opposite of what we assumed in the previous section.

**Pre-industrial equilibrium with labour-intensive production.** Can there be an equilibrium where all goods are produced with the pre-industrial technology using labour only? The marginal cost of production with the pre-industrial technology in the labour-intensive mode is unity. Hence, the price of all goods would be equal to unity. To see that these prices make it optimal to use the labour-intensive mode of production, note that they imply that the marginal cost of intermediate-input composites in (7) is unity. The marginal cost of production using the intermediate-input intensive mode compared with the labour-intensive mode is therefore \(1 + \varepsilon > 1\) (in the pre-industrial as well as the industrial sector). Hence, all firms will find it optimal to use the labour-intensive mode of production.

In a pre-industrial equilibrium, the adoption of the increasing-returns-to-scale technology by a single firm must lead to losses. If industrial firms can count on consumption-goods demand only, this will be the case if \(L(1 - c) < F\). But an industrial firm may be able to generate additional demand by getting industries just downstream to switch to an intermediate-input intensive mode of production. While this can happen in principle, it will not happen if \(H\) is sufficiently large. To see this, consider the case where a single industrial firm supplies its good to downstream industries at
marginal cost. In this case, (7) yields that the marginal cost of the intermediate input-intensive mode of production relative to the labour-intensive mode becomes \( c^\frac{1}{1+\varepsilon}(1+\varepsilon) \), which will be greater than unity when \( H \) is sufficiently large (recall that \( 1+\varepsilon > 1 \)). Hence, a single industrial firm cannot generate downstream intermediate-input demand even if it reduces its price to marginal cost. For \( H \) sufficiently large, a pre-industrial labour-intensive equilibrium will therefore exist if \( L(1-c) < F \).

**Industrial equilibrium with intermediate-input intensive production.** When is there an industrial equilibrium where all firms use the intermediate-input intensive mode of production? To simplify the analysis, suppose that industrial firms can price discriminate between households and industrial users of their goods. As before, industrial firms will find it optimal to follow a limit pricing strategy when it comes to sales to households. Industrial firms will therefore price consumption goods at unity. When it comes to intermediate-input sales to downstream industries, industrial firms must also take into account that users will switch to the labour-intensive mode of production if the cost of the intermediate-input composite is greater than \( 1/(1+\varepsilon) \). Hence, each industrial firm will find it optimal to set a limit price of \( 1/(1+\varepsilon) \) for intermediate inputs if other industrial intermediate-input suppliers do the same.

Aggregate income in the industrial equilibrium where all firms use the intermediate-input intensive mode of production can be determined following the argument that led to (10). The only difference is that an additional unit of all goods at location \( s > 1 \) now translates into a demand of \( c(1+\varepsilon) \) units of each good at location \( s - 1 \). Aggregate income when all goods are produced with the industrial technology in the intermediate-input intensive mode is therefore \( Y[S] = (L-F)/c\hat{\theta}[S] \) where

\[
\hat{\theta}[S] = \frac{1 - (c(1+\varepsilon))^s}{(1 - (c(1+\varepsilon))^s}.
\]

(12)

An industrial equilibrium exists if the firm furthest downstream can break even given the demand for consumption goods, \( \pi_s = (1-c)(1+\varepsilon)(Y[S]/N - f \geq 0 \) or, to make use of the expression for aggregate income just above,

\[
(1-c(1+\varepsilon)L \geq \left((c\hat{\theta}[S] + (1-c(1+\varepsilon))\right)F).
\]

**Multiple equilibria with vertical demand and cost linkages.** There will be multiple equilibria if both \( L(1-c) < F \) and \( (1-c(1+\varepsilon)L \geq \left((c\hat{\theta}[S] + (1-c(1+\varepsilon))\right)F\). This implies that the pre-industrial equilibrium with labour-intensive production and the industrial equilibrium with intermediate-input intensive production may exist side by side if and only if there are input chains \( (\hat{\theta}[S] < 1) \). The virtuous circle sustaining industrial equilibria now consists of an interplay between vertical demand and cost linkages. The increase in the intermediate-input intensity of production necessary for increasing-returns-to-scale technologies to be profitable (vertical demand linkages) comes about because the adoption of increasing-returns-to-scale technologies translates into falling intermediate-input prices (vertical cost linkages). Note that, for this virtuous circle to be operative, the elasticity of substitution between intermediate inputs and labour in industrial production must be greater than unity (our model assumed that this elasticity is infinity for simplicity). In a pre-industrial equilibrium, on the other hand, pre-industrial technologies are both the cause and the consequence of labour-intensive modes of production.

**Conclusion**

Neither horizontal nor vertical demand linkages across industries lead to underdevelopment traps if increasing-returns-to-scale technologies differ from pre-industrial technologies only in that they are more productive at large scale. Nevertheless, theories of underdevelopment based on vicious circles of low demand and low productivity are consistent with economic principles. For example, in the case of vertical demand linkages, there can be development traps if increasing-returns-to-scale technologies use intermediate inputs more intensively than the technologies they replace.
More generally, multiple equilibria in our models exist under assumptions that do not appear to be in contradiction by empirical evidence. The exception is that all our model economies were taken to be closed to international trade, but we could have assumed instead that only some goods are non-tradable or that all goods are tradable at some cost (for example, Okuno-Fujiwara 1988; Rodriguez-Clare 1996; Krugman and Venables 1995). Still, it remains to be seen what part of international income differences can be attributed to development traps (for steps in this direction, see Fafchamps and Helms 1996; Graham and Temple 2006).

See Also

► Balanced Growth
► Development Economics
► External Economies
► Externalities
► Multiple Equilibria in Macroeconomics
► New Economic Geography
► Returns to Scale
► Supermodularity and Supermodular Games

Bibliography