Appendix A. Dealing with Mismeasured Export Shares

for

International Commodity Prices and Civil War Outbreak: New Evidence for Sub-Saharan Africa and Beyond

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Appendix A. Dealing with Mismeasured Export Shares

Suppose the risk of civil war is determined by

$$y_{c,t} = \alpha_c + \alpha_t + \beta \left(\sum_i s^*_{i,c,t-1} \pi_{i,t} \right) + u_{c,t} \tag{A1}$$

where $y_{c,t}$ is the risk of civil war in country c in years $t = 1, ..., T$; $s^*_{i,c,t-1}$ is the share of commodity i in total commodity exports of country c in year $t-1$; and $\pi_{i,t} = \log p_{i,t} - \log p_{i,t-1}$ the log-change in the international price of commodity i. The summation is across all possible commodities i. The true commodity export shares entering the commodity price shock on the right-hand side of (A1) are taken to be the export shares in $t-1$ (which results in the commodity price shock being equal to the log growth rate of a Laspeyres price index). But the results below would remain unchanged if the relevant export shares were the shares in t or the average of the export shares in $t-1$ and t (which would result in the commodity price shock being equal to, respectively, the log growth rates of a Paasche or Fisher price index). The α-terms capture the country-specific and time-specific risk of civil war, and $u_{c,t}$ captures other factors affecting the risk of civil war. I take $u_{c,t}$ to be independent of the international commodity price shocks $\pi_{i,t}$ and of the average true export shares over the sample period. International commodity price shocks are assumed to be exogenous and bounded, $|\pi_{i,t}\pi_{j,t}| \leq \mu$ for some $0 < \mu < \infty$. The parameter of interest in (A1) is β, the causal effect of the true commodity price shock $\sum_i s^*_{i,c,t-1}\pi_{i,t}$ on the risk of civil war.

The model for the risk of civil war in (A1) assumes that the risk of civil war only depends on the contemporaneous commodity price shock. It is therefore simpler than the model in (11) in the main text, which also includes lagged and twice-lagged commodity price shocks. As the main issues do not depend on whether or not the risk of civil war also depends on lagged commodity price shocks, it is convenient to proceed with (A1) above as it avoids having to use matrix notation in the proofs below. Another simplifying assumption I make compared to the main text, is that I take the risk of civil war on the left-hand side of (A1) to be observable. In practice it isn't and researchers instead use indicator variables that capture whether a civil war has broken out or not (as I do in the main text). I simplify here as the issues around this approach are well understood and unrelated to what I want to discuss here.

Suppose the researcher does not observe the true commodity export shares $s^*_{i,c,t}$ but export shares measured with some error

$$s^m_{i,c,t} = s^*_{i,c,t} + \varepsilon_{i,c,t} \tag{A2}$$

where $\varepsilon_{i,c,t}$ is a measurement error that is independent of all the elements of the true model in (A1), has bounded moments of all orders across commodities and over time, and satisfies $E(\varepsilon_{i,c,t}|i,c) = 0$ and $E(\varepsilon_{i,c,t}|i,t) = 0$. The measurement error is also assumed to be independent across countries, but $\varepsilon_{i,c,t}$ may be serially correlated and correlated across different commodities i, j for a given country c.

As is well understood, estimating the model in (A1) using the mismeasured export shares in (A2) instead of the true export shares gives rise to measurement error bias. The issue I want to examine here is whether mismeasured export shares can still be used to obtain a consistent estimator of the causal effect of commodity price shocks on civil war risk, β in (A1). Intuitively, this might be possible as long as the mismeasured export shares averaged over time

$$\overline{s}^m_{i,c} = \frac{1}{T}\sum_{t=0}^{T} s^m_{i,c,t} \tag{A3}$$

converge in probability to the average over time of the true export shares $\overline{s}^*_{i,c} = \frac{1}{T}\sum s^*_{i,c,t}$. The remainder of this appendix shows that consistent estimation of β in (A1) is in fact possible as long as the serial correlation in the measurement error is, loosely formulated, limited.

There are several ways to make statements about limited serial correlation precise. The specific condition I will use is so-called K-dependence (Amemiya, 1985, p.175).

Definition 1 ***(K-dependence of a single stochastic process)*** *A stochastic process ω_t is K-dependent if $(\omega_{t_1}, \omega_{t_2}, ..., \omega_{t_n})$ is independent of $(\omega_{\tau_1}, \omega_{\tau_2}, ..., \omega_{\tau_m})$ for any set of integers satisfying $t_1 < t_2 ... < t_n < \tau_1 < \tau_2 ... < \tau_m$ and $t_n + K < \tau_1$.*

Intuitively, K-dependence allows for correlation within time windows of width K but not outside time windows of width K, that is $Cov(\omega_\tau, \omega_t) \neq 0$ if $|t - \tau| \leq K$ but $Cov(\omega_\tau, \omega_t) = 0$ if $|t - \tau| > K$. For example, a moving-average process of order smaller or equal K based in i.i.d. shocks is K-dependent. An advantage of the K-dependence condition is that it does not impose restrictions on the functional form of serial correlation. Such conditions would be unlikely to capture the measurement error in the Bazzi and Blattman dataset as missing data is interpolated.

The next definition is a generalization of K-dependence to the case of multiple stochastic processes. This definition will be useful as the mismeasured commodity price shocks for a country will generally involve the commodity export shares, and hence measurement errors, of more than one commodity.

Definition 2 ***(K-dependence with multiple stochastic processes)*** *The D stochastic processes ω_t^1, ω_t^2, ..., ω_t^D are K-dependent if $(\omega_{t_1}^i, \omega_{t_2}^i, ..., \omega_{t_n}^i)$ is independent of $(\omega_{\tau_1}^j, \omega_{\tau_2}^j, ..., \omega_{\tau_m}^j)$ for any two stochastic processes i, j and any set of integers satisfying $t_1 < t_2... < t_n < \tau_1 < \tau_2... < \tau_m$ and $t_n + K < \tau_1$.*

Intuitively, for any two of the D stochastic processes, this generalization of K-dependence allows for correlation within time windows of width K but not outside time windows of width K. That is, $Cov(\omega_\tau^i, \omega_t^j) \neq 0$ if $|t - \tau| \leq K$ but $Cov(\omega_\tau^i, \omega_t^j)= 0$ if $|t - \tau| > K$ for any i, j. For example, the moving-average processes $\omega_t^i = (\varepsilon_t^i + \eta_t) + \theta_1(\varepsilon_{t-1}^i + \eta_{t-1}) + .. + \theta_K(\varepsilon_{t-K}^i + \eta_{t-K})$ and with all the ε and η i.i.d. would satisfy K-dependence as defined above.

The next proposition presents an instrumental-variables estimator that relies on mismeasured export shares only but still converges to the parameter β in (A1) if the measurement errors for any country are K-dependent.

Proposition 1 *Suppose that measurement errors $\varepsilon_{i,c,t}$ are K-dependent for any country c. Consider the following model obtained by replacing the true time-varying export shares on the right-hand side of (A1) by the mismeasured time-varying export shares*

$$y_{c,t} = a_c + a_t + b \left(\sum_i s_{i,c,t-1}^m \pi_{i,t} \right) + v_{c,t}. \tag{A4}$$

Estimating this model with an instrumental-variables approach using as an instrument the fixed-weight price shock $\sum_i \overline{s}_{i,c}^m \pi_{i,t}$, with the time-averaged mismeasured export shares $\overline{s}_{i,c}^m$ defined in (A3), yields a consistent estimate of β in (11) as T goes to infinity.

The proposition is proven further below. To get an intuitive idea of what is driving this result, note that the mismeasured commodity price shock employed as instrument uses the average mismeasured export shares over the sample period as commodity weights. The measurement error in these time-averaged mismeasured export shares goes to zero as the sample period goes to infinity. As a result, the measurement error in the fixed-weight price-shock instrument ends up being uncorrelated with the measurement error in the mismeasured commodity price shock on the right-hand-side of (A4) as the sample period goes to infinity.

It is also interesting to examine the reduced-form approach corresponding to the instrumental-variables approach in Proposition 1. The next proposition summarizes the key result.

Proposition 2 *Suppose that measurement errors $\varepsilon_{i,c,t}$ are K-dependent for any country c. Consider the following model obtained by replacing the true time-varying export shares on the right-hand side of (A1) by the mismeasured time-averaged export shares $\overline{s}^m_{i,c}$ defined in (A3)*

$$y_{c,t} = a_c + a_t + b^r \left(\sum_i \overline{s}^m_{i,c} \pi_{i,t} \right) + v_{c,t}. \tag{A5}$$

The least-squares slope estimate of this model satisfies

$$plim_{T\to\infty} \widehat{b^r} = \beta(1+\gamma) \tag{A6}$$

where β is defined in (11) and γ is

$$\gamma = plim_{T\to\infty} \frac{\frac{1}{T}\frac{1}{N}\sum_t \sum_c z_{c,t}(x^m_{c,t} - \overline{x}^m_c - \overline{x}^m_t + \overline{x}^m)}{\frac{1}{T}\frac{1}{N}\sum_t \sum_c (x^m_{c,t} - \overline{x}^m_c - \overline{x}^m_t + \overline{x}^m)^2} \tag{A7}$$

with $x^m_{c,t} = \sum_i \overline{s}^m_{i,c}\pi_{i,t}$ and $z_{c,t} = \sum_i (s^m_{i,c,t-1} - \overline{s}^m_{i,c})\pi_{i,t}$. The variables with upper bars in (A7) denote averages. The upper bar with subscript c denotes the average over time for country c; the upper bar with subscript t denotes the average across countries for year t; and the upper bar without any subscript is the average both over time and across countries.

This proposition is also proven further below. The intuition is that there are two issues with the reduced-form model in (A5) compared to the true model in (A1). First, there is measurement error in the right-hand-side variable $x^m_{c,t} = \sum_i \overline{s}^m_{i,c}\pi_{i,t}$ as the export shares used as commodity weights are mismeasured. Second, while the model in (A1) implies that the true risk of civil war also reflects the time variation in the export shares, the reduced-form model in (A5) only captures the effect of average export shares over the sample period. This generally leads to an omitted-variable bias when estimating (A5). When measurement errors in mismeasured export shares are K-dependent, the measurement error issue is resolved as the sample period goes to infinity (because the measurement error in the time-averaged mismeasured export shares used as commodity weights in the right-hand-side variable $x^m_{c,t} = \sum_i \overline{s}^m_{i,c}\pi_{i,t}$ in (A5) goes to zero as the sample period goes to infinity). But the omitted-variable issue remains. This is why the probability limit of the least-squares slope in (A6) is equal to $\beta(1+\gamma)$ with γ capturing the omitted-variable bias in percent of β when estimating the reduced-form model in (A5).

Proposition 2 also characterizes γ in terms of mismeasured observables. The expression on the right-hand side of (A7) is the probability limit of the least-squares slope when regressing $z_{c,t} = \sum_i (s^m_{i,c,t-1} - \overline{s}^m_{i,c})\pi_{i,t}$ on $x^m_{c,t} = \sum_i \overline{s}^m_{i,c}\pi_{i,t}$ and country plus year fixed effects. Hence, γ can be estimated consistently as the least-squares slope $\widehat{\gamma}$ when regressing $z_{c,t}$ on $x^m_{c,t}$ and country plus year fixed effects. Intuitively, $z_{c,t} = \sum_i (s^m_{i,c,t-1} - \overline{s}^m_{i,c})\pi_{i,t}$ is the mismeasured version of the omitted variable, that is, the mismeasured difference between the commodity price shock using time-varying export shares and the commodity price shock using time-averaged export shares. When measurement errors in mismeasured export shares are K-dependent, this mismeasured version of the omitted variable can be used to obtain a consistent estimate of γ and hence the omitted-variable bias.

An interesting special case of Proposition 2 is when $\gamma = 0$. In this case, the reduced-form estimate obtained by applying least squares to (A5) converges to β in probability. The reason is that there is no omitted-variable bias asymptotically in this case, because the omitted variable is uncorrelated with the fixed-weight commodity price shock using time-averaged export shares as the sample period goes to infinity. The condition $\gamma = 0$ can be checked by testing the hypothesis $\widehat{\gamma} = 0$.

When the risk of civil war in (A1) depends on lagged and twice-lagged commodity price shocks also, as assumed in the main text in equation (11), (A6) has to be replaced by equation (15) in the main text. The intuition and proof remain unchanged however.

Proof of Proposition 1 To prove Proposition 1 it is useful to write the true model in (A1) in terms of the mismeasured commodity price shock, which using (A2) implies that the measurement error becomes part of the residual

$$y_{c,t} = \alpha_c + \alpha_t + \beta \left(\sum_i s^m_{i,c,t-1} \pi_{i,t} \right) - \beta \left(\sum_i \varepsilon_{i,c,t-1} \pi_{i,t} \right) + u_{c,t}. \tag{A8}$$

The standard measurement error bias arises when (A8) is estimated using least squares because the measurement error enters both the mismeasured commodity price shock $\sum_i s^m_{i,c,t-1}\pi_{i,t}$ and the residual.

When the mismeasured commodity price shock in (A8) is instrumented with $\sum_i \overline{s}^m_{i,c}\pi_{i,t}$, the resulting instrumental-variables estimator will be consistent as T goes to infinity, $plim_{T\to\infty}\, \widehat{b}_{IV} = \beta$, if and only if the residual $-\beta\left(\sum_i \varepsilon_{i,c,t-1}\pi_{i,t}\right) + u_{c,t}$ is uncorrelated with the instrument $\sum_i \overline{s}^m_{i,c}\pi_{i,t}$ as T goes to infinity. As $u_{c,t}$ is taken to be independent of the instrument, this condition is equivalent to $\sum_i \varepsilon_{i,c,t-1}\pi_{i,t}$ being uncorrelated with the instrument $\sum_i \overline{s}^m_{i,c}\pi_{i,t}$ as T goes to infinity. Formally

$$plim_{T\to\infty} \frac{1}{T}\frac{1}{N} \sum_c \sum_t \left(\sum_i \varepsilon_{i,c,t-1}\pi_{i,t} \right) \left(\sum_i \overline{s}^m_{i,c}\pi_{i,t} \right) = 0. \tag{A9}$$

Writing the time-averaged mismeasured export share in (A9) as the true time-averaged export share plus the time-averaged measurement error yields

$$A = \frac{1}{T}\frac{1}{N} \sum_c \sum_t \left(\sum_i \varepsilon_{i,c,t-1}\pi_{i,t} \right) \left(\sum_i \overline{s}^*_{i,c}\pi_{i,t} + \overline{\varepsilon}_{i,c}\pi_{i,t} \right). \tag{A10}$$

The term A in (A10) can be written as the sum of the two following terms

$$B = \sum_{i,j} \left(\frac{1}{NT} \sum_c \sum_t \overline{s}^*_{j,c}\varepsilon_{i,c,t-1}\pi_{i,t}\pi_{j,t} \right) = \sum_{i,j} B_{i,j} \tag{A11}$$

and

$$C = \sum_{i,j} \left(\frac{1}{NT} \sum_c \sum_t \varepsilon_{i,c,t-1}\overline{\varepsilon}_{j,c}\pi_{i,t}\pi_{j,t} \right) = \sum_{i,j} C_{i,j}. \tag{A12}$$

Both terms go to zero in probability as T goes to infinity if the measurement errors $\varepsilon_{i,c,t}$ are K-dependent for any country c and independent across countries.

To see this, start with B. It follows from (A11) that this term goes to zero in probability as T goes to infinity if $B_{i,j}$ goes to zero for all i, j. The zero-expectation properties of the measurement error implies that

$$EB_{i,j} = 0. \tag{A13}$$

Moreover, the assumption that $\varepsilon_{i,c,t}$ is K-dependent implies that there is a positive, finite ϕ_1 such that

$$EB_{i,j}^2 \leq \frac{1}{NT}(2K+1)\mu\phi_1 \tag{A14}$$

where μ is the bound on $|\pi_{i,t}\pi_{j,t}|$ and I have used that export shares are bounded above by one. For example, in the case where the measurement error is i.i.d., ϕ_1 could be set equal to the variance of the measurement error. As $EB_{i,j} = 0$ and $EB_{i,j}^2$ goes to zero as T goes to infinity, the probability limit of $B_{i,j}$ goes to zero as T goes to infinity. This implies that the probability limit of B in (A11) goes to zero as T goes to infinity.

The C term in (A12) goes to zero in probability as T goes to infinity if $C_{i,j}$ goes to zero for all i, j. To examine the $C_{i,j}$ terms it is useful to rewrite them as

$$C_{i,j} = \frac{1}{NT^2}\sum_{c}\sum_{t,\tau}\varepsilon_{i,c,t-1}\varepsilon_{j,c,\tau}\pi_{i,t}\pi_{j,t} \tag{A15}$$

where $\sum_{t,\tau} \equiv \sum_t \sum_\tau$. The K-dependence assumption implies that there are finite ϕ_2, ϕ_3 with $\phi_2 \leq 0 < \phi_3$ such that

$$\frac{1}{T}(2K+1)\mu\phi_2 \leq EC_{i,j} \leq \frac{1}{T}(2K+1)\mu\phi_3. \tag{A16}$$

For example, in the case where the measurement error is i.i.d., ϕ_2 could be set equal zero and ϕ_3 equal to the variance of the measurement error. More generally, ϕ_2 could be picked to be the minimum of zero and the smallest possible value of the $E(\varepsilon_{i,c,t-1}\varepsilon_{j,c,\tau})$ across all commodities and years, and ϕ_3 could be picked to be the largest possible value of $E(\varepsilon_{i,c,t-1}\varepsilon_{j,c,\tau})$. An immediate implication of (A16) is that the expectation of $C_{i,j}$ in (A15) goes to zero as T goes to infinity.

The next step is to examine $EC_{i,j}^2$. To do so, it is useful to write this term as

$$EC_{i,j}^2 = \frac{1}{N^2T^4}\sum_{c,q}\sum_{t,\tau,p,w} E(\varepsilon_{i,c,t-1}\varepsilon_{j,c,\tau}\pi_{i,t}\pi_{j,t}\varepsilon_{i,q,p-1}\varepsilon_{j,q,w}\pi_{i,p}\pi_{j,p}\Big) \tag{A17}$$

where $\sum_{t,\tau,p,w} \equiv \sum_t \sum_\tau \sum_p \sum_w$. The K-dependence assumption implies that there are positive, finite numbers ϕ_4, ϕ_5 such that

$$EC_{i,j}^2 \leq \frac{1}{T^2}(2K+1)^2\mu^2\phi_4 + \frac{1}{NT^2}(2K+1)^2\mu^2\phi_5 \tag{A18}$$

where μ^2 is the squared bound on $|\pi_{i,t}\pi_{j,t}|$. For example, in the case where the measurement errors are i.i.d., ϕ_4 and ϕ_5 could be picked to be the maximum of $(E\varepsilon^2)^2$ and $E\varepsilon^4$. More generally, ϕ_4 and ϕ_5 could be picked to be the largest possible value of the $E(\varepsilon_{i,c,t-1}\varepsilon_{j,c,\tau}\varepsilon_{i,q,r-1}\varepsilon_{j,q,s})$ across all countries, commodities, and years. The inequality in (A18) implies that $EC_{i,j}^2$ goes to zero as T goes to infinity. Hence, both $EC_{i,j}$ and $EC_{i,j}^2$ go to zero as T goes to infinity, which implies that the probability limit of $C_{i,j}$ goes to zero as T goes to infinity. As a result, the probability limit of C in (A12) goes to zero as T goes to infinity, which completes the proof that the probability limit of A in (A10) goes to zero as T goes to infinity. Hence, (A9) holds and $plim_{T\to\infty}\,\widehat{b}_{IV} = \beta$.

Proof of Proposition 2 To prove Proposition 2 it is useful to express the least-squares estimator of b^r in (A5) in terms of demeaned data

$$\widehat{b^r} = \frac{\frac{1}{T}\frac{1}{N}\sum_c\sum_t(y_{c,t}-\overline{y}_c-\overline{y}_t+\overline{y})(x_{c,t}^m-\overline{x}_c^m-\overline{x}_t^m+\overline{x}^m)}{\frac{1}{T}\frac{1}{N}\sum_c\sum_t(x_{c,t}^m-\overline{x}_c^m-\overline{x}_t^m+\overline{x}^m)^2} \tag{A19}$$

where $x_{c,t}^m = \sum_i \overline{s}_{i,c}^m\pi_{i,t}$ and the variables with upper bars denote averages. The upper bar with subscript c denotes the average over time for country c, the upper bar with subscript t denotes the average across countries for year t, and the upper bar without any subscript denotes the average both over time and across countries.

The true model in (A1) implies that the demeaned risk of civil war can also be written in terms of demeaned variables

$$y_{c,t} - \overline{y}_c - \overline{y}_t + \overline{y} = \beta(x_{c,t} - \overline{x}_c - \overline{x}_t + \overline{x}) + (u_{c,t} - \overline{u}_c - \overline{u}_t + \overline{u}) \tag{A20}$$

where $x_{c,t} = \sum_i s^*_{i,c,t-1}\pi_{i,t}$ is the true commodity price shock. The true commodity price shock can be written as the commodity price shock using mismeasured time-averaged commodity weights, $s^m_{c,t} = \sum_i \overline{s}^m_{i,c}\pi_{i,t}$, plus two additional terms

$$x_{c,t} = x^m_{c,t} + z_{c,t} + e_{c,t} \tag{A21}$$

where $z_{c,t}$ is the differences between the commodity price shock using mismeasured time-varying commodity weights and mismeasured time-averaged commodity weights

$$z_{c,t} = \sum_i (s^m_{i,c,t-1} - \overline{s}^m_c)\pi_{i,t} \tag{A22}$$

and $e_{c,t}$ captures the difference between the commodity price shock using true and mismeasured time-varying commodity weights

$$e_{c,t} = \sum_i (s^*_{i,c,t-1} - s^m_{i,c,t-1})\pi_{i,t} = -\sum_i \varepsilon_{i,c,t-1}\pi_{i,t}. \tag{A23}$$

Substituting (A21) in (A20) yields that the least-squares estimator in (A19) can be written as

$$\widehat{b^r} = \beta + \beta\widehat{\gamma} + \beta\widehat{b^r_e} + \widehat{b^r_u} \tag{A24}$$

where

$$\widehat{\gamma} = \frac{\frac{1}{T}\frac{1}{N}\sum_c\sum_t z_{c,t}(x^m_{c,t} - \overline{x}^m_c - \overline{x}^m_t + \overline{x}^m)}{\frac{1}{T}\frac{1}{N}\sum_c\sum_t (x^m_{c,t} - \overline{x}^m_c - \overline{x}^m_t + \overline{x}^m)^2} \tag{A25}$$

$$\widehat{b^r_e} = \frac{\frac{1}{T}\frac{1}{N}\sum_c\sum_t e_{c,t}(x^m_{c,t} - \overline{x}^m_c - \overline{x}^m_t + \overline{x}^m)}{\frac{1}{T}\frac{1}{N}\sum_c\sum_t (x^m_{c,t} - \overline{x}^m_c - \overline{x}^m_t + \overline{x}^m)^2} \tag{A26}$$

and

$$\widehat{b^r_u} = \frac{\frac{1}{T}\frac{1}{N}\sum_c\sum_t u_{c,t}(x^m_{c,t} - \overline{x}^m_c - \overline{x}^m_t + \overline{x}^m)}{\frac{1}{T}\frac{1}{N}\sum_c\sum_t (x^m_{c,t} - \overline{x}^m_c - \overline{x}^m_t + \overline{x}^m)^2}. \tag{A27}$$

As $u_{c,t}$ is independent of true export shares, international commodity price shocks, and measurement errors, $\widehat{b_u^r}$ goes to zero in probability as T goes to infinity, $plim_{T\to\infty}\widehat{b_u^r} = 0$. Moreover, the K-dependence assumption implies that $\widehat{b_e^r}$ goes to zero in probability as T goes to infinity. To see this, note that using (A23) and $x_{c,t}^m = \sum_i \overline{s}_{i,c}^m \pi_{i,t}$, the term $\frac{1}{T}\frac{1}{N}\sum_c \sum_t e_{c,t} x_{c,t}^m$ in the numerator of $\widehat{b_e^r}$ can be written as

$$E = -\frac{1}{T}\frac{1}{N}\sum_c \sum_t \sum_i \varepsilon_{i,c,t-1}\pi_{i,t}\left(\sum_i \overline{s}_{i,c}^{*}\pi_{i,t} + \overline{\varepsilon}_{i,c}\pi_{i,t}\right). \qquad \text{(A28)}$$

This term is equal to the negative of A in (A10) which I have shown to go to zero in probability as T goes to infinity. The terms $\frac{1}{T}\frac{1}{N}\sum_c \sum_t e_{c,t}\overline{x}_c^m$ and $\frac{1}{T}\frac{1}{N}\sum_c \sum_t e_{c,t}\overline{x}_t^m$ in the numerator of $\widehat{b_e^r}$ in (A26) can be shown to be equal, respectively, to $-\frac{1}{T}\frac{1}{N}\sum_c \sum_t \sum_i \varepsilon_{i,c,t-1}\pi_{i,t}\left(\sum_i \overline{s}_{i,c}^{*}\overline{\pi}_i + \overline{\varepsilon}_{i,c}\overline{\pi}_i\right)$ and to $-\frac{1}{T}\frac{1}{N}\sum_c \sum_t \sum_i \varepsilon_{i,c,t-1}\pi_{i,t}\left(\sum_i \overline{\overline{s}}_i^{*}\pi_{i,t} + \overline{\overline{\varepsilon}}_i\pi_{i,t}\right)$ where $\overline{\pi}_i = \frac{1}{T}\sum_t \pi_{i,t}$, $\overline{\overline{s}}_i^{*} = \frac{1}{N}\sum_c \overline{s}_{i,c}^{*}$, and $\overline{\overline{\varepsilon}}_i = \frac{1}{N}\sum_c \overline{\varepsilon}_{i,c}$. As a result, they can be shown go to zero in probability as T goes to infinity using the same type of argument used for E in (A28) (and A in (A10)). Hence, $plim_{T\to\infty}\widehat{b_e^r} = 0$ and (A24) implies that

$$plim_{T\to\infty}\widehat{b^r} = \beta + \beta plim_{T\to\infty}\widehat{\gamma}. \qquad \text{(A29)}$$