

Price Trends over the Product Life Cycle and the Optimal Inflation Target*

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Abstract

We document a new stylized fact for the life-cycle behavior of consumer prices: relative to a narrowly defined set of competing products, the price of individual products tends to fall over the product lifetime. This holds true for more than 90% of the expenditure items underlying the U.K. consumer price index and has important normative implications. Constructing a sticky price model featuring a product life cycle and rich amounts of heterogeneity, we explain how the optimal inflation target can be estimated from the observed trends in relative prices. The optimal inflation target for the U.K. is found to range between 2.6% and 3.2% and to have steadily increased over the period 1996 to 2016. We show how changes in relative price trends contributed to this development.

JEL Class. No.: E31

1 Introduction

A defining feature of modern economies is the high rate of product turnover in the market place. This feature is documented in a number of micro studies (Nakamura and Steinsson (2008), Broda and Weinstein (2010)) and is a key focus of the Schumpeterian literature on creative destruction (Aghion and Howitt (1992)). It is, however, routinely abstracted from in the monetary policy literature. This relative neglect of the product life cycle in the monetary literature is surprising, but not innocuous from the perspective of monetary policy design: we show that features of the product life cycle turn

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out to be important for determining the optimal inflation rate that a welfare maximizing central bank should target.

We start our analysis by documenting a new set of stylized facts for the behavior of product prices over the product lifetime. We do this by considering the official micro price data that underlies the construction of the consumer price index in the United Kingdom. Our monthly data covers the years 1996-2016. It features more than 1200 narrowly defined expenditure items and contains close to 29 million individual price observations.

Using this data set, we document that for more than 90% of the expenditure items, the price of individual products declines over the product lifetime, relative to the average price of products in the item.¹ New products thus tend to be initially expensive, while becoming cheaper over their lifetime in relative terms. There is also considerable heterogeneity in the average rate of relative price decline across items. Items featuring some kind of ‘news value’, e.g., fashion and entertainment products, display very high rates of price decline, while for the vast majority of items the rates of relative price decline range between zero and five percent per year. For a small set of items, relative prices are estimated to increase over the lifetime of the product, but most of the estimated positive price trends are quantitatively close to zero.

We also document that the downward trend in relative prices has significantly accelerated over the past two decades. Expenditure items that dropped out of the consumption basket displayed smaller relative price declines than the average expenditure item. Newly entering items displayed above average relative price declines. Furthermore, within the set of continuing items, the expenditure weights have shifted away from items displaying low rates of price decline towards items that display stronger rates of price decline.

Taken together, these empirical facts have strong normative implications for the inflation target that a welfare maximizing central bank should pursue. We arrive at this conclusion through a number of steps. We start by showing that sticky price models imply that the documented relative price declines are actually efficient. This is the case because price rigidities and historically suboptimal rates of inflation distort only the *level* of relative prices, but leave the time trend of relative prices unchanged. As a result, the observed time trends of relative prices in the micro price data are identical to the ones one would observe in a setting with perfectly flexible prices.

In light of this insight, the question of finding the optimal inflation rate is equivalent to determining how to best implement the documented relative price declines in a setting where prices are sticky. While the decline in relative prices is consistent with many alternative rates of inflation, as relative price trends are simply invariant to inflation, different inflation rates nevertheless have welfare implications because they imply different *level distortions* for relative prices.

To understand why this is the case, consider two alternative approaches for implementing declining

¹Relative prices can decline on average because there is constant product turnover. Absent turnover, this is hardly possible.

relative prices. One approach lets all newly entering products charge a fixed nominal price P at the entry date and subsequently lets them cut the nominal price at some constant rate over the product lifetime. With constant product entry and exit rates, the product price distribution and thus the average product price is constant over time: there is zero inflation, even though all individual prices decline over their respective lifetimes.² Importantly, this setting requires constant adjustments of existing prices. When prices are rigid, these price adjustments tend to happen inefficiently.

An alternative - and as we show - preferable approach is to have constant nominal prices for existing products over time. One can nevertheless implement a decline in relative prices, simply by having newly entering products charge a higher (but also constant) price than the average existing product. This way, relative prices decline because the average product price keeps rising over time: there is positive inflation. Since this setting requires no adjustment of individual product prices, it is generally preferred when prices are sticky.

Since the strength of the relative price decline is found to vary considerably across expenditure items, the optimal inflation rate also varies across different expenditure items. It is thus impossible to implement - with the help of just one policy instrument (aggregate inflation) - perfectly constant nominal product prices in all expenditure items. The optimal inflation target must thus trade off the relative-price and mark-up distortions that are generated by different aggregate inflation rates across different expenditure items.

To determine how this trade-off is optimally resolved, we construct a sticky price model that incorporates a product life cycle and rich forms of product heterogeneity. To obtain a model that can capture key characteristics of micro price behavior, we augment the theoretical setup of Adam and Weber (2019) by adding many expenditure items, each of which consists of a continuum of individual products that get continuously replaced over time, and by adding important dimensions of heterogeneity across these items. In particular, we introduce (i) heterogeneity in the productivity and product quality growth rates across expenditure items, to capture the observed heterogeneity in relative price trends; (ii) heterogeneity in the degree of price rigidity and the rate of product turnover, to capture the observed differences along these dimensions; and (iii) idiosyncratic components to product quality and productivity, to capture the large amount of observed price dispersion. In a further extension, we also incorporate sales prices.

Despite the richness of the model, we can derive a closed-form expression for the optimal steady-state inflation rate, i.e., for the inflation target that a welfare-maximizing central bank should adopt. We show how the optimal inflation target trades-off the mark-up and price distortions generated across different expenditure items, but overall fails to implement the efficient price distribution in steady state.³ Analytical aggregation is partly feasible because we abstain from explicitly modeling

²This is possible because exiting products have low prices, while entering products have high prices, which generates a force towards inflation that compensates for the negative product price inflation over the lifetime of products.

³This differs notably from the findings for the simpler setup used in Adam and Weber (2019), where monetary policy can implement efficient relative prices, even in response to economic disturbances.

the product replacement process, instead treat it as an exogenous (albeit heterogeneous) stochastic process. In fact, the precise economic forces driving product replacement are not important for our results, as long as these forces are independent of the inflation target pursued by the central bank.⁴

We then use our analytic result to estimate the optimal inflation rate for the U.K. economy. We start by showing that to a first-order approximation only three features of heterogeneity matter for the optimal inflation target: (1) heterogeneity in productivity and quality growth across expenditure categories, which we show to be identified by the estimated relative price trends in the micro data; (2) heterogeneity in expenditure weights across expenditure categories, and (3) heterogeneity in the steady-state real growth rates of (quality-adjusted) output across expenditure categories. All remaining dimensions of heterogeneity, e.g., the heterogeneity in price stickiness as emphasized in the sticky price literature (e.g., Aoki (2001), Benigno (2004)) or heterogeneity in product entry and exit rates, generate only second-order effects for the optimal inflation target.

The analytic first-order result has considerable empirical appeal, because it allows estimating the optimal inflation target using micro price data only. We thus use the U.K. price data to estimate the optimal U.K. inflation target. For the year 2016, the optimal target ranges between 2.6% and 3.2%, depending on how exactly one treats sales prices in the data set. Independently of the treatment of sales prices, we robustly find that the optimal inflation target has increased by around 1.2% over the period 1996 to 2016. This reflects the fact that negative price trends have become stronger over time through the introduction of new expenditure items with stronger negative trends and the removal of items with less negative or positive trends.

The remainder of this paper is structured as follows. The next section presents the micro price data set and a new set of stylized facts on relative price trends. Section 4 introduces a sticky price model featuring a product life cycle and rich amounts of heterogeneity, which allow capturing the documented heterogeneity in micro price data. Section 5 characterizes the steady state outcome by aggregating the nonlinear model. Section 6 derives the closed-form result for the optimal inflation target and section 7 explains how one can estimate the optimal inflation target from micro price data. Section 8 shows that our estimation approach remains valid even if statistical agencies account only imperfectly for quality progress. Section 9 presents our baseline estimation for the U.K. and section 10 offers various robustness checks. A conclusion briefly summarizes. A series of appendices present our theoretical aggregation result, various proofs and details of our empirical approach.

⁴A number of potential forces driving product replacement dynamics naturally satisfy the independence requirement: product replacement could be driven by changing consumer tastes that cause some products to fall out of fashion and others to become fashionable; alternatively, replacement could be driven by negative productivity shocks that cause the producer of an existing product to discontinue production and have the next best producer enter the market with a new product.

2 Related Literature

The literature discussing the role of the product life cycle in connection with monetary policy is sparse and the present paper appears to be the first one drawing normative conclusions from the product life cycle for monetary policy design.

The early product life cycle literature presented theoretical models of the evolution of firm entry, exit and product innovation, but abstracted from nominal rigidities and monetary issues (Shleifer (1986), Aghion and Howitt (1992), Klepper (1996)).

Nakamura and Steinsson (2008) present empirical evidence on product turnover in the BLS consumer and producer price data sets. Broda and Weinstein (2010) present empirical evidence on product creation and destruction for an important consumer good segment and quantify the quality bias in consumer price indices. Bils (2009) decomposes aggregate price changes into changes originating from new products and changes from existing products, with the aim of improving estimated quality growth. Aghion et al. (2019) also estimate the missing growth arising from incomplete adjustments associated with the quality gains triggered by creative destruction. The issue of mismeasured quality growth is orthogonal to the issue studied in this paper. In fact, as we show in section 8, our results apply even when statistical agencies mismeasure quality growth and thus the inflation rate.

Argente, Lee and Moreira (2018) provide empirical evidence on how firms grow through the introduction of new products and Argente and Yeh (2018) determine to what extent product replacement and perpetual demand learning by firms contributes to monetary non-neutrality. To the best of our knowledge, the latter paper is the only one incorporating a product life cycle into a setting with nominal rigidities, but it does not study monetary policy implications.

The monetary policy literature has considered settings with endogenous firm entry and exit (Bergin and Corsetti (2008), Bilbiie et al. (2008) and Bilbiie, Fujiwara and Ghironi (2014)), which could be re-interpreted as models of endogenous product entry and exit.⁵ These papers study a complementary setup in which monetary policy affects the entry decisions of firms/products, while abstracting from firm/product heterogeneity. Product heterogeneity is, however, key to be able to account for the observed relative price trends.

Also related is the optimal inflation literature, see Schmitt-Grohé and Uribe (2010) for an overview. This literature has identified a number of complementary economic forces affecting the optimal rate of inflation. Concerns about an occasionally binding lower bound constraint on nominal interest rates, for instance, tend to generate a force towards positive inflation on average (Adam and Billi (2006, 2007), Coibion, Gorodnichenko and Wieland (2012)). The same tends to be true when wages are downwardly rigid (Carlsson and Westermarck (2016), Benigno and Ricci (2011), Carlsson Kim and Ruge-Murcia (2009)). Conversely, the optimal inflation rate tends to become negative when taking into account cash distortions (Kahn, King and Wolman (2003)).

⁵Broda and Weinstein (2010) emphasize that product entry and exit dynamics differ considerably from firm or establishment entry and exit dynamics.

Table 1: Number of Price Quotes and ONS Product Identifiers

Price quotes in raw data	28.995.064
ONS product identifiers	736078
Price quotes excluding duplicate quotes	24.525.632
ONS product identifiers	687212
Price quotes excluding duplicate & invalid quotes	22.825.052
ONS product identifiers	682747
Price quotes w/o duplicate & invalid quote for replicated items	21.215.430
ONS product identifiers	613031

3 U.K. Micro Price Data: New Evidence

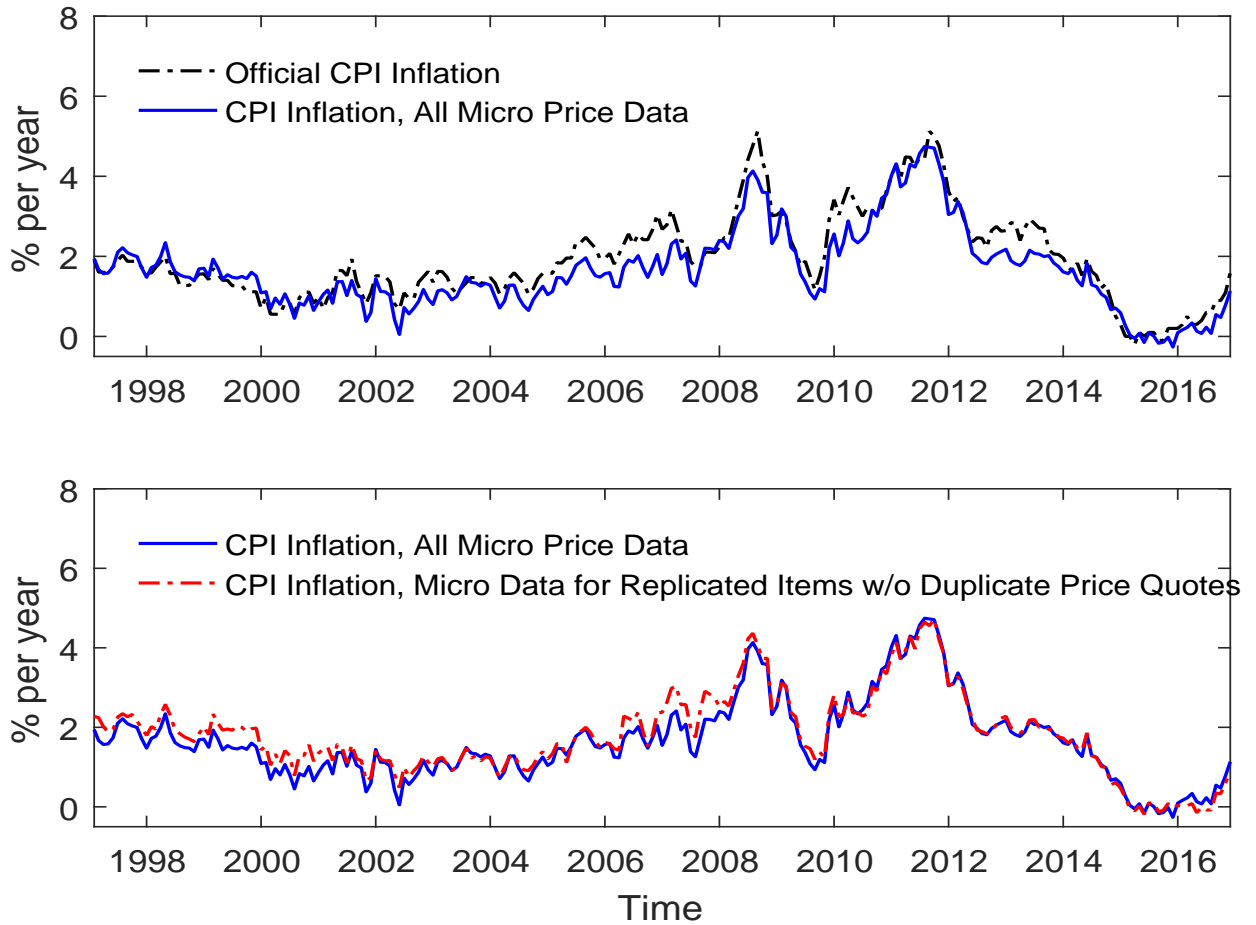
We consider the micro price data that the Office of National Statistics (ONS) collects on a monthly basis to compile the official consumer price index (CPI) for the United Kingdom (ONS (2014)). While the data has previously been analyzed in Bunn and Ellis (2012), Kryvtsov and Vincent (2017), Blanco (2018) and Hahn and Marencak (2018), none of these papers considers price trends over the product lifetime. More generally, it appears that the only other paper studying life-cycle price trends is Melser and Syed (2016), who consider supermarket prices in Chicago. They focus on trends in nominal prices and show that nominal prices of supermarket goods have a tendency to fall over the product life, but that there is considerable heterogeneity across products, with many goods' prices actually increasing over the lifetime. We focus on life-cycle trends in relative prices and find very consistent evidence of declining prices for a much broader set of goods and services. When considering trends in nominal prices in our data set, we similarly find inconclusive evidence.

3.1 Data Description and Product Definition

We consider goods and service prices for the sample period February 1996 to December 2016. The data covers the economic territory of the U.K., excluding offshore islands. For any given sales outlet, data collectors find the most popular and regularly available products (or services), record price information, as well as information for uniquely identifying the product and categorizing it into the Classification of Individual Consumption by Purpose (COICOP). The raw data comprise almost 29 million individual price quotes, see table 1, and all prices are sampled on a monthly basis.

The publicly available micro price data set does not contain all price information underlying the construction of the official CPI. For instance, it does not contain most of the housing related expenditure components and also does not report so-called 'centrally collected items', such as 'Golf green fees', 'Horseracing admissions' or 'Air fares'. Despite this, the inflation rate obtained from aggregating the price indices for which micro price data is available is very similar to the official CPI inflation rate, see the top panel of figure 1.

Figure 1: U.K. CPI Inflation, Various Measures



Our analysis of relative price trends over the product life cycle requires us to track the same product over time. Using the available product and outlet characteristics, we can construct around 736k unique product identifiers for the raw data. For confidentiality reasons, however, ONS does not disclose all available location information. As a result, we have some product identifiers where our data contains more duplicate price quotes for the same month, so that we cannot perfectly distinguish between products in these cases. We therefore discard all price quotes belonging to the identifiers with duplicate price quotes. As table 1 shows, this leaves us with a slightly lower number of product identifiers and about 24.5 million price quotes.

Following ONS practice, we also remove so-called "invalid" price quotes, which are price quote that do not pass ONS cross-checking procedures (see ONS (2014) for details). Table 1 shows that removing duplicate and invalid price quotes leaves us with 22.8 million price quotes.

We estimate life-cycle trends in relative product prices at the finest available level of product disaggregation. In the ONS data set, this is the level of so-called expenditure items and there are

1233 such item categories in the data set. The large number of expenditure items insures that we convincingly capture heterogeneity across the product spectrum.

We compute relative prices by deflating nominal product prices with a quality-adjusted item price index. To make sure that we understand the ONS methodology for expenditure weighting and quality adjustment, we first replicate the official ONS item price indices using their methodology and all prices (i.e., including the eliminated duplicate price quotes but without the invalid price quotes). In a second step, we compute the item price indices without duplicate price quotes. In a third step, we make sure that excluding duplicate price quotes does not materially affect the item price index and thus the estimated relative price trends. This is insured by keeping only expenditure items for which the difference in the ONS item price index with and without duplicate prices is small in a root mean square error sense, see appendix ?? for details. This leaves us with 1093 of the 1233 item categories. Table 1 shows that restricting our sample to successfully replicated items leaves us with 21.2 million price quotes and 613031 ONS product identifiers. Throughout the paper, this is the baseline sample we work with.

The bottom panel of figure 1 shows that the aggregate inflation rate obtained from aggregating all available micro price data is very similar to the rate obtained from our baseline sample.⁶

We then split the observed time series of price quotes for each product identifier at months in which product changes occur. In a first step, we exploit ONS information on (comparable and non-comparable) product substitutions that are reported by price collectors. Table 2 reports the monthly substitution rates: at the level of the identifier there is a lot of product churning in terms of comparable substitutions but relatively low turnover in terms of non-comparable substitutions. Non-comparable products thus appear to mainly enter via new product identifiers. In fact, as table 2 shows, the monthly entry and exit rate for product identifiers is fairly high and such that the average number of product identifiers is constant over time.

Table 2: Substitution & Turnover Rates: Products and Product Identifiers

Substitution Rates within Product Identifiers	Monthly Rate in %
Comparable substitutions	5.74
Non-comparable substitutions	0.31
Turnover Rates for Product Identifiers	
Entry rate	2.44
Exit rate	2.44

In a final step, we further refine our product definition by splitting the time series of product prices whenever there are missing price quotes for more than one month. This insures that we do

⁶Figure 1 adjusts for two outliers in January 1999 and May 2005 when computing the inflation rate for the replicated item indices and without duplicate price quotes.

not accidentally lump products together for which the price collector failed to record a product substitution simply because no prices were recorded in the months prior to the month of price collection. We are aware that this approach may accidentally split product price observations that are in fact coming from the same product. According to the theory that we develop later on, however, accidentally splitting price observations that come from the same product is innocuous, while lumping price observations together that are in fact coming from different products would lead to biased estimates.

Our refined product definition leaves the total number of price observations at the item level unchanged, even if it reduces (potentially artificially) the length of the price spells of individual products. Since we estimate relative price slopes at the level of the item category, the latter is largely irrelevant. Table 3 reports descriptive statistics for the 1093 analyzed items in our baseline sample, in terms of the mean and median of (refined) products per items, price quotes per item and the length of price spells per (refined) product.

Table 3: Analyzed Expenditure Items and Products (Refined Definition)

Number of Products per Item	
Median	925
Mean	1523.5
Number of Price Quotes per Item	
Median	14846
Mean	18739
Length of Price Spell per Product (Months)	
Median	9
Mean	14.5

3.2 Relative Price Trends over the Product Life

This section presents empirical evidence on the behavior of relative product prices over the product lifetime.

Let \tilde{P}_{jzt} denote the nominal (not-quality-adjusted) price of product j in expenditure category z at time t and let P_{zt} denote the expenditure-weighted and quality-adjusted average price of all products present in item z at time t . We are interested in following the relative product price \tilde{P}_{jzt}/P_{zt} over the lifetime of product j . To this end, we consider linear panel regressions of the form

$$\ln \frac{\tilde{P}_{jzt}}{P_{zt}} = f_{jz} + \ln(b_z) \cdot s_{jzt} + u_{jzt}, \quad (1)$$

where f_{jz} is a product and item-specific intercept term, s_{jzt} the in-sample age of the product (normalized to zero at the date of product entry), and u_{jzt} a mean zero residual potentially displaying

serial and cross-sectional dependence. The coefficient of interest is the slope coefficient b_z , which measures the average growth rate of the relative product price over the product lifetime in item z . Since regression (1) includes a product-specific intercept (f_{jz}), the coefficient of interest (b_z) remains unaltered when using the quality-adjusted product price P_{jzt} instead of the non-adjusted price \tilde{P}_{jzt} in the numerator on the left-hand side.⁷

If the set of products were constant over time, i.e., in the absence of product entry and exit, we would have $b_z = 1$, as not all products can simultaneously become cheaper or more expensive relative to each other with product age.⁸ However, with product turnover, the price of each product relative to the price of existing products can rise or fall over time because the existing set of products keeps changing over time. This is the case, for instance, when products enter at a high price and leave at a low price, in a way that the average price in the cross-section of products remains constant over time. Each product's relative price is then falling with product age.

We consider only linear trends in product age in equation (1) for two important reasons. First, we observe only a censored measure of true product age: we see the in-sample age of a product but not its true age. This distinction is relevant because products enter the ONS basket with a considerable time delay, i.e., months or sometimes even years after their market introduction. The extent of the time delay is also likely going to vary across products and items, which makes it impossible to identify any non-linear age effects without observing the true product age. Second, the linear specification will have a direct structural interpretation that is relevant for determining the optimal inflation target in the sticky price model that we introduce later on.

We estimate the slope coefficient b_z by running the fixed-effect panel regression (1) for each of the more than one thousand expenditure items in our baseline sample, treating the intercept f_{jz} as unobserved components.⁹ Figure 2 displays the distribution of estimated slope coefficients, weighting coefficients by their average expenditure weight in the sample.¹⁰ To facilitate interpretation, figure 2 reports the estimated b_z coefficients in terms of annualized net growth rates in percent ($100((b_z)^{12} - 1)$).

The distribution of estimated coefficients in figure 2 reveals that the age trend is negative for the vast majority of expenditure items. This shows that relative product prices tend to decline with

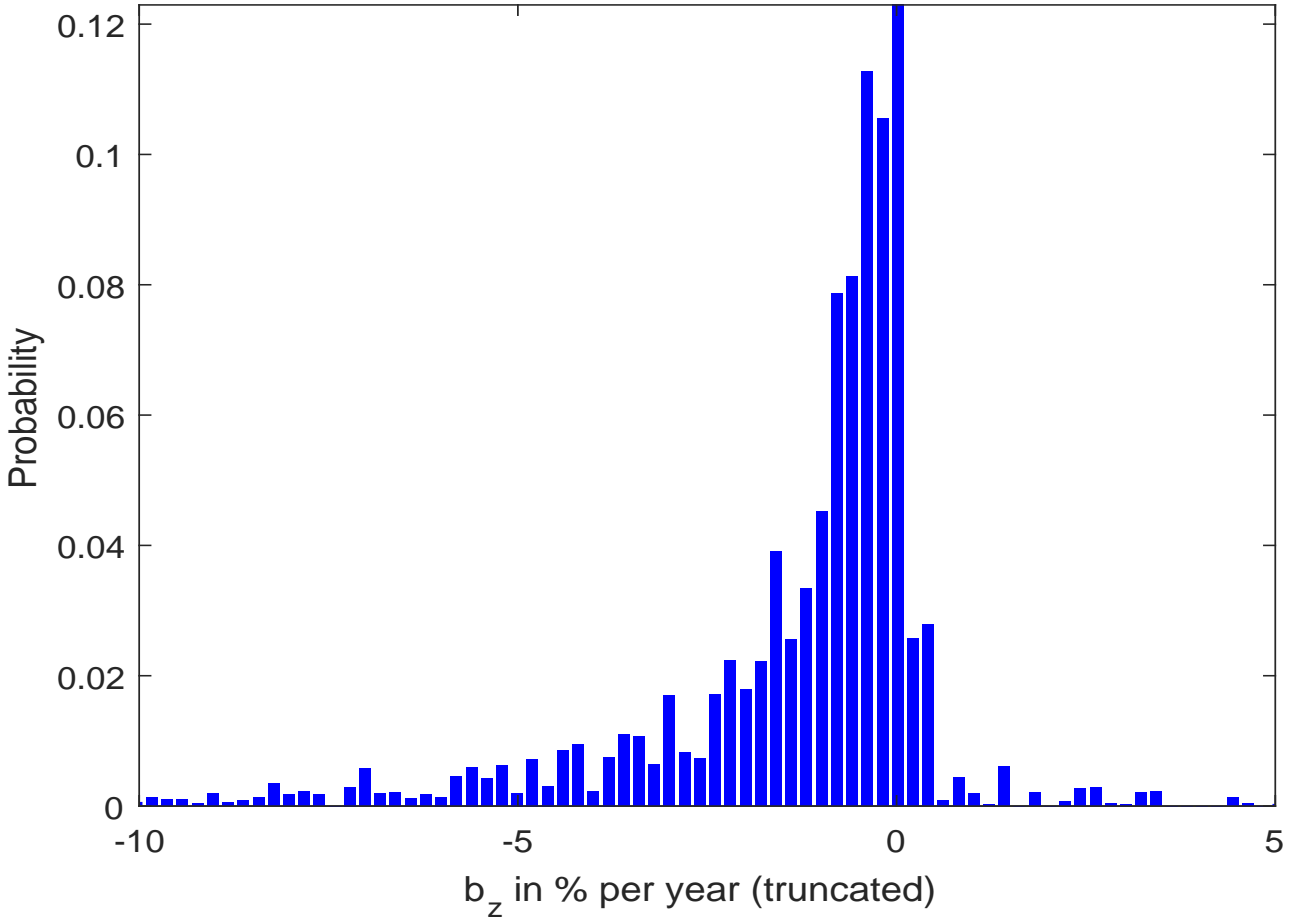
⁷This is so because product quality is constant over the product lifetime, given our refined product definition. We use the not-quality-adjusted price because this allows for some further interpretation of the intercepts f_{jz} in the next section.

⁸For the case without product turnover, our theoretical model in fact predicts $b_z = 1$.

⁹We also estimated equation (1) using a random effects estimator. This delivers very similar results. Using a first-difference specification, estimation results turn out to be less robust, in particular for items with a short average length for price spells. However, this non-robustness disappears if one appropriately treats temporary price changes, e.g., sales prices. For instance, running the first-difference estimation, using the regular price filter of Kehoe and Midrigan (2015) to adjust for temporary price changes, yields very similar results as our fixed effects regression.

¹⁰We average by first computing for each item the average weight over the sample period (1996-2016) and then rescale the item weights such that they sum to unity across items. The unweighted distribution looks very similar to the weighted one shown in figure 2.

Figure 2: Distribution of Estimated Slope Coefficients Across Items from Equation (1)



product age, so that new products tend to be initially expensive, but become cheaper over their lifetime (in relative terms). Figure 2 also shows that there is pronounced item-level heterogeneity in the rate at which relative prices tend to decline over the product life. Most weight of the estimated distribution falls into the range between minus five and zero percent per year, whereas the more extreme parts of the distribution, below minus five and above zero, receive considerably smaller weight.

Table 4 presents information on the tails of the relative price trend distribution. It lists the 15 items with the most positive and most negative relative price trends that have at least an expenditure weight of 0.15%. The table shows that the largest rates of price declines are recorded for products that display a certain news value, i.e., fashion and entertainment products, as well as consumer electronics. For most of the items displaying positive relative price trends, the relative price increase remains well below 1% per year. The most positive relative price trend is observed for a luxury product.

Table 4: Top and Bottom Rates of Relative Price Change

Item Description	Relative Price Change (in % per year)	Exp. Weight (in %)
Relative Price Increase		
HIFI - 2007	3.28	0.15
WIDESCREEN TV - 2005	2.55	0.31
CAMCORDER-8MM OR VHS-C	2.34	0.16
WASHING MACHINE - 2008	1.82	0.16
WASHING MACH NO DRYER MAX 1800	1.48	0.17
LEISURE CENTRE ANNUAL MSHIP	1.34	0.16
COOKED HAM PREPACKED/SLICED	0.84	0.17
PRIV RENTD UNFURNISHD PROPERTY	0.41	1.02
AUTOMATIC WASHING MACHINE 2009	0.35	0.16
MILK SEMI-PER 2 PINTS/1.136 L	0.34	0.26
CIGARETTES 5	0.33	0.25
VEGETARIAN MAIN COURSE	0.24	0.17
DOMESTIC CLEANER HOURLY RATE	0.22	0.23
HOME REMOVAL- 1 VAN	0.17	0.18
STAFF RESTAURANT SANDWICH	0.17	0.20
Relative Price Decline		
NEWSPAPER AD NON TRADE 20 WORD	-3.66	0.19
COFFEE TABLE -2	-3.68	0.16
FLAT PANEL TV 33" +	-3.84	0.16
KITCHEN WALL UNIT SELF ASSMBLY	-3.94	0.16
FLAT PANEL TV 26" - 42"	-4.26	0.29
WIDESCREEN TV (24-32 INCH)	-4.50	0.19
AUTOMATIC WASHING MACHINE	-4.76	0.18
WOMENS TROUSERS-FORMAL	-7.12	0.17
MENS SHOES TRAINERS	-7.84	0.18
PRE-RECORDED DVD TOP 20	-8.14	0.23
WOMENS SUIT	-8.95	0.17
LADYS SCARF	-20.19	0.17
COMPUTER GAME TOP 20 CHART	-21.69	0.31
WOMENS DRESS-CASUAL 1	-25.55	0.17
PRE-RECORDED DVD (FILM)	-35.03	0.16

Notes: The table reports the fifteen top and bottom rates of relative price change for items with expenditure weight greater than 0.15%. Weights are average expenditure weights for the full sample period.

Table 5 aggregates item-level price trends to the level of so-called ONS product divisions, using item-level expenditure weights for the year 2016. The table shows that the weighted average rate of relative price changes over the product lifetime is negative in all product divisions. Yet, even for this relatively high level of aggregation, there exists a considerable amount of heterogeneity in the rates of relative price decline: the observed rates range from close to zero to almost minus ten percent per year. While 8 out of the 11 reported rates fall into the range between minus two and zero percent, there are two outstanding divisions, ‘Clothing & Footwear’ and ‘Recreation & Culture’, which both display a strong rate of price decline and a high expenditure weight.

Table 5: Relative Price Changes over the Product Lifetime for ONS Divisions

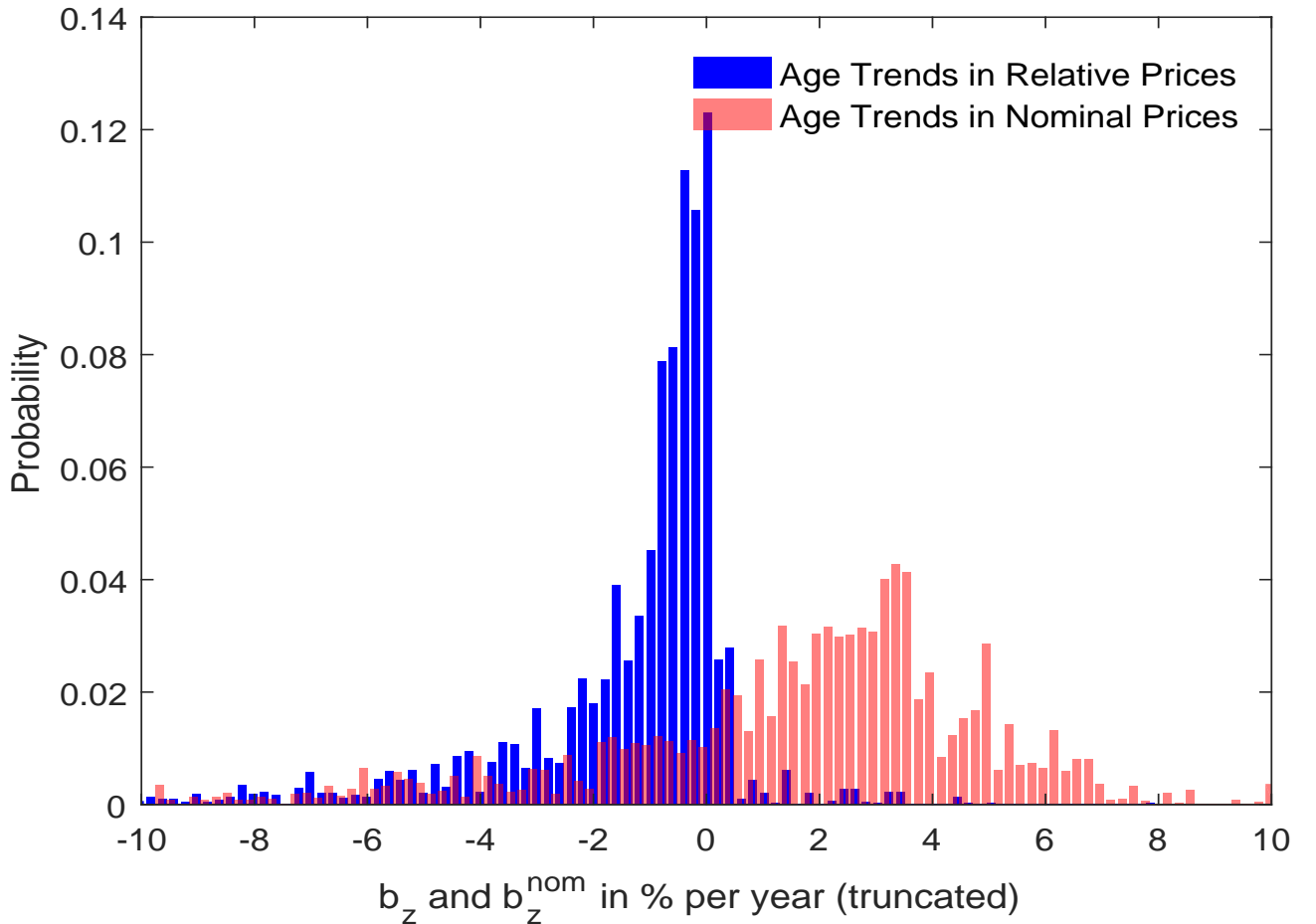
Division Description	Relative Price Trend (in % per year)	Exp. Weight in 2016 (in %)	Number of Items (full sample)
Food & Non-Alcoholic Beverages	-1.00	18.07	282
Alcoholic Beverages & Tobacco	-0.41	8.03	66
Clothing & Footwear	-9.36	11.92	149
Housing, Water, Electricity & Gas	-0.83	0.75	38
Furniture, Equipment & Maintenance	-1.67	9.98	146
Health	-0.73	3.82	26
Transport	-0.79	6.99	41
Communications	-6.97	0.11	7
Recreation & Culture	-3.98	9.44	157
Restaurants & Hotels	-0.36	18.82	79
Miscellaneous Goods & Services	-1.68	12.54	90

Notes: The number of items does not sum to 1093 because not all items are assigned to a division.

Estimating equation (1) using nominal prices ($\ln \tilde{P}_{jzt}$) instead of relative prices ($\ln \tilde{P}_{jzt}/P_{zt}$) as left-hand side variable, we also find more mixed evidence regarding the sign of price trends, in line with evidence by Melser and Syed (2016) for U.S. supermarket products. This is illustrated in figure 3, which depicts the coefficient estimates obtained from both regressions. With nominal prices, the coefficient distribution is shifted to the right and also more dispersed. The rightward-shift of the (expenditure-weighted) mean of the distribution by approximately 2.3% largely reflects aggregate inflation, which averaged almost 2% over the sample period. The increase in the dispersion of the distribution shows that there is considerable heterogeneity in (same-good) inflation rates across expenditure items¹¹, in addition to the heterogeneity in relative price trends documented above. Our theoretical model will be able to capture both of these data features. Yet, only heterogeneity in relative price trends will turn out to be relevant for the optimal inflation target.

¹¹Bils (2009) decomposes inflation at the item level into inflation from existing goods and inflation from the entry and exit of goods. Following Bils (2009), we refer to the former as same-good inflation.

Figure 3: Distribution of Age Trend Coefficients from Regression (1) with Relative Prices ($\ln \tilde{P}_{jzt}/P_{zt}$) and Nominal Prices ($\ln \tilde{P}_{jzt}$) as l.h.s. Variable.



3.3 Additional Dimensions of Heterogeneity

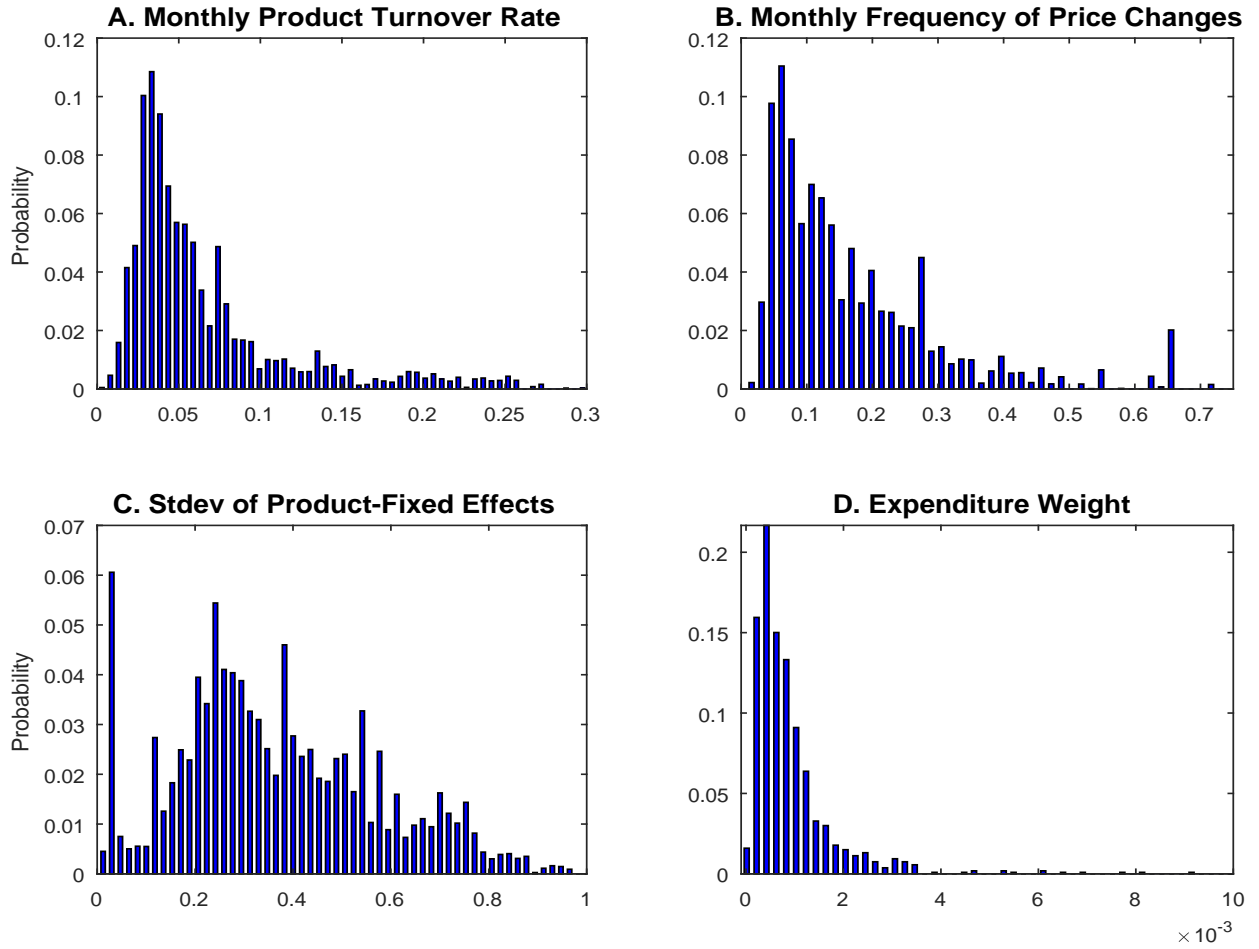
Besides heterogeneity in relative price trends, the U.K. price data features important heterogeneity along a number of other dimensions. Part of this heterogeneity is already well-known, other parts are new. This section briefly outlines the key dimensions that we incorporate into our theoretical model in the next section.

Panel A in figure 4 presents the item-level distribution of product turnover rates.¹² Turnover is defined as the unweighted average of the product entry and exit rates.¹³ The median monthly turnover rate is 4.8% and thus fairly high. This partly reflects our refined product definition, which

¹²The distributions in panels A-C of figure 4 are expenditure-weighted in the same way as in figure 2, see footnote 10. Unweighted distributions look very similar, except for Panel C, which in unweighted terms does not have the large spike on the left-hand side of the distribution. Panel D shows the (unweighted) distribution of expenditure weights.

¹³Product entry and exit rates have similar levels and are highly correlated across items.

Figure 4: Distribution of Various Variables across Items



Notes: In all panels, x-axes are truncated to enhance readability.

treats two or more missing price quotes as product exit events. More importantly, panel A shows that there is a lot of dispersion in turnover rates across items: the cross-sectional standard deviation of turnover is 5.5%. Our theoretical model will, therefore, allow for heterogeneity in product turnover rates.

It is important to note that the data not only features turnover of products, but also turnover of expenditure items. Certain items become obsolete over time (e.g., CD players) and are replaced by new items (e.g. flash drive devices). Yet, relative to the high turnover rates at the product level, item turnover is a relatively slow process. On average only about 0.5% of items are being replaced in any given month, which is less than one tenth of the product turnover rate.

Panel B in figure 4 reports the monthly price change frequencies across expenditure items. The median frequency is 12.6% and the standard deviation across items is 13.6%. These numbers include temporary price changes such as sales or discounts. Nakamura and Steinsson (2008) show that

excluding temporary price changes reduces the frequency of price changes considerably. In fact, when we exclude price quotes that ONS flags as sales prices, the median frequency drops to 7.8%. Overall, we find price change statistics that are very similar to the ones reported in Kryvtsov and Vincent (2017).¹⁴ Given the large amount of heterogeneity documented in panel B, our model will allow for heterogeneous degrees of price rigidity across items.

Panel C in figure 4 reports the weighted distribution of the standard deviation of the item-level intercept term f_{jz} from regression (1). It shows that items differ vastly in terms of the dispersion of relative price intercepts. The dispersion reflects differences along two important dimension within each item: quality differences and productivity differences, both of which increase intercept dispersion.¹⁵ In fact, the big spike of around 6% on the left-hand side of the distribution shown in panel C is due to cigarettes (various types) and gasoline (petrol and diesel). Both of these items have a high expenditure weight (around 3% each), but also - due to low degrees of quality and productivity differences - very homogeneous prices. Overall, panel C reveals that for many items, intercept dispersion and thus productivity and/or quality dispersion is very large in the cross section of products. In light of this finding, our model will allow for idiosyncratic productivity and quality differences across products within each item.

Panel D in figure 4 displays the distribution of expenditure weights across items, which is the distribution that has been used to compute the weighted distributions in the other panels of the figure. Panel D shows that most items have an expenditure weight around one tenth of a percent, but that there is a relatively long right tail to the distribution. To reflect this dimension of heterogeneity, our model will allow for different expenditure weights across items.

4 Sticky Price Model with a Product Life Cycle

This section introduces a sticky price model featuring a product life cycle. The model contains a range of new elements that allow capturing the key dimensions of product and price heterogeneity documented in section 3. In particular, it features multiple expenditure items, each of which is populated by a continuum of heterogeneous products. Expenditure items are allowed to have different degrees of price stickiness and different product-entry and exit rates. The model also allows for heterogeneity in productivity and quality trends across items, which is key for being able to capture the heterogeneity in relative price trends documented before. Finally, the model allows for idiosyncratic elements in product quality and productivity, which allows capturing the large and heterogeneous amounts of dispersion in relative prices observed in the data. The setup in this section non-trivially

¹⁴They consider ONS micro price data for a slightly shorter sample period (February 1996 to September 2013) and report the weighted mean of price changes frequencies to be equal to 15.8% including price sales price (we find 16.9% for our sample) and equal to 13.2% excluding sales prices (we find 12.5% for our sample).

¹⁵Mark-up dispersion also also generate intercept dispersion. Since we do not observe information on production costs and thus mark-ups, we abstract from this dimension of heterogeneity.

generalizes the one studied in Adam and Weber (2019), which does not feature heterogeneity along any of these dimensions.

The next sections present the model, derive the steady state of the economy and a closed-form expression for the optimal steady-state inflation rate.

4.1 Demand Side and Production Side

The demand side of the model is standard and consists of a representative consumer with balanced-growth consistent preferences over an aggregate consumption good C_t and hours worked L_t , described by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{[C_t V(L_t)]^{1-\sigma} - 1}{1-\sigma} \right), \quad (2)$$

where $\beta \in (0, 1)$ is a discount factor and $\sigma > 0$.¹⁶ The household faces the flow budget constraint

$$C_t + K_{t+1} + \frac{B_t}{P_t} = (r_t + 1 - d)K_t + \frac{W_t L_t}{P_t} + \frac{B_{t-1}}{P_t} (1 + i_{t-1}) - \tilde{T}_t, \quad (3)$$

where K_{t+1} denotes the capital stock, B_t nominal government bond holdings, P_t the nominal price of the aggregate consumption good, i_{t-1} the nominal interest rate, W_t the nominal wage rate, r_t the real rental rate of capital, d the depreciation rate of capital, and \tilde{T}_t a summary variable that contains lump sum taxes and firm profits, which the household takes as given. Household borrowing is subject to a no-Ponzi scheme constraint. The first-order conditions characterizing optimal household behavior are standard and derived in appendix A.1. To insure that utility remains bounded, we assume

$$\beta (\gamma^e)^{1-\sigma} < 1,$$

where $\gamma^e \geq 1$ denotes the steady-state growth rate of the aggregate economy under balanced growth, as defined in equation (29) below.

The aggregate consumption good C_t is made up of Z_t different consumption items (in the language of the ONS). A consumption item is a product category, e.g., "Flatscreen TV, 30-inch display" or "CD-player, portable", which itself contains a range of individual products. Letting C_{zt} ($z = 1, \dots, Z_t$) denote consumption of item z in period t , we have

$$C_t = \prod_{z=1}^{Z_t} (C_{zt})^{\psi_{zt}}, \quad (4)$$

where $\psi_{zt} \geq 0$ denotes the expenditure weight for item z at time t and $\sum_{z=1}^{Z_t} \psi_{zt} = 1$. We allow the set of items Z_t and the expenditure weights ψ_{zt} to be time-varying, so as to capture the fact that

¹⁶We assume $\sigma > 0$ and that $V(\cdot)$ is such that period utility is strictly concave in (C_t, L_t) and that Inada conditions are satisfied. We also assume that $V(\cdot)$ is such that the steady state amount of labor is positive.

ONS regularly drops and adds items to its consumption basket and adjusts the expenditure weights over time.¹⁷

For simplicity, we interpret item entry and exit or changing expenditure weights for items as simply being due to changing consumer tastes. Obviously, item substitution could be due to a variety of other factors, such as increased competition from a different item, e.g., flash-drive devices becoming increasingly competitive relative to portable CD players and thus leading to the exit of the latter. We refrain from explicitly modeling competition across items, instead take changes in the item structure as exogenous. In the U.K. data, the item structure changes only slowly over time, with on average 0.5% of items leaving the sample every month.

Every item contains a large number of differentiated products. To capture this fact, item level consumption C_{zt} is a Dixit-Stiglitz aggregate of individual products $j \in [0, 1]$, so that

$$C_{zt} = \left(\int_0^1 \left(Q_{jzt} \tilde{C}_{jzt} \right)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad (5)$$

where \tilde{C}_{jzt} denotes the consumed physical units of product j in item z in period t , Q_{jzt} the quality level of the product and $\theta > 1$ the elasticity of substitution between products. We consider a constant product variety over time, simply because our data does not offer any information about variety trends.¹⁸ The aggregation assumes that consumption goods with higher product quality deliver proportionately higher consumption services relative to consumption goods with a lower quality level. This a standard approach for modeling the quality content of goods, see Schmitt-Grohé and Uribe (2012).

In equilibrium, the quantity of products \tilde{C}_{jzt} consumed must be equal to the quantity \tilde{Y}_{jzt} produced. Individual products are produced using a Cobb-Douglas production function

$$\tilde{Y}_{jzt} = A_{zt} G_{jzt} (K_{jzt})^{1-\frac{1}{\phi}} (L_{jzt})^{\frac{1}{\phi}} \quad (6)$$

where A_{zt} denotes the level of productivity common to all producers of products in item z and G_{jzt} a product-specific productivity factor that captures idiosyncratic productivity components, as well as productivity dynamics associated with experience accumulation in the manufacturing of the product. The variables K_{jzt} and L_{jzt} , respectively, denote the capital and labor inputs into production.

In line with the evidence in micro price data, there will be constant churning of products j at the level of each expenditure item z . In the U.K. data, product entry and exit is - unlike the entry and exit of expenditure items z - a fast moving process, see section 3.3. In practice, product turnover may take place for a variety of reasons: (1) consumers may simply no longer demand a specific product and demand other products instead, (2) the producers of a particular product may receive a

¹⁷We assume Z_t to be a stationary stochastic process that assumes an integer value $Z > 0$ in the steady state.

¹⁸The number of products sampled by ONS at the item level is not a function of true underlying product variety, instead governed by the desire to minimize measurement error. Product inclusion decisions thus reflect the variability of underlying product prices and by the item's expenditure weight, see chapter 4 in ONS (2014).

sufficiently negative productivity shock that causes the product to become uncompetitive and being replaced by a new product, or (3) a newly available product is in quality-adjusted terms simply more attractive. Whatever is the precise cause for product turnover, we assume that it can be described by an product-specific, idiosyncratic and exogenous Poisson process with arrival rate $\delta_z \in (0, 1)$. We thus assume that monetary policy does not affect the product turnover dynamics.

For simplicity, we assign to the newly entering product the same product index j as to the exiting product. Let s_{jzt} denote the age of product j in item z at time t , with $s_{jzt} = 0$ in the period of entry. Given this definition, the time $t - s_{jzt}$ denotes - at any period t - the date at which product j entered into the economy. Initial productivity at the time of product entry is assumed to be independently drawn from some distribution Ξ_z^G , that is

$$\epsilon_{jzt}^G \sim \Xi_z^G, \quad (7)$$

with $\epsilon_{jzt}^G > 0$ and $E[(\epsilon_{jzt}^G)^{\theta-1}] = 1$ and to remain constant over the lifetime of the product. By incorporating item-specific idiosyncratic elements to productivity, the model can replicate the standard-deviation of product fixed effects, as well as its heterogeneity across items, as documented in figure 4. Letting \bar{G}_{jzt} denote the experience-related productivity component for product j , the product-specific productivity component is given by

$$G_{jzt} = \bar{G}_{jzt} \cdot \epsilon_{jzt}^G, \quad (8)$$

where \bar{G}_{jzt} evolves over time according to

$$\bar{G}_{jzt} = \begin{cases} 1 & \text{for } s_{jzt} = 0, \\ g_{zt} \bar{G}_{jz,t-1} & \text{otherwise,} \end{cases} \quad (9)$$

with

$$g_{zt} = g_z \epsilon_{zt}^g, \quad (10)$$

where $g_z \geq 1$ denotes the average growth rate of this productivity component and captures the average rate of experience accumulation in the production of products in item z . The disturbance ϵ_{zt}^g is an arbitrary stationary process satisfying $E \ln \epsilon_{zt}^g = 0$. Heterogeneity in the experience growth rates g_z allows the model to match different rates of relative price decline across items, see figure 2.

The common level of productivity for item z evolves according to

$$\begin{aligned} A_{zt} &= a_{zt} A_{zt-1} \\ a_{zt} &= a_z \epsilon_{zt}^a, \end{aligned}$$

where $a_z \geq 1$ denotes the average productivity growth rate and ϵ_{zt}^a is an arbitrary stationary processes satisfying $E \ln \epsilon_{zt}^a = 0$. While accumulated experience G_{jzt} associated with product j in item z is lost upon exit of the product, the growth rate in the common productivity level A_{zt} allows for permanent productivity gains in item z . Heterogeneity in the productivity growth rates a_z across items thus allows the model to account for heterogeneity in product price trends of purely nominal prices, i.e.,

of prices that are not scaled by the item price index, see figure 3. The item-level growth trends a_z also allow to generate a disconnect between aggregate productivity growth, which is affected by a_z , and relative price trends, which will remain unaffected by a_z .

It now remains to describe the product quality dynamics at the item level. Let \tilde{P}_{jzt} denote the price at which one unit of output \tilde{Y}_{jzt} is sold at time t . The price \tilde{P}_{jzt} will be set by the producer optimally subject to price-adjustment frictions. The quality-adjusted price of the product is then given by

$$P_{jzt} = \frac{\tilde{P}_{jzt}}{Q_{jzt}}, \quad (11)$$

where Q_{jzt} denotes the product-specific quality level that is assumed constant over the lifetime of the product.¹⁹ The quality level of products j entering in period t is given by

$$Q_{jzt} = Q_{zt} \cdot \epsilon_{jzt}^Q, \quad (12)$$

where ϵ_{jzt}^Q captures an idiosyncratic quality component and Q_{zt} a common item-level quality component. The idiosyncratic component is an independent draw from some distribution Ξ_z^Q , that is

$$\epsilon_{jzt}^Q \sim \Xi_z^Q,$$

with $\epsilon_{jzt}^Q > 0$ and $E[(\epsilon_{jzt}^Q)^{\theta-1}] = 1$. Idiosyncratic quality components allow the model to match the standard-deviation of product fixed effects, as documented in figure 4.²⁰ The common quality component evolves according to

$$Q_{zt} = q_{zt} Q_{zt-1} \quad (13)$$

$$q_{zt} = q_z \epsilon_{zt}^q, \quad (14)$$

where $q_z \geq 1$ denotes the average quality progress in item z and ϵ_{zt}^q a random component of quality growth, which is an arbitrary stationary process satisfying $E \ln \epsilon_{zt}^q = 0$. Heterogeneity in average quality growth (q_z) allows the model to generate different rates of relative price increase over the product lifetime, see figure 2.

Since the quality of a product remains constant over its lifetime, we have

$$Q_{jzt} = Q_{jzt-s_{jzt}} \text{ for all } (j, z, t),$$

¹⁹In model and in the data, we interpret a new quality level of the same product as being a new product. Given our model convention that new products inherit the product indices of exiting products, Q_{jzt} must have a time index, even though product quality remains constant throughout the product lifetime.

Since we assign the same index j to the exiting and newly entering product, Q_{jzt} must have a time subscript, as the entering and exiting product can have different quality levels.

²⁰We incorporate both idiosyncratic productivity components (ϵ_{jzt}^G) and idiosyncratic quality components (ϵ_{jzt}^Q) because we do not want to take a stance on what generates the observed dispersion of product fixed effects in figure 4.

where s_{jzt} denotes the age of product j in item z at time t .

Using the quality-adjusted product price (11), we can then define the quality-adjusted price level for item z according to

$$P_{zt} = \left(\int_0^1 \left(\frac{\tilde{P}_{jzt}}{Q_{jzt}} \right)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (15)$$

Aggregation across items z delivers the quality-adjusted overall price level

$$P_t = \prod_{z=1}^{Z_t} \left(\frac{P_{zt}}{\psi_{zt}} \right)^{\psi_{zt}}. \quad (16)$$

The optimal inflation rate will be defined in terms of this (perfectly) quality-adjusted price level, i.e., inflation is defined as

$$\Pi_t = \frac{P_t}{P_{t-1}}. \quad (17)$$

We show in section 8 that our results are robust to the presence of imperfect quality adjustment.

Optimal product demand by consumers and market clearing implies that product demand satisfies

$$Y_{jzt} = Y_{zt} \left(\frac{P_{jzt}}{P_{zt}} \right)^{-\theta} \quad (18)$$

$$Y_{zt} = \psi_{zt} \left(\frac{P_{zt}}{P_t} \right)^{-1} Y_t \quad (19)$$

where

$$Y_{jzt} \equiv Q_{jzt} \tilde{Y}_{jzt} \quad (20)$$

denotes output in constant quality units.

4.2 Optimal Price Setting

We now consider the producers' price setting problem. We assume that the price of a product can be chosen freely at the time of product entry, but that price adjustments at the product level are subsequently subject to Calvo-type adjustment frictions.

Let $\alpha_z \in [0, 1)$ denote the time-invariant idiosyncratic probability that the price of some product j in item z can not be adjusted in any given period.²¹ Since product quality is constant over the product lifetime (new qualities are treated as new products), we let producers directly choose the quality-adjusted product price P_{jzt} . Let W_t denote the nominal wage and r_t the real rental rate of capital. The factor input mix (K_{jzt}, L_{jzt}) is then chosen to minimize production costs $K_{jzt}P_t r_t + L_{jzt}W_t$ subject to the constraint imposed by the production function (6).

²¹We abstract here from the possibility that firms can charge temporary prices or sales prices. We discuss this feature in our robustness section 10.2.

From standard cost minimization follows, see appendix A.2, that the nominal marginal costs (normalized by productivity and quality) are given by

$$MC_t = \left(\frac{W_t}{1/\phi} \right)^{\frac{1}{\phi}} \left(\frac{P_t r_t}{1 - 1/\phi} \right)^{1 - \frac{1}{\phi}}. \quad (21)$$

We can then express the price-setting problem for product j in a price-adjustment period t as follows:

$$\begin{aligned} \max_{P_{jzt}} E_t \sum_{i=0}^{\infty} (\alpha_z (1 - \delta_z))^i \frac{\Omega_{t,t+i}}{P_{t+i}} \left[(1 + \tau) P_{jzt} Y_{jzt+i} - \frac{MC_{t+i}}{A_{zt+i} Q_{jzt+i} G_{jzt+i}} Y_{jzt+i} \right] \\ \text{s.t. } Y_{jzt+i} = \psi_{zt} \left(\frac{P_{jzt}}{P_{zt+i}} \right)^{-\theta} \left(\frac{P_{zt+i}}{P_{t+i}} \right)^{-1} Y_{t+i}, \end{aligned} \quad (22)$$

where $MC_{t+i}/(A_{zt+i} Q_{jzt+i} G_{jzt+i})$ denotes the effective nominal marginal costs when productivity is $A_{zt+i} G_{jzt+i}$ and product quality equal to Q_{jzt} , which is constant over the product lifetime. The variable $\Omega_{t,t+i}$ denotes the representative household's discount factor between periods t and $t+i$, and τ denotes a sales subsidy (tax) if $\tau > 0$ ($\tau < 0$). We assume

$$-1 < \tau \leq 1/(\theta - 1), \quad (23)$$

so that the sales subsidy cannot be higher than what is required to eliminate the monopoly distortion in the flexible price equilibrium.

The first constraint in problem (22) captures the firm's technology (6); the second constraint captures consumers' optimal product demand functions (18)-(19). Appendix A.3 shows that the optimal price P_{jzt}^* satisfies

$$\frac{P_{jzt}^*}{P_t} \left(\frac{Q_{jzt} G_{jzt}}{Q_{zt}} \right) = \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) \frac{N_{zt}}{D_{zt}}, \quad (24)$$

where the item-level variables N_{zt} and D_{zt} are independent of the firm index j and defined in the appendix. The previous equation shows that the optimal relative reset price of a firm (P_{jzt}^*/P_t) depends only on item-level variables (N_{zt}/D_{zt}) and on how its own productivity ($A_{zt} Q_{jzt} G_{jzt}$) relates to the average productivity of newly entering products ($A_{zt} Q_{zt}$), where productivity is measured in quality-adjusted terms. In appendix A.4 we use this insight to derive a recursive representation for the evolution of the quality-adjusted item price level P_{zt} .

5 Characterizing the Steady State Outcome

This section presents the key equations determining the economy's deterministic balanced growth path equilibrium.²² We obtain these equations by aggregating the nonlinear sticky price model in

²²The equilibrium definition is entirely standard. It requires firms to maximize profits, the representative household to maximize utility and market clearing. The government passively adjusts lump-sum taxed to balance the government budget, keeping the real value of government bonds constant in detrended terms.

closed form and by detrending variables by their respective balanced growth path trends. The derivations are quite involved and are performed in appendices A, B, C and D. Nevertheless, the resulting equilibrium equations are intuitively accessible and reveal how mark-up distortions and relative price distortions across expenditure items move the economy away from its first-best allocation. They also reveal how the aggregate inflation rate affects these distortions.

We start by defining the steady state as a situation without aggregate shocks and without item turnover, in which idiosyncratic shocks continue to operate:

Definition 1 *A steady state is a situation with a fixed set of items $Z_t = Z$, constant expenditure weights $\psi_{zt} = \psi_z$, no aggregate item-level disturbances ($g_{zt} = g_z, q_{zt} = q_z, a_{zt} = a_z$), and a constant (but potentially suboptimal) inflation rate Π . The following idiosyncratic shocks continue to operate in the steady state: product entry and exit shocks, shocks to price adjustment opportunities, and product-specific shocks to quality and productivity that realize at the time of product entry.*

Appendix D shows how aggregate inflation Π , the detrended values of aggregate output y , consumption c and capital k , and hours worked L satisfy the following four simple equations:²³

$$y = \left(\frac{\rho(\Pi)}{\Delta^e} \right) \left(k^{1-\frac{1}{\phi}} L^{\frac{1}{\phi}} \right) \quad (25)$$

$$c \left(-\frac{\partial V(L)/\partial L}{V(L)} \right) = \frac{1}{\mu(\Pi)} \frac{1}{\Delta^e} \left(\frac{k}{L} \right)^{1-\frac{1}{\phi}} \left(\frac{1}{\phi} \right) \quad (26)$$

$$\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d = \frac{1}{\mu(\Pi)} \frac{1}{\Delta^e} \left(\frac{k}{L} \right)^{-\frac{1}{\phi}} \left(1 - \frac{1}{\phi} \right) \quad (27)$$

$$y = c + (\gamma^e - 1 + d)k. \quad (28)$$

Equation (25) is the aggregate production function that determines output as a function of the aggregate capital and labor inputs. The variable Δ^e is a productivity parameter that captures the (detrended) steady state distribution of productivities and qualities across products and item categories.²⁴ The term $\rho(\Pi) \leq 1$ captures the distortions that arise from *inefficient* relative price distortions, as defined in detail below.²⁵ The size of these distortions depends on the inflation rate, except in the special case with flexible prices, where we have $\rho(\Pi) = 1$ for all Π .

Equation (26) equates the marginal rate of substitution between consumption and work on the l.h.s. to the marginal rate of transformation on the r.h.s. of the equation. The latter is multiplied by the aggregate mark-up distortion $1/\mu(\Pi)$, as defined further below. The mark-up distortion depends on the degree of monopolistic competition, the level of output subsidies/taxes and the inflation rate. Again, in the special case with flexible prices, the mark-up distortion is independent of the inflation rate.

²³The existence conditions for a steady state are discussed in appendix D.3.

²⁴See appendix B for a definition.

²⁵Price dispersion that reflects differences in productivity or product quality is efficient.

Equation (27) determines the optimal capital-to-labor ratio: it equates the marginal product of capital on the r.h.s., again adjusted for potential mark-up-distortions, to the sum of the steady-state real interest rate $(1/\beta(\gamma^e)^{-\sigma} - 1)$ and the capital depreciation rate (d) on the l.h.s. The parameter

$$\gamma^e \equiv \prod_{z=1}^Z (a_z q_z)^{\psi_z \phi} \geq 1 \quad (29)$$

denotes the steady-state growth rate of quality-adjusted aggregate output y and affects (via consumption growth) the steady-state real interest rate.

Finally, equation (28) is the resource constraint, which says that output is consumed and invested to keep the capital stock constant in detrended terms.

The aggregate mark-up distortion $\mu(\Pi)$ is an expenditure-weighted average of the item-level mark-up distortions $\mu_z(\Pi)$ and given by

$$\mu(\Pi) = \prod_{z=1}^Z \mu_z(\Pi)^{\psi_z}, \quad (30)$$

where the item-level distortions are given by

$$\mu_z(\Pi) \equiv \left(\frac{1}{1 + \tau} \frac{\theta}{\theta - 1} \right) M_z \left(\frac{1 - \alpha_z(1 - \delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1 - \delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^{\theta}(g_z/q_z)^{-1}} \right), \quad (31)$$

for all $z = 1, \dots, Z$, with

$$M_z \equiv \left(\frac{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1 - \delta_z)(g_z/q_z)^{\theta-1}} \right)^{\frac{1}{\theta-1}},$$

and

$$\gamma_z^e \equiv (a_z q_z)(\gamma^e)^{1-\frac{1}{\phi}}.$$

The relative price distortion $\rho(\Pi)$ is given by

$$(\rho(\Pi)\mu(\Pi))^{-1} = \sum_{z=1}^Z \psi_z (\mu_z(\Pi)\rho_z(\Pi))^{-1}, \quad (32)$$

where for all $z = 1, \dots, Z$ the item-level relative price distortions $\rho_z(\Pi)$ are given by

$$\rho_z(\Pi)^{-1} = M_z^\theta \left(\frac{1 - \alpha_z(1 - \delta_z)(g_z/q_z)^{\theta-1}}{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta}(g_z/q_z)^{-1}} \right). \quad (33)$$

As is easy to see, for the limiting case without price stickiness ($\alpha_z \rightarrow 0$ for all z), we have

$$\begin{aligned} \mu &= \left(\frac{1}{1 + \tau} \frac{\theta}{\theta - 1} \right) \\ \rho &= 1, \end{aligned}$$

independently of Π . This shows that the flexible price equilibrium is efficient, whenever the output subsidy τ is such that it eliminates the monopoly distortion ($\tau = 1/(\theta - 1)$, so that $\mu = 1$). This

mirrors results of standard New Keynesian models that do not feature product heterogeneity and a product life cycle.

For the general case with price stickiness and suboptimal output subsidies, there exists a trade-off between reducing mark-up distortions and reducing relative price distortions. In particular, the steady-state inflation rate that minimizes the mark-up distortion, i.e., moves $1/\mu(\Pi)$ closest to one, is generally different from the steady-state inflation rate that minimizes the effects of relative price distortions, i.e., moves $\rho(\Pi)$ closest to one. Dealing analytically with this trade-off is challenging, which prompts us to consider the following limiting case:²⁶

Lemma 1 *Consider a steady state with a potentially suboptimal inflation rate Π . For the limiting case $\beta(\gamma^e)^{1-\sigma} \rightarrow 1$, we have*

$$\rho(\Pi) = \left(\frac{1}{1+\tau} \frac{\theta}{\theta-1} \right) \frac{1}{\mu(\Pi)}.$$

In the considered limit, mark-up distortions are proportional to relative price distortions. As a result, both distortions are minimized by choosing the maximum feasible value for $\rho(\Pi)$. The next section derives the optimal inflation rate, i.e., the inflation rate that achieves this maximum.

6 The Optimal Inflation Target: Theory

This section presents our main theoretical result about the optimal inflation target. The optimal target is defined as the inflation rate that maximizes steady state utility. The idea underlying this approach is that economic shocks generate only temporary deviations of the optimal inflation rate from its steady-state value, so that the average inflation rate that a welfare-maximizing central bank should target is in fact the optimal steady-state inflation rate.²⁷ The following proposition states our main result:

Proposition 1 *Consider an arbitrary output subsidy/tax satisfying (23) and the limit $\beta(\gamma^e)^{1-\sigma} \rightarrow 1$. The welfare maximizing steady-state inflation rate Π^* is given by*

$$\Pi^* = \sum_{z=1}^Z \omega_z \frac{\gamma_z^e g_z}{\gamma^e q_z}, \quad (34)$$

where γ_z^e is the output growth rate of item z and γ^e the aggregate growth rate, with

$$\frac{\gamma_z^e}{\gamma^e} = \frac{a_z q_z}{\prod_{z=1}^Z (a_z q_z)^{\psi_z}}.$$

²⁶See appendix E.1 for the proof.

²⁷This holds true to a first-order approximation in the aggregate shocks. Nonlinear terms can cause the time average of the optimal stochastic inflation rate to differ from its steady-state value, but such terms tend to be quantitatively small in sticky-price models, as long as the lower bound on nominal rates is not binding.

The item weights $\omega_z \geq 0$ are given by

$$\omega_z \equiv \frac{\tilde{\omega}_z}{\sum_{z=1}^Z \tilde{\omega}_z},$$

with

$$\tilde{\omega}_z \equiv \frac{\psi_z \theta \alpha_z (1 - \delta_z) (\gamma^e / \gamma_z^e)^\theta (\Pi^*)^\theta (q_z / g_z)}{[1 - \alpha_z (1 - \delta_z) (\gamma^e / \gamma_z^e)^\theta (\Pi^*)^\theta (q_z / g_z)] [1 - \alpha_z (1 - \delta_z) (\gamma^e / \gamma_z^e)^{\theta-1} (\Pi^*)^{\theta-1}]}.$$

The proof of the proposition is contained in appendix E.2. It shows that the steady-state amount of labor (L) does not depend on the inflation target (Π), so that the target is optimally chosen to maximize steady-state consumption. Consumption is shown to depend on the inflation target only via the aggregate markup distortion. The inflation rate minimizing the markup distortion is the one stated in the proposition.

Equation (34) shows that the optimal inflation target is a doubly-weighted average of the item-level terms g_z/q_z . To interpret this finding, we start by discussing the role of the item-level terms g_z/q_z . Thereafter, we assess the role of the two weights ω_z and γ_z^e/γ^e . Finally, we discuss the relevance of the imposed limiting condition $\beta(\gamma^e)^{1-\sigma} \rightarrow 1$.

Items with $g_z > q_z$ generate a force towards positive inflation ($\Pi^* > 1$), while items with $g_z < q_z$ generate a force towards deflation ($\Pi^* < 1$). To understand why this the case, abstract for a moment from quality progress ($q_z = 1$) and suppose $g_z > 1$. Productivity then increases with the lifetime of the product, so that old products should become increasingly cheaper relative to newly entering products. In the presence of price setting frictions, this relative price decline of old products is best implemented by having new products charge higher prices, i.e., by positive amounts of inflation, rather than by having old products continuously adjust prices downward. This is so because price cuts cannot be synchronized across firms due to Calvo frictions and thereby give rise to inefficient price dispersion.²⁸ Now consider the polar case without age-dependent productivity ($g_z = 1$) and positive quality progress ($q_z > 1$). New products can then be produced at increasingly higher quality, without having to use more inputs into their production. New products should thus become cheaper (in quality-adjusted terms), relative to old products. Again, in the presence of price setting frictions, this is best achieved by having new products charge lower prices, i.e., via deflation, rather than by having old product increase prices.

The proof of the proposition implies that the item-level term g_z/q_z captures the value of the inflation target that eliminates inefficient price dispersion in item z . To the extent that g_z/q_z varies across items, the optimal inflation rate must thus trade-off between the distortions across different items. The optimal resolution of this trade-off is captured by the item weights γ_z^e/γ_z^e and ω_z .

The first set of weights, γ_z^e/γ_z^e , captures the (quality-adjusted) output growth in item z relative to the growth rate of the aggregate economy. This leads to an overweighting of items with fast output growth and an underweighting of items with slow growth.

²⁸With menu cost frictions, continuous price cuts would be equally undesirable because price adjustment is costly.

The second set of weights, ω_z , are nonlinear functions of item-level fundamentals, i.e., the expenditure weight ψ_z , the product turnover rate δ_z , the price stickiness α_z , and the demand elasticity θ . The weights also depend on the item-level terms q_z/g_z and on the optimal inflation rate Π^* itself. Admittedly, the dependence on Π^* makes it hard to interpret the item weights ω_z . Yet, for the special case where an item features no price stickiness ($\alpha_z = 0$), the optimal item weight is zero ($\omega_z = 0$). This is in line with the insights provided in Aoki (2001). Similarly, with only a single item ($Z = 1$) and thus no trade-off between items, all weights are equal to one, so that we obtain $\Pi^* = g_1/q_1$. This is the special case with a single relative price trend considered in Adam and Weber (2019).

For the case where the condition $\beta(\gamma^e)^{1-\sigma}$ is not close to one, the inflation rate stated in proposition 1 still minimizes the effects arising from price distortions, i.e. maximizes the term $\rho(\Pi)$ in equation (25). The inflation rate that minimizes markup distortions, i.e., maximizes the term $1/\mu(\Pi)$ in equations (26) and (28), is then generally a different one. Yet, for reasonable model parameterizations, we find that the difference between these two inflation rates is quantitatively small. This suggests that our quantitative results are not driven by imposing $\beta(\gamma^e)^{1-\sigma} \rightarrow 1$.

7 Optimal Inflation Target: Empirical Approach

This section explains how one can estimate the optimal inflation target using micro price data. Perhaps not surprisingly, a fully-fledged estimation based on the nonlinear analytic expression in proposition 1 is challenging, because this requires empirically identifying a large range of structural parameters in a model-consistent way.²⁹ We proceed by considering a first-order approximation to the optimal inflation rate in proposition 1. We then show how one can empirically identify all elements that matter for inflation to first order using official micro price data only. We leave the quantification of the effects of higher-order terms to future research.

The lemma below shows - rather surprisingly - that only very few dimensions of heterogeneity matter for the inflation target to first order:

Lemma 2 *The optimal steady-state inflation rate is equal to*

$$\Pi^* = \sum_{z=1}^Z \psi_z \frac{\gamma_z^e g_z}{\gamma^e q_z} + O(2), \quad (35)$$

where $O(2)$ denotes a second-order approximation error and where the approximation to equation (34) has been taken around a point, in which $\frac{g_z \gamma_z^e}{q_z \gamma^e}$ and $\alpha_z(1 - \delta_z)(\gamma^e/\gamma_z^e)^{\theta-1}$ are constant across sectors item categories $z = 1, \dots, Z$.

²⁹For instance, using empirical price adjustment frequencies to estimate the price rigidity parameters α_z , as is commonly done in the literature, requires assuming that marginal production costs are not constant. The estimation approach we adopt below does not rely on this assumption.

Three dimensions of heterogeneity are relevant to first order: heterogeneity in expenditure weights ψ_z , heterogeneity in growth rates γ_z^e/γ^e and heterogeneity in the item-level terms g_z/q_z .

The first two dimensions can be readily identified from official micro price data sets. In particular, the expenditure weights are naturally part of micro price data sets that are used to compute an aggregate price index. The heterogeneity in growth rates can be identified using the model-implied relationship $\gamma_z^e/\gamma^e = \Pi/\Pi_z$. We use the sample means for Π and Π_z to estimate γ_z^e/γ^e .

Identifying the item-level terms g_z/q_z is more challenging. A main contribution of the paper is to derive a model-consistent estimation approach that directly yields estimates of the item-level terms g_z/q_z under fairly general conditions. The following proposition states our main insight:³⁰

Proposition 2 *Consider a stochastic sticky price economy with a stationary (and potentially sub-optimal) inflation rate Π_t . Let T_{jz}^* denote the set of periods in which the price of product j in item z can be adjusted and let s_{jzt} denote the product age. The optimal reset price P_{jzt}^* in adjustment periods, defined in equation (24), satisfies*

$$\ln \frac{P_{jzt}^*}{P_{zt}} = f_{jz}^* - \ln \left(\frac{g_z}{q_z} \right) \cdot s_{jzt} + u_{jzt}^*, \quad \text{for all } t \in T_{jz}^*, \quad (36)$$

where the residual satisfies $E[u_{jzt}^*] = 0$.

The proposition shows that the optimal reset price displays an age trend at the rate g_z/q_z , which is our parameter of interest. The result applies, whenever there is a strictly positive rate of product turnover ($\delta_z > 0$), as assumed, but is otherwise independent of the turnover rate. For the case without item turnover ($\delta_z = 0$), the model implies that the age trend is discontinuously different and equal to zero.³¹

It may appear surprising that the age-trend coefficient in equation (36) reveals our parameter of interest in a setting with sticky prices and potentially suboptimal inflation rates. To understand why this is the case, we note that with flexible prices ($\alpha_z = 0$), the same relative price trend would emerge. Specifically, the proof of proposition 2 implies that flexible prices satisfy

$$\ln \frac{P_{jzt}^f}{P_{zt}} = f_{jz}^f - \ln \left(\frac{g_z}{q_z} \right) \cdot s_{jzt} + u_{jzt}^f, \quad (37)$$

for all periods t in which the product is on offer. Price stickiness and suboptimal inflation rates thus only affect the level of relative reset prices, i.e., the intercept term f_{jz}^* and the residual u_{jzt}^* , but leave the time trend of relative prices invariant. This invariance property is key for the ability to identify the structural parameters g_z/q_z . In fact, it implies that the relative price trends that are present in micro price data are efficient, i.e., reflect economic fundamentals.

³⁰See appendix E.4 for the proof.

³¹This seeming discontinuity at $\delta_z = 0$ arises only because steady state considerations also involve taking a limit, so that one effectively considers a double limit: the limit $\delta_z \rightarrow 0$ and additionally the limit distribution for steady state productivities implied by δ_z .

The invariance property of relative price trends is by no means special to the Calvo setting. It would similarly be present, if price rigidities were driven by menu costs instead. Menu costs generate inaction bands around the frictionless optimal relative price. If this frictionless price is trending over time, then reset prices must display the same time trend.³²

Equation (36) thus provides a highly tractable approach for empirically identifying g_z/q_z .³³ This is the case, despite the fact that the true product age s_{jzt} is typically not observed. As is easily seen, using the number of months since the product has been included into the price data set as the ‘age’ regressor, instead of the true product age, affects only the estimated intercept term, but leaves the coefficient of interest multiplying the ‘age’ term unchanged. One can thus estimate the parameter of interest even without observing true product age.

Since product prices are reset only infrequently in adjustment periods but stay constant otherwise, the entire price path of a product displays the very same time trend as the reset prices. We thus have that the entire price path satisfies

$$\ln \frac{P_{jzt}}{P_{zt}} = f_{jz} - \ln \left(\frac{g_z}{q_z} \right) \cdot s_{jzt} + u_{jzt}, \quad (38)$$

for some alternative intercept term and a residual that again satisfies $E[u_{jzt}] = 0$.³⁴ In our empirical approach, we shall use equation (38) as our baseline equation, which uses all price observations. We will consider estimates based on the reset prices only, see equation (36), in a robustness exercise.

Regression (38) is almost the one we have estimated in our empirical section 3. The only differences are that in section 3 we used the non-quality adjusted relative product price ($\ln(\tilde{P}_{jzt}/P_{zt})$) on the left-hand side of equation (1) and that there was a different sign for the regression coefficient. With product quality being constant across the product lifetime, the first difference does not affect the estimated age coefficient, as gets absorbed in the estimated constant; the second difference only implies a trivial sign reversal. The estimates of slope coefficients b_z displayed in figure 2 thus already reveal the (negative) of the optimal item-level inflation rates.

8 Imperfect Quality Adjustment

The estimation approach and the underlying theory developed in the previous sections assume that statistical agencies perfectly adjust prices for quality. This is clearly an idealized assumption, as a number of studies show that quality adjustment is far from perfect (Bils (2009), Broda and Weinstein (2010), Aghion et al. (2019)).

³²This assumes - as is standard in menu cost models - that the width of the inaction bands does not itself display a time trend.

³³This is true even though the residual u_{jzt}^* can potentially contain a unit root, see the proof of proposition 2. Since product lives tend to be short, the asymptotics of interest are the ones where the number of products gets large, not the ones where the lifetime of the products get large. Non-stationarity is thus not an issue for consistency and asymptotic normality of OLS estimates.

³⁴Appendix E.5 derives the intercept term and the properties of the modified residual in the previous equation

This section shows that failure to perfectly adjust prices for quality generates biases in the slope coefficients identified in regression (36), i.e., the coefficient in front of the age trend will then *not* be equal to $\ln g_z/q_z$, unlike in the case with perfect quality adjustment. This is the case because the item price level P_{zt} showing up on the left-hand side of the regression equation displays a different trend when quality adjustment is imperfect. While this may be a source of concern, we show below that the optimal inflation target computed according to lemma 2 based on these biased slope estimates nevertheless delivers the welfare maximizing target for the imperfectly quality-adjusted price index. In other words, the approach developed in the previous sections works perfectly well, even if quality adjustment is imperfect, as is likely the case in practice.

To make this point most forcefully, we consider an extreme setting in which the statistical agency makes no quality adjustments whatsoever. The item price level is thus computed using not-quality-adjusted prices \tilde{P}_{jzt} and given by

$$\tilde{P}_{zt} \equiv \left(\int_0^1 (\tilde{P}_{jzt})^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \quad (39)$$

with the associated item-level inflation rate given by

$$\tilde{\Pi}_{zt} = \tilde{P}_{zt}/\tilde{P}_{zt-1}.$$

Appendix E.6 derives the recursive law of motion for the not-quality adjusted item price level and shows that in steady state the following holds:

$$\tilde{\Pi}_z = q_z \Pi_z. \quad (40)$$

The item-level inflation rate without quality adjustment $\tilde{\Pi}_z$ thus exceeds the quality-adjusted inflation rate whenever there is quality growth ($q_z > 1$). Similarly, the aggregate steady-state inflation rate without quality adjustment $\tilde{\Pi}$ is given by³⁵

$$\ln \tilde{\Pi} = \ln \Pi + \sum_{z=1}^Z \psi_z \ln q_z, \quad (41)$$

and exceeds the quality-adjusted rate by a weighted average of the item-level quality growth rates. This feature is well understood in the literature. The key new observation in this section is that in the absence of quality adjustment, the regression coefficient on the age trend in equation (36) is equally distorted.³⁶

Proposition 3 *Consider a steady state with a potentially suboptimal inflation rate Π , and let T_{jz}^* denote the set of periods in which the price of product j in item z can be adjusted. Then,*

$$\ln \frac{\tilde{P}_{jzt}^*}{\tilde{P}_{zt}} = \tilde{f}_{jz}^* - \ln(g_z) \cdot s_{jzt}, \quad \text{for all } t \in T_{jz}^*, \quad (42)$$

where s_{jzt} denotes the age of product j in item z at time t .

³⁵See appendix E.8 for the derivation.

³⁶See appendix E.7 for the proof.

This shows that with positive quality progress ($q_z > 1$), the regression estimates are also upwardly distorted by the amount of quality progress and given by g_z instead of g_z/q_z . Therefore, when computing the optimal inflation target for the not-quality adjusted inflation rate using the distorted regression coefficients, one unwittingly implements the optimal target rate for the quality-adjusted rate of inflation.

To formally show this, note that the optimal inflation target from lemma 2 can alternatively be expressed as³⁷

$$\ln \Pi^* = \sum_{z=1}^Z \psi_z \ln \left(b_z \frac{\gamma_z^e}{\gamma^e} \right) + O(2). \quad (43)$$

where b_z is the regression coefficient on the age trend in equation (36). We have $b_z = g_z/q_z$ for perfect quality adjustment and $b_z = g_z$ in the absence of quality adjustment. Using the distorted regression coefficients ($b_z = g_z$) and equation (43), we arrive at an optimal inflation target for the not-quality-adjusted inflation rate given by

$$\ln \tilde{\Pi}^* = \sum_{z=1}^Z \psi_z \ln \left(g_z \frac{\gamma_z^e}{\gamma^e} \right).$$

From equation (41) then follows that the quality-adjusted inflation rate satisfies

$$\ln \Pi = \ln \tilde{\Pi}^* - \sum_{z=1}^Z \psi_z \ln q_z = \sum_{z=1}^Z \psi_z \ln \left(\frac{g_z \gamma_z^e}{q_z \gamma^e} \right) = \ln \Pi^*.$$

This shows that the inflation target in terms of quality-adjusted prices is in fact optimal. Imperfect quality adjustment is thus not a source of concern for the approach developed in this paper.

9 The Optimal Inflation Target for the U.K.

We now use the approach developed in the previous sections to estimate the optimal inflation target for the U.K. economy.

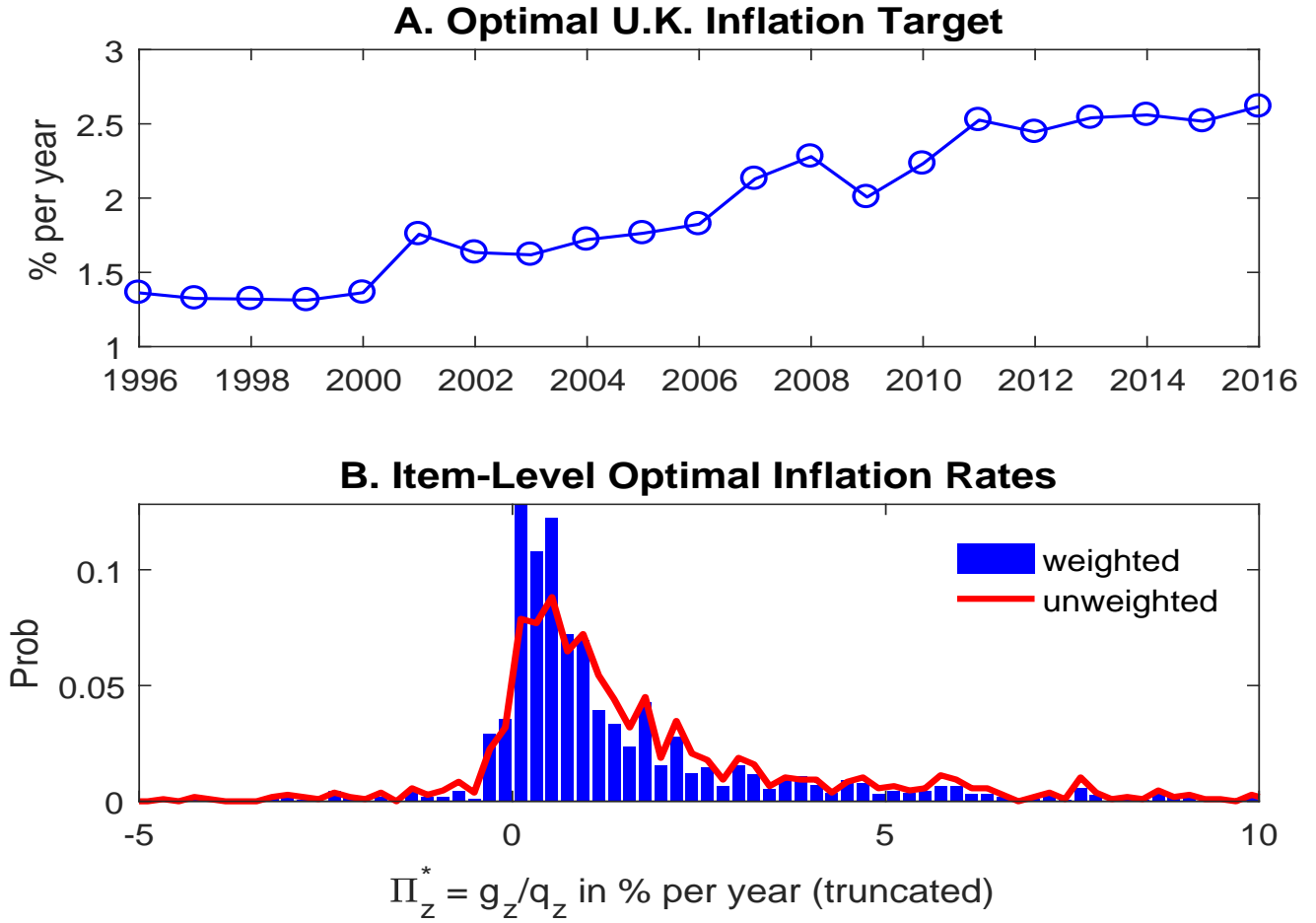
The top panel of figure 5 presents our baseline estimate. The baseline estimation approach uses all price observations in the baseline sample to estimate the item-level relative price trends g_z/q_z , see equation (38). It then uses the expenditure items present at any considered date, the corresponding ONS expenditure weights, as well as the estimated values for γ_z/γ , to compute the optimal inflation target at this date according to lemma 2.

The top panel of figure 5 shows that the optimal inflation target is significantly positive and stands at approximately 2.6% in the year 2016. The optimal target also steadily increased over the period 1996-2016. The observed increase is about 1.2% and thus quantitatively significant.

Clearly, the observed gradual increase in figure 5 does not imply that the Bank of England should have continuously revised its inflation target upward in line with the estimates shown in the graph. If

³⁷See appendix E.8 for a derivation

Figure 5: Optimal U.K. Target - Baseline Results



target adjustments are costly, e.g., because they require costly reputation building, then the optimal adjustments to the target would happen through lumpy and infrequent adjustments and not via small continuous adjustments.

The optimal inflation target in figure 5 deviates from zero in a quantitative significant way because of the strong negative relative price trends (section 3). Panel B in figure 5 depicts the distribution of item-level optimal inflation rates $\Pi_z^* \equiv g_z/q_z$ (in annualized terms) for all items present over the period 1996-2016. The panel depicts the distribution once in expenditure weighted form (blue bars) and once using item frequencies (red line).³⁸ Both of these distribution show that the optimal inflation rate is positive for the vast majority of items. This is the case for 90.5 % of the expenditure-weighted items and 89.8% of raw items in the sample.

Panel B in figure 5 also highlights that the aggregate inflation result is not driven by outliers,

³⁸The expenditure-weighted distribution is the mirror image of the relative price trend distribution shown in figure 2.

instead there is a large mass of items for which the item-level optimal inflation rate is close to the estimated optimal inflation target. There is, however, considerable heterogeneity in relative price trends in the economy, causing some expenditure items to have substantially positive rates of optimal item-level inflation.

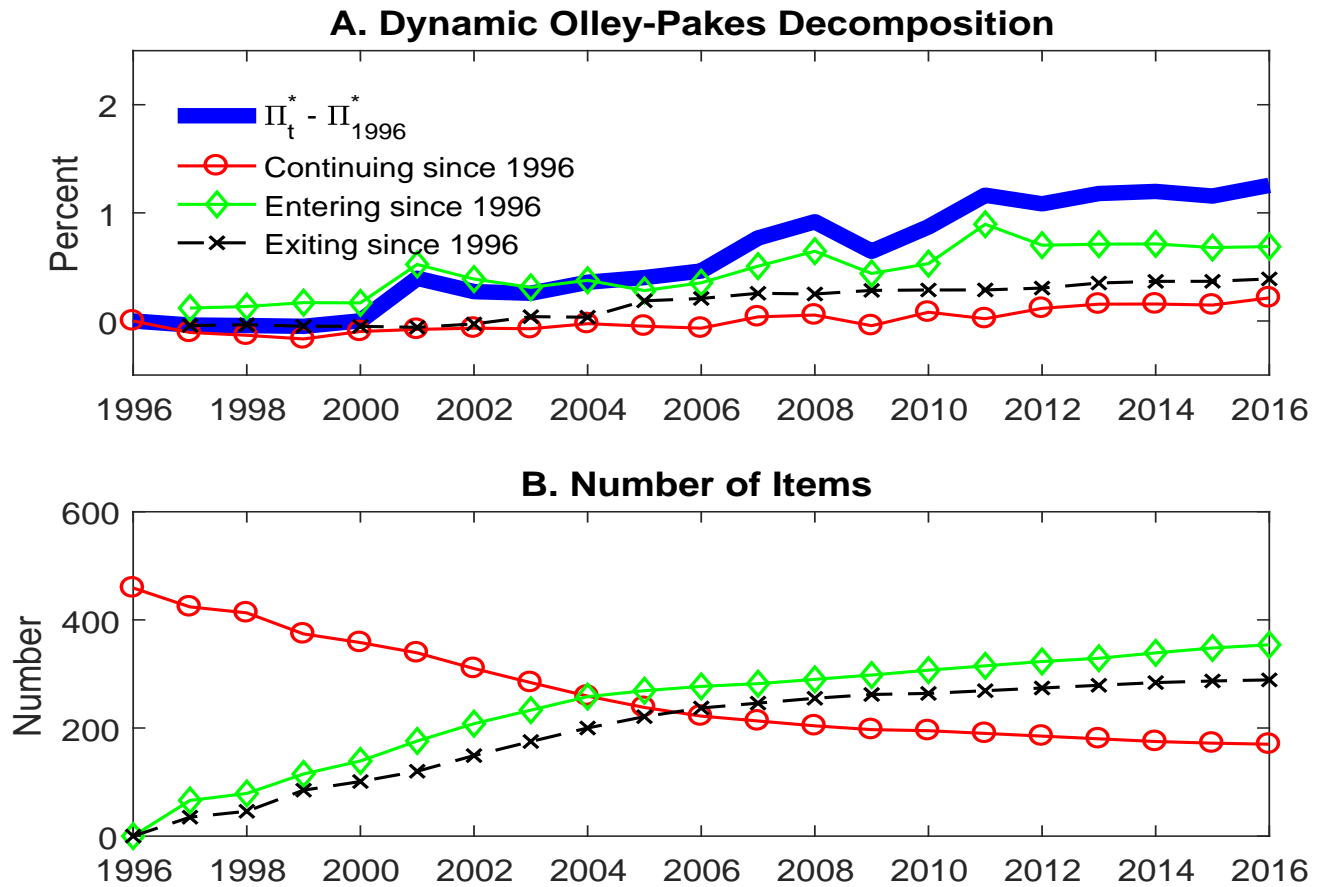
To understand the source of the upward trend in the optimal inflation target in figure 5, we perform a dynamic Olley-Pakes decomposition, following the approach of Melitz and Polanec (2015). Specifically, we decompose the increase in the inflation target for any year of interest relative to the base year 1996 into three components: the effect of newly added items up to the year of interest, the effect of items that have exited up to the year of interest, and the effects of changing expenditure weights among continuing items up to the year of interest.

The result of this decomposition is depicted in figure 6. The bottom panel of the figure shows the number of continuing, exiting and entering items at any given date (all relative to the base year 1996). The top panel decomposes the total increase in the inflation target (the solid blue line) into the three elements. It shows that all elements contribute to the observed increase in the optimal inflation target. The largest upward force comes from newly entering items, which display (on average) a larger rate of relative price decline and thus a higher optimal item-level inflation rate than the items present in 1996. The second largest upward force comes from exiting items: exiting items display a rate of relative price decline that was on average below the one displayed by items that were present in 1996. Finally, a small positive force is due to a reshuffling of expenditure weights among the set of continuing items towards items displaying a larger rate of relative price decline.

Figure 7 compares the expenditure-weighted distribution of item-level inflation rates in 1996 and 2016. It shows how item entry and exit, as well as expenditure reweighting among continuing items have shifted the distribution of optimal item-level inflation rates towards the right over these two decades. The figure makes it clear that there was a notable shift in the center of the distribution and that results are not driven by outliers.

Figure 8 explores the quantitative relevance of the weighting schemes for the estimated optimal inflation target. The figure compares the baseline estimate to an inflation target estimate that ignores the growth rate weights ($\sum_{z=1}^{Z_t} \psi_{zt} \frac{g_z}{q_z}$) and an estimate that ignores all weights altogether ($\frac{1}{Z_t} \sum_{z=1}^{Z_t} \frac{g_z}{q_z}$). The figure shows that the growth rate weights have quantitatively only small effects on the estimated inflation target. The situation is different for the expenditure weights: without the appropriate expenditure weights, the optimal inflation target is estimated to be around 0.25-0.5% higher. While the upward trend of the optimal inflation remains unchanged, the level increase highlights the fact that expenditure weights covary negatively with the downward trend in relative prices, i.e., expenditure items with less pronounced relative price declines (and thus lower optimal item-level inflation rates) tend to have higher expenditure weights.

Figure 6: Decomposing the upward trend in the optimal inflation target



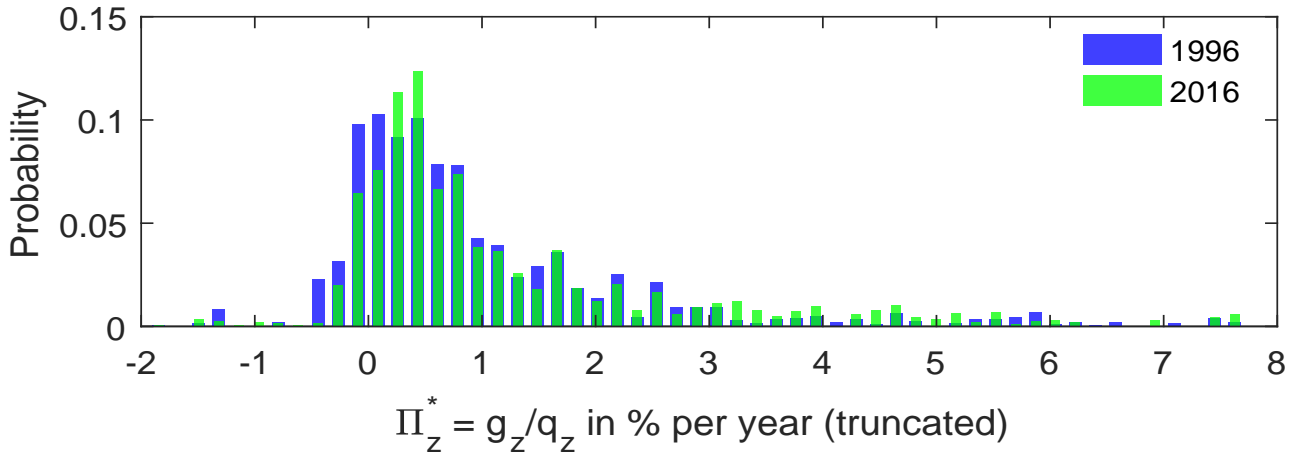
10 Optimal U.K. Inflation - Robustness Checks

This section explores the robustness of the baseline results along two dimensions. We first show in section 10.1 that results are robust towards using reset prices only (rather than all prices) in the relative price trend regressions. We then consider in section 10.2 alternative approaches for dealing with sales prices and show that these tend to generate even higher estimates for the optimal inflation target.

10.1 Using Reset Prices Only to Estimate Price Trends

The baseline approach uses all price observations available in our baseline sample to estimate relative price trends. According to our theory, the price trend can alternatively be recovered using reset prices only (proposition 2). Given this, we rerun our relative price trend regressions using only price observations for which the monthly price deviated from the previous month's price. Clearly, this

Figure 7: Item-level optimal inflation rates: 2016 versus 1996, expenditure-weighted distributions



leads to a much smaller number of price observations used in the age trend regressions: estimates are then based on just 2.6m price observations compared to the 20.4m observations used in the baseline approach.³⁹ Figure 9 shows that the inflation target recovered via this alternative estimation approach differs only in quantitatively minor ways from our baseline findings. We find this result reassuring, as it effectively represents a test of an overidentifying restriction implied by the underlying price setting model.

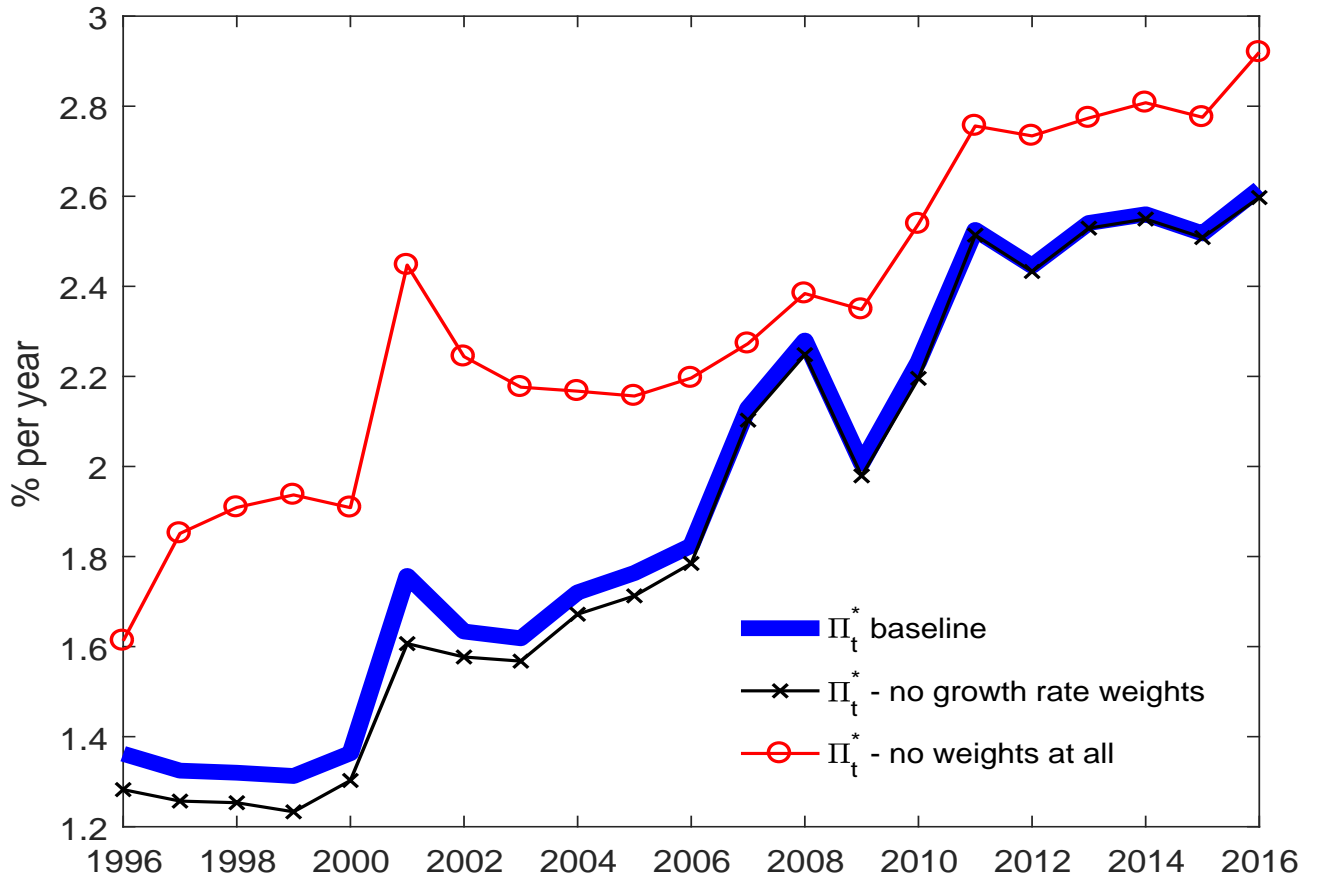
10.2 Alternative Treatment of Sales Prices

An important feature of micro price data is that it features many short-lived price changes that are subsequently reversed. These typically take the form of temporary price reductions (sales), but also occasionally the form of temporary price increases. The sticky price model outlined in the previous sections does not allow for such temporary price changes. We show below that the model can be augmented, following the lines of Kehoe and Midrigan (2015), and that doing so leaves our empirical estimation approach unchanged. We furthermore explore the quantitative effects of alternative treatments of sales prices for our results.

Consider for a moment the following augmented sticky-price setup featuring also temporary prices. Firms choose a regular list price P_{jzt}^L , which is subject to the same price adjustment frictions as the prices in the pure Calvo model presented before. After learning about the adjustment opportunity for the list price, a share $\alpha_T \in [0, 1)$ of producers gets to choose freely a temporary price P_{jzt}^T at which

³⁹The latter number is slightly below the number of price observations reported in the second to last row in table 1 because we exclude products featuring just one price observation from the estimation, as these do not provide information on the age coefficient.

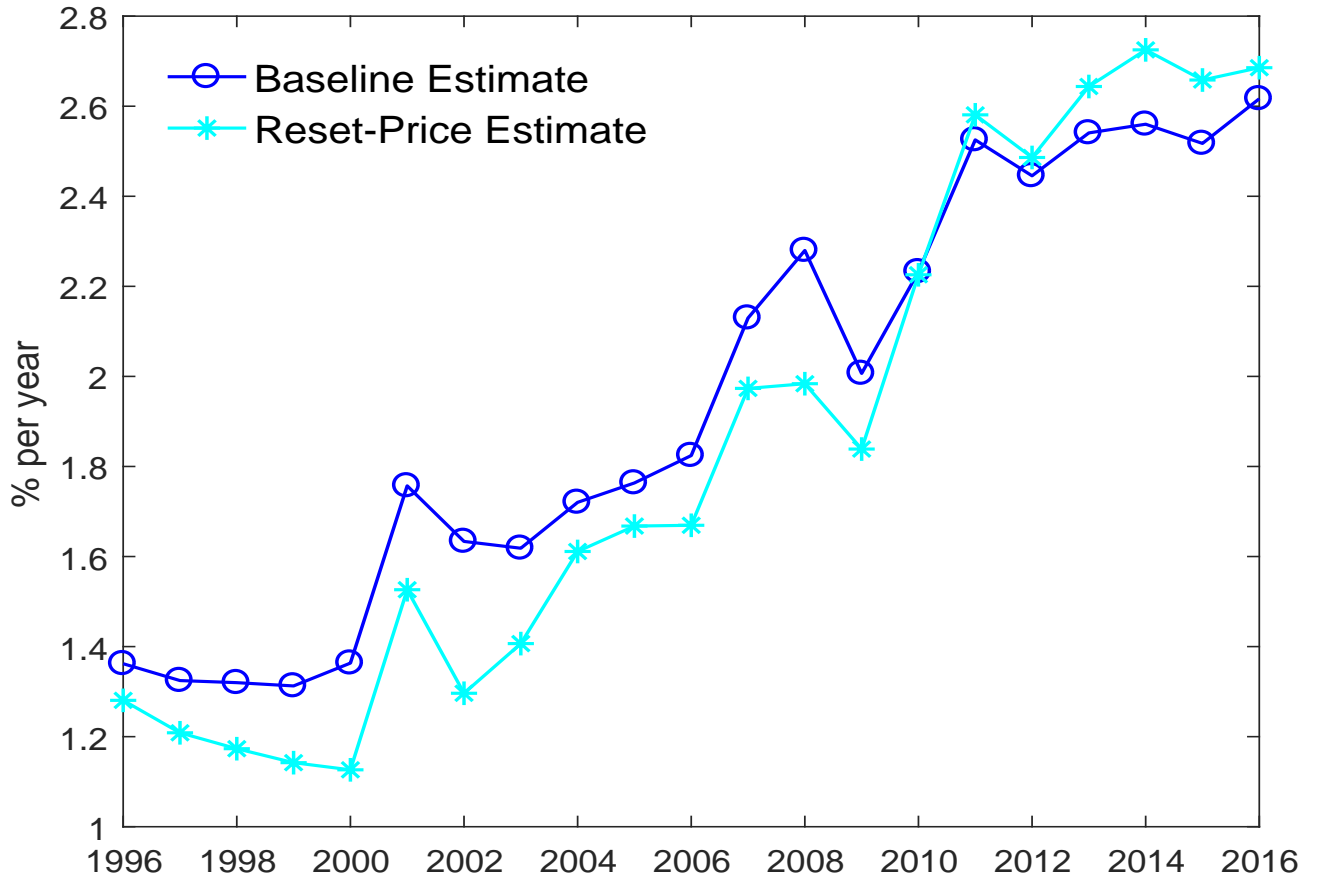
Figure 8: Relevance of the weighting schemes for the estimated optimal inflation target



they can sell the product in the current period. The temporary price is valid for one period only and does not affect the list price. Furthermore, absent further temporary price adjustment opportunities in the next period, prices revert to the list price in the next period. With this setup, the optimal temporary price P_{jzt}^{T*} is equal to the static optimal price in the period, i.e., equal to the flexible price P_{jzt}^f . It follows from equation (37) that the relative price trend of temporary (or flexible) prices is no different from that of all prices, so that the inclusion of temporary prices in the relative price trend regressions should make no difference for our results.

Nevertheless, sales prices can make a difference for the estimated relative price trends due to a number of reasons. Sales prices might, for instance, not be evenly distributed over the product life cycle, unlike assumed in the augmented theoretical setup sketched in the previous paragraph. Sales may happen, for instance, predominantly at the beginning (or at the end) of the product lifetime. If this were the case, then our baseline regressions would probably underestimate (overestimate) relative price declines and thereby underestimate (overestimate) the optimal inflation target. In light of this,

Figure 9: Optimal inflation target: baseline versus reset price based estimation



it appears of interest to investigate the robustness of our baseline results towards using alternative approaches for treating sales prices in the data.

Figure 10 displays the baseline estimate of the optimal inflation target together with various alternative estimates for the optimal inflation target. A first approach (baseline w/o sales prices) uses the ONS sales flag to exclude all sales prices from regression (38).⁴⁰ The figure shows that the optimal inflation target increases by around 0.3% per year as a result. A quantitatively similar result is obtained, if only the so-called "regular prices" are used in the regression (Kehoe-Midrigan, regular prices only), where regular prices are defined according to the regular price filter of Kehoe and Midrigan (2015).

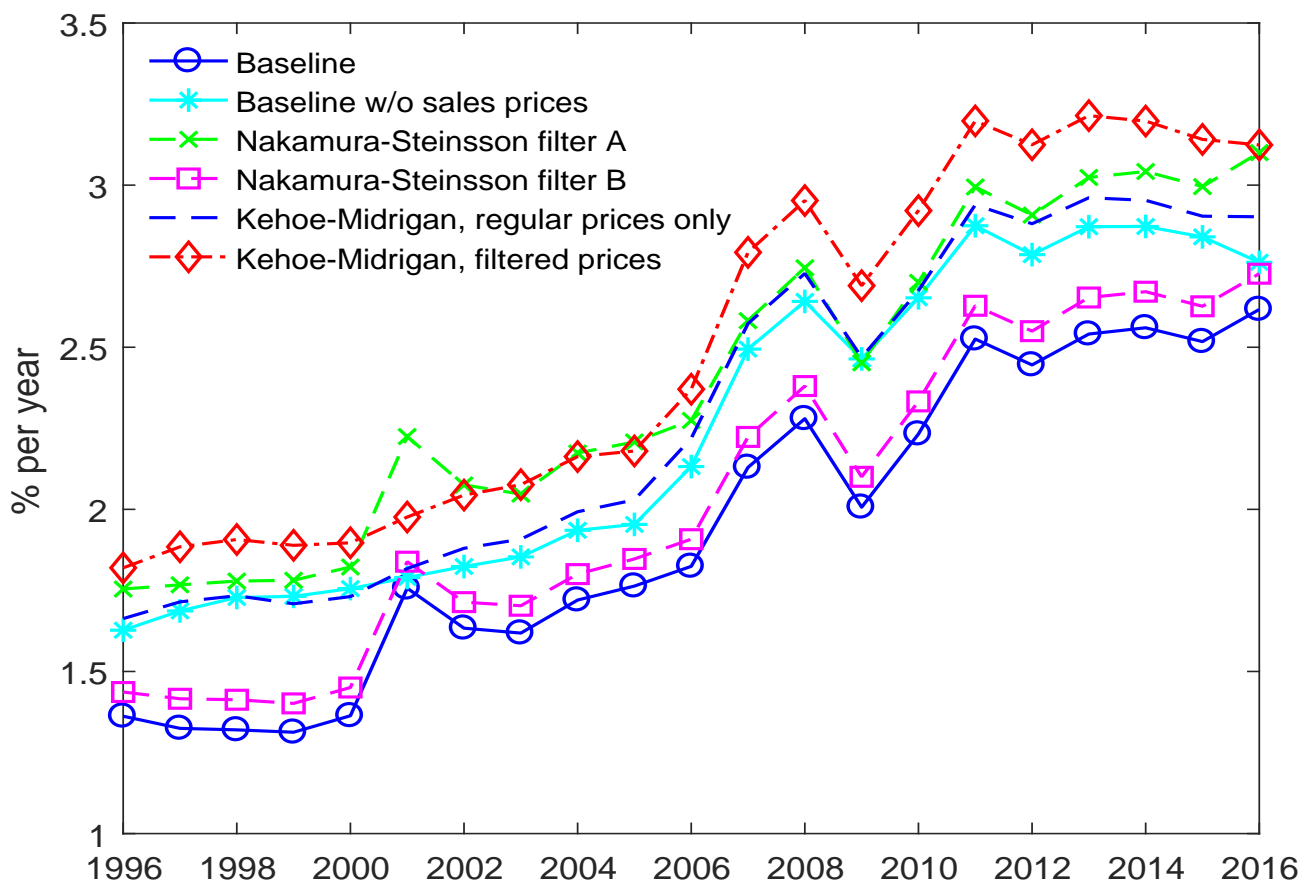
Instead of simply excluding sales prices from the regression, one can adjust sales prices based

⁴⁰A sales flag is an indicator variable that the price collector records, whenever she/he finds the product to be on sale. In this and subsequent robustness checks, we always recompute the item price levels after excluding or adjusting sales prices.

on various adjustment techniques and continue using them in the estimation. Figure 10 reports the outcome when making adjustments using the sales filters A and B from Nakamura and Steinsson (2008) and the regular price filter of Kehoe-Midrigan (2015) (Kehoe-Midrigan, filtered prices). The outcomes across these filtering approaches vary quite substantially. While the Nakamura-Steinsson filter B leads to only small adjustments relative to the baseline estimation, filter A leads to adjustments of the same order of magnitude as when dropping sales prices from the regression. The largest upward revision of the inflation target is observed for the regular price filter of Kehoe and Midrigan: the inflation target is then on average about 0.5% higher than the baseline estimate.

Overall, we can conclude that a different treatment of sales prices can lead to considerably higher optimal inflation targets than the ones obtained via our baseline approach.

Figure 10: Optimal inflation target for alternative treatments of sales prices



11 Conclusions

The present paper documents relative price trends at the product level and shows that they are informative about the inflation target that a welfare-maximizing central bank should pursue. The estimated relative price trends make it optimal to target significantly positive rates of inflation, something that the monetary policy literature has struggled to rationalize.

While our empirical approach allows for a rich set of heterogeneity across products, we have abstracted from a number of features that are worthwhile investigating in future work. Given our focus on consumer products, we have abstracted from intermediate and imported goods, both of which would appear to be relevant for the U.K. economy. Introducing heterogeneity in product trends into sticky price models featuring sectoral input-output structures, e.g., Nakamura and Steinsson (2010), Pasten, Schoenle and Weber (2018), appears to be an interesting avenue for future research.

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A Key Model Derivations

A.1 First-Order Conditions of the Household Problem

The representative household maximizes expected discounted utility in equation (2) subject to the budget constraint (3). The first-order conditions to this maximization problem comprise

$$\frac{W_t}{P_t} = -C_t \frac{\partial V(L_t)/\partial L_t}{V(L_t)} \quad (44)$$

$$\Omega_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{V(L_{t+1})}{V(L_t)} \right)^{1-\sigma} \quad (45)$$

$$1 = E_t \left[\Omega_{t,t+1} \left(\frac{1 + i_t}{P_{t+1}/P_t} \right) \right] \quad (46)$$

$$1 = E_t [\Omega_{t,t+1} (r_{t+1} + 1 - d)], \quad (47)$$

a no-Ponzi scheme condition, the transversality condition and the household's budget constraint.

A.2 Derivation of Firms' Marginal Cost Expression (21)

Let

$$I_{jzt} \equiv Y_{jzt} / (A_{zt} Q_{jzt} G_{jzt})$$

denote the units of factor inputs $(K_{jzt}^{1-\frac{1}{\phi}} L_{jzt}^{\frac{1}{\phi}})$ required to produce Y_{jzt} units of (quality-adjusted) output. We now show that cost minimization yields the expression for nominal marginal costs of I_{jzt} provided in equation (21). Firm j chooses the factor input mix to minimize production costs subject to the constraint imposed by the production function (6),

$$\min_{K_{jzt}, L_{jzt}} K_{jzt} r_t + L_{jzt} W_t / P_t \quad s.t. \quad Y_{jzt} = A_{zt} Q_{jzt} G_{jzt} K_{jzt}^{1-\frac{1}{\phi}} L_{jzt}^{\frac{1}{\phi}} .$$

Denoting the Lagrange multiplier by λ_t , this cost minimization problem yields first-order conditions

$$\begin{aligned} 0 &= r_t + \left(1 - \frac{1}{\phi} \right) \lambda_t A_{zt} Q_{jzt} G_{jzt} \left(\frac{L_{jzt}}{K_{jzt}} \right)^{\frac{1}{\phi}} \\ 0 &= W_t / P_t + \frac{1}{\phi} \lambda_t A_{zt} Q_{jzt} G_{jzt} \left(\frac{L_{jzt}}{K_{jzt}} \right)^{\frac{1}{\phi}-1} . \end{aligned}$$

These conditions imply that the optimal capital labor ratio is the same for all firms $j \in [0, 1]$ and all items $z = 1, \dots, Z_t$, i.e.,

$$\frac{K_{jzt}}{L_{jzt}} = \frac{W_t}{P_t r_t} (\phi - 1). \quad (48)$$

Substituting the optimal factor input mix into the production function (6) and solving for the factor inputs yields the factor demand functions

$$L_{jzt} = \left(\frac{W_t}{P_t r_t} (\phi - 1) \right)^{\frac{1}{\phi} - 1} I_{jzt} \quad (49)$$

$$K_{jzt} = \left(\frac{W_t}{P_t r_t} (\phi - 1) \right)^{\frac{1}{\phi}} I_{jzt}, \quad (50)$$

where I_{jzt} is defined in the text. Firm j demands these amounts of labor and capital, respectively, to combine them to Y_{jzt} units of (quality-adjusted) output. Thus, the firm's cost function is

$$MC_t I_{jzt} = W_t \left(\frac{W_t}{P_t r_t} (\phi - 1) \right)^{\frac{1}{\phi} - 1} I_{jzt} + P_t r_t \left(\frac{W_t}{P_t r_t} (\phi - 1) \right)^{\frac{1}{\phi}} I_{jzt}, \quad (51)$$

where MC_t denotes nominal marginal (or average) costs. The previous equation can be rearranged to obtain equation (21).

A.3 Derivation of the Optimal Price Setting Equation (24)

The first order condition to the firm's price setting problem (22) yields

$$0 = E_t \sum_{i=0}^{\infty} (\alpha_z (1 - \delta_z))^i \frac{\Omega_{t,t+i}}{P_{t+i}} Y_{jzt+i} \left[P_{jzt}^* - \frac{\theta_z}{(1 + \tau_z)(\theta_z - 1)} \left(\frac{MC_{t+i}}{A_{zt+i} Q_{zt+i} Q_{jzt+i}} \right) \right],$$

where we use the short-hand notation $Q_{jzt} = Q_{jzt} G_{jzt} / Q_{zt}$. Solving this equation for P_{jzt}^* yields

$$\begin{aligned} \frac{P_{jzt}^*}{P_t} Q_{jzt} &= \left(\frac{1}{1 + \tau} \frac{\theta}{\theta - 1} \right) \\ &= \frac{E_t \sum_{i=0}^{\infty} (\alpha_z (1 - \delta_z))^i \Omega_{t,t+i} \left(\frac{P_{zt+i}}{P_{zt}} \right)^{\theta-1} \left(\frac{P_{t+i} Y_{t+i}}{P_t Y_t} \right) \left(\frac{MC_{t+i}}{P_{t+i} A_{zt+i} Q_{zt+i}} \right) \left(\frac{Q_{jzt}}{Q_{jzt+i}} \right)}{E_t \sum_{i=0}^{\infty} (\alpha_z (1 - \delta_z))^i \Omega_{t,t+i} \left(\frac{P_{zt+i}}{P_{zt}} \right)^{\theta-1} \left(\frac{Y_{t+i}}{Y_t} \right)}. \end{aligned} \quad (52)$$

We can express the ratio Q_{jzt}/Q_{jzt+i} in the previous equation as

$$\frac{Q_{jzt}}{Q_{jzt+i}} = \frac{G_{jzt} Q_{zt+i}}{G_{jzt+i} Q_{zt}},$$

because quality remains constant over the lifetime of product j , so that $Q_{jzt} = Q_{jzt+i}$. Using equation (8) to substitute for productivity G_{jzt} and the fact that the idiosyncratic component ϵ_{jzt}^G remains constant of the product lifetime further yields

$$\frac{Q_{jzt}}{Q_{jzt+i}} = \frac{\bar{G}_{jzt}}{\bar{G}_{jzt+i}} \frac{Q_{zt+i}}{Q_{zt}}.$$

Given the evolution of \bar{G}_{jzt} implied by equation (9), this equation can be rearranged to obtain

$$\frac{Q_{jzt}}{Q_{jzt+i}} = \frac{\prod_{k=1}^i q_{zt+k}}{\prod_{k=1}^i g_{zt+k}},$$

which is independent of the product index j and reduces to $Q_{jzt}/Q_{jzt+i} = 1$ for $i = 0$. Using the previous equation, we can express the numerator on the r.h.s. of equation (52), denoted by N_{zt} , recursively as

$$N_{zt} = \frac{MC_t}{P_t A_{zt} Q_{zt}} + \alpha_z(1 - \delta_z) E_t \left[\Omega_{t,t+1} \left(\frac{P_{zt+1}}{P_{zt}} \right)^{\theta-1} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{q_{zt+1}}{g_{zt+1}} \right) N_{zt+1} \right]. \quad (53)$$

We can also express the denominator on the r.h.s. of equation (52), denoted by D_{zt} , recursively as

$$D_{zt} = 1 + \alpha_z(1 - \delta_z) E_t \left[\Omega_{t,t+1} \left(\frac{P_{zt+1}}{P_{zt}} \right)^{\theta-1} \left(\frac{Y_{t+1}}{Y_t} \right) D_{zt+1} \right], \quad (54)$$

which then leads to equation (24) for the optimal price.

A.4 Item Price Level and Its Recursive Evolution Equation

We derive a recursive representation of the item price level P_{zt} in two steps. First, we decompose the price level into the prices of newly entering products, the prices of existing products that are optimally reset in period t , and all remaining prices. Second, we show that optimal reset prices for existing products with age $s \geq 1$ can be expressed as a function of the optimal prices of newly entering products. This relationship allows us to derive the recursive price-level representation. The derivation in the present section follows similar steps as in Adam and Weber (2019) but generalizes it by allowing for idiosyncratic components in productivity and product quality.

From equation (15), we have

$$P_{zt}^{1-\theta} = \int_0^1 P_{jzt}^{1-\theta} dj,$$

where $P_{jzt} = \tilde{P}_{jzt}/Q_{jzt}$ denotes the quality-adjusted price of product j in item z . We decompose this price level into (i) all prices that are adjusted in period t , including prices for newly entering products; (ii) the sticky prices of continuing products. The share of the latter is equal to $\alpha_z(1 - \delta_z)$ and their average price is equal to the lagged item price level. Thus, applying this decomposition to the previous equation yields

$$P_{zt}^{1-\theta} = \sum_{s=0}^{\infty} \int_{J_{t-s,t}^*} (P_{jzt}^*)^{1-\theta} dj + \alpha_z(1 - \delta_z) (P_{zt-1})^{1-\theta}, \quad (55)$$

where $J_{t-s,t}^*$ denotes the set of products with age s in period t that can adjust prices in t . The share of products that can adjust prices in t is equal to $\delta_z + (1 - \delta_z)(1 - \alpha_z)$, where δ_z is the share of newly entering products (all with optimal prices) and $(1 - \delta_z)(1 - \alpha_z)$ is the share of continuing products that can adjust prices. We can define the average optimal price of products newly entering in t as

$$P_{z,t,t}^* \equiv \left(\frac{1}{\delta_z} \int_{J_{t,t}^*} (P_{jzt}^*)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \quad (56)$$

and the average optimal price of products that entered in $t - s$ (for $s \geq 1$) and reset prices in t as

$$P_{z,t-s,t}^* \equiv \left(\frac{1}{(1 - \alpha_z)\delta_z(1 - \delta_z)^s} \int_{J_{t-s,t}^*} (P_{jzt}^*)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (57)$$

Substituting the previous two definitions into equation (55) yields

$$P_{zt}^{1-\theta} = \delta_z (P_{z,t,t}^*)^{1-\theta} + (1 - \alpha_z)\delta_z \sum_{s=1}^{\infty} (1 - \delta_z)^s (P_{z,t-s,t}^*)^{1-\theta} + \alpha_z(1 - \delta_z)(P_{zt-1})^{1-\theta}, \quad (58)$$

where $(1 - \alpha_z)\delta_z \sum_{s=1}^{\infty} (1 - \delta_z)^s + \alpha_z(1 - \delta_z) = 1 - \delta_z$ is equal to the share of continuing products.

In the second step, we use the optimal price setting equation (24) to express the item price level in the previous equation recursively. Consider the pricing equation for product j with age $s_{jzt} = s \geq 1$ and rewrite (24) by substituting G_{jzt} using equation (8) and substituting Q_{jzt} using equation (12). This yields

$$\frac{P_{jzt}^*}{P_t} \left(\frac{Q_{zt-s} \bar{G}_{jzt}}{Q_{zt}} \right) [\epsilon_{jz,t-s}^Q \epsilon_{jz,t-s}^G] = \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) \frac{N_{zt}}{D_{zt}}, \quad (59)$$

where the term in brackets captures the idiosyncratic component of the optimal price, which is constant over the product's lifetime. Since the previous equation refers to products with the same age, we can use equation (9) to rewrite \bar{G}_{jzt} and equation (13) to rewrite Q_{zt-s}/Q_{zt} . This yields

$$\frac{P_{jzt}^*}{P_t} \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right) [\epsilon_{jz,t-s}^Q \epsilon_{jz,t-s}^G] = \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) \frac{N_{zt}}{D_{zt}}. \quad (60)$$

Rearranging the previous equation to obtain the average of the optimal prices of products with the same age s , as defined in equation (57), yields

$$P_{z,t-s,t}^* = \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{-1} \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) \frac{N_{zt} P_t}{D_{zt}}, \quad (61)$$

where we used $E[(\epsilon_{jzt}^G)^{\theta-1}] = 1$ and $E[(\epsilon_{jzt}^Q)^{\theta-1}] = 1$ and the fact that ϵ_{jzt}^G and ϵ_{jzt}^Q are independent.

Analogous steps for the case of products that newly entering in period t deliver the following expression for the optimal average price $P_{z,t,t}^*$ of these products, as defined in equation (56):

$$P_{z,t,t}^* = \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) \frac{N_{zt} P_t}{D_{zt}}. \quad (62)$$

Equations (61) and (62) jointly deliver

$$P_{z,t-s,t}^* = P_{z,t,t}^* \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{-1}, \quad (63)$$

for $s \geq 1$. This equation shows how the optimal average price of older products is related to the optimal average price of newly entering products. Using the previous equation to substitute for

$P_{z,t-s,t}^*$ in equation (58) and rearranging the result yields

$$P_{zt}^{1-\theta} = (P_{z,t,t}^*)^{1-\theta} \left\{ \alpha_z \delta_z + (1 - \alpha_z) \left[\delta_z + \sum_{s=1}^{\infty} \delta_z (1 - \delta_z)^s \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{\theta-1} \right] \right\} + \alpha_z (1 - \delta_z) (P_{zt-1})^{1-\theta}. \quad (64)$$

Now define

$$(\Delta_{zt}^e)^{1-\theta} \equiv \delta_z + \sum_{s=1}^{\infty} \delta_z (1 - \delta_z)^s \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{\theta-1}, \quad (65)$$

and substitute this definition into equation (64). This delivers the recursive representation of the item price level:

$$P_{zt}^{1-\theta} = \{ \alpha_z \delta_z + (1 - \alpha_z) (\Delta_{zt}^e)^{1-\theta} \} (P_{z,t,t}^*)^{1-\theta} + \alpha_z (1 - \delta_z) (P_{zt-1})^{1-\theta}, \quad (66)$$

where $P_{z,t,t}^*$ is defined in equation (56). Finally, we rewrite the definition of Δ_{zt}^e according to

$$\begin{aligned} (\Delta_{zt}^e)^{1-\theta} &= \delta_z + (1 - \delta_z) \left(\frac{g_{zt}}{q_{zt}} \right)^{\theta-1} \left(\delta_z + \sum_{s=1}^{\infty} \delta_z (1 - \delta_z)^s \left(\frac{\prod_{k=0}^{s-1} g_{zt-1-k}}{\prod_{k=0}^{s-1} q_{zt-1-k}} \right)^{\theta-1} \right) \\ &= \delta_z + (1 - \delta_z) (\Delta_{zt-1}^e q_{zt} / g_{zt})^{1-\theta}, \end{aligned} \quad (67)$$

which shows that $(\Delta_{zt}^e)^{1-\theta}$ is a stationary variable that evolves recursively. We define the item-level (gross) inflation rate as

$$\Pi_{zt} \equiv P_{zt} / P_{zt-1}$$

and the relative price p_{zt}^* as

$$p_{zt}^* \equiv P_{z,t,t}^* / P_{zt}. \quad (68)$$

Using these definitions, we rearrange equation (66) to obtain

$$1 = \{ \alpha_z \delta_z + (1 - \alpha_z) (\Delta_{zt}^e)^{1-\theta} \} (p_{zt}^*)^{1-\theta} + \alpha_z (1 - \delta_z) (\Pi_{zt})^{\theta-1}. \quad (69)$$

The previous equation shows that in a balanced growth path with a constant item-level inflation Π_z , the relative price p_z^* is also constant.

A.5 Item-Level and Economy-Wide Aggregate Production Functions

We aggregate the model in two steps. In a first step, we aggregate firm-specific production functions to item-level production functions. In a second step, we aggregate the item-level production functions to a economy-wide production function.

To obtain the item-level production function, we substitute (quality-adjusted) output of product j in item z in the production function (6) using the demand function (18). This yields

$$\frac{Y_{zt}}{A_{zt} Q_{jzt} G_{jzt}} \left(\frac{P_{jzt}}{P_{zt}} \right)^{-\theta_z} = \left(\frac{K_{jzt}}{L_{jzt}} \right)^{1-\frac{1}{\phi}} L_{jzt}.$$

Integrating the previous equation over all firms $j \in [0, 1]$ in item z , using the definition

$$L_{zt} \equiv \int L_{jzt} \, dj,$$

and equation (48), which shows that capital-to-labor ratio is identical for all products, we obtain the item-level production function for quality-adjusted output in item z

$$Y_{zt} = \frac{A_{zt}Q_{zt}}{\Delta_{zt}} \left(K_{zt}^{1-\frac{1}{\phi}} L_{zt}^{\frac{1}{\phi}} \right), \quad (70)$$

where

$$K_{zt} \equiv \int L_{jzt} \, dj$$

and where we have defined the productivity parameter $1/\Delta_{zt}$ as

$$\Delta_{zt} \equiv \int_0^1 \left(\frac{Q_{zt}}{Q_{jzt}G_{jzt}} \right) \left(\frac{P_{jzt}}{P_{zt}} \right)^{-\theta_z} \, dj, \quad (71)$$

which captures the (detrended) distribution of productivities and qualities across products in item z . The recursive evolution equation for Δ_{zt} is derived in appendix A.6.

To obtain the economy-wide aggregate production function, we rewrite equation (70) to obtain

$$Y_{zt} \frac{\Delta_{zt}}{A_{zt}Q_{zt}} = \left(\frac{K_t}{L_t} \right)^{1-\frac{1}{\phi}} L_{zt}.$$

where we used the fact that the capital-to-labor ratio is the same across items, see equation (48). Summing the previous equation over all items $z = 1, \dots, Z$, and using labor market clearing across items, $L_t = \sum_z L_{zt}$, and the demand function (19) to substitute for item-level output Y_{zt} , we obtain

$$Y_t \sum_{z=1}^{Z_t} \psi_{zt} \left(\frac{P_{zt}}{P_t} \right)^{-1} \left(\frac{\Delta_{zt}}{A_{zt}Q_{zt}} \right) = K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}}.$$

The aggregate economy-wide production function for quality-adjusted output is thus given by

$$Y_t = \frac{(\Gamma_t^e)^{1/\phi}}{\Delta_t} \left(K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} \right), \quad (72)$$

where the aggregate economy-wide productivity parameter $1/\Delta_t$ is defined according to

$$\Delta_t \equiv (\Gamma_t^e)^{1/\phi} \sum_{z=1}^{Z_t} \psi_{zt} \left(\frac{P_{zt}}{P_t} \right)^{-1} \left(\frac{\Delta_{zt}}{A_{zt}Q_{zt}} \right), \quad (73)$$

and where Γ_t^e denotes the trend-growth factor defined in Appendix B.4 and ensures that Δ_t a stationary variable.

A.6 Derivation of the Recursive Evolution Equation for Δ_{zt}

To derive a recursive representation for the productivity shifter Δ_{zt} , defined in equation (71), we decompose it in a way that resembles the decomposition of the item price level in Appendix A.4. This yields

$$\frac{\Delta_{zt}}{P_{zt}^\theta} = \sum_{s=0}^{\infty} \int_{J_{t-s,t}^*} \left(\frac{Q_{zt}}{Q_{jzt} G_{jzt}} \right) (P_{jzt}^*)^{-\theta} dj + \frac{q_{zt}}{g_{zt}} \int_{J_t} \left(\frac{Q_{zt-1}}{Q_{jzt-1} G_{jzt-1}} \right) (P_{jzt-1})^{-\theta} dj, \quad (74)$$

where, as before, $J_{t-s,t}^*$ denotes the set of products with age $s \geq 0$ at time t that can adjust prices in t . Let J_t denote the set of all products that can not adjust prices in t . To derive equation (74), we have used the fact that all products in J_t have age $s \geq 1$. We have also used the fact that the productivity component \bar{G}_{jzt} for the products in $J_{t-1,t}$ continues to evolve over time, which yields

$$\begin{aligned} G_{jzt} &= \bar{G}_{jzt} \cdot \epsilon_{jzt-1}^G \\ &= \left(\frac{\bar{G}_{jzt}}{\bar{G}_{jzt-1}} \right) (\bar{G}_{jzt-1} \cdot \epsilon_{jzt-1}^G) \\ &= g_{zt} G_{jzt-1}, \end{aligned} \quad (75)$$

where the last line follows from equations (8) and (9) for the case with $s \geq 1$.

Since products in J_t are a representative subset of all products in the economy at date $t-1$ and since J_t has mass $\alpha_z(1-\delta_z)$, we can rewrite equation (74) by shifting equation (71) one period into the past, which yields

$$\frac{\Delta_{zt}}{P_{zt}^\theta} = \sum_{s=0}^{\infty} \int_{J_{t-s,t}^*} \left(\frac{Q_{zt}}{Q_{jzt} G_{jzt}} \right) (P_{jzt}^*)^{-\theta} dj + \alpha_z(1-\delta_z) \frac{q_{zt}}{g_{zt}} \frac{\Delta_{zt-1}}{P_{zt-1}^\theta}. \quad (76)$$

We now rearrange the infinite sum in the previous equation. The steps involved in this resemble the steps used in the derivation of the item price level in Appendix A.4, but with slight modifications. We first show how the integrals appearing in the infinite sum on the r.h.s. of equation (76) are related to the average optimal price of newly entering products $P_{z,t,t}^*$. For $s \geq 1$, we obtain

$$\int_{J_{t-s,t}^*} \left(\frac{Q_{zt}}{Q_{jzt} G_{jzt}} \right) (P_{jzt}^*)^{-\theta} dj = \left(\frac{\prod_{k=0}^{s-1} q_{zt-k}}{\prod_{k=0}^{s-1} g_{zt-k}} \right) \int_{J_{t-s,t}^*} \left[\frac{Q_{zt-s}}{Q_{jzt-s} G_{jzt-s}} \right] (P_{jzt}^*)^{-\theta} dj, \quad (77)$$

using $Q_{zt} = (\prod_{k=0}^{s-1} q_{zt-k}) Q_{zt-s}$ and the fact that products in $J_{t-s,t}^*$ have age greater or equal to s . We can rearrange the r.h.s. of equation (77) further using

$$G_{jzt} = \left(\prod_{k=0}^{s-1} g_{zt-k} \right) G_{jzt-s},$$

which follows from (75). The brackets in equation (77) contain only idiosyncratic components and thus simplify as

$$\frac{Q_{zt-s}}{Q_{jzt-s} G_{jzt-s}} = [\epsilon_{jz,t-s}^Q \epsilon_{jz,t-s}^G]^{-1}.$$

Substituting the previous two equations into equation (77) and integrating the result over the products in $J_{t-s,t}^*$ yields

$$\int_{J_{t-s,t}^*} \left(\frac{Q_{zt}}{Q_{jzt}G_{jzt}} \right) (P_{jzt}^*)^{-\theta} dj = \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{-1} \int_{J_{t-s,t}^*} [\epsilon_{jz,t-s}^Q \epsilon_{jz,t-s}^G]^{-1} (P_{jzt}^*)^{-\theta} dj. \quad (78)$$

To link the previous equation to the average optimal price of newly entering products $P_{z,t,t}^*$, we rearrange equation (61) to obtain

$$[\epsilon_{jz,t-s}^Q \epsilon_{jz,t-s}^G]^{-1} (P_{jzt}^*)^{-\theta} = [\epsilon_{jz,t-s}^Q \epsilon_{jz,t-s}^G]^{\theta-1} \left[\left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{-1} \left(\frac{\theta}{\theta-1} \frac{1}{1+\tau} \right) \frac{N_{zt} P_t}{D_{zt}} \right]^{-\theta}.$$

Integrating the previous equation over the set of products in $J_{t-s,t}^*$ and normalizing the result yields

$$\int_{J_{t-s,t}^*} \frac{[\epsilon_{jz,t-s}^Q \epsilon_{jz,t-s}^G]^{-1}}{(1-\alpha_z)\delta_z(1-\delta_z)^s} (P_{jzt}^*)^{-\theta} dj = \left[\left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{-1} \left(\frac{\theta}{\theta-1} \frac{1}{1+\tau} \right) \frac{N_{zt} P_t}{D_{zt}} \right]^{-\theta},$$

where we used $E[(\epsilon_{jzt}^G)^{\theta-1}] = 1$ and $E[(\epsilon_{jzt}^Q)^{\theta-1}] = 1$ and the fact that ϵ_{jzt}^G and ϵ_{jzt}^Q are independent. We can now use equation (62) to substitute $P_{z,t,t}^*$ into the previous equation, which yields

$$\int_{J_{t-s,t}^*} \frac{[\epsilon_{jz,t-s}^Q \epsilon_{jz,t-s}^G]^{-1}}{(1-\alpha_z)\delta_z(1-\delta_z)^s} (P_{jzt}^*)^{-\theta} dj = \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{\theta} (P_{z,t,t}^*)^{-\theta}.$$

Furthermore, substituting the previous equation for the r.h.s. of equation (78) yields

$$\int_{J_{t-s,t}^*} \left(\frac{Q_{zt}}{Q_{jzt}G_{jzt}} \right) (P_{jzt}^*)^{-\theta} dj = (1-\alpha_z)\delta_z(1-\delta_z)^s \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{\theta-1} (P_{z,t,t}^*)^{-\theta},$$

which shows how the integral terms on the r.h.s. of equation (76) are related to the average optimal price of newly entering products $P_{z,t,t}^*$ for $s \geq 1$. For the case with $s = 0$, analogous steps yield

$$\int_{J_{t,t}^*} [\epsilon_{jzt}^Q \epsilon_{jzt}^G]^{-1} (P_{jzt}^*)^{-\theta} dj = \delta_z (P_{z,t,t}^*)^{-\theta}.$$

Using the preceding two equations to substitute for the integrals in the infinite sum on the r.h.s. of equation (76), we obtain

$$\frac{\Delta_{zt}}{P_{zt}^\theta} = (P_{z,t,t}^*)^{-\theta} \left\{ \delta_z + (1-\alpha_z) \sum_{s=1}^{\infty} \delta_z (1-\delta_z)^s \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{\theta-1} \right\} + \alpha_z (1-\delta_z) \frac{q_{zt}}{g_{zt}} \frac{\Delta_{zt-1}}{P_{zt-1}^\theta},$$

where the term in curly brackets is the same as the term in curly brackets in equation (64). Accordingly, rearranging the previous equation yields the recursive representation

$$\Delta_{zt} = (p_{zt}^*)^{-\theta} \{ \alpha_z \delta_z + (1-\alpha_z) (\Delta_{zt}^e)^{1-\theta} \} + \alpha_z (1-\delta_z) (\Pi_{zt})^\theta (g_{zt}/q_{zt})^{-1} \Delta_{zt-1},$$

where $\Pi_{zt} = P_{zt}/P_{zt-1}$. The stationary variable Δ_{zt}^e evolves as described in equation (67) and p_{zt}^* is defined in equation (68). The previous equation shows that Δ_{zt} is constant in the balanced growth path, because p_{zt}^* is constant in this path due to equation (69).

B Efficient Allocation and Efficient Growth Trends

As a reference point and to better understand the distortions emerging in the decentralized economy, this section derives the first-best allocation. This involves deriving the allocation of factor inputs across products with different levels of product quality and productivity at the level of each expenditure item z , in addition to the allocation of factor inputs across items z with different average quality and productivity. It also requires determining the optimal intertemporal paths of aggregate variables. This appendix also derives the growth trend of variables in the efficient allocation. Using the efficient trends we derive expressions for the efficient allocation in terms of detrended variables. Throughout the appendix, variables carrying the superscript 'e' denote efficient quantities.

B.1 Efficient Allocation at the Item-Level

Consider a setting where it is efficient to allocate L_{zt}^e units of labor and K_{zt}^e units of capital to the production of products in item z . The optimal allocation of capital and labor across products j in item z maximizes then (quality-adjusted) item-level output/consumption in equation (5), subject to the production function (6) and the feasibility constraints $L_{zt}^e = \int_z L_{jzt}^e dj$ and $K_{zt}^e = \int_z K_{jzt}^e dj$. This allocation problem yields the efficient item-level output

$$Y_{zt}^e = \frac{A_{zt}Q_{zt}}{\Delta_{zt}^e} (K_{zt}^e)^{1-\frac{1}{\phi}} (L_{zt}^e)^{\frac{1}{\phi}}, \quad (79)$$

where the efficient productivity parameters $1/\Delta_{zt}^e$ is defined as

$$1/\Delta_{zt}^e \equiv \left(\int_0^1 (G_{jzt}Q_{jzt}/Q_{zt})^{\theta-1} dj \right)^{\frac{1}{\theta-1}}.$$

To derive a recursive representation for $1/\Delta_{zt}^e$, we rearrange the previous equation to obtain

$$(\Delta_{zt}^e)^{1-\theta} = \delta_z \sum_{s=0}^{\infty} (1-\delta_z)^s \frac{1}{\delta_z(1-\delta_z)^s} \int_{J_{t-s,t}} (G_{jzt}Q_{jzt}/Q_{zt})^{\theta-1} dj, \quad (80)$$

where $J_{t-s,t}$ denotes the set of products with age $s \geq 0$ in period t . The integrals appearing on the r.h.s. of the infinite sum in the previous equation can be expressed as

$$\frac{1}{\delta_z(1-\delta_z)^s} \int_{J_{t-s,t}} (G_{jzt}Q_{jzt}/Q_{zt})^{\theta-1} dj = \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{\theta-1},$$

since $E[(\epsilon_{jzt}^Q)^{\theta-1}] = 1$ and $E[(\epsilon_{jzt}^G)^{\theta-1}] = 1$ and ϵ_{jzt}^Q and ϵ_{jzt}^G are independent. Plugging the previous equation into equation (80) yields equation (65) which as is shown in appendix A.4, has the recursive representation described in equation (67).

B.2 Efficient Allocation Across Items

The optimal allocation of capital and labor between items maximizes (quality-adjusted) aggregate output/consumption in equation (4), subject to the efficient item-level production function (79) and the feasibility conditions $L_t^e = \sum_z L_{zt}^e$ and $K_t^e = \sum_z K_{zt}^e$, for given levels of L_t^e and K_t^e . Solving this allocation problem delivers the aggregate economy-wide efficient production function

$$Y_t^e = \frac{(\Gamma_t^e)^{1/\phi}}{\Delta_t^e} (K_t^e)^{1-\frac{1}{\phi}} (L_t^e)^{\frac{1}{\phi}}, \quad (81)$$

where the efficient productivity level $1/\Delta_t^e$ is defined as

$$\frac{1}{\Delta_t^e} \equiv (\Gamma_t^e)^{-\frac{1}{\phi}} \left(\prod_{z=1}^{Z_t} \psi_{zt}^{\psi_{zt}} \left(\frac{A_{zt} Q_{zt}}{\Delta_{zt}^e} \right)^{\psi_{zt}} \right), \quad (82)$$

and Γ_t^e denotes the aggregate growth rate defined in Appendix B.4 and ensures that Δ_t^e a stationary variable.

B.3 Efficient Intertemporal Allocation

The intertemporal allocation maximizes expected discounted utility of the representative household, equation (2), subject to the intertemporal feasibility condition

$$C_t^e + K_{t+1}^e = (1-d)K_t^e + Y_t^e \quad (83)$$

and the aggregate economy-wide production function (81). The first order conditions to this problem comprise the feasibility condition (83) and

$$Y_{Lt}^e = -\frac{U_{Lt}^e}{U_{Ct}^e}, \quad (84)$$

$$1 = \beta E_t \left[\frac{U_{Ct+1}^e}{U_{Ct}^e} (Y_{Kt+1}^e + 1 - d) \right], \quad (85)$$

where U_{Ct} denotes the marginal utility of consumption in t , U_{Lt} the marginal disutility from labor, Y_{Kt}^e the marginal product of capital and Y_{Lt}^e the marginal product of labor.

B.4 Efficient Item-Level and Aggregate Growth Trends

This section determined the efficient growth for the balanced growth path equilibrium in which aggregate hours worked L_t^e and item-level hours worked L_{zt}^e are stationary for all z . The variables C_t^e , K_t^e and Y_t^e all display the same growth trend, which we denote by Γ_t^e . Since the capital-to-labor ratio is constant across products, it then follows the item-level capital stocks K_{zt} have the same growth trend Γ_t^e for all z .

We can then derive the item-level output growth trend by rewriting equation (79) as

$$Y_{zt}^e = \frac{A_{zt}Q_{zt}}{\Delta_{zt}^e} (\Gamma_t^e)^{1-\frac{1}{\phi}} \left(\frac{K_{zt}^e}{\Gamma_t^e} \right)^{1-\frac{1}{\phi}} (L_{zt}^e)^{\frac{1}{\phi}},$$

which shows that Y_{zt}^e grows at the same rate as $\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e} (\Gamma_t^e)^{1-\frac{1}{\phi}}$ because all other variables are stationary. We can thus define the item-level growth trend as

$$\Gamma_{zt}^e \equiv \frac{A_{zt}Q_{zt}}{\Delta_{zt}^e} (\Gamma_t^e)^{1-\frac{1}{\phi}}. \quad (86)$$

To derive the aggregate growth trend Γ_t^e , we substitute equilibrium output for equilibrium consumption in equation (4) and detrend all output variables in the resulting equation by their respective growth trends, which yields

$$\frac{Y_t^e}{\Gamma_t^e} = \left[\frac{\prod_{z=1}^{Z_t} (\Gamma_{zt}^e)^{\psi_{zt}}}{\Gamma_t^e} \right] \prod_{z=1}^{Z_t} \left(\frac{Y_{zt}^e}{\Gamma_{zt}^e} \right)^{\psi_{zt}}.$$

Since Y_{zt}^e/Γ_{zt}^e is stationary, the the aggregate growth trend is given by

$$\Gamma_t^e \equiv \prod_{z=1}^{Z_t} (\Gamma_{zt}^e)^{\psi_{zt}}. \quad (87)$$

Using definition (86) to substitute for Γ_{zt}^e in the previous equation and solving for Γ_t^e yields

$$\Gamma_t^e = \prod_{z=1}^{Z_t} \left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e} \right)^{\phi\psi_{zt}}, \quad (88)$$

which determines the aggregate growth trend in terms of model primitives. Substituting the previous equation for Γ_t^e into equation (86) shows that the item-level growth trend relative to the aggregate growth trend is independent of the parameter ϕ and given by

$$\frac{\Gamma_{zt}^e}{\Gamma_t^e} = \frac{\left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e} \right)}{\prod_{z=1}^{Z_t} \left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e} \right)^{\psi_{zt}}}. \quad (89)$$

We also define the aggregate growth rate as

$$\gamma_t^e \equiv \Gamma_t^e / \Gamma_{t-1}^e. \quad (90)$$

Using equation (88) to substitute for Γ_t^e and Γ_{t-1}^e we obtain:

$$\gamma^e = \prod_{z=1}^Z (a_z q_z)^{\psi_z \phi} \quad (91)$$

in the steady state. Furthermore, we define the item-level growth rate as

$$\gamma_{zt}^e \equiv \Gamma_{zt}^e / \Gamma_{zt-1}^e, \quad (92)$$

and using equation (89), we obtain that in steady state,

$$\frac{\gamma_z^e}{\gamma^e} = \frac{a_z q_z}{\prod_{z=1}^Z (a_z q_z)^{\psi_z}}.$$

B.5 Efficient Production in Terms of Detrended Variables

We now express the item-level and aggregate production functions in the planned economy in terms of detrended output and capital variables. Letting lower case letters denote stationary variables, we can define $y_t^e \equiv Y_t^e/\Gamma_t^e$, $k_t^e \equiv K_t^e/\Gamma_t^e$, $k_{zt}^e \equiv K_{zt}^e/\Gamma_t^e$ and $y_{zt}^e \equiv Y_{zt}^e/\Gamma_{zt}^e$. To obtain the production function in item z in terms of detrended variables, we divide equation (79) by equation (86) and use the definitions of item-level detrended variables. This yields

$$y_{zt}^e = (k_{zt}^e)^{1-\frac{1}{\phi}} (L_{zt}^e)^{\frac{1}{\phi}}. \quad (93)$$

To obtain the aggregate production function in terms of detrended variables, we divide equation (81) by Γ_t^e and use the definitions of aggregate detrended variables, which yields

$$y_t^e = \frac{1}{\Delta_t^e} (k_t^e)^{1-\frac{1}{\phi}} (L_t^e)^{\frac{1}{\phi}}. \quad (94)$$

Here, $1/\Delta_t^e$ is defined in equation (82), and this definition simplifies to

$$\frac{1}{\Delta_t^e} = \prod_{z=1}^{Z_t} \psi_{zt}^{\psi_{zt}}, \quad (95)$$

after substituting the equation (88) for Γ_t^e into the definition.

C The Decentralized Economy and its Distortions

We now express the prices and allocations in the decentralized economy in terms of detrended variables, using the efficient growth trends derived in the previous appendix to detrend quantities. We then relate the allocation in the decentralized economy to the first-best allocation derived in the previous section using two key distortions (or wedges), namely a mark-up distortion and a relative-price distortion.

Appendices C.1 and C.2 start by deriving the growth trends of relative prices and express optimal reset prices in terms of detrended variables. Appendix C.3 introduces the mark-up distortion and uses it to rewrite various first-order conditions of households and firms. Appendix C.4 derives the item-level and aggregate production functions for the decentralized economy and relates them to the efficient allocation by introducing a relative-price distortion term. Appendix C.5 summarizes the equations characterizing the decentralized economy in detrended variables.

C.1 Relative Price Trends and Relative Inflation Rates

To detrend the relative price of item z , P_{zt}/P_t , we multiply the demand function (19) by the (inverse of the) relative growth factor Γ_{zt}^e/Γ_t^e , which yields

$$y_{zt}/y_t = \psi_{zt} p_{zt}^{-1}, \quad (96)$$

where we have defined

$$p_{zt} \equiv (P_{zt}/P_t) (\Gamma_{zt}^e/\Gamma_t^e), \quad (97)$$

which is constant in steady state. The demand function (19) also implies

$$\frac{\Pi_{zt}}{\Pi_t} = \left(\frac{\psi_{zt}}{\psi_{zt-1}} \right) \left(\frac{\gamma_{zt}^e y_{zt}/y_{zt-1}}{\gamma_t^e y_t/y_{t-1}} \right)^{-1},$$

which shows that items with stronger price increases face stronger output declines, which is a result of Cobb-Douglas aggregation across expenditure items.

C.2 Optimal Price in Terms of Detrended Variables

To express the optimal reset price in equation (24) in terms of detrended variables, we multiply the equation by the relative sectoral growth trend, Γ_{zt}^e/Γ_t^e (see Appendix B) and divide by item price level P_{zt} . This yields

$$\frac{P_{jzt}^*}{P_{zt}} \left(\frac{Q_{jzt} G_{jzt}}{Q_{zt}} \right) p_{zt} = \left(\frac{1}{1 + \tau} \frac{\theta}{\theta - 1} \right) \frac{N_{zt}}{D_{zt}} \left(\frac{\Gamma_{zt}^e}{\Gamma_t^e} \right), \quad (98)$$

where p_{zt} is defined in equation (97). Since D_{zt} is stationary, see equation (54), we can define

$$d_{zt} \equiv D_{zt}. \quad (99)$$

The variable N_{zt} in equation (98) grows over time, but the variable

$$n_{zt} \equiv N_{zt} \left(\frac{\Gamma_{zt}^e}{\Gamma_t^e} \right) \quad (100)$$

is again stationary, as we show below. Using these definitions, we can thus write equation (98) in terms of stationary variables according to

$$\frac{P_{jzt}^*}{P_{zt}} \left(\frac{Q_{jzt} G_{jzt}}{Q_{zt}} \right) p_{zt} = \left(\frac{1}{1 + \tau} \frac{\theta}{\theta - 1} \right) \frac{n_{zt}}{d_{zt}}. \quad (101)$$

It remains to prove the stationarity of n_{zt} . Using the definition of n_{zt} and equation (53) delivers

$$\begin{aligned} n_{zt} &= \left(\frac{MC_t}{P_t A_{zt} Q_{zt}} \right) \left(\frac{\Gamma_{zt}^e}{\Gamma_t^e} \right) \\ &+ \alpha_z (1 - \delta_z) E_t \left[\Omega_{t,t+1} \left(\frac{P_{zt+1}}{P_{zt}} \right)^{\theta-1} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{q_{zt+1}}{g_{zt+1}} \right) \left(\frac{\Gamma_{zt}^e}{\Gamma_t^e} \right) \left(\frac{\Gamma_{zt+1}^e}{\Gamma_{t+1}^e} \right)^{-1} n_{zt+1} \right] \end{aligned}$$

or equivalently

$$\begin{aligned} n_{zt} &= \left(\frac{MC_t}{P_t A_{zt} Q_{zt}} \right) \left(\frac{\Gamma_{zt}^e}{\Gamma_t^e} \right) \\ &+ \alpha_z (1 - \delta_z) E_t \left[\Omega_{t,t+1} \Pi_{zt+1}^{\theta-1} \Pi_{t+1} (y_{t+1}/y_t) \gamma_{t+1}^e \left(\frac{q_{zt+1}}{g_{zt+1}} \right) \left(\frac{\gamma_{t+1}^e}{\gamma_{zt+1}^e} \right) n_{zt+1} \right]. \quad (102) \end{aligned}$$

We can rewrite equation (86) to obtain

$$\frac{\Gamma_{zt}^e}{\Gamma_t^e} = (\Gamma_t^e)^{-\frac{1}{\phi}} \left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e} \right),$$

and use this equation to rearrange the term involving marginal costs in equation (102) according to

$$\left(\frac{MC_t}{P_t A_{zt} Q_{zt}} \right) \left(\frac{\Gamma_{zt}^e}{\Gamma_t^e} \right) = \left(\frac{MC_t}{P_t A_{zt} Q_{zt}} \right) (\Gamma_t^e)^{-\frac{1}{\phi}} \left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e} \right) = \left(\frac{MC_t}{P_t (\Gamma_t^e)^{1/\phi}} \right) \left(\frac{1}{\Delta_{zt}^e} \right).$$

We then define real detrended marginal costs as

$$mc_t \equiv \frac{MC_t}{P_t (\Gamma_t^e)^{1/\phi}}, \quad (103)$$

where MC_t is defined in equation (21). Substituting the previous equation into equation (102) yields

$$n_{zt} = \frac{mc_t}{\Delta_{zt}^e} + \alpha_z (1 - \delta_z) E_t \left[\Omega_{t,t+1} \Pi_{zt+1}^{\theta-1} \Pi_{t+1} (y_{t+1}/y_t) \gamma_{t+1}^e \left(\frac{q_{zt+1}}{g_{zt+1}} \right) \left(\frac{\gamma_{t+1}^e}{\gamma_{zt+1}^e} \right) n_{zt+1} \right],$$

which contains only stationary variables. From equation (54) and the definition of d_{zt} we likewise obtain

$$d_{zt} = 1 + \alpha_z (1 - \delta_z) E_t \left[\Omega_{t,t+1} \Pi_{zt+1}^{\theta-1} (y_{t+1}/y_t) \gamma_{t+1}^e d_{zt+1} \right].$$

To obtain a detrended expression for the average optimal price of new products, we integrate equation (101) over the set of newly entering products in t , normalize the resulting equation and use the assumptions $E[(\epsilon_{jzt}^G)^{\theta-1}] = 1$ and $E[(\epsilon_{jzt}^Q)^{\theta-1}] = 1$ and independence of ϵ_{jzt}^G and ϵ_{jzt}^Q . This yields

$$p_{zt}^* p_{zt} = \left(\frac{1}{1 + \tau \theta - 1} \right) \frac{n_{zt}}{d_{zt}}, \quad (104)$$

where we have also used the definition (68).

C.3 Aggregate Mark-Up Distortions

We define the average markup μ_{zt} at the item level as the relative price of item z over real marginal costs (all in detrended terms),

$$\mu_{zt} \equiv \frac{p_{zt}}{mc_t}, \quad (105)$$

and the aggregate markup as

$$\mu_t \equiv \prod_{z=1}^{Z_t} \mu_{zt}^{\psi_{zt}}. \quad (106)$$

Substituting equation (106) for μ_{zt} into the previous equation, we obtain

$$\mu_t = mc_t^{-1} \prod_{z=1}^{Z_t} p_{zt}^{\psi_{zt}}.$$

Expressing the aggregate price in equation (16) in terms of detrended relative prices and also using equation (95), we obtain from the previous equation

$$\mu_t = \frac{1}{mc_t \Delta_t^e}. \quad (107)$$

Using the definition (103) and equation (21), we obtain

$$mc_t = \left(\frac{k_t}{L_t} \right)^{\frac{1}{\phi}} \left(\frac{r_t}{1 - 1/\phi} \right),$$

where we have also used equation (48) determining the optimal input mix. Substituting into the previous equation the expression for the markup and rearranging yields

$$r_t = \mu_t^{-1} \left(1 - \frac{1}{\phi} \right) \frac{1}{\Delta_t^e} \left(\frac{k_t}{L_t} \right)^{-\frac{1}{\phi}}. \quad (108)$$

Analogous steps deliver

$$w_t = \mu_t^{-1} \left(\frac{1}{\phi} \right) \frac{1}{\Delta_t^e} \left(\frac{k_t}{L_t} \right)^{1-\frac{1}{\phi}}. \quad (109)$$

The previous two equations show how the capital-to-labor ratio gets distorted by the aggregate markup μ_t .

C.4 Relative Price Distortions

We define detrended variables according to $y_t \equiv Y_t/\Gamma_t^e$, $k_t \equiv K_t/\Gamma_t^e$, $k_{zt} \equiv K_{zt}/\Gamma_t^e$ and $y_{zt} \equiv Y_{zt}/\Gamma_{zt}^e$. To obtain the production function in item z in terms of detrended variables, we rewrite equation (70) as

$$\frac{Y_{zt}}{\Gamma_{zt}^e} = \left[\frac{(\Gamma_t^e)^{1-\frac{1}{\phi}} A_{zt} Q_{zt}}{\Gamma_{zt}^e \Delta_{zt}} \right] \left(\frac{K_{zt}}{\Gamma_t^e} \right)^{1-\frac{1}{\phi}} L_{zt}^{\frac{1}{\phi}}.$$

Using the definitions for detrended variables and the definition of the item-level growth trend in equation (86), we obtain a production function in detrended variables:

$$y_{zt} = \left(\frac{\Delta_{zt}^e}{\Delta_{zt}} \right) k_{zt}^{1-\frac{1}{\phi}} L_{zt}^{\frac{1}{\phi}}. \quad (110)$$

In a situation in which relative prices in the decentralized economy are efficient, we have

$$\Delta_{zt} = \Delta_{zt}^e,$$

such that equation (110) becomes equal to the efficient production function in the planner solution, see equation (93). Item-level distortions arising from inefficient price dispersion can thus be captured by the item-level distortion factor

$$\rho_{zt} \equiv \Delta_{zt}^e / \Delta_{zt} \leq 1 \quad (111)$$

We obtain the aggregate production function in detrended variables for the decentralized economy by dividing equation (72) by Γ_t^e and using the definitions of aggregate detrended variables:

$$y_t = \left(\frac{\Delta_t^e}{\Delta_t} \right) \left(\frac{1}{\Delta_t^e} \right) k_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}}. \quad (112)$$

We can then define an aggregate distortion factor capturing inefficiencies associated with relative price distortions across all items:

$$\rho_t \equiv \Delta_t^e / \Delta_t \leq 1. \quad (113)$$

When relative prices are efficient, we have $\rho_t = 1$, so that the aggregate production function in the decentralized economy (112) becomes equal to the aggregate production function in the planner allocation (94).

We take the inverse of equation (73) and multiply it by Δ_t^e . We simplify the resulting equation by substituting for $(\Gamma_t^e)^{1/\phi}$ using equation (86) and using the definition of p_{zt} in equation (97). This yields

$$\frac{\Delta_t^e}{\Delta_t} = \Delta_t^e \left(\sum_{z=1}^{Z_t} \psi_{zt} p_{zt}^{-1} (\Delta_{zt} / \Delta_{zt}^e) \right)^{-1},$$

and shows that the relative price distortion at the aggregate level is a weighted sum over item-level relative price distortions with weights equal to the item's relative output (recall $y_{zt}/y_t = \psi_{zt} p_{zt}^{-1}$ from equation (96)). We can rearrange the previous equation by using the definition (106) to substitute for p_{zt} and equation (107) to substitute for mc_t in this definition. This yields

$$(\rho_t \mu_t)^{-1} = \sum_{z=1}^{Z_t} \psi_{zt} (\mu_{zt} \rho_{zt})^{-1} \quad (114)$$

and shows that the product of (inverse) aggregate distortion corresponds to the weighted sum of the product of (inverse) item-level distortions.

C.5 Summary of Equations Characterizing the Decentralized Economy

At the aggregate level, the decentralized and detrended economy is summarized by the following four equations:

$$y_t = \left(\frac{\rho_t}{\Delta_t^e} \right) k_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} \quad (115)$$

$$\mu_t^{-1} \left(\frac{1}{\phi} \right) \frac{1}{\Delta_t^e} \left(\frac{k_t}{L_t} \right)^{1-\frac{1}{\phi}} = -c_t \left(\frac{\partial V(L_t) / \partial L_t}{V(L_t)} \right) \quad (116)$$

$$1 = E_t \left[\Omega_{t,t+1} \left\{ \mu_{t+1}^{-1} \left(1 - \frac{1}{\phi} \right) \frac{1}{\Delta_{t+1}^e} \left(\frac{k_{t+1}}{L_{t+1}} \right)^{-\frac{1}{\phi}} + 1 - d \right\} \right] \quad (117)$$

$$\gamma_{t+1}^e k_{t+1} = (1-d)k_t + y_t - c_t. \quad (118)$$

Equation (115) follows from substituting the definition of the relative price distortion (113) into the aggregate production function (112). Equation (116) follows from substituting equation (108) for the wage into the household's first-order condition (47). Equation (117) follows from substituting equation (109) for the real rate into the first-order condition (44). Equations (116) and (117) show how the markup distorts the intra and intertemporal optimal household choices compared to the first-best allocation, see equations (84) and (85). Equation (118) is derived from consolidating the budget constraints of the representative household and the government and expressing the resulting equation in terms of detrended variables.

Equations (115)–(118) determine the variables y_t, k_t, L_t and c_t given values for the aggregate distortions ρ_t and μ_t , which depend on the inflation rate, aggregate growth γ_t^e , the productivity parameter Δ_t^e determined by equation (95) and given the equation for the discount factor

$$\Omega_{t,t+1} = \beta \left(\frac{\gamma_{t+1}^e c_{t+1}}{c_t} \right)^{-\sigma} \left(\frac{V(L_{t+1})}{V(L_t)} \right)^{1-\sigma}.$$

Furthermore, we previously determined in equation (114) and definition (106) that the aggregate markup and relative price distortions are functions of the item-level markup and relative price distortions. These equations are repeated here, jointly with the definitions of item-level markup and relative price distortions (106) and (111), respectively:

$$\begin{aligned} (\rho_t \mu_t)^{-1} &= \sum_{z=1}^{Z_t} \psi_{zt} (\mu_{zt} \rho_{zt})^{-1} \\ \mu_t &= \prod_{z=1}^{Z_t} \mu_{zt}^{\psi_{zt}} \\ \rho_{zt} &= \Delta_{zt}^e / \Delta_{zt} \\ \mu_{zt} &= p_{zt} / m c_t. \end{aligned}$$

Note that the distortions depend on the inflation rate.

The item-level outcomes are described by the following set of equations:

$$1 = \{\alpha_z \delta_z + (1 - \alpha_z)(\Delta_{zt}^e)^{1-\theta}\} (p_{zt}^*)^{1-\theta} + \alpha_z(1 - \delta_z)(\Pi_{zt})^{\theta-1} \quad (119)$$

$$p_{zt}^* p_{zt} = \left(\frac{1}{1 + \tau \theta - 1} \right) \frac{n_{zt}}{d_{zt}} \quad (120)$$

$$n_{zt} = \frac{mC_t}{\Delta_{zt}^e} + \alpha_z(1 - \delta_z) E_t \left[\Omega_{t,t+1} \Pi_{zt+1}^{\theta-1} \Pi_{t+1} (y_{t+1}/y_t) \gamma_{t+1}^e \left(\frac{q_{zt+1}}{g_{zt+1}} \right) \left(\frac{\gamma_{t+1}^e}{\gamma_{zt+1}^e} \right) n_{zt+1} \right] \quad (121)$$

$$d_{zt} = 1 + \alpha_z(1 - \delta_z) E_t \left[\Omega_{t,t+1} \Pi_{zt+1}^{\theta-1} (y_{t+1}/y_t) \gamma_{t+1}^e d_{zt+1} \right] \quad (122)$$

$$\left(\frac{\gamma_{zt}^e}{\gamma_t^e} \right) \Pi_{zt} = \left(\frac{\psi_{zt}}{\psi_{zt-1}} \frac{p_{zt}}{p_{zt-1}} \right) \Pi_t \quad (123)$$

$$\Delta_{zt} = (p_{zt}^*)^{-\theta} \{\alpha_z \delta_z + (1 - \alpha_z)(\Delta_{zt}^e)^{1-\theta}\} + \alpha_z(1 - \delta_z)(\Pi_{zt})^\theta (g_{zt}/q_{zt})^{-1} \Delta_{zt-1} \quad (124)$$

$$(\Delta_{zt}^e)^{1-\theta} = \delta_z + (1 - \delta_z) (\Delta_{zt-1}^e q_{zt}/g_{zt})^{1-\theta} \quad (125)$$

$$mC_t = \left(\frac{w_t}{1/\phi} \right)^{\frac{1}{\phi}} \left(\frac{r_t}{1 - 1/\phi} \right)^{1 - \frac{1}{\phi}} \quad (126)$$

$$r_t k_t = (\phi - 1) w_t L_t \quad (127)$$

$$\gamma_{zt}^e = (\gamma_t^e)^{1 - \frac{1}{\phi}} (a_{zt} q_{zt} \Delta_{zt-1}^e / \Delta_{zt}^e), \quad (128)$$

where inflation Π_t is defined in equation (17) and the aggregate price level in equation (16). Furthermore, the aggregate growth rate γ_t^e is defined in equation (90) and the aggregate growth trend is determined by equation (88).

D Derivation of the Steady State Equations in Section 5

In the steady state, the one-period discount factor in equation (45) is

$$\Omega = \beta(\gamma^e)^{-\sigma}.$$

Using this, equations (115)–(118) simplify to the equations (25)–(28) in the steady state. Furthermore, in the steady state, the aggregate markup in equation (106) and the relative price distortion in equation (114) simplify to equations (30) and (32), respectively. These aggregate distortions are functions of the item-level distortions, which are functions of the aggregate inflation rate. We now derive the steady-state expressions for the item-level distortions μ_z in equation (31) and ρ_z in equation (33).

D.1 Item-Level Relative Price Distortion

To express ρ_z as function of inflation, we consider the equations (119) and (124) in the steady state. This yields

$$\begin{aligned} 1 - \alpha_z(1 - \delta_z)\Pi_z^{\theta-1} &= \{\alpha_z \delta_z + (1 - \alpha_z)(\Delta_z^e)^{1-\theta}\} (p_z^*)^{1-\theta} \\ (1 - \alpha_z(1 - \delta_z)\Pi_z^\theta (g_z/q_z)^{-1}) \Delta_z &= \{\alpha_z \delta_z + (1 - \alpha_z)(\Delta_z^e)^{1-\theta}\} (p_z^*)^{-\theta}. \end{aligned} \quad (129)$$

Dividing both equations by each other yields

$$p_z^* = \Delta_z^{-1} \left(\frac{1 - \alpha_z(1 - \delta_z)\Pi_z^{\theta-1}}{1 - \alpha_z(1 - \delta_z)\Pi_z^\theta(g_z/q_z)^{-1}} \right). \quad (130)$$

Substituting this expression for p_z^* into equation (129) yields

$$\left(\frac{\Delta_z}{\Delta_z^e} \right)^{1-\theta} = \left(\frac{\alpha_z \delta_z (\Delta_z^e)^{\theta-1} + (1 - \alpha_z)}{1 - \alpha_z(1 - \delta_z)\Pi_z^\theta(g_z/q_z)^{-1}} \right) \left(\frac{1 - \alpha_z(1 - \delta_z)\Pi_z^{\theta-1}}{1 - \alpha_z(1 - \delta_z)\Pi_z^\theta(g_z/q_z)^{-1}} \right)^{-\theta}.$$

We substitute for Δ_z^e on the r.h.s. of the previous equation using the steady-state version of equation (125), which yields

$$\frac{\Delta_z}{\Delta_z^e} = \left(\frac{1 - \alpha_z(1 - \delta_z)(g_z/q_z)^{\theta-1}}{1 - \alpha_z(1 - \delta_z)\Pi_z^\theta(g_z/q_z)^{-1}} \right)^{\frac{1}{1-\theta}} \left(\frac{1 - \alpha_z(1 - \delta_z)\Pi_z^{\theta-1}}{1 - \alpha_z(1 - \delta_z)\Pi_z^\theta(g_z/q_z)^{-1}} \right)^{\frac{\theta}{\theta-1}}.$$

Simplifying the previous equation, using the definition (111) and substituting for Π_z using equation (123) in the steady state yields

$$\rho_z(\Pi)^{-1} = \left(\frac{1 - \alpha_z(1 - \delta_z)(g_z/q_z)^{\theta-1}}{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\Pi]^\theta(g_z/q_z)^{-1}} \right) \left(\frac{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1 - \delta_z)(g_z/q_z)^{\theta-1}} \right)^{\frac{\theta}{\theta-1}}, \quad (131)$$

which shows that the item-level relative price distortion can be expressed as function of Π only. Rearranging the previous equation yields equation (33).

D.2 Item-Level Markup Distortion

To express μ_z as function of inflation, we consider the pricing equation (120) in the steady state and substitute for n and d using the equations (121) and (122) in the steady state. This yields

$$\frac{p_z}{mc} = \left(\frac{1}{1 + \tau\theta - 1} \right) \frac{1}{p_z^* \Delta_z^e} \left(\frac{1 - \alpha_z(1 - \delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1 - \delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^\theta(g_z/q_z)^{-1}} \right), \quad (132)$$

where we have also substituted for Π_z using equation (123) in the steady state. Using equation (130), the definition (111) and equation (123) to substitute for Π_z , we obtain

$$\frac{1}{p_z^* \Delta_z^e} = \rho_z(\Pi)^{-1} \left(\frac{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\Pi]^\theta(g_z/q_z)^{-1}} \right)^{-1}.$$

Using the previous equation to substitute for $(p_z^* \Delta_z^e)^{-1}$ on the r.h.s. in equation (132) yields

$$\begin{aligned} \mu_z(\Pi) &= \left(\frac{1}{1 + \tau\theta - 1} \right) \rho_z(\Pi)^{-1} \left(\frac{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\Pi]^\theta(g_z/q_z)^{-1}} \right)^{-1} \\ &\times \left(\frac{1 - \alpha_z(1 - \delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1 - \delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^\theta(g_z/q_z)^{-1}} \right). \end{aligned} \quad (133)$$

Using equation (131) to substitute for $\rho_z(\Pi)^{-1}$ and the definition (105) to substitute for p_z/mc in the previous equation yields equation (31) determining the item-level markup as function of inflation.

D.3 Steady State: Existence Conditions

We now derive the existence conditions for a steady state (or deterministic balanced growth path). First, we need to impose

$$1 > (1 - \delta_z) (g_z/q_z)^{\theta-1}, \quad (134)$$

for all z , so that $1/\Delta_z^e$, which measures quality-adjusted productivity in the efficient economy, see equation (125), has a well-defined steady-state value:

$$\left(\frac{1}{\Delta_z^e}\right)^{\theta-1} = \frac{\delta_z}{1 - (1 - \delta_z) (g_z/q_z)^{\theta-1}},$$

Given the substantial amount of product turnover ($\delta_z \gg 0$), see panel A of Figure 4, and the relatively low rates of relative price decline (g_z/q_z), see figure 2, condition (134) is likely to be fulfilled for reasonable values for the demand elasticity parameter θ .

To insure that the item-level distortions $\rho_z(\Pi)$ and $\mu_z(\Pi)$ in equations (33) and (31) have well-defined steady state values, we furthermore impose

$$1 > \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\Pi]^\theta(g_z/q_z)^{-1} \quad (135)$$

$$1 > \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}, \quad (136)$$

for all z . Since $\alpha_z \ll 1$ and $\delta_z \gg 0$, it follows from the fact that γ^e/γ_z^e and g_z/q_z take on values fairly close to one, that these conditions are easily fulfilled for reasonable values for the demand elasticity parameter θ and plausible (gross) steady-state inflation rates Π .

E Proofs

E.1 Proof of Lemma 1

For the limiting case $\beta(\gamma^e)^{1-\sigma} \rightarrow 1$, we have from item-level distortions in equations (31) and (33) that

$$\mu_z(\Pi) = \left(\frac{1}{1 + \tau\theta - 1} \frac{\theta}{\theta - 1}\right) \rho_z(\Pi)^{-1}. \quad (137)$$

Multiplying the previous equation by $\rho_z(\Pi)$ and substituting the result into equation (32) yields

$$(\rho(\Pi)\mu(\Pi))^{-1} = \left(\frac{1}{1 + \tau\theta - 1} \frac{\theta}{\theta - 1}\right)^{-1},$$

so that

$$\mu(\Pi) = \left(\frac{1}{1 + \tau\theta - 1} \frac{\theta}{\theta - 1}\right) \rho(\Pi)^{-1}.$$

E.2 Proof of Proposition 1

The proof proceed as follows: section E.2.1 derives a convenient formulation for the steady-state solution for general values of $\beta(\gamma^e)^{1-\sigma} < 1$; section 1 considers this formulation for the limiting case $\beta(\gamma^e)^{1-\sigma} \rightarrow 1$ and shows that labor is independent of the inflation rate, whereas consumption depends on the inflation rate only via the aggregate markup distortion; section E.2.3 derives the inflation rate that minimizes the aggregate markup distortion and thus maximizes consumption.

E.2.1 Steady State Solution

We rewrite equations (25) to (28) by expressing the variables y, c and k relative to hours worked L , which yields

$$\frac{y}{L} = \left(\frac{\rho(\Pi)}{\Delta^e} \right) \left(\frac{k}{L} \right)^{1-\frac{1}{\phi}} \quad (138)$$

$$\frac{c}{L} = \frac{1}{\mu(\Pi)} \frac{1}{\Delta^e} \left(\frac{1}{\phi} \right) \left(\frac{k}{L} \right)^{1-\frac{1}{\phi}} \left(-\frac{V(L)}{L \partial V(L)/\partial L} \right) \quad (139)$$

$$\frac{k}{L} = \frac{1}{\mu(\Pi)} \frac{1}{\Delta^e} \left(1 - \frac{1}{\phi} \right) \left(\frac{k}{L} \right)^{1-\frac{1}{\phi}} \left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d \right)^{-1} \quad (140)$$

$$\frac{y}{L} = \frac{c}{L} + (\gamma^e - 1 + d) \frac{k}{L}. \quad (141)$$

We now show that these four equations determine the four variables y, c, L, k , given a steady-state inflation rate Π . For given Π , one can solve for hours worked L by substituting the equations (138) to (140) into equation (141). This yields

$$\left(-\frac{V(L)}{L \partial V(L)/\partial L} \right) = \phi \mu(\Pi) \rho(\Pi) - (\phi - 1) \left(\frac{\gamma^e - 1 + d}{\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d} \right). \quad (142)$$

Given Π and L , the solutions for k, c , and y can then be recursively computed from the equations (138) to (140). These solutions are

$$k(\Pi) = \left(\frac{1}{\mu(\Pi)} \frac{1}{\Delta^e} \right)^\phi \left(1 - \frac{1}{\phi} \right)^\phi \left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d \right)^{-\phi} L \quad (143)$$

$$c(\Pi) = \frac{1}{\mu(\Pi)} \frac{1}{\Delta^e} \left(\frac{1}{\phi} \right) \left(\frac{k}{L} \right)^{1-\frac{1}{\phi}} \left(-\frac{V(L)}{\partial V(L)/\partial L} \right) \quad (144)$$

$$y(\Pi) = c + (\gamma^e - 1 + d)k. \quad (145)$$

E.2.2 Steady-state solution for the limiting case in proposition 1:

We now consider the steady-state solution from the previous section for the limiting case $\beta(\gamma^e)^{1-\sigma} \rightarrow 1$. Using lemma 1 equation (142) simplifies to

$$\left(-\frac{V(L)}{L \partial V(L)/\partial L} \right) = \left(\frac{1}{1 + \tau \theta - 1} \right) \phi - (\phi - 1). \quad (146)$$

This shows that the steady state amount of labor does not depend on Π . Next, rewrite equation (143) as

$$\left(\frac{k(\Pi)}{L}\right)^{1-\frac{1}{\phi}} = \left(\frac{1}{\mu(\Pi)} \frac{1}{\Delta^e}\right)^{\phi-1} \left(1 - \frac{1}{\phi}\right)^{\phi-1} (\gamma^e - 1 + d)^{1-\phi}.$$

Substitute this equation and equation (146) into equation (144), this delivers

$$c(\Pi) = \left(\frac{1}{\mu(\Pi)}\right)^\phi \left\{ L \left(\frac{1}{\Delta^e}\right)^\phi (\gamma^e - 1 + d)^{1-\phi} \left(\left(\frac{1}{1+\tau} \frac{\theta}{\theta-1}\right) \phi - (\phi-1) \right) \phi^{-\phi} (\phi-1)^{\phi-1} \right\},$$

where the term in parentheses depends is independent of inflation Π . We thus have

$$c(\Pi) \propto \left(\frac{1}{\mu(\Pi)}\right)^\phi.$$

The inflation rate that minimizes the aggregate markup distortion thus maximizes steady-state consumption and thereby welfare, given that labor is fixed.

E.2.3 Minimizing The Aggregate Markup Distortion

From equation (30), minimizing the aggregate markup distortion in the steady state implies

$$\frac{\partial \mu(\Pi)}{\partial \Pi} = \sum_{z=1}^Z \psi_z \mu_z(\Pi)^{\psi_z-1} [\partial \mu_z(\Pi) / \partial \Pi] \left(\prod_{z^C} \mu_z(\Pi)^{\psi_z} \right) = 0,$$

where z^C to denote the set of all items except for item z . The equation holds if and only if

$$\sum_{z=1}^Z \psi_z \frac{\partial \mu_z(\Pi) / \partial \Pi}{\mu_z(\Pi)} = 0. \quad (147)$$

Using equation (137), the expression for $\rho_z(\pi)$ in equation (33) and the shorthand notation $\tilde{\alpha}_z = \alpha_z(1 - \delta_z)(\gamma^e / \gamma_z^e)^{\theta-1}$, we obtain

$$\frac{\partial \mu_z(\Pi) / \partial \Pi}{\mu_z(\Pi)} = \frac{\theta \tilde{\alpha}_z \Pi^{\theta-2} \left(\frac{q_z \gamma_z^e}{g_z \gamma_z^e}\right)}{\left(1 - \tilde{\alpha}_z \Pi^\theta \left(\frac{q_z \gamma_z^e}{g_z \gamma_z^e}\right)\right) (1 - \tilde{\alpha}_z \Pi^{\theta-1})} \left[\Pi - \frac{g_z \gamma_z^e}{q_z \gamma_z^e} \right].$$

Plugging this expression into equation (147) and multiplying by Π^2 yields

$$\sum_{z=1}^Z \left\{ \frac{\psi_z \theta \tilde{\alpha}_z \Pi^\theta \left(\frac{q_z \gamma_z^e}{g_z \gamma_z^e}\right)}{\left(1 - \tilde{\alpha}_z \Pi^\theta \left(\frac{q_z \gamma_z^e}{g_z \gamma_z^e}\right)\right) (1 - \tilde{\alpha}_z \Pi^{\theta-1})} \right\} \left[\Pi - \frac{g_z \gamma_z^e}{q_z \gamma_z^e} \right] = 0. \quad (148)$$

The expression in the parentheses is the weight $\tilde{\omega}_z$ in proposition 1. We normalize the weights so that they sum to unity over all $z = 1, \dots, Z$. This yields normalized weights $\omega_z = \tilde{\omega}_z / \sum_{z=1}^Z \tilde{\omega}_z$, with $\sum_{z=1}^Z \omega_z = 1$. Using these, we can rewrite equation (148) according to

$$\sum_{z=1}^Z \omega_z \left[\Pi^* - \frac{g_z \gamma_z^e}{q_z \gamma_z^e} \right] = 0, \quad (149)$$

where ω_z is given by the expression in the proposition and Π^* denotes the optimal solution. Solving equation (149) for Π^* yields the expression for the optimal inflation target in proposition 1.

E.3 Proof of Lemma 2

Defining $m_z = \frac{g_z \gamma_z^e}{q_z \gamma^e}$ one can express equation (148) as

$$\sum_{z=1}^Z \tilde{\omega}_z(\Pi, m_z) [\Pi - m_z] = 0, \quad (150)$$

where $\tilde{\omega}_z(\Pi, m_z) = \frac{\psi_z \theta \tilde{\alpha}_z \Pi^\theta / m_z}{(1 - \tilde{\alpha}_z \Pi^\theta / m_z)(1 - \tilde{\alpha}_z \Pi^{\theta-1})}$ and $\tilde{\alpha}_z = \alpha_z (1 - \delta_z) (\gamma^e / \gamma_z^e)^{\theta-1}$. Linearizing equation (150) at a point where $\bar{\Pi} = \bar{m}_z$ for all z , yields

$$\sum_{z=1}^Z \tilde{\omega}_z(\bar{\Pi}, \bar{m}_z) [\Pi - m_z] = 0 + O(2).$$

Letting again Π^* denote the optimal solution, we can rewrite the previous equation as

$$\Pi^* = \sum_{z=1}^Z \frac{\tilde{\omega}_z(\bar{\Pi}, \bar{m}_z)}{\left(\sum_{z'=1}^Z \tilde{\omega}_{z'}(\bar{\Pi}, \bar{m}_{z'})\right)^{-1}} m_z + O(2), \quad (151)$$

which shows that Π^* is a weighted average of m_z 's for all item categories z and with weights evaluated at the expansion point and normalized to unity. The normalized weight of item z evaluated at $\bar{\Pi} = \bar{m}_z$ is given by

$$\begin{aligned} \frac{\tilde{\omega}_z(\bar{\Pi}, \bar{m}_z)}{\sum_{z=1}^Z \tilde{\omega}_z(\bar{\Pi}, \bar{m}_z)} &= \psi_z \left[\frac{\theta \tilde{\alpha}_z \bar{\Pi}^{\theta-1}}{(1 - \tilde{\alpha}_z \bar{\Pi}^{\theta-1})^2} \right] \left(\sum_{z=1}^Z \psi_z \left[\frac{\theta \tilde{\alpha}_z \bar{\Pi}^{\theta-1}}{(1 - \tilde{\alpha}_z \bar{\Pi}^{\theta-1})^2} \right] \right)^{-1}, \\ &= \psi_z, \end{aligned}$$

where the second equality follows from the fact that $\tilde{\alpha}_z$ is constant across item categories $z = 1, \dots, Z$ and the fact that $\sum_{z=1}^Z \psi_z = 1$. Equation (151) can be rearranged to obtain

$$\Pi^* = \sum_{z=1}^Z \psi_z m_z + O(2),$$

which is the equation stated in lemma 2, when using $m_z = \frac{g_z \gamma_z^e}{q_z \gamma^e}$.

E.4 Proof of Proposition 2

Taking the natural logarithm of the equation (101), which describes the optimal reset price, yields

$$\ln \frac{P_{jzt}^*}{P_{zt}} = \ln \left(\frac{1}{1 + \tau \theta - 1} \right) - \ln \left(\frac{Q_{jzt} G_{jzt}}{Q_{zt}} \right) + \ln \left(\frac{n_{zt}}{p_{zt} d_{zt}} \right). \quad (152)$$

We rearrange the term $\ln(Q_{jzt}G_{jzt}/Q_{zt})$ in the previous equation for $s_{jzt} \geq 1$ as

$$\begin{aligned} \ln\left(\frac{Q_{jzt}G_{jzt}}{Q_{zt}}\right) &= \ln(\epsilon_{jzt}^G \epsilon_{jzt}^Q) + \ln\left(\frac{Q_{zt-s} \bar{G}_{jzt}}{Q_{zt}}\right) \\ &= \ln(\epsilon_{jzt}^G \epsilon_{jzt}^Q) + \ln\left(\frac{\prod_{k=0}^{s_{jzt}-1} g_{zt-k}}{\prod_{k=0}^{s_{jzt}-1} q_{zt-k}}\right) \\ &= \ln(\epsilon_{jzt}^G \epsilon_{jzt}^Q) + \ln\left(\frac{g_z}{q_z}\right) \cdot s_{jzt} + \sum_{i=t-s_{jzt}+1}^t (\ln \epsilon_{zi}^g - \ln \epsilon_{zi}^q). \end{aligned} \quad (153)$$

where the first equality follows from using equations (8) and (12), the second equality follows from using equations (9) and (13), the third equality follows from using equations (10) and (14), and where $\ln(\epsilon_{jzt}^G \epsilon_{jzt}^Q)$ denotes the product-fixed effect. For the case with $s_{jzt} = 0$, we obtain $\ln(Q_{jzt}G_{jzt}/Q_{zt}) = \ln(\epsilon_{jzt}^G \epsilon_{jzt}^Q)$. Substituting the equation (153) into equation (152) yields equation (36) in the proposition, where we have defined

$$f_{jz}^* \equiv \ln\left(\frac{1}{1+\tau} \frac{\theta}{\theta-1} \frac{n_z}{p_z d_z}\right) - \ln(\epsilon_{jzt}^G \epsilon_{jzt}^Q) \quad (154)$$

$$u_{jzt}^* \equiv \ln\left(\frac{n_{zt}}{p_{zt} d_{zt}} \frac{p_z d_z}{n_z}\right) - \sum_{i=t-s_{jzt}+1}^t (\ln \epsilon_{zi}^g - \ln \epsilon_{zi}^q), \quad (155)$$

and $E[u_{jzt}^*] = 0$ holds because by assumption $E \ln \epsilon_{zt}^g = 0$ and $E \ln \epsilon_{zt}^q = 0$ and $\ln\left(\frac{n_{zt}}{p_{zt} d_{zt}} \frac{p_z d_z}{n_z}\right)$ denotes the percentage deviation of stationary variables from their steady state values.

E.5 Relative Price Regression Using all Prices (Equation 38)

As proven below, the intercepts and residuals of regression (38) satisfy the following properties:

Proposition 4 *The evolution of the relative product price in all periods, including adjustment periods, is described by equation (38), where*

$$f_{jz} = f_{jz}^* + \bar{u}_z,$$

with f_{jz}^* being defined in equation (154) and

$$\bar{u}_z = -\frac{\alpha_z}{1-\alpha_z} [E \ln \Pi_{zt} - \ln(g_z/q_z)]. \quad (156)$$

For products with age $s_{jzt} > 0$, we have

$$u_{jzt} = \begin{cases} u_{jzt}^* - \bar{u}_z & \text{in price adjustment periods,} \\ u_{jz,t-1} + \ln(g_z/q_z) - \ln \Pi_{zt} & \text{otherwise,} \end{cases} \quad (157)$$

where u_{jzt}^* is defined in equation (155). For new products with $s_{jzt} = 0$, we have

$$u_{jzt} = u_{jzt}^* - \bar{u}_z,$$

where

$$u_{jzt}^* \equiv \ln \left(\frac{n_{zt}}{p_{zt}d_{zt}} \frac{p_z d_z}{n_z} \right).$$

Given the results in the previous proposition, we can compute the unconditional mean of u_{jzt} . Rewrite equation (157) as

$$u_{jzt} = \xi_{jzt}[u_{jz,t-1} + \ln(g_z/q_z) - \ln \Pi_{zt}] + (1 - \xi_{jzt})(u_{jzt}^* - \bar{u}_z),$$

where the product-specific, idiosyncratic, and independent Poisson process ξ_{jzt} captures the price adjustment process: ξ_{jzt} is equal to zero with probability $1 - \alpha_z$ and equal to one otherwise. Given the independence of ξ_{jzt} from $u_{jz,t-1}$, Π_{zt} and u_{jzt}^* , we obtain

$$\begin{aligned} E[u_{jzt}] &= E[\xi_{jzt}]E[u_{jz,t-1} + \ln(g_z/q_z) - \ln \Pi_{zt}] + E[u_{jzt}^* - \bar{u}_z] - E[\xi_{jzt}]E[u_{jzt}^* - \bar{u}_z] \\ &= \alpha_z (E[u_{jz,t-1}] + \ln(g_z/q_z) - E[\ln \Pi_{zt}]) + (1 - \alpha_z)E[u_{jzt}^* - \bar{u}_z]. \end{aligned}$$

Since u_{jzt} is a stationary process, we have $E[u_{jzt}] = E[u_{jz,t-1}]$. Since $E[u_{jzt}^*] = 0$, see proposition 2, we obtain from the previous equation and equation (156) that

$$E[u_{jzt}] = -\frac{\alpha_z}{1 - \alpha_z} [E \ln \Pi_{zt} - \ln(g_z/q_z)] - \bar{u}_z = 0,$$

as claimed in the text.

Proof. We start by deriving the evolution of the modified residual u_{jzt} . Let the sticky price in t be equal to the optimal price set $k \geq 0$ periods ago, $P_{jzt} = P_{jz,t-k}^*$, where $k \leq s_{jzt}$. Then, we can rewrite equation (38) as

$$\ln \frac{P_{jz,t-k}^*}{P_{z,t-k}} + \ln \frac{P_{z,t-k}}{P_{zt}} = f_{jz} - \ln \left(\frac{g_z}{q_z} \right) \cdot (k + s_{jz,t-k}) + u_{jzt},$$

or equivalently

$$\ln \frac{P_{jz,t-k}^*}{P_{z,t-k}} + \ln \frac{P_{z,t-k}}{P_{zt}} = f_{jz} - \bar{u}_z - \ln \left(\frac{g_z}{q_z} \right) \cdot (k + s_{jz,t-k}) + u_{jzt} + \bar{u}_z.$$

Defining $f_{jz} - \bar{u}_z = f_{jz}^*$, the previous equation is equal to the reset price equation (36) shifted k periods into the past, i.e.,

$$\ln \frac{P_{jz,t-k}^*}{P_{z,t-k}} = f_{jz}^* - \ln \left(\frac{g_z}{q_z} \right) \cdot s_{jz,t-k} + u_{jz,t-k}^*,$$

where u_{jzt} is given by

$$u_{jzt} = u_{jz,t-k}^* - \bar{u}_z + \ln \left(\frac{g_z}{q_z} \right) \cdot k - \ln \frac{P_{zt}}{P_{z,t-k}}. \quad (158)$$

For $k = 0$, we have $u_{jzt} = u_{jzt}^* - \bar{u}_z$. For $k \geq 1$, we can derive a recursive representation. Equation (158) then also holds in period $t - 1$, where the age of the price is $k - 1$, so that

$$\begin{aligned} u_{jz,t-1} &= u_{jz,t-k}^* - \bar{u}_z + \ln\left(\frac{g_z}{q_z}\right) \cdot (k-1) - \ln\frac{P_{z,t-1}}{P_{z,t-k}} \\ &= u_{jzt} - \ln\left(\frac{g_z}{q_z}\right) - \ln\frac{P_{z,t-1}}{P_{zt}}. \end{aligned}$$

The last line follows from equation (158). Rewriting the previous equation yields the postulated recursive law of motion of the residual u_{jzt} for non-adjustment periods:

$$u_{jzt} = u_{jz,t-1} + \ln(g_z/q_z) - \ln \Pi_{zt}.$$

■

E.6 Derivation of Equation (40)

The not-quality adjusted price level of item z , defined in equation (39), can be decomposed as follows:

$$\tilde{P}_{zt}^{1-\theta} = \delta_z (\tilde{P}_{z,t,t}^*)^{1-\theta} + (1 - \alpha_z) \delta_z \sum_{s=1}^{\infty} (1 - \delta_z)^s (\tilde{P}_{z,t-s,t}^*)^{1-\theta} + \alpha_z (1 - \delta_z) (\tilde{P}_{zt-1})^{1-\theta}, \quad (159)$$

where the average optimal (not-quality adjusted) price of new products entering in t is given by

$$\tilde{P}_{z,t,t}^* \equiv \left(\frac{1}{\delta_z} \int_{J_{t,t}^*} (\tilde{P}_{jzt}^*)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \quad (160)$$

and the average optimal (not-quality adjusted) price of continuing products with age $s \geq 1$ is given by

$$\tilde{P}_{z,t-s,t}^* \equiv \left(\frac{1}{(1 - \alpha_z) \delta_z (1 - \delta_z)^s} \int_{J_{t-s,t}^*} (\tilde{P}_{jzt}^*)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (161)$$

To obtain a recursive representation of equation (159), we derive the equation corresponding to equation (61) for the case without quality adjustment. This yields

$$\tilde{P}_{z,t-s,t}^* = \left(\prod_{k=0}^{s-1} g_{zt-k} \right)^{-1} \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) \frac{N_{zt} P_t}{D_{zt}} Q_{zt}. \quad (162)$$

For the special case $s = 0$, we have

$$\tilde{P}_{z,t,t}^* = \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) \frac{N_{zt} P_t}{D_{zt}} Q_{zt}. \quad (163)$$

Dividing equation (162) by equation (163) yields

$$\tilde{P}_{z,t-s,t}^* = \tilde{P}_{z,t,t}^* \left(\prod_{k=0}^{s-1} g_{zt-k} \right)^{-1}. \quad (164)$$

Using the previous equation to substitute for $P_{z,t-s,t}^*$ in equation (159) yields

$$\tilde{P}_{zt}^{1-\theta} = (\tilde{P}_{z,t,t}^*)^{1-\theta} \left\{ \delta_z + (1 - \alpha_z) \sum_{s=1}^{\infty} \delta_z (1 - \delta_z)^s \left(\prod_{k=0}^{s-1} g_{zt-k} \right)^{\theta-1} \right\} + \alpha_z (1 - \delta_z) (\tilde{P}_{zt-1})^{1-\theta},$$

which can be rearranged to obtain

$$\tilde{P}_{zt}^{1-\theta} = \left\{ \alpha_z \delta_z + (1 - \alpha_z) (\tilde{\Delta}_{zt}^e)^{1-\theta} \right\} (\tilde{P}_{z,t,t}^*)^{1-\theta} + \alpha_z (1 - \delta_z) (\tilde{P}_{zt-1})^{1-\theta}, \quad (165)$$

where the stationary variable $\tilde{\Delta}_{zt}^e$ is given by

$$(\tilde{\Delta}_{zt}^e)^{1-\theta} = \delta_z + (1 - \delta_z) (\tilde{\Delta}_{zt-1}^e / g_{zt})^{1-\theta}. \quad (166)$$

In order to relate P_{zt} in equation (66) to \tilde{P}_{zt} in equation (165), we derive the mapping between $P_{z,t,t}^*$ and $\tilde{P}_{z,t,t}^*$. In particular, dividing equation (163) by equation (62) and taking growth rates yields

$$\frac{\tilde{P}_{z,t,t}^*}{\tilde{P}_{z,t-1,t-1}^*} = \frac{Q_{zt}}{Q_{z,t-1}} \frac{P_{z,t,t}^*}{P_{z,t-1,t-1}^*}, \quad (167)$$

which shows that in item z , the growth rates of the average optimal price of newly entering products with and without quality adjustment are related via quality growth.

The steady-state version of equation (165) can be rearranged to obtain

$$\begin{aligned} (\tilde{\Pi}_z \tilde{P}_{z,t-1})^{1-\theta} &= \left\{ \alpha_z \delta_z + (1 - \alpha_z) (\tilde{\Delta}_z^e)^{1-\theta} \right\} \left(\frac{\tilde{P}_{z,t,t}^*}{\tilde{P}_{z,t-1,t-1}^*} \tilde{P}_{z,t-1,t-1}^* \right)^{1-\theta} \\ &\quad + \alpha_z (1 - \delta_z) (\tilde{\Pi}_z \tilde{P}_{z,t-2})^{1-\theta}, \end{aligned}$$

For equation (168) to be consistent with equation (165), it must hold that

$$\tilde{\Pi}_z = \tilde{P}_{z,t,t}^* / \tilde{P}_{z,t-1,t-1}^*. \quad (168)$$

Similar reasoning for the item price level with quality adjustment yields

$$\Pi_z = P_{z,t,t}^* / P_{z,t-1,t-1}^*. \quad (169)$$

Using equations (168) and (169) to rewrite equation (167) in the steady state yields equation (40) in the main text.

E.7 Proof of Proposition 3

Consider a steady state and use equation (11) to replace in equation (101) the quality-adjusted reset price P_{jzt}^* by $\tilde{P}_{jzt}^* / Q_{jzt}$. This yields

$$\frac{\tilde{P}_{jzt}^*}{\tilde{P}_{zt}} \frac{\tilde{P}_{zt}}{P_{zt}} \frac{1}{Q_{jzt}} \left(\frac{Q_{jzt} G_{jzt}}{Q_{zt}} \right) = \left(\frac{1}{1 + \tau} \frac{\theta}{\theta - 1} \right) \frac{n_z}{p_z d_z}.$$

Taking the natural logarithm of the previous equation and using equation (153) to substitute for $\ln(Q_{jzt}G_{jzt}/Q_{zt})$ in the steady state yields

$$\ln \frac{\tilde{P}_{jzt}^*}{\tilde{P}_{zt}} = \ln \left(\frac{1}{1 + \tau} \frac{\theta}{\theta - 1} \frac{n_z}{p_z d_z} \right) + \ln \left(\frac{Q_{jzt}}{\epsilon_{jzt}^G \epsilon_{jzt}^Q} \right) - \ln \left(\frac{g_z}{q_z} \right) \cdot s_{jzt} + \ln \left(\frac{P_{zt}}{\tilde{P}_{zt}} \right). \quad (170)$$

Steady-state relative item price levels evolve as

$$\begin{aligned} \ln(P_{zt}/\tilde{P}_{zt}) &= (t + 1) \cdot \ln(\Pi_z/\tilde{\Pi}_z) + \ln(P_{z,-1}/\tilde{P}_{z,-1}) \\ &= -(t + 1) \cdot \ln(q_z) + \ln(P_{z,-1}/\tilde{P}_{z,-1}) \\ &= -\ln(q_z) \cdot s_{jzt} - (t - s_{jzt} + 1) \cdot \ln(q_z), \end{aligned}$$

where the second equality follows from equation (40) and the third equality uses the initial condition $P_{z,-1}/\tilde{P}_{z,-1} = 1$, without loss of generality. Using the previous equation to substitute for the ratio of item price levels in equation (170) yields

$$\ln \frac{\tilde{P}_{jzt}^*}{\tilde{P}_{zt}} = \ln \left(\frac{1}{1 + \tau} \frac{\theta}{\theta - 1} \frac{n_z}{p_z d_z} \right) + \ln \left(\frac{Q_{jzt}}{\epsilon_{jzt}^G \epsilon_{jzt}^Q} \right) - (t - s_{jzt} + 1) \cdot \ln(q_z) - \ln \left(\frac{g_z}{q_z} \right) \cdot s_{jzt} - \ln(q_z) \cdot s_{jzt}. \quad (171)$$

Defining the product-fixed effect as⁴¹

$$\tilde{f}_{jz}^* \equiv \ln \left(\frac{1}{1 + \tau} \frac{\theta}{\theta - 1} \frac{n_z}{p_z d_z} \right) + \ln \left(\frac{Q_{jzt}}{\epsilon_{jzt}^G \epsilon_{jzt}^Q} \right) - (t - s_{jzt} + 1) \cdot \ln(q_z)$$

shows that equation (171) is equivalent to equation (42) in the proposition.

E.8 Imperfect Quality Adjustment: Derivation of Equations (41) and (43)

To derive equation (41), we define the price level for the case without quality adjustment as

$$\tilde{P}_t = \prod_{z=1}^{Z_t} \left(\tilde{P}_{zt}/\psi_{zt} \right)^{\psi_{zt}},$$

analogously to equation (16). Taking growth rates of the previous equation and using equation (40) to substitute for $\tilde{\Pi}_z$ in the steady state yields

$$\tilde{\Pi} = \prod_{z=1}^Z (q_z \Pi_z)^{\psi_z}.$$

Taking the natural logarithm of the previous equation and using $\ln \Pi = \sum_{z=1}^Z \psi_z \ln \Pi_z$, which follows from equation (16), yields equation (41).

⁴¹Recall that $t - s_{jzt}$ is constant over the product lifetime.

To derive equation (43), we rewrite the equation in Lemma 2, which holds to first order at the approximation point $(\bar{\Pi}, \bar{m}_z)$, with $\bar{\Pi} = \bar{m}_z$ and $m_z = \frac{g_z \gamma_z^e}{q_z \gamma^e}$, using

$$\begin{aligned}\Pi^* &= \bar{\Pi} + \bar{\Pi}(\ln \Pi^* - \ln \bar{\Pi}) + O(2) \\ m_z &= \bar{m}_z + \bar{m}_z(\ln m_z - \ln \bar{m}_z) + O(2),\end{aligned}$$

to substitute for Π^* and m_z , respectively. This yields

$$\ln \Pi^* - \ln \bar{\Pi} = \sum_{z=1}^Z \psi_z (\ln m_z - \ln \bar{m}_z) + O(2),$$

which after simplifying is equivalent to equation (43).

F Data Appendix

[to be added]