Optimal Sovereign Default

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When is it optimal for a fully committed government to default on its legal repayment obligations? Considering a small open economy with domestic production risk and non-contingent government debt, we show that it is ex-ante optimal to occasionally deviate from the legal repayment obligation and to repay debt only partially. This holds true even if default generates significant deadweight costs ex-post. A quantitative analysis reveals that default is optimal only in response to persistent disaster-like shocks to domestic output. Applying the framework to the situation in Greece, we find that optimal default policies suggest a considerably larger and more timely default than the one actually implemented in the year 2012.
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When is it optimal for a sovereign to default on its outstanding debt? We analyze this normative question within a quantitative equilibrium framework. Specifically, we consider a country that seeks to smooth the consumption implications of adverse shocks to domestic productivity and that can do so by (1) borrowing internationally, (2) by adjusting domestic investment and (3) by defaulting on outstanding foreign debt. We determine the optimal borrowing, investment and default policies that maximize the country's *ex ante* welfare, i.e. derive the Ramsey optimal policy under full commitment, and show that these policies occasionally prescribe that the government repays less than what is legally required to serve the outstanding debt contracts. This is optimal even if default events give rise to sizeable deadweight costs ex-post. A quantitative analysis suggests that optimal default policies can significantly increase *ex ante* welfare, relative to a situation where sovereign default is simply ruled out under the optimal commitment plan.

The fact that sizeable welfare gains can arise from sovereign default may appear surprising, given that policy discussions and also the academic literature tend to emphasize the inefficiencies associated with sovereign default events. Popular discussions, for example, tend to focus on the potential *ex post* costs associated with a sovereign default, such as the adverse consequences for the functioning of the banking sector or the econ-

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omy as a whole. While certainly relevant, we show that sovereign default can remain optimal, even if default costs of an empirically plausible magnitude arise. Likewise, following the seminal contribution of Eaton and Gersovitz 1981, the academic literature equally tends to emphasize the inefficiencies created by default decisions: anticipation of default in the future limits the ability to issue debt today, thereby constrains the ability to smooth out adverse shocks.

Our analysis emphasizes that sovereign default also fulfills a very useful economic function, even in a setting with a fully committed government: a default engineers a resource transfer from lenders to the sovereign debtor in times when resources are scarce on the sovereign's side. The option to default thus provides insurance against adverse economic developments in domestic income. Grossmann and Van Huyck 1988 previously emphasized this point using a static framework and abstracting from default costs. Here we reconsider this issue using a fully dynamic setup and for the empirically more relevant case with positive default costs. As we show analytically, taking into account default costs has a strong effect on optimal default policies.

Sovereign default is Ramsey optimal in our setting because government bond markets are incomplete, such that bond markets do not provide any explicit insurance against domestic income shocks. The incompleteness of government bond markets is in line with empirical evidence, which shows that existing government debt consists predominantly of non-contingent debt instruments.[1] We provide microfoundations for the market incompleteness in Appendix A of the paper but in the main text take the incompleteness as given, following much of the existing Ramsey literature (Sims 2001, Angeletos 2002, Ayiagari et al. 2002, Buera and Nicolini 2004, Adam 2011). Using a setting with non-contingent sovereign debt and default costs, we thus extend the existing Ramsey policy literature by treating repayment of debt as a (continuous) decision variable. We show analytically that the assumption of full debt repayment entertained in this literature is typically inconsistent with fully optimal behavior. While full repayment is optimal whenever the country has accumulated a sufficient amount of international wealth, which then serves as a buffer against adverse shocks to domestic income, full repayment is suboptimal when the country's wealth level is sufficiently low or when adverse domestic shocks are sufficiently large and persistent.[2,3]

Besides providing analytical characterizations of the optimal default policies, including a characterization of the country's borrowing limits, we assess quantitatively the economic conditions under which sovereign default is part of Ramsey optimal policy. For this purpose, we provide a lower bound estimate for the costs of default implied by our structural model and use it as an input for our quantitative analysis. We show that plau-

[1] Most sovereign debt is non-contingent only in nominal terms, and could be made contingent by adjusting the price level, a point emphasized by Chari, Christiano and Kehoe 1991. As shown in Schmitt-Grohe and Uribe 2004, however, such price level adjustments are suboptimal in the presence of even modest nominal rigidities. Morevoer, for countries that are members of a monetary union, non-contingent nominal debt is effectively non-contingent in real terms, since the country cannot control the price level.

[2] This assumes that default costs do not take on prohibitive values, i.e. a level equal to or higher than the amount of resources not repaid to lenders.

[3] The presence of non-zero default costs is key for this finding, because optimal default patterns would be independent of the country's wealth position whenever default does not generate costs.

sible levels of default costs make it optimal for the government *not* to default following business cycle sized shocks to productivity and large but temporary disaster-like shocks to productivity, provided the country's net foreign asset position is in the range of empirically observed values. These results thus vindicate the full repayment assumption often entertained in the Ramsey policy literature with incomplete markets. Indeed, sovereign default becomes optimal only when the country's net foreign asset position approaches its maximum sustainable level, which - in a setting with commitment - turns out to be very negative[4], or when adverse domestic shocks are sufficiently large and persistent.

We then apply our quantitative framework to the case of Greece, which - following the year 2008 - has experienced a very large but also very persistent contraction in domestic output that lasts until today. Greece has also partially defaulted on its sovereign debt in the year 2012. We find that - given the output contractions experienced in Greece - Ramsey optimal policies would have called for a significantly larger and more timely default over a wide range of empirically plausible default cost levels. In particular, our model suggests that default should have occurred already following the output contractions experienced in the years 2010 and 2011 and possibly again following the contraction in the year 2012. When applying the same quantitative framework to some of the other European countries facing sovereign debt problems in the recent past (Italy, Ireland, Portugal and Spain), we find that default in these countries would have been Ramsey optimal only when assuming considerably lower default costs.

We also determine the welfare consequences of optimal default policies, relative to as situation where the government is simply *assumed* to repay debt unconditionally.[5] We find that the consumption equivalent welfare gains associated with optimal default decisions in Greece lead to a permanent increase of consumption of up to 0.5 percentage points and that from an ex-ante viewpoint the actual default policies implemented in Greece generated only negligible welfare gains. Overall, our model suggests that - from the standpoint of Greece - the actual default policies have been suboptimal.

In related work, Chari, Christiano, and Kehoe 1994 consider Ramsey optimal distortionary taxation in a closed economy setting. They allow returns on bonds to be state-contingent, so that government bonds contribute to insuring the government budget against adverse economic shocks. In a world without default costs, such state contingent returns could be reinterpreted as Ramsey optimal government default on non-contingent repayment promises. For the case with costly default considered in the present paper, this equivalence breaks down.

Sims 2001 discusses fiscal insurance in the context of whether or not Mexico should dollarize its economy. Considering a setting where the government is assumed to issue only non-contingent nominal debt that is assumed always to be repaid, he shows how giving up the domestic currency allows for less insurance, as it deprives the government of the possibility of using price adjustments to alter the real value of outstanding debt. The

[4]Under commitment, the sustainable net foreign asset position can easily reach minus twenty times average GDP, when assuming that it is possible to transfer all of domestic output to foreign lenders, so as to serve foreign debt. Debt capacity is significantly lower when domestic consumption cannot be driven down to zero, as we discuss in section III.

[5]In the latter setting, adjustments in the international wealth position and of the domestic investment margin are the only channels available for smoothing domestic consumption.

present paper considers a model with non-contingent real bonds and allows for outright government debt default. Our setting could thus be reinterpreted as one where bonds are effectively non-contingent in nominal terms, but where the country has delegated the control of the price level to a monetary authority that pursues price stability, say by dollarizing or by joining a monetary union. As we then show, in such a setting the default option still provides the country with a quantitatively relevant insurance mechanism.

Angeletos 2002 explores fiscal insurance in a closed economy and incomplete government bond markets, assuming full repayment of debt. He shows how a government might use the maturity structure of domestic government bonds to insure against domestic shocks, exploiting the fact that bond yields of different maturities react differently to shocks. In our small open economy setting, this channel is unavailable because the international yield curve does not react to domestic events. Moreover, Buera and Nicolini 2004 show that the optimal maturity structure in Angeletos 2002 involves very large long and short positions at different maturities and that these positions are rather sensitive to small variations in model parameters, calling into question the practical relevance of the approach.

The remainder of the paper is structured as follows. Section I introduces the economic environment, formulates the Ramsey policy problem, and derives the necessary and sufficient conditions characterizing optimal policy. In doing so, we also introduce a new approach for proving concavity of Ramsey problems involving default decisions and show how to properly impose ‘natural borrowing limits’ in quantitative model applications. Both issues should be of interest in a variety of other applications. Section II derives a number of analytical results characterizing optimal default policies and section III quantitatively evaluates the model predictions in a setting with business cycle sized and disaster-like shocks to productivity. Section IV presents our analysis of the Greek situation after the year 2008 and section V discusses an extension of the model to a setting with bonds of longer maturity. A conclusion briefly summarizes. Technical material is contained in the appendices.

I. A Small Open Economy Model

Consider a small open economy with shocks to domestic productivity where the government can borrow and invest internationally to insure domestic consumption against fluctuations in domestic income. The economy is populated by a representative consumer with expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where $c \geq 0$ denotes consumption and $\beta \in (0, 1)$ the discount factor. We assume $u(\cdot)$ to be twice continuously differentiable with $u' > 0$ and $u'' < 0$, and that Inada conditions hold. Domestic output is produced by a representative firm using the production function

$$y_t = z_t k_{t-1}^{\alpha},$$

where y_t denotes output of consumption goods in period t, k_{t-1} the capital stock from the previous period, $\alpha \in (0,1)$ the capital share, and $z_t > 0$ an exogenous stochastic productivity shock. The capital stock depreciates over time at rate $d \in (0,1]$ and productivity shocks are the only source of randomness in the model, causing domestic income to be risky. Productivity assumes values from some finite set $Z = \left\{z^1, ..., z^N\right\}$ with $N \in \mathbb{N}$ and the transition probabilities across periods are described by some measure $\pi(z'|z)$ for all $z', z \in Z$. Without loss of generality, we order productivity states such that

$$z^1 > z^2 > ... > z^N.$$

A. The Government

The government seeks to maximize the utility of the representative domestic household (1) and is fully committed to its plans. It can insure consumption against domestic income risk by investing in foreign bonds, i.e. by building up a buffer stock of foreign wealth, and by issuing own bonds, i.e. by borrowing internationally.

Without loss of generality, we consider a setting in which foreign bonds are zero coupon bonds with a maturity of one period.[6] Foreign bonds are assumed to be risk-free and the interest rate r on these bonds satisfies $1 + r = 1/\beta$. We let $F_t \geq 0$ denote the government's holdings of foreign bonds in period t. These bonds mature in period $t + 1$ and repay F_t units of consumption at maturity.

We shall assume that the domestic government has a particular 'technology' available for issuing domestic bonds, i.e., for borrowing internationally. Specifically, the government can issue non-contingent zero coupon bonds which promise - as part of their legally stated payment obligation - to repay unconditionally one unit of consumption in the period after they have been issued.[7] The government can choose to deviate from the legal payment obligation at maturity, i.e. can choose to default, but such deviations are costly. The 'default costs' take the form of a dead-weight resource cost that is proportional to the size of the default, capturing the intuitive facts that sizeable *ex post* costs can be associated with sovereign default events and that these costs are likely increasing in the default size. Appendix A provides microfoundations for the assumptions that the government issues non-contingent debt only and faces proportional costs of default based on a setting with contracting frictions.

Let $D_t \geq 0$ denote the amount of domestic bonds issued by the government in period t. These bonds legally promise to repay D_t units of consumption in period $t + 1$.

[6] Allowing for a richer maturity structure for foreign bonds makes no difference to the analysis: the small open economy setting implies that foreign interest rates are independent of domestic conditions, such that the government cannot use the maturity structure of foreign bonds to insure against domestic productivity shocks. Foreign bonds thus only serve as a store of value.

[7] The effects of introducing domestic bonds with longer maturity will be discussed separately in section V.

When issuing these bonds in period t, the government also decides on a default profile $\Delta_t \in [0,1]^N$, which is a vector determining, for each future productivity state z^n $(n = 1, ..., N)$, the share of the legal payment promise the government will default on:

$$\Delta_t = (\delta_t^1, ..., \delta_t^N).$$

An entry of one indicates a state in which full default occurs, an entry of zero a state with full repayment, and intermediate values capture partial default events. Let $\delta_t(z_{t+1})$ denote the entry in the default profile Δ_t pertaining to productivity state $z_{t+1} \in Z$. Total repayment on domestic bonds maturing in period $t+1$ is then given by

$$D_t(1 - \delta_t(z_{t+1})) + \lambda D_t \delta_t(z_{t+1}). \tag{2}$$

The first term captures the amount of domestic debt that is repaid to lenders, net of the default share $\delta_t(z_{t+1})$; the second term captures the default costs accruing to the sovereign borrower, where $\lambda \geq 0$ is a cost parameter. Default costs only emerge if $\delta_t(z_{t+1}) > 0$, occur in the period in which the default takes place, and are assumed to be proportional to the default amount $D_t \delta_t(z_{t+1})$.[8] Proportional default costs are considered mainly for analytical convenience: such a specification allows us to prove concavity of the Ramsey problem later on. While it may be plausible that sovereign default events also give rise to fixed costs that are independent of the default amount, such specifications generate non-convexities in the constraint set of the Ramsey problem, which considerably complicate the optimal policy analysis.[9] Our proportional specification is furthermore identical to the one used in Calvo 1988 and similar to the specifications used in Zame 1993 and Dubey, Geanakoplos and Shubik 2005 to study default on private contracts.[10]

In the setting just described, the government can insure domestic consumption against productivity risk either by adjusting its holdings of foreign and domestic bonds, i.e. by adjusting its buffer stock of savings or debt, by adjusting domestic investment, and by choosing to default on its bonds. The optimal mix between these insurance mechanisms will depend on the level of the default costs λ.

B. The Ramsey Problem

To derive the Ramsey problem determining optimal government policies, it turns out to be useful to define the amount of resources available to the domestic government at the beginning of the period, i.e. before issuing new domestic debt, before making investment decisions, and before consumption takes place, but after (partial) repayment of maturing

[8] The analysis would remain unchanged if default costs were instead spread out over time, as long as the present value of the costs still sums to $\lambda D_t \delta_t(z_{t+1})$, when discounting costs at the international interest rate $1 + r$.

[9] Section II.D discusses an extension to the case with small fixed costs of default.

[10] Default costs in our setting represent a resource cost, while the general equilibrium literature with incomplete markets and private default, referenced above, introduces default cost in the form of a direct utility cost, which enters separably into the borrower's utility function.

bonds. We refer to these resources as beginning-of-period wealth and define them as

$$w_t \equiv z_t k_{t-1}^{\alpha} + (1-d)k_{t-1} + F_{t-1} - D_{t-1}(1-(1-\lambda)\delta_{t-1}(z_t)) \tag{3}$$

Beginning-of-period wealth w_t is thereby a function of past decisions and current exogenous shocks only. The government can raise additional resources in period t by issuing new domestic bonds and use the available funds to invest in foreign riskless bonds, in the domestic capital stock and to finance consumption. The economy's budget constraint is thus given by[11]

$$w_t + \frac{D_t}{1+R(z_t,\Delta_t)} = c_t + \overline{c} + k_t + \frac{F_t}{1+r},$$

where $1/(1+r)$ and $1/(1+R(z_t,\Delta_t))$ denote the issue price of the foreign and domestic bond, respectively, $\overline{c} \geq 0$ a subsistence level of consumption and c_t consumption in excess of this subsistence level.[12] As a baseline, we choose $\overline{c} = 0$ in our quantitative analysis but we also consider the robustness of our results to assuming positive values. The analytical results derived below hold independently of the value chosen for $\overline{c} \geq 0$.

The interest rate on domestic debt $R(z_t,\Delta_t)$ depends on the default profile Δ_t chosen by the government and on the current productivity state z_t, as the latter generally affects the likelihood of entering different states tomorrow. Since the government takes default decisions under commitment, it takes into account in its optimization problem how future default decisions affect the pricing of bonds today, as described by the function $R(\cdot,\cdot)$. Assuming that international investors are risk-neutral, this pricing function is given by

$$\frac{1}{1+R(z_t,\Delta_t)} = \frac{1}{1+r}\sum_{n=1}^{N}(1-\delta_t(z^n))\cdot\pi(z^n|z_t), \tag{4}$$

which equates the expected returns on the domestic bond and the foreign bond.

Using the previous notation, the Ramsey problem characterizing optimal government

[11] Note that we do not distinguish between the government and household budget constraints, instead consider directly the economy wide budget constraint. This implicitly assumes that the government can costlessly transfer resources between these two budgets via lump sum taxes. The economy wide constraint is derived by using the private sector budget constraint to substitute taxes in the government budget constraint. The Ramsey problem that we formulate then imposes this economy wide constraint, i.e. is formulated in so-called primal form, where taxes have already been substituted out.

[12] The presence of subsistence consumption is consistent with the utility specification in equation (1) if we set $u(c) = -\infty$ for all $c < 0$, i.e. whenever total consumption falls short of its subsistence level.

policy is then given by

$$\max_{\{F_t\geq 0, D_t\geq 0, \Delta_t\in[0,1]^N, k_t\geq 0, c_t\geq 0\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{5a}$$

$$s.t.:$$

$$c_t = w_t - \bar{c} - k_t + \frac{D_t}{1+R(z_t, \Delta_t)} - \frac{F_t}{1+r} \tag{5b}$$

$$w_{t+1} = z_{t+1}k_t^{\alpha} + (1-d)k_t + F_t - D_t(1-(1-\lambda)\delta_t(z_{t+1})) \tag{5c}$$

$$w_{t+1} \geq NBL(z_{t+1}) \quad \forall z_{t+1} \in Z \tag{5d}$$

$$w_0, z_0 : given.$$

We impose the natural borrowing limits (5d) on the problem to prevent the possibility of explosive debt dynamics (Ponzi schemes). We allow the natural borrowing limits to be potentially state contingent and assume that the initial condition satisfies $w_0 \geq NBL(z_0)$. The time-zero Ramsey optimal policy involves defaulting on all outstanding debt at time zero, a feature that should be reflected in the initial value for w_0.[13]

While intuitive, the Ramsey problem (5) is characterized by two features that complicate its solution. First, the price of the domestic government bond in the constraint (5b) depends on the chosen default profile, with the result that the constraint fails to be linear in the government's choice variables. It is thus unclear whether problem (5) is concave, which prevents us from working with first order conditions. Second, the presence of the natural borrowing limits (5d) creates problems for numerical solution algorithms. Specifically, imposing sufficiently lax natural borrowing limits, as is usually recommended if one wants to rule out Ponzi schemes only, gives rise to a non-existence problem: sufficiently lax borrowing limits imply that there exist beginning-of-period wealth levels above these limits, for which no policy can insure that the borrowing limits are respected under all contingencies. This non-existence of optimal policies creates problems for numerical solution approaches and thus for a quantitative evaluation of the model. While one could remedy the existence problem by imposing sufficiently tight borrowing limits, this may imply ruling out feasible and potentially optimal policies that would be consistent with non-explosive debt dynamics.

The next sections address both of these issues in turn. We first prove concavity by reformulating problem (5) into a specific variant of a complete markets model, which can be shown to be concave and equivalent to the original problem. This approach to proving concavity is - to the best of our knowledge - new to the literature and should be useful in a range of other applications involving default decisions. We then proceed by showing how to deal properly with the presence of natural borrowing limits in numerical applications. Again, this approach seems new to the literature and is of interest for a range of other applications. In a final step, we show that the concave and equivalent

[13] This is due to our specification of proportional default costs and holds true whenever $\lambda \leq 1$. In a setting with fixed costs of default, it only holds true when the level of foreign debt exceeds the total costs of default.

formulation of the Ramsey problem has a recursive structure, which facilitates numerical solution.

CONCAVITY OF THE RAMSEY PROBLEM. — We now define an alternative Ramsey problem with a different asset market structure and show that this alternative problem is equivalent to the original problem (5). Since the alternative problem is concave, it permits working with first order conditions.

We can define a safe asset b_t and a set of risky assets $a_t \in R^N$, where each unit of the safe asset b_t pays one unit of consumption in period $t+1$ with certainty, while a unit of the n-th risky asset ($n = 1, ..., N$) pays out $(1-\lambda)$ units of the consumption good in $t+1$ if only if $z_{t+1} = z^n$. Assuming that the price of the safe asset is equal to $\frac{1}{1+r}$ and the price of the n-th risky asset given by

$$p_t(z^n) = \frac{1}{1+r}\pi(z^n|z_t), \tag{6}$$

we can define an alternative Ramsey problem with alternative consumption and capital stock choices $(\widetilde{c}_t, \widetilde{k})$:

$$\max_{\{b_t, a_t \geq 0, \widetilde{k}_t \geq 0, \widetilde{c}_t \geq 0\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(\widetilde{c}_t) \tag{7a}$$

$$s.t.$$

$$\widetilde{c}_t = \widetilde{w}_t - \bar{c} - \widetilde{k}_t - \frac{1}{1+r} b_t - p_t' a_t \tag{7b}$$

$$\widetilde{w}_{t+1} \geq NBL(z_{t+1}) \quad \forall z_{t+1} \in Z$$

$$\widetilde{w}_0 = w_0,\ z_0\ given,$$

where beginning-of-period wealth is defined as

$$\widetilde{w}_t \equiv z_t \widetilde{k}_{t-1}^{\alpha} + (1-d)\widetilde{k}_{t-1} + b_{t-1} + (1-\lambda)a_{t-1}(z_t). \tag{8}$$

Appendix C.C2 then proves the following equivalence result:

PROPOSITION 1: *A consumption path $\{c_t\}_{t=0}^{\infty}$ is feasible in problem (5) if and only if the same consumption path $\{\widetilde{c}_t = c_t\}_{t=0}^{\infty}$ is feasible in problem (7).*

Proposition 1 shows that the optimal consumption path for (7) is the same as the one solving (5). This is of interest because the constraint (7b) in problem (7) is linear in the choice variables, as the price vector p_t is independent of policy choices, unlike with constraint (5b) in problem (5), where $R(z_t, \Delta_t)$ depends on the default choices Δ_t. First order conditions (FOCs) of problem (7) thus provide necessary and sufficient conditions for optimality.[14] The FOCs are explicitly stated in Appendix C.C1.

The proof of proposition 1 furthermore shows that the optimal asset choices in the alternative Ramsey problem are related to the asset and default choices in the original problem via the relations

$$b_t = F_t - D_t \tag{9}$$

$$a_t = D_t \Delta_t \geq 0 \tag{10}$$

and that the optimal investment choices in both problems satisfy $k_t = \widetilde{k}_t$.[15] With these results at hand, we can thus solve for the Ramsey optimal consumption path using the concave problem (7), but interpret the solution in terms of the financial market and investment choices supporting this path in the original problem (5).

Dealing with Natural Borrowing Limits. — In our quantitative evaluation of the model, we wish to impose borrowing limits that insure the existence of optimal policies, but which are sufficiently lax so as to not rule out policies that would be consistent with non-explosive debt dynamics. We call such borrowing limits the 'marginally binding natural borrowing limits'. We explain below how they can be computed for our setting with incomplete asset markets and derive their properties.

Let $NBL(z^n)$ denote the marginally binding natural borrowing limit (NBL) in productivity state z^n $(n = 1, \ldots, N)$.[16] These limits are implicitly defined by the following optimization problems for $n = 1, \ldots, N$:

(11)

$$NBL(z^n) = \min_{a^n, b^n, k^n} \overline{c} + \widetilde{k}^n + \frac{1}{1+r} b^n + \sum_{j=1}^{N} a^n(z^j) p^n(z^j)$$

$$s.t.$$

$$z^j \left(\widetilde{k}^n\right)^{\alpha} + (1-d)\widetilde{k}^n + b^n + (1-\lambda) a^n(z^j) \geq NBL(z^j) \text{ for } j = 1, ..., N$$

$$a^n(z^j) \geq 0 \ \text{ for } j = 1, ..., N,$$

where $p^n(z^j) \equiv p_t(z^j)$ for $z_t = z^n$. Problem (11) defines the marginally binding NBL as the minimum amount of wealth (the objective function) required for insuring that beginning-of-period wealth in all future states remains above the respective NBL limits (the first N constraints). In doing so, we assume that consumption is driven to its lower bound.

[14] This follows from the additional observation that future beginning of period wealth, as defined in equation (8), is a linear function of the financial market choices (a, b) and a convex function of investment $\widetilde{k}$.

[15] While the gross positions D_t and F_t are indeterminate in the original problem, net debt positions and default amounts are uniquely determined.

[16] The NBLs depend only on the current productivity shock because the shock process is Markov and because beginning-of-period wealth is the only other state variable, as will become clear in section I.B.

Since the NBLs in problem (11) also appear in the constraints, the optimization problem defines a fixed point problem. Appendix C.C3 derives the following result:

PROPOSITION 2: *Suppose higher productivity states induce a first order stochastic dominance ordering about subsequent productivity states, i.e., for* $z \geq z'$

$$\pi\left(z_{t+1} \leq z^n | z_t = z\right) \leq \pi\left(z_{t+1} \leq z^n | z_t = z'\right) \text{ for all } n = 1, ..., N. \tag{12}$$

Then there exists, almost surely for all model parameterizations[17], *a unique fixed point solution to (11) satisfying*

$$NBL(z^1) \leq NBL(z^2) \leq ... \leq NBL(z^N). \tag{13}$$

The proposition shows that under the stated assumption about the productivity process there exists a unique solution satisfying the intuitively plausible property that better productivity states are associated with laxer borrowing limits.[18] The proof of the proposition also provides a method for explicitly solving for the NBLs in numerical applications. The subsequent proposition establishes a key property of the fixed point solution, its proof is in appendix C.C4:

PROPOSITION 3: *Consider the fixed point solution from proposition 2 satisfying (13). For any productivity state* z^n $(n = 1, \ldots, N)$, *if the beginning-of-period wealth level satisfies* $\widetilde{w} < NBL(z^n)$, *then all policies violate any finite debt limit with positive probability.*

The previous result shows that the fixed point solution from proposition 2 provides indeed the loosest borrowing limits consistent with non-explosive debt dynamics.[19]

RECURSIVE FORMULATION OF THE RAMSEY PROBLEM. — We now present a recursive formulation of the Ramsey problem (7) to simplify the representation and characterization of optimal policies. Let $V(\widetilde{w}_t, z_t)$ denote the value function associated with optimal continuation policies when starting with beginning of period wealth $\widetilde{w}_t$ and productivity

[17] Almost surely refers to all probability distributions over the parameter space that do not assign positive mass to a specific parameter combination.

[18] Since problems (11) define only a monotone mapping from the NBLs in the constraints to the NBLs in the minimized objective, one generally can not rule out the existence of other fixed points that do not satisfy property (13). In our numerical applications we have never encountered such alternative fixed points, though.

[19] Clearly, policies leading to non-explosive debt dynamics always exist for wealth levels satisfying $\widetilde{w} \geq NBL(z^n)$. In state z^n, for example, one can pursue the policies a^n, b^n and k^n solving (11) and consume any possibly remaining consumption goods. This insures that debt always stays above the NBLs and thus finite.

state z_t. The Ramsey problem (7) then has a recursive representation given by

$$V(\widetilde{\omega}_t, z_t) = \max_{b_t, a_t \geq 0, \widetilde{k}_t \geq 0} u(\widetilde{\omega}_t - \bar{c} - \widetilde{k}_t - \frac{1}{1+r} b_t - p_t' a_t) + \beta E_t[V(\widetilde{\omega}_{t+1}, z_{t+1})]$$
$$s.t. \;\; \widetilde{\omega}_{t+1} = z_{t+1}\widetilde{k}_t^{\alpha} + b_t + (1-\lambda) a_t(z_{t+1})$$
$$\widetilde{\omega}_{t+1} \geq NBL(z_{t+1}) \quad \forall z_{t+1} \in Z,$$

so that optimal policies can be expressed as functions of just two state variables $(\widetilde{\omega}_t, z_t)$.

II. Optimal Sovereign Default: Analytic Results

This section presents a number of analytic results characterizing the optimal default policies that solve the Ramsey problem (7). We start by considering a setting without default costs ($\lambda = 0$), showing that sovereign default is then Ramsey optimal for virtually all productivity realizations. Conversely, for 'prohibitive' default cost levels ($\lambda \geq 1$), default is never optimal. For the most relevant case with intermediate levels of default costs ($0 < \lambda < 1$), we show how Ramsey optimal default decisions depend on the country's wealth level and on the productivity realization.

A. Zero Default Costs

In the absence of default costs ($\lambda = 0$) the original Ramsey problem (5) reduces to a generalized version of the problem analyzed in section II in Grossman and Van Huyck 1988.[20] The proposition below shows that - as in Grossman and Van Huyck - full consumption smoothing and frequent default are then optimal. The proof of the proposition can be found in Appendix C.C5.

PROPOSITION 4: *Suppose $\lambda = 0$ and the productivity process satisfies (13). The solution to the Ramsey problem (7) involves constant consumption equal to*

$$\widetilde{c} = (1-\beta)(\Pi(z_0) + \widetilde{\omega}_0) \tag{14}$$

where $\Pi(\cdot)$ denotes the maximized expected discounted profits from production, defined as

$$\Pi(z_t) \equiv E_t\left[\sum_{j=0}^{\infty} \beta^j \left(-k^*(z_{t+j}) + \beta z_{t+j+1}\left(k^*(z_{t+j})\right)^{\alpha} + \beta(1-d)k^*(z_{t+j}) - \bar{c}\right)\right]$$

with

$$k^*(z_t) = \left(\frac{\alpha\beta E(z_{t+1}|z_t)}{1-\beta(1-d)}\right)^{\frac{1}{1-\alpha}} \tag{15}$$

[20]Grossman and Van Huyck 1988 consider an economy without wealth accumulation and iid income shocks.

denoting the optimal investment policy. For any period t, the optimal default level satisfies

$$a_{t-1}(z_t) \propto -\left(\Pi(z_t) + z_t \left(k^*(z_{t-1})\right)^{\alpha}\right) \tag{16}$$

From equation (16) follows that the optimal commitment policy involves a strictly positive amount of default in all but the best possible productivity state that can be reached in period t.[21] Moreover, the optimal default levels are inversely related to the current level of productivity, i.e., the amount of non-repaid claims strictly increases with the distance of current productivity from its maximally possible level.[22]

Default fully insures the domestic economy against two sources of risk: first, it insures against a low realization of current output due to a low value of current productivity, as captured by the term $z_t\,(k^*(z_{t-1}))^{\alpha}$ in equation (16), a risk that is present in similar form in the endowment setting of Grossman and Van Huyck 1988; second, it insures the domestic economy against (adverse) news regarding the expected profitability of future investments, as captured by the term $\Pi(z_t)$. As a result of this policy, 'total wealth' of the economy, defined as the sum of expected future profits $\Pi(z_t)$ and accumulated past wealth $\widetilde{\omega}_t$, remains constant over time and equal to its initial value $\Pi(z_0) + \widetilde{\omega}_0$, causing consumption in equation (14) to be equally constant over time. In the absence of default costs, risk sharing thus fully and exclusively occurs via optimal sovereign default, with domestic investment being at its expected profit-maximizing level (15).

In the special case where future net worth $\Pi(z)$ is state independent, e.g., in the case where productivity is iid, the previous result replicates the one derived by Grossman and Van Huyck 1988, even though their setting does not consider intertemporal wealth accumulation and features only one internationally traded asset. This occurs because $\lambda = 0$ makes it optimal to have constant wealth and because the shape of the production function considered in Grossmann and Van Huyck becomes linear at the point where the marginal product of capital equals the international interest rate, thereby effectively playing the role of the foreign bond in the present setting. For $\lambda > 0$, this close relationship disappears and intertemporal wealth dynamics start to play a role for risk-absorption.

The finding that optimal default policies achieve complete risk-sharing in the absence of default costs is also related to results in Bulow and Rogoff 1989, who show how even a non-committed government can achieve complete risk-sharing when confronted with a complete set of fairly priced international assets.

B. Prohibitive Default Costs

We now consider 'prohibitive' default cost levels with $\lambda \geq 1$. Default then induces deadweight costs that (weakly) exceed the amount of resources not repaid to lenders. Net

[21]The optimal default policy in the best productivity state is actually indeterminate for $\lambda = 0$. The indeterminacy disappears for $\lambda > 0$, where default is always zero in the highest state.

[22]All this follows from equation (16) noting that (1) $a_{t-1} \geq 0$, (2) realized output $\left(z_t \left(k^*(z_{t-1})\right)^{\alpha}\right)$ is strictly increasing in z_t, and (3) expected future profits $\Pi(z_t)$ weakly increasing in z_t.

of default costs, the sovereign thus cannot gain resources by defaulting. This leads to the following obvious result:

LEMMA 1: *For $\lambda \geq 1$ it is optimal to choose $a_t = 0$ for all t.*

Since default is never optimal, consumption smoothing in response to productivity shocks occurs via the accumulation and decumulation of the international net wealth position and potentially via adjustments of the domestic investment margin.[23] This shows that an interesting trade-off between insurance via default and insurance via international wealth adjustments emerges only for intermediate default cost levels ($0 < \lambda < 1$), as considered in the next section.

C. Intermediate Default Cost Levels

Deriving general analytic results for the Ramsey policy problem (7) is difficult when $0 < \lambda < 1$ because the Ramsey problem is non-linear, features endogenous state variables, and a number of occasionally binding constraints. Nevertheless, one can derive such results for the special cases with very low or very high beginning-of-period wealth levels, as discussed below. The presented results show that the country's wealth position is an important determinant of optimal default policies, in contrast to what is suggested by the limiting cases with zero or prohibitive default costs.

INITIAL WEALTH AT ITS LOWER LIMIT. — Consider first a situation where productivity is given by $z_t = z^n$ ($n \in \{1, ..., N\}$) and beginning-of-period wealth is at its lower bound ($\widetilde{\omega}_t = NBL(z^n)$). We can then define a critical productivity index n^*:

$$n^* = \arg\max_{i \in [1,...,N]} i \quad s.t. \sum_{j=i}^{N} \pi(z^j|z^n) \geq 1 - \lambda. \tag{17}$$

The critical index n^* is defined as the highest productivity index, such that reaching states z^j tomorrow with $j \geq n^*$ still has a likelihood larger than $1-\lambda$. It turns out that the index n^* divides states for which default is Ramsey optimal tomorrow from states for which full repayment is optimal, given that the current state is z^n. The critical index also affects the optimal investment level and the optimal amount of bond holdings. The following proposition characterizes optimal policy in period t:[24]

PROPOSITION 5: *Suppose the productivity process satisfies (13) and consider the NBLs from proposition 2. In addition, suppose $z_t = z^n$ ($n \in \{1, ...N\}$) and $\widetilde{\omega}_t = NBL(z^n)$.*

[23] The latter is discussed in detail in the next section.

[24] For λ sufficiently close to or above 1,we have $n_t^* = N$, such that all expressions in proposition 5 remain well-defined.

Then the optimal policy solving the Ramsey policy problem (7) in period t is given by

$$\widetilde{k}_t = \left(\frac{\alpha\beta}{1+(1-d)\beta} \left(\frac{\left(\sum_{j=1}^{n^*} \pi(z^j|z^n)\right) - \lambda}{1-\lambda} z^{n^*} + \frac{\sum_{j=n^*+1}^{N} \pi(z^j|z^n) z^j}{1-\lambda} \right) \right)^{\frac{1}{1-\alpha}} \tag{18}$$

$$b_t = NBL(z^{n^*}) - z^{n^*} \left(\widetilde{k}_t\right)^{\alpha} - (1-d)\widetilde{k}_t \tag{19}$$

$$\widetilde{c}_t = 0 \tag{20}$$

$$a_t(z^j) = 0 \text{ for } j \leq n^* \tag{21}$$

$$a_t(z^j) = \frac{NBL(z^j) - z^j \left(\widetilde{k}_t\right)^{\alpha} - (1-d)\widetilde{k}_t - b_t}{(1-\lambda)} > 0 \text{ for } j > n^* \tag{22}$$

The proof of the proposition is given in Appendix C.C6. Equations (21) and (22) show that default is suboptimal for all sufficiently good productivity states z^j with $j \leq n^*$; for $j > n^*$ strictly positive default is optimal and the default amount is strictly increasing in j, i.e., there is more default the lower is realized productivity. Consistent with earlier results, it will never be optimal to default as $\lambda \to 1$, as then $n^* \to N$. Conversely, it is optimal to default in all but one state when $\lambda \to 0$, as then $n^* \to 1$. Moreover, it follows from problem (17) that an increase in default costs (weakly) reduces the set of states for which default is optimal.

Proposition 5 also shows that consumption is at its lower bound once wealth is at the marginally binding borrowing limit.[25] Consumption will stay at its lower bound in the next period whenever a sufficiently bad productivity state is reached, i.e. a state z^j with index $j \geq n^*$. This is so because tomorrow's beginning-of-period wealth levels are exactly at their state-contingent marginally binding natural borrowing limit for all productivity states z^j with $j \geq n^*$, such that proposition 5 applies again in the next period. Yet, if a productivity state z^j with $j < n^*$ is reached, then beginning-of-period wealth will strictly exceed the natural borrowing limit and consumption will move back to strictly positive values. The latter event can only occur if λ is sufficiently large, as otherwise $n_t^* = 1$.

Equation (18) shows that the presence of default costs also distorts the optimal investment decision downward relative to the expected profit-maximizing investment level $k^*(z_t)$ defined in (15). This downward distortion is (ceteris paribus) increasing in n^* and in the default costs λ. Since n^* is itself an increasing function of λ, the total effect of increased default cost is to decrease investment relative to the efficient level. Intuitively, lower investment reduces the output risk generated by productivity shocks, as these affect the existing capital stock multiplicatively, and this helps to satisfy the marginally binding natural borrowing limits. In a setting where income risk is purely due to endowment risk

[25]Total consumption can still be positive, as we allow for a positive subsistence level of consumption expenditures $\bar{c} \geq 0$.

and where the production technology is not risky, as in Grossmann and Van Huyck 1988, investment would continue to be optimal.

LARGE WEALTH LEVELS. — This section derives analytical expressions that approximate the Ramsey optimal policies for sufficiently large beginning-of-period wealth levels. The following position summarizes the main result:

PROPOSITION 6: *Suppose* $\lambda > 0$ *and consumption preferences satisfy* $u''(c) \to 0$ *as* $c \to \infty$. *Consider a time horizon* $T < \infty$ *and for* $i = 0, ..., T$ *the policies*

$$\begin{aligned} \widetilde{c}_{t+i} &= (1-\beta)(\Pi(z_{t+i}) + \widetilde{\omega}_{t+i}) \\ \widetilde{k}_{t+i} &= k^*(z_{t+i}) \\ b_{t+i} &= (1+r)\left(\widetilde{\omega}_{t+i} - k^*(z_{t+i}) - \widetilde{c}_{t+i} - \overline{c}\right) \\ a_{t+i}(z^j) &= 0 \text{ for all } j = 1, ..., N, \end{aligned}$$

where $\Pi(z_{t+i})$ *and* $k^*(z_{t+i})$ *are as defined in proposition 4. For any* $\epsilon > 0$ *we can find a wealth level* $\bar{\omega} < \infty$ *so that for all finite initial wealth levels* $\widetilde{\omega}_t \geq \bar{\omega}$, *the Euler equation errors* e_{t+i} *implied by the policies above satisfy* $e_{t+i} < \epsilon$ *for all periods* $j = 0, ..., T-1$.

The proof of the proposition is contained in Appendix C.C7. The fact that the Euler equation error vanishes for sufficiently large initial wealth levels $\widetilde{\omega}_t$ implies that the policies stated in the proposition approximate the truly optimal policies increasingly well; see Santos 2000, for example. For a sufficiently large wealth level, domestic investment thus remains undistorted, i.e. maximizes expected discounted profits. In addition, it is optimal to always fully repay debt. This is in stark contrast to the case with $\lambda = 0$ where frequent default is optimal, independently of the wealth level, see proposition 4. For sufficiently high wealth levels, the presence of even tiny default costs thus implies a complete shift from frequent default to no default. For such wealth levels, risk sharing occurs optimally only via self-insurance, i.e. via the accumulation and decumulation of international wealth, as investment is at its efficient level. This holds true provided u'' decreases towards zero as consumption increases without bond, as is the case for commonly used preference specifications (constant relative or absolute risk aversion). Intuitively, vanishing curvature causes the output risk implied by optimal investment levels to have only negligible influence on consumption utility, whenever consumption (and thus wealth) are sufficiently high. With strictly positive default costs, it is then suboptimal to use default to insure against these output fluctuations.

D. Comparison to a Setting without Commitment

This section compares Ramsey optimal default to the default policies that are optimal in the absence of commitment. Specifically, we consider a non-committed government deciding at the start of any period t whether or not to honour its international repayment obligations.

For the considered setting with proportional default costs, a non-committed government will never find it optimal to repay international debt $D_t > 0$ at the beginning of period t as along as $\lambda < 1$. A default delivers $(1-\lambda)D_t > 0$ units of additional consumption goods relative to the case with repayment and does not adversely affect future choice sets.[26] Lack of commitment (and reputation) thus imposes the restriction that the government cannot issue international debt at all ($D_t = 0$ for all t).[27]

The previous outcome highlights the fact that for $\lambda < 1$ the ability to sustain international debt in the absence of commitment critically hinges on assuming that debt default is associated with fixed costs. The limited commitment literature, e.g. Arellano 2008, typically assumes such costs in the form of market exclusion and direct output costs. These cost are incurred independently of the size of the debt default.

To compare our setting to that studied in the limited commitment literature, we add fixed default costs $\overline{\lambda}(z_t) \geq 0$ into the optimization problem (7). Following the limited commitment literature, we thereby assume that these costs are (weakly) higher for higher productivity states, i.e.

$$\overline{\lambda}(z^1) \geq \overline{\lambda}(z^2) \geq .... \geq \overline{\lambda}(z^N). \tag{23}$$

Furthermore, to facilitate comparison to the limited commitment literature, which typically does not consider endogenous investment choices, we move to an endowment setup by letting $\alpha \to 0$ and $d = 1$. It is then optimal to choose $\widetilde{k}_t \to 0$ for all $t \geq 0$, so that the investment choice can be dropped from the Ramsey problem (7). The only additional change to the Ramsey problem (7) is that one needs to replace the definition of $\widetilde{\omega}_t$, previously given by equation (8), by

$$\widetilde{\omega}_t \equiv z_t + b_{t-1} + a_{t-1}(z_t)(1-\lambda) - \overline{\lambda}(z_t) I_{\{a_{t-1}(z_t)>0\}}, \tag{24}$$

so as to account for the fixed costs, where $I_{\{\cdot\}}$ denotes the indicator function. Note that the productivity shock z_t now assumes the role of an endowment shock.

Provided the fixed default costs $\overline{\lambda}(z^1)$ are sufficiently small, proposition 5 continues to apply, except that for $j > n^*$ one has to add the terms $\frac{\overline{\lambda}(z_t)}{1-\lambda}$ to the default values $a_t(z^j)$ stated in the proposition.[28] Under commitment, default thus continues to be optimal in the very same states as without fixed costs and the optimal default amounts increase as productivity falls.

As we show below, these outcomes differ considerably from those obtained within a

[26] We implicitly assume that the government has no reputation for debt repayment amongst creditors. As is well known, reputation may allow a non-committed government to replicate the commitment solution.

[27] Clearly, for the case with $\lambda > 1$, debt repayment will always be optimal and lack of commitment is not an issue.

[28] The NBLs in the proposition depend on the presence of fixed costs of default and become tighter. The result in the main text follows from noting that (24) can be written as $\widetilde{\omega}_t \equiv z_t + b_{t-1} + \min\{a_{t-1}(z_t)(1-\lambda) - \overline{\lambda}(z_t), 0\}$, and that only $a_{t-1}(z_t) = 0$ or $a_{t-1}(z_t) > \overline{\lambda}(z_t)/(1-\lambda)$ can be optimal choices. As a result, the first order conditions for the stated default choices continue to be satisfied. Furthermore, because fixed costs are assumed to be sufficiently small, strictly positive default for $j > n^*$ continues to dominate no default for these states.

setting with limited commitment.[29] Consider the case $\lambda < 1$ and let $V^c(z_t, b_{t-1})$ denote the value function for a non-committed government which fully honors maturing debt and $V^d(z_{t-1}, b_{t-1})$ the value associated with some default ex-post. We then have

$$V^c(z_t, b_{t-1}) = \max_{c_t, b_t} u(c_t) + \beta E_t[\max\{V^c(z_{t+1}, b_t), V^d(z_{t+1}, b_t)\}]$$
$$s.t.: c_t = z_t + b_{t-1} - \frac{1}{1 + R(z_t, b_t)} b_t$$

where $R(z_t, b_t)$ is the international interest rate charged by lenders, when the productivity state is z_t and the country's international wealth position b_t.[30] The value of defaulting is

$$V^d(b_{t-1}, z_t) = \max_{c_t, b_t, \delta_t} u(c_t) + \beta E_t[\max\{V^c(z_t, b_{t-1}), V^d(z_t, b_{t-1})\}]$$
$$s.t.: c_t = z_t - \frac{1}{1 + R(z_t, b_t)} b_t + \lambda b_{t-1}\delta_t + (1 - \delta_t) b_{t-1} - \overline{\lambda}(z_t)$$

where $\delta_t \in (0, 1]$ denotes the default share. Since the schedule $R(z_t, b_t)$ is independent of past default decisions, default is optimal ($V^d(b_{t-1}, z_t) > V^c(b_{t-1}, z_t)$) whenever

$$b_{t-1} \leq -\frac{\overline{\lambda}(z_t)}{1 - \lambda}. \tag{25}$$

From condition (23) follows that under limited commitment default occurs for low productivity states, as with commitment. Yet, unlike in the commitment setting, it is always optimal to completely default on all outstanding debt ($\delta_t = 1$), whenever condition (25) is met. Furthermore, the maximum amount of debt that can be sustained under limited commitment is equal to $\max_j \overline{\lambda}(z^j)/(1 - \lambda)$, which implies that debt capacity under limited commitment is increasing with the fixed and variable costs of default. As we shall see in the subsequent sections, debt capacity under commitment, as defined by the NBLs, tightens as default costs increase.

III. Quantitative Exploration of Optimal Default Policy

This section investigates whether sovereign default can be optimal in realistically calibrated versions of the model. We thereby consider a setting with only business cycle sized shocks to domestic productivity, as well as a setting in which there is the possibility for a temporary output disaster. The theoretical results established in the previous

[29] In line with the limited commitment literature, we assume no minimum consumption level ($\overline{c} = 0$). The results for the case with commitment stated in this section apply for all $\overline{c} \geq 0$.

[30] It is given by

$$\frac{1}{1 + R(z_t, b_t)} = \frac{1}{1 + r} \sum_{j=1}^{N} I_{\{V^d(z_{z+1}, b_t) > V^c(z_{z+1}, b_t)\}} \pi(z_{t+1}|z_t)$$

and reflects default premia associated with the possibility of non-repayment in future productivty states.

section show that sovereign default is suboptimal for sufficiently high wealth levels, but likely to be optimal for low wealth levels. This section quantitatively explores over what wealth ranges default is Ramsey optimal.

The next section presents the model calibration, including our estimation of default costs. The optimal default policies are discussed in the subsequent sections.

A. Model Calibration

We interpret a model period as one year and use a standard parameterization for business cycle shocks to productivity, setting annual persistence of technology to $(0.9)^4$ and the annual standard deviation of the innovation to 1%.[31] Using Tauchen's 1986 procedure to discretize into a process with two states, one obtains a high productivity state $z^h = 1.0133$, a low productivity state $z^l = 0.9868$ and a transition matrix for the productivity states $\left(z^h, z^l\right)$ given by

$$\pi\left(\cdot|\cdot\right) = \begin{pmatrix} 0.8077 & 0.1923 \\ 0.1923 & 0.8077 \end{pmatrix}. \tag{26}$$

When incorporating also a disaster state, we calibrate using the data provided by Barro and Jin 2011 who consider a sample of 157 GDP disasters and report a mean reduction in GDP of 20.4% during disaster states. Based on these findings we set disaster productivity to $z^d = 0.796$. The transition probability from the business cycle states into the disaster state is chosen to match the unconditional disaster probability of 3.8% per year, as reported in Barro and Jin 2011, and the disaster persistence is set to match the average duration of GDP disasters, which equals 3.5 years. Rescaling the transition probabilities of the business cycle states (26) to reflect the transition probability into a disaster state, we obtain the following transition matrix for the productivity states (z^h, z^l, z^d)

$$\pi\left(\cdot|\cdot\right) = \begin{pmatrix} 0.770 & 0.1850 & 0.0380 \\ 0.1850 & 0.770 & 0.0380 \\ 0.1429 & 0.1429 & 0.7143 \end{pmatrix},$$

where we assumed that an output disaster is equally likely in both business cycle states and that once the disaster ends, both business cycle states can be reached with equal likelihood.

The capital share parameter in the production function is set to $\alpha = 0.34$, the annual depreciation rate to $d = 10\%$ and the discount factor to $\beta = 0.97$. The latter implies an annual real interest rate of approximately 3% for risk-free debt instruments. We choose

[31] The quantitative results reported below are not very sensitive to the precise numbers used. A corresponding calibration at a quarterly frequency is employed in Adam 2011.

consumption preferences with constant relative risk aversion

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

and a moderate degree of risk aversion of $\sigma = 2$. The preference specification satisfies the assumption in proposition 6. For our baseline analysis, we set subsistence consumption to $\overline{c} = 0$. Table 1 summarizes the model parameterization.

Variable	Value	Variable Description
β	0.97	Discount factor
σ	2	Relative risk aversion
α	0.34	Capital share
d	10%	Depreciation rate
$\overline{c}$	0	Subsistence consumption
z^h	1.0133	Productivity in high bus. cycle state
z^l	0.9868	Productivity in low bus. cycle state
z^d	0.796	Productivity in disaster state

Table 1: Baseline parameterization of the annual model

It only remains to determine the default cost parameter λ. While default costs are notoriously difficult to estimate, it is possible to exploit restrictions from our structural model to obtain an estimated lower bound for the costs of default. The idea underlying our estimation approach is to exploit the fact that default costs that accrue to the lender (but not those borne by the borrower) can be estimated from financial market prices and information on default events. This is feasible because the borrower has to compensate the lender *ex ante* for the default costs arising on the lender's side, such that lenders' default costs are reflected in financial market prices.

To exploit this idea, we consider a slightly more general setting than that considered in the Ramsey problem (5), namely one where the lender also bears default costs ($\lambda^l > 0$). Total default costs are then given by $\lambda = \lambda^l + \lambda^b$ with λ^b denoting the borrower's default cost. The structure of the Ramsey problem (5) then remains unchanged, except for the bond pricing equation (4), which has to be adapted so as to reflect the presence of the lenders' default costs λ^l:[32,33]

$$\frac{1}{1+R(z_t,\Delta)} = \frac{1}{1+r}\sum_{n=1}^{N}\left(1-(1+\lambda^l)\delta^n\right)\cdot\pi\left(z^n|z_t\right). \tag{27}$$

[32] In the definition of the beginning of period wealth level (3), λ has to replaced by λ^b, where the latter captures the borrower's proportional default costs.

[33] Appendix C.C8 shows that if a consumption allocation is feasible in a setting where default costs are borne exclusively by the borrower, then it is also feasible if some or all of these costs are borne by the lender instead, as long as the total amount of default costs λ remains unchanged.

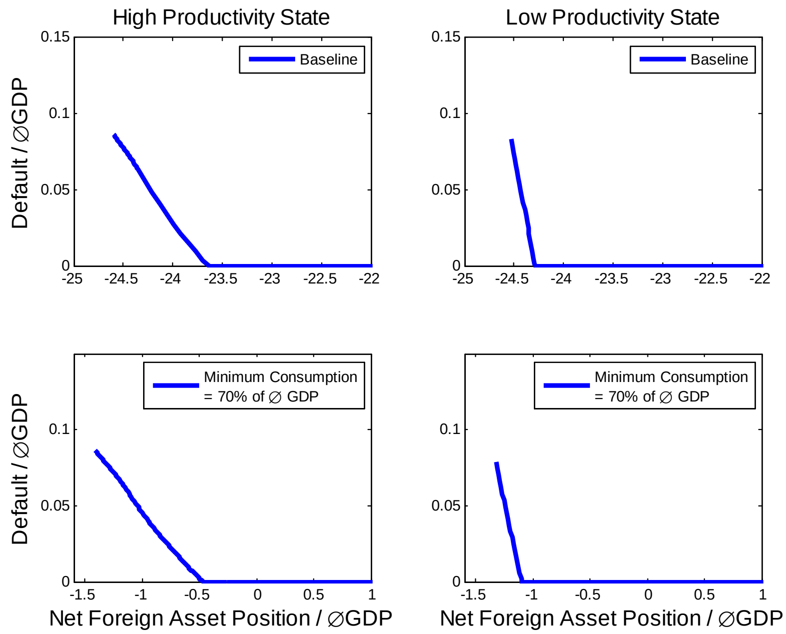

FIGURE 1. OPTIMAL DEFAULT POLICIES FOR $\lambda = 10\%$ (TOP ROW: BASELINE CALIBRATION; BOTTOM ROW: $\overline{c} = 70\%$ OF $\varnothing GDP$)

Appendix C.C9 shows how the previous equation can be combined with data on *ex post* returns from Klingen, Weder, and Zettelmeyer 2004, who consider 21 countries over the period 1970-2000, and data on default events for the corresponding set of countries and years from Cruces and Trebesch 2011, to obtain an estimate of the lender's default cost.[34] This yields

$$\lambda^l = 6.1\%$$

and suggests that lenders suffer a loss of about 6% of the default amount in a sovereign default event.[35] Note that this loss is in addition to the direct losses that result from incomplete repayment by the sovereign debtor.

The total costs of default include the costs accruing to the lender and to the borrower. In our quantitative analysis, we therefore consider default cost levels λ that exceed the estimated value of λ^l. As a baseline, the next subsections consider default cost levels of 10%, which implies that about 60% of the overall default costs are borne by the lender, but we also explore the effects of higher or lower values.

[34]Cruces and Trebesch kindly provided us with required information for the set of countries and years considered in Klingen, Weder, and Zettelmeyer 2004.

[35]If lenders charge a risk-premium, say because their stochastic discount factor covaries with the equilibrium default decisions of the sovereign, then the default cost estimate could ultimately be due risk aversion on the lender's side.

B. Optimal Default with Business Cycle Shocks

This section determines the optimal default policies for the calibrated model with business cycle sized shocks only ($z_t \in \{z^h, z^l\}$). The top panels of figure 1 depict the optimal default policies for the baseline calibration when default costs are given by $\lambda = 10\%$. The left graph shows the optimal default policy when current productivity is high (z^h), while the graph on the right depicts the policy if current productivity is low (z^l). Each graph reports the optimal default amount in the next period if productivity in that period happens to be low (z^l). Default is optimally zero, whenever the high productivity state (z^h) realizes tomorrow. To facilitate interpretation, the optimal default policies are depicted as a function of the net foreign asset position of the country, which is shown on the x-axis.[36] Moreover, the default amounts (y-axis) and the net foreign asset positions (x-axis) are both normalized by average GDP.[37] A value of -1 on the x-axis, for example, corresponds to a situation where the government has issued repayment claims such that its net foreign asset position equals -100% of average GDP.

The lowest value on the x-axis for which the policies are depicted in figure 1 is the point where the country's beginning-of-period wealth level has reached its marginally binding natural borrowing limit (NBL).[38] Default policies extent to the 'right', i.e. to higher net foreign asset positions, without bound. Optimal default thereby remains at a zero level once zero default is optimal for a lower net foreign asset position, in line with proposition 6.

The top panel of figure 1 shows that a fully committed government can sustain a huge amount of debt, close to 25 times average GDP. This is feasible because we assume that domestic consumption can be driven to zero in order to service foreign debt and because the government has commitment power. The top panels of figure 1 also show that default is suboptimal over a wide range of net foreign asset positions. In fact, default happens only once the asset position approaches its lower sustainable bound. Close to that bound default is optimally increasing as the country's net foreign asset (or wealth) position falls.

These outcomes contrast markedly to the case with zero default costs ($\lambda = 0$). Optimal default is then strictly positive and independent of the net foreign asset or wealth position, see proposition 4. Indeed, for $\lambda = 0$, default in the future low state (z^l) always optimally amounts to 6.59% (6.57%) of average GDP if today's productivity state is high (low). This shows that even moderately sized default costs shift optimal default policies strongly in favor of full repayment.

The lower panels of figure 1 explore the robustness of these findings to assuming a non-zero value for subsistence consumption $\bar{c}$. In particular, the lower panels set $\bar{c}$ equal to 70% of average GDP and display the optimal default policies for $\lambda = 10\%$. Comparing the result to the top panels shows that the presence of a minimum consumption threshold tightens the natural borrowing limits and shifts the optimal default policies to the 'right',

[36] The net foreign asset position is the value of b_t, as defined in section 5.

[37] We compute average GDP assuming efficient investment levels, as given by equation (15), and the ergodic productivity state distribution.

[38] This is so because the net foreign asset position b_t and beginning-of-period wealth $\widetilde{w}_t$ commove positively in the optimal solution.

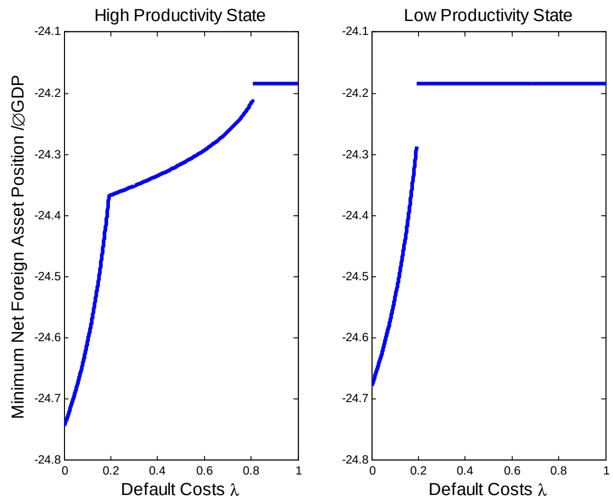

FIGURE 2. EFFECT OF DEFAULT COSTS λ ON THE SUSTAINABLE NET FOREIGN ASSET POSITIONS

i.e., causes default to occur for higher net foreign asset positions. It leaves the shape of the optimal default policies largely unchanged.

Figure 2 depicts - again for both productivity states - how the net foreign asset position at the marginally binding borrowing constraint depends on the level of default costs (depicted on the x-axis).[39] It shows that lower default costs considerably reduce the sustainable net foreign asset position. This contrasts markedly from the case without commitment, discussed in section II.D, where lower default costs reduce the sustainable debt levels.

The two discontinuities in the net foreign asset positions present in figure 2 relate to two default cost thresholds, above which default in the low state tomorrow becomes suboptimal. Specifically, the first discontinuity arises at $\lambda = 19.23\%$ in the panel on the right, arises at the point where default costs equal one minus the transition probability from z^l to z^l. Above this cost level, it becomes suboptimal to default in the next period if current productivity is low. The second discontinuity in the left panel arises at $\lambda = 80.77\%$, which equals one minus the probability of transiting from z^h into z^l. Above this cost level, it also becomes suboptimal to default if the current state is high. For our calibrated model, the no-default assumption is thus fully optimal whenever $\lambda \geq 80.77\%$. More generally, the no-default assumption is consistent with optimality whenever default costs exceed a level of one minus the probability of reaching an undesirable productivity state. In settings where the probability of reaching such states is very low, default costs thus need to be close to the prohibitive level of 100% to make full repayment Ramsey optimal at all times.

[39]The figure is using the baseline calibration with $\bar{c} = 0$.

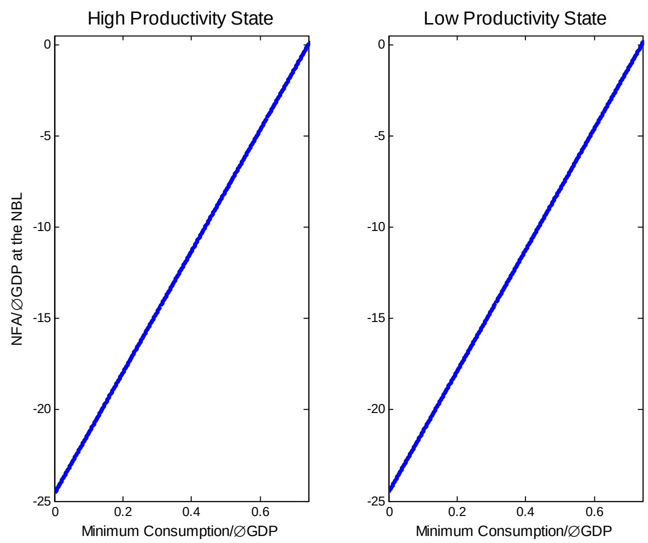

FIGURE 3. EFFECT OF THE MINIMUM CONSUMPTION THRESHOLD ($\bar{c}$) ON THE SUSTAINABLE NET FOREIGN ASSET POSITION

Figure 3 shows how the sustainable net foreign asset position depends on the assumed value for the minimum consumption threshold $\bar{c}$. The strongly positive slopes in the figure indicate that minimum consumption levels have a considerable impact on the net foreign asset positions that are sustainable under commitment. This illustrates that the commitment solution is not necessarily inconsistent with what appear to be plausible borrowing limits from a positive perspective.

Overall, the results from this section show that with business cycle shocks, even moderate levels of default costs cause default to be suboptimal over a large range of net foreign asset positions. The assumption of full repayment, standardly entertained in the Ramsey literature with incomplete markets, thus provides a reasonable approximation to the fully optimal Ramsey policy.

C. Optimal Default with Temporary Output Disasters

Figure 4 depicts the optimal default policies for the economy featuring also a disaster state ($z_t \in \{z^h, z^l, z^d\}$). The figure uses the baseline calibration from section III.A and default costs of $\lambda = 10\%$. Each panel in the figure corresponds to a different productivity state today and reports the optimal default amounts when tomorrow's states are given by z^l and z^d.[40] The default amounts (on the y-axis) are depicted as a function of the country's net foreign asset position (on the x-axis), with both variables being normalized

[40] As before, default is never optimal if the highest productivity state z^h realizes in the next period.

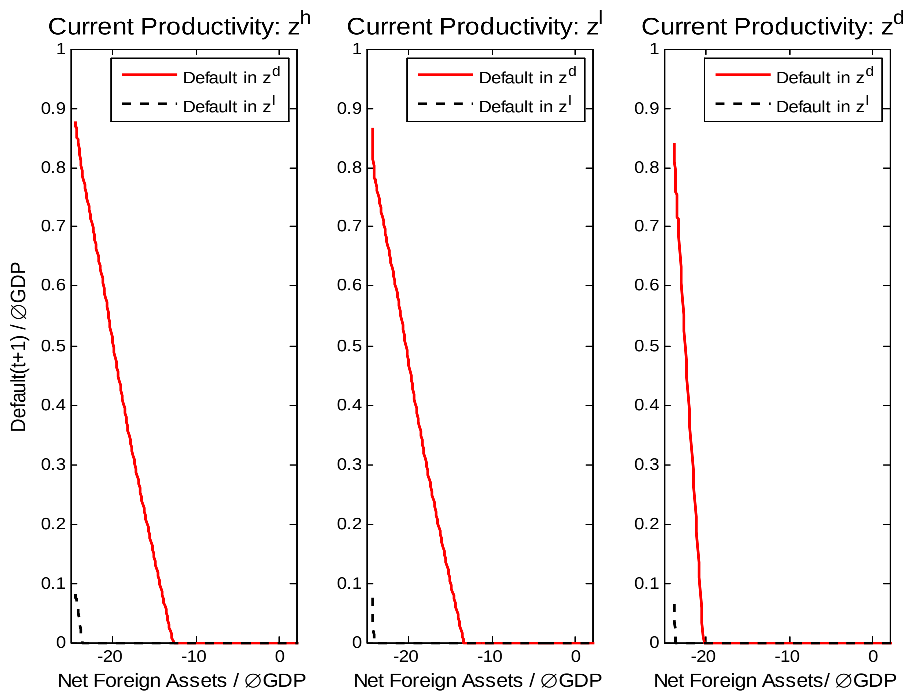

FIGURE 4. OPTIMAL DEFAULT POLICIES WITH A TEMPORARY OUTPUT DISASTER ($\lambda = 10\%$)

by average GDP.

Figure 4 reveals that the default policies for the low business cycle state are only weakly affected by the presence of a disaster state (z^d). In particular, it is optimal to default in the low business cycle state (z^l) only if the net foreign asset position comes close to its maximally sustainable level and default amounts are always below 10% of GDP. For the disaster state, default can become considerably larger and optimally occurs way before the borrowing limit is reached. For a given level of the net foreign asset position, default is thereby larger when transiting from a business cycle state into the disaster state than when simply staying in the disaster state. This is so because the likelihood of reaching the disaster state next period is higher when the economy is already in the disaster state in the current period, which makes it less attractive to purchase insurance against this state because insurance is not fairly priced due to the presence of default costs.

Overall, this section shows that even a temporary output disaster fails to make it Ramsey optimal to default on sovereign debt for empirically encountered values of the net foreign asset position. The next section explores to what extend this continues to be true for the more permanent and larger output disaster experienced in recent times in Greece.

IV. Ramsey Optimal Default in Greece

We now use our framework to derive the model implied timing and the size of sovereign default during the Greek economic crisis following the year 2008.

The Greek economic crisis started towards the end of the year 2008 when output started to slightly contract after having grown at an average annual rate of 3.6% in the years 1995-2007. Following the year 2008, output collapsed. It stood almost 40% below its pre-crisis trend path in 2014. The output collapse experienced by Greece is thus almost twice as large and lasting considerably longer than the average GDP disaster in the Barro and Jin 2011 data set.

As a result of these developments, Greece announced on February 21, 2012 a debt restructuring plan for approximately Euro 200 billion of privately held Greek bonds.[41] Investors received restructuring information and could submit their bonds for restructuring. At the same time, collective action clauses (CACs) were retroactively introduced into all Greek Law Bonds on a joint basis and without explicit consent from creditors. CACs were also applied individually to Foreign Law Bonds. Greece announced on April 25, 2012 that 97 per cent of the targeted bonds had been successfully restructured. According to Zettelmeyer, Trebesch and Gulati 2013 the haircut amounted - depending on the discount rate applied - to 55-65 per cent, making the Greek default the largest in history in terms of absolute size. The actual default size against foreign lenders thereby equaled approximately Euro 68.5 billion.[42]

[41] Official holdings of ECB and Euro Area Central Banks were exempted from the restructuring.

[42] Of the Euro 200 billion of debt targeted in the restructuring since only about Euro 151 billion was held abroad. About 3% of the total was not restructured, i.e., about Euro 6 bln; a further Euro 30.8 bln was held by the ECB within its Security Markets Program (SMP), which was also not restructured. This leaves Euro 114.2 bln of restructured debt held abroad. Using a the midpoint of the haircut estimate of Zettelmeyer, Trebesch and Gulati 2013 of 60%, one obtains

The timing of the default decision and the size of haircut were, however, controversial. A range of analysts and commentators have informally argued that Greece would have been better off with a more timely and larger default.[43] We take up this issue in the next sections, using our optimal default model.

A. Calibrating the Greek Output Process

We determine optimal default policy by calibrating the output process in a way that is consistent with the output forecasts available for Greece at the time and with the realized GDP outcomes, so as to accurately capture output risk in the Greek economy.[44]

We start by estimating the trend growth rate, using GDP data for 1995 - 2007. This yields a pre-crisis real GDP growth trend of 3.6%. We then define all output levels in terms of deviation from the estimated trend line, normalizing trend output to $z_t = 1$. Figure 5 depicts the realized output levels in terms of deviations from trend for the period 1995-2014. The figure illustrates the severity of the output contraction experienced in Greece after the year 2008.

Figure 5 also depicts the OECD's one and two-year ahead output forecasts at various points in time.[45] These forecasts are based on information available at the end of the year preceding the forecasted two year horizon and all predict an increasing deviation from the pre-crisis trend over time. For forecasts made before the year 2012, the realized deviations from trend far exceed the forecasted ones, highlighting the fact that the experienced output drop was largely unexpected.

We capture this output risk in our model by allowing in any year t after the year 2008 for one of the following scenarios: First, there is the possibility of a benign scenario in which the growth rate returns to its pre-crisis level, so that in detrended terms $z_{t+j} = z_t$ for all $j \geq 1$; in this benign scenario there is no further output risk but output will remain permanently below its pre-crises trend path to the extent that an output shortfall has already occurred ($z_t < 1$). Second, the historically realized output shortfall depicted in figure 5 occurs; if this adverse contingency realizes, then there is again the possibility for the same two scenarios occurring in the subsequent year.

For any year $t \geq 2008$ we then choose the ex-ante likelihood for the second scenario such that the model matches the one-year-ahead output forecast of the OECD for the year $t + 1$ along the historically realized output path.[46] Appendix C.C11 provides details of

a default amount of Euro 68.5 bln.

[43]In July 2011, Ken Rogoff stated in a BBC interview: 'I don't think there is any question that if you look at it narrowly from Greece's point of view, it would be better to default now, clean it up and move on.' Following the Greek default in 2012, Martin Feldstein stated in Sep. 2013 in a CNN interview: 'The only way out is for Greece to default on its sovereign debt. When it does, it must write down the principal value of that debt by at least 50 per cent.'

[44]We calibrate directly the output process rather than the technology process because Greek consumption and investment data display somewhat unusual patterns. In particular, the consumption output ratio in Greek data is stable at 90% from 2002-2014, so that variations in the investment to output ratio correlate almost perfectly with variations in the net export to output ratio. The model becomes an endowment economy in the limiting case with $\alpha \to 0$; the optimal investment level k_t then converges to zero and output equals the exogenous productivity process z_t.

[45]Forecasts that are connected with a line refer to forecasts made in the same year, namely the one preceding the forecasted years. We use OECD forecasts because the OECD has not been involved in any bailout operation in Greece, so that forecasts do not need to be consistent with the assumption underlying these operations.

[46]For the years 2013 and 2014, realized output is slightly above the OECD forecast, so that we cannot match the

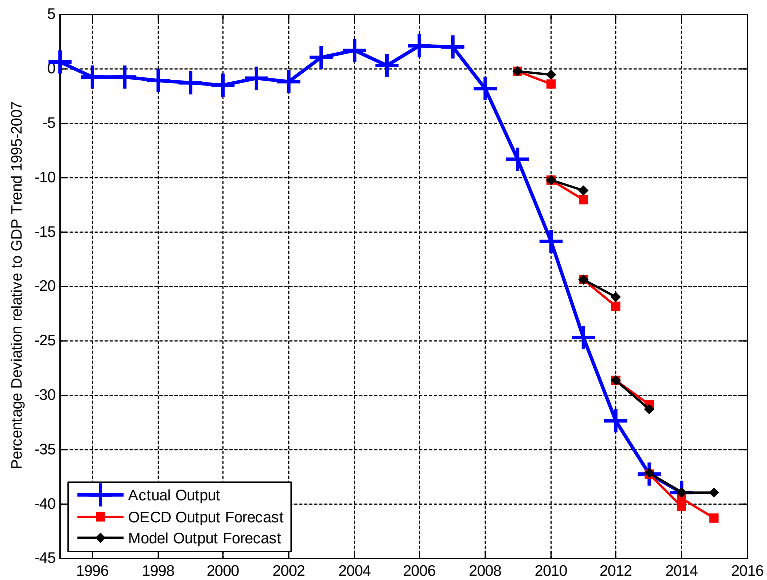

FIGURE 5. GREEK GDP: ACTUAL AND FORECASTS

the resulting output process and the resulting state transition probabilities.

We then check the plausibility of the resulting output process by comparing the two-year-ahead output forecasts of the OECD with that implied by our model, which did not use information contained in these forecasts. Figure 5 depicts the model implied one and two year-ahead forecasts and shows that these replicate very well the two-year-ahead OECD forecasts for the years 2010-2014. The gap for the two-year-ahead forecast for the year 2015 emerges only because the 2015 output level is still unknown at this point, leading us to assume that growth will resume to trend with probability one in the year 2015.[47]

The values for the remaining model parameters (β,σ,α,d and $\bar{c}$) are set equal to their baseline values, as reported in table 1.

B. Optimal Default Policies

We now report the Ramsey optimal default levels for Greece, considering the path of actually realized output levels for the years 2009-2014 and starting the economy in 2008

forecast exactly. We deal with this by setting the probability for the realized scenario to 99 per cent. Quantitatively, this leads to only minor discrepancies.

[47]Ignoring the possibility of an another output shortfall in 2015 biases results against default. Including a further output state to match the 2-year forecast for 2015 would thus give rise to even higher default levels than reported in the subsequent findings.

with the external debt position obtained from Eurostat data.[48]

Figure 6 depicts the Ramsey optimal default amounts against foreign lenders for the years 2009-2012, which are the years in which there were large negative output surprises. The figure reports the Ramsey optimal default amounts (on the y-axis) as a function of the default costs (on the x-axis) and reports for each year the optimal default amounts assuming that default was also optimal in previous years.[49] The total default amount implied by Ramsey optimal policy for the productivity path actually experienced by Greece is thus the sum of the yearly amount shown in the figure. Figure 6 also reports the actual default amount against foreign lenders in the year 2012 (the horizontal line).[50]

Depending on the precise value of default costs assumed, figure 6 shows that default can be Ramsey optimal in each of the considered years. While default in the years 2009 and 2012 is optimal only if default costs are below 12% and 15%, respectively, optimal default levels are much larger for the years 2010 and 2011. Indeed, for default cost levels up to 22%, the optimal default amounts in 2010 and 2011 exceed in both years Greece's actual default for the year 2012. For default cost levels up to this threshold, a larger and more timely default would thus have been optimal for Greece. We take this as evidence, that - from the viewpoint of Greece - the actually implemented default may have been too small, given the size of the output contractions experienced, and may also have come too late.

Note that the Ramsey optimal default amounts for the years 2010 and 2011 often exceed the total amount of Greek debt held by foreigners at the time. This shows that Greece should have - according to our model - issued even more domestic debt (and invest the proceeds in foreign safe debt), so as to be able to insure against adverse shocks of the kind experienced. Indeed, for low enough default cost levels the default amounts become very large because in the limit with zero default costs, the optimal default amount equals the present value of the output shortfall.

Optimal default policies for the years 2013 and 2014, which are not shown in figure 6, imply that default would not have been optimal for the considered range of default costs (again assuming that default in preceding years was Ramsey optimal). This is due to the fact that realized output in those years slightly exceeds the previously forecasted levels.

Figure 7 reports the expected welfare implications of implementing Ramsey optimal default policies for Greece.[51] In particular, the figure reports the welfare equivalent consumption change (y-axis) associated with optimal default policies, relative to the actual default policy (dashed line) and relative to a situation in which default never occurs (solid

[48]The Eurostat balance of payments statistics record bonds at market prices, so that the net external debt position is not identical to our model definition, which is based on face values. We correct this by recomputing the net debt positions, using the net external debt position in 2007, when government bond spreads where largely absent, and adding to this the current account balance of the year 2007. We thus obtain a starting wealth level (GDP+net debt position) of 0.337.

[49]Motivated by our finding from section III.A, which shows that default costs for lenders alone exceed 6%, the figure depicts default polices for default cost levels above 5%.

[50]Actual default against foreign lenders was approximately equal to Euro 68 billion. See footnote 42 for how this can be computed.

[51]Welfare is computed as expected welfare from the year 2008 onwards using 10000 paths for the realization of the productivity process and truncating the infinite sum when welfare contributions of additional periods fall below 10^{-6}.

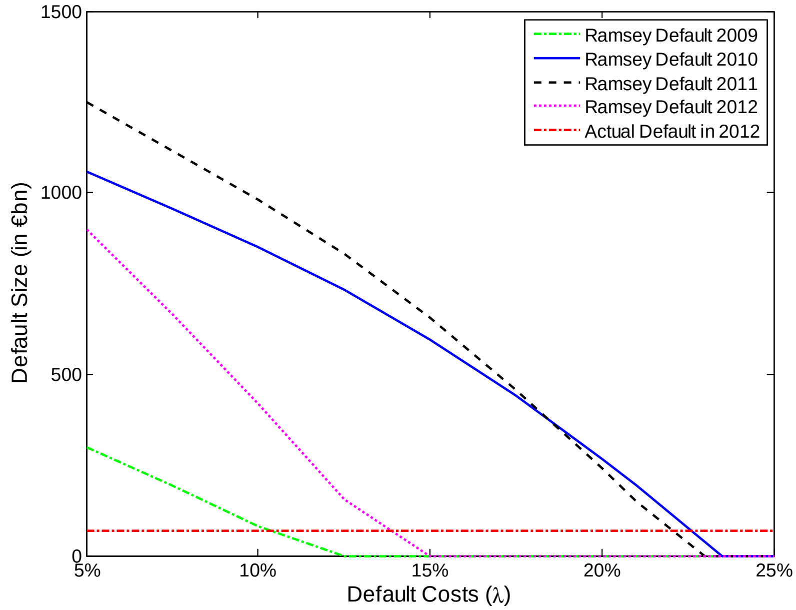

FIGURE 6. RAMSEY OPTIMAL AND ACTUAL DEFAULT AMOUNTS FOR GREECE

line).[52] The welfare gains are shown as a function of the level of the default costs (x-axis) and indicate that - depending on the level of default costs - optimal default policies could have permanently increased consumption by up to one half of a percentage point. The ex-ante welfare gains of Ramsey optimal default relative to the actual default policy are thereby almost equal to those experienced relative to a situation with no default. This shows that the actual default policy did - according to our model - not capture much of the ex-ante achievable welfare gains.

C. Other European Crisis Countries

Italy, Ireland, Portugal and Spain have also experienced sovereign debt crisis, albeit with different degrees of severity. While Portugal and Ireland required formal bail-outs, Italy and Spain did not lose market access. A natural question in this context is whether our model predicts default to be Ramsey optimal in any of these countries. Using the same approach as for Greece to calibrate the output processes in these countries[53], our model suggests that default in these countries occurs only for much smaller default cost

[52]The actual default policy is one where default only happens if the productivity state z_{2012} is reached, in which case default is of the size actually observed.

[53]For Ireland we use 2007 as the starting point for our analysis, owing to the fact that Ireland experienced already in 2008 a large drop in GDP.

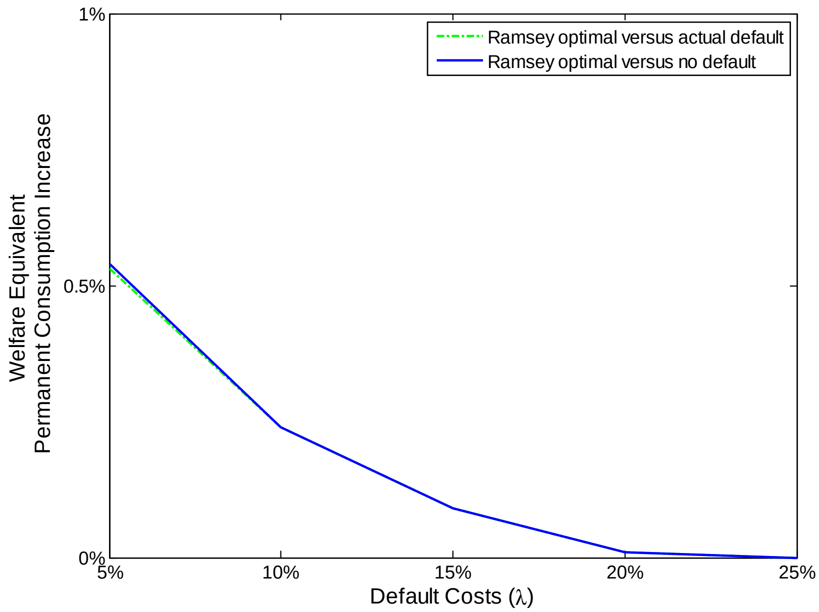

FIGURE 7. WELFARE IMPLICATIONS OF RAMSEY OPTIMAL GREEK DEFAULT

levels than in Greece. For Spain, Portugal and Italy, for example, default only occurs if default cost fall below 10%. For Ireland, the threshold is 12.5%. This shows that it is considerably harder to justify default as Ramsey optimal for these countries.

V. Long Maturities and Optimal Bond Repurchase Programs

This section briefly discusses the effects of also considering bonds with longer maturities in a setting with government commitment. For the case without commitment, Dovis 2013 analyzes how longer maturities can be used to implement a constrained efficient outcome and Amador and Aguiar 2014 determine the optimal deleveraging strategy if a government can issue multiple maturities.

Within the present setup with commitment, introducing longer maturities of risk-free foreign debt has no implications for the analysis. Such bonds cannot be used to better insure against domestic output risk and - like short-dated foreign bonds - merely provide a store of value. Being able to issue long-dated domestic bonds, however, can offer an advantage over just issuing short bonds because long-term bonds may allow the government to economize on the default costs.

Suppose for a moment that the government can repurchase long-dated domestic bonds at the frictionless price.[54] The government can then fully insure domestic consumption against adverse productivity shocks materializing tomorrow by issuing a bond that matures in the period after tomorrow but whose repayment profile depends only on the productivity realization tomorrow. This is true whenever the government can fully repurchase this bond tomorrow once productivity has realized. By repurchasing all bonds tomorrow there will then not be any default in the period after tomorrow when these bond mature, simply because these bonds will not be outstanding anymore, so that the government will not incur any default costs. This way the government can replicate the (efficient) consumption and real investment allocation that is feasible in the absence of default costs (see proposition 4). It is possible because - unlike in the case without commitment - the default probabilities on long-term government bonds do not depend on the repurchase decision per se, instead are functions of the productivity realizations only. The well-known adverse effects of repurchase programs under limited commitment, described for instance in Amador and Aguiar 2014, where long-term bond repurchases lead to an increase in the price of these bonds because the repurchases reduce the default probability, do thus not occur under commitment.

Clearly, market frictions may prevent the government from being able to repurchase long-dated bonds at the frictionless price. Still, issuing long-dated bonds will offer an advantage over outright default on short-term bonds, whenever the surcharge that needs to be paid over the frictionless price when repurchasing bonds is lower than the costs associated with an outright default. If both costs are identical or if the costs of a repurchase exceed those of an outright default, then there exists no advantage from issuing long-dated bonds within the present framework.

[54] This is the price implied by risk-neutral pricing at the international interest rate.

VI. Conclusions and Outlook

In a setting with non-contingent government bonds, debt default is part of the optimal policy of a fully committed government. In a setting with government commitment, the possibility to default relaxes the country's borrowing limits, increases international risk sharing and the efficiency of domestic investment. This increases welfare ex-ante, even if a default is associated with considerable deadweight costs, and shows that even a perfectly committed government may choose not to repay debt in a situation where it has sufficient resources to do so. Our quantitative analysis demonstrates, however, that with plausibly sized default cost levels, default is part of Ramsey optimal policy only in response to very large and persistent adverse domestic shocks such as the ones recently experienced by Greece.

The normative benchmark derived within the present paper may also offer a plausible positive description of the actually observed debt and default patterns. The commitment solution has - for example - no difficulties with rationalizing the existing sovereign debt levels. If anything, the commitment solution can rationalize too large debt levels relative to those observed in the data, at least when assuming that domestic consumption can be driven down to zero so as to serve foreign debt. Similarly, the present approach has no difficulties with explaining why default may be only partial, why default occurs following negative output realizations and why more negative shocks lead to larger default amounts. Exploring the positive relevance of the presented normative theory thus appears to be an interesting avenue for future research.

MICROFOUNDATIONS FOR BOND MARKET STRUCTURE AND DEFAULT COSTS

This appendix provides microfoundations for the assumptions made in the main text that the government issues only non-contingent debt and that deviations from the legally stated repayment promise give rise to proportional default costs. We do so by considering a setting where the government can issue arbitrary state contingent debt contracts, but where contracting frictions make it optimal for the government to issue debt with only a non-contingent legal repayment promise. The same frictions also give rise to default costs. The microfoundations we provide below provide a specific example justifying the setup specified in the previous section, but a range of other conceivable microfoundations may exist.

A1. *Explicit and Implicit Contract Components*

We consider a setting where a government debt contract consists of two contract components. The first component is the explicit contract, which is written down in the form of a legal text. In its most general form, the legal text consists of a description of the contingencies z^n and of the legal repayment obligations $l^n \geq 0$ associated with each contingency $n \in \{1, ..., N\}$.[55] We normalize the size of the legal contract by assuming $\max_n l^n = 1$. The second component is an implicit contract component. This component is not formalized in explicit terms but is commonly understood by the contracting parties. We capture such implicit contract components by a state contingent 'default profile' $\Delta = (\delta^1, ..., \delta^N) \in [0, 1]^N$, which specifies for each possible contingency the share of the legal payment obligation that is *not* fulfilled by the government.[56] Actual repayment at maturity is then jointly determined by the explicit and implicit contract components and given by

$$l^n(1 - \delta^n)$$

for each contingency $n \in \{1, ..., N\}$. If a contingency arises for which $\delta^n > 0$, the countries pays back less than the legally (or explicitly) specified amount l^n and we shall say that 'the country is in default'. The explicit and implicit contract components are perfectly known to agents.

In the setting just described, a desired state-contingent repayment profile can be implemented by incorporating it either into the explicit legal repayment profile l^n or into the implicit profile δ^n. In the absence of further frictions, these two components would be perfect substitutes and the optimal form of the government debt contract thus indeterminate.

[55]The fact that $l^n \geq 0$ can be justified by assuming a lack of commitment on the lenders' side. Such a lack of commitment appears reasonable, given the existence of secondary markets on which government debt can be traded.

[56]The fact that $\delta^n \leq 1$ can again be justified by a lack of commitment on the lenders' side, which makes it impossible to write contracts that specify additional transfers to the borrower at maturity. The assumption that $\delta^n \geq 0$ facilitates interpretation in terms of default, but is never binding in our numerical applications.

A2. Contracting Frictions

We now introduce two simple contracting frictions. First, we assume that explicit legal contracting is costly. This reflects the fact that writing down an explicit legal text describing a financial contract requires the input of lawyers and bankers, thus consuming resources. Second, we assume that implicit contracting, while not creating similar resource costs, gives rise to the risk that the common understanding about the implicit contract component may be lost *after the maturity date* of the contract. This reflects the possibility that agents may have difficulties recalling the implicit contract agreements after a long period of time, especially after the maturity date of the contract. This differs for the explicit contract, where agents can always go back and read about their contract obligations, such that common understanding is insured independently of time. The fact that the common understanding about the implicit contract components may disappear is perfectly and rationally anticipated by all agents.

We now describe these two frictions in greater detail. We normalize the costs of writing a non-contingent legal contract ($l^n = 1$ for $n = 1, ..., N$) to zero and assume that incorporating a contingency gives rise to a proportional legal fee $\lambda \geq 0$ that is charged against the value of the contingent agreement. This is in line with the casual empirical observation that lawyers typically charge fees that are proportional to the value of the agreements they formulate. In particular, legally incorporating a payment $l^n \leq 1$ for some contingency z^n in the explicit contract, involves the costs

$$\lambda \left(1 - l^n\right)$$

per contract issued, where $1 - l^n$ denotes the value of the deviation from the baseline payment of 1.

While incorporating a state contingency in the repayment structure via the implicit contract component $\delta^n > 0$ does not give rise to costs, it exposes the government to the risk that the common understanding about a default event may be lost after the maturity date of the contract. This is relevant for the borrower because in the absence of a recallable implicit contract component, courts base their decisions on a comparison of the explicit contract obligation with the actual actions (payments) that occurred. Default events that are followed by a lack of common understanding about the implicit contract thus provide strong incentives for lenders to sue the government for fulfilment of the explicit contract, i.e. to sue the government for repayment of the legally stated amount.[57] Anticipating such behavior, the government will engage - at the time the default occurs - in a negotiation process with the lender, with the objective to reach an *explicit* legal settlement that protects it from being sued in the future.

[57] As documented in Panizza, Sturzenegger and Zettelmeyer 2009, legal changes in a range of countries in the late 1970s and early 1980s eliminated the legal principle of 'sovereign immunity' when it came to sovereign borrowing. Specifically, in the U.S. and the U.K. private parties can sue foreign governments in courts if the complaint relates to a commercial activity, amongst which courts regularly count the issuance of sovereign bonds. We implicitly assume that lenders cannot commit to not sue the government. Again, this appears plausible, given that secondary markets allow initial buyers of government debt to sell the debt instruments to other agents.

The settlement agreement transforms the previous implicit contract component into an explicit one, by stating that the debt contract is regarded as fulfilled, even if the actual payment amount fell short of the amount specified in the legal text of the contract. The threat of going to court to obtain such an explicit settlement via a court ruling in the period where the default happens and where a common understanding about the implicit components still exists, will induce the lender to agree to such an agreement.[58] Since we assume explicit legal contracting to be costly, the settlement agreement following a default event gives rise to the legal costs (or default costs)

$$\lambda l^n \delta^n$$

per contract, where $l^n \delta^n$ denotes the value of the settlement agreement, i.e. the defaulted amount on each contract. For simplicity, we assume here that the same proportional fee λ is charged in the ex-post settlement stage as applies when writing an explicit contingent contract *ex ante*. We discuss below the case where the ex-post settlement costs are higher. While the legal fees associated with writing a legal contract are assumed to be borne by the government, we allow for the possibility that the settlement fees are shared between the lender and the borrower, with the lender paying $\lambda^l \geq 0$, the borrower paying $\lambda^b \geq 0$, and $\lambda^l + \lambda^b = \lambda$.

A3. Optimal Government Debt Contract

Consider a government that wishes to implement a contingent payment $p(z) \leq 1$ for some contingency $z \in Z$. Specifying the contingency as part of the legal contract involves the contract writing costs

$$\lambda \left(1 - p(z)\right)$$

per contract and no *ex post* settlement costs in case the default contingency arises in the future. Alternatively, not specifying the contingent payment as part of the legal contract gives rise to expected default costs of[59]

$$\Pr(z|z_0)\lambda \left(1 - p(z)\right),$$

where z_0 is the contingency prevailing at the time when the contract is issued. Since $\Pr(z|z_0) \leq 1$ and since default costs are borne at a later stage, i.e. when the contract matures, the government will always strictly prefer to issue a non-contingent explicit contract and to shift contingencies into the implicit contract profile. This continues to be

[58]The fact that - due to the large number of actors involved - the implicit contract component of government debt can be verified in court makes government debt contracts special. Implicit components of private contracts, for example, are often private information available to the contracting parties only, and thus cannot be verified in court, not even over the lifetime of the contract. The optimal form of private contracts, therefore, generally differs from the optimal form of government debt contracts.

[59]The expected settlement cost for the lender enters the borrower's optimization reasoning because the borrower has to compensate the lender *ex ante* for the expected costs borne by the lender.

true even in the more general case where the *ex post* settlement costs are much higher than the cost associated with incorporating the contingency *ex ante* into the legal contract, provided the probability $\Pr(z|z_0)$ of reaching the default event is sufficiently small.

Summing up, it is optimal for the government to issue debt that is non-contingent in explicit legal terms. At the same time, the government has the option to deviate from the legally specified payment amount, but such actions give rise to proportional default costs λ. We thus provided microfoundations for the assumptions entertained in the previous section.

A4. Government versus Private Debt

The contracting framework can also be used to justify why the government optimally borrows on behalf of private agents in the international market. This is highly relevant because it implies that private and sovereign default cannot be used interchangeably to insure domestic consumption. The optimality of sovereign borrowing arises because of a fundamental difference between sovereign and private debt contracts: the implicit contract components of private contracts are private information to the contracting parties, and thus cannot be verified in court, not even over the lifetime of the contract. This is different for a sovereign debt contract that is widely shared among many individuals. Achieving state contingency in private contracts thus has to rely on explicit contracting, which creates costs that dominate those incurred in sovereign borrowing, as the non-contingency of sovereign bonds is optimal precisely to avoid the costs of explicit legal contracting.

*

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ONLINE APPENDIX

Paper: Optimal Sovereign Default

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C1. First Order Equilibrium Conditions

This appendix derives the necessary and sufficient first order conditions for problem (7). Using equation (8) to express beginning-of-period wealth, the problem is given by

$$\max_{\{b_t, a_t \geq 0, \widetilde{k}_t \geq 0, \widetilde{c}_t \geq 0\}} E_0 \sum_{t=0}^{\infty} \beta^t u(\widetilde{c}_t)$$

$$s.t.:$$

$$\widetilde{c}_t = z_t \widetilde{k}_{t-1}^{\alpha} + (1-d)k_{t-1} + b_{t-1} + (1-\lambda)a_{t-1}(z_t) - \bar{c} - \widetilde{k}_t - \frac{1}{1+r} b_t - p_t' a_t$$

$$z_{t+1}\widetilde{k}_t^{\alpha} + (1-d)k_t + b_t + (1-\lambda)a_t(z_{t+1}) \geq NBL(z_{t+1}) \quad \forall z_{t+1} \in Z$$

$$\widetilde{w}_0 = w_0,\ z_0 \ given,$$

We now formulate the Lagrangian Λ, letting η_t denote the multiplier on the budget constraint in period t, $\nu_t(z^j)$ the multiplier for the short-selling constraint on the Arrow security that pays off in state z^j in $t+1$, and $\omega_t \in R^N$ the vector of multipliers associated with the natural borrowing limits for each possible realization of productivity in $t+1$, where $\omega_t(z^j)$ denotes the entry of the vector pertaining to productivity state z^j:

$$\Lambda = E_0 \Bigg[\sum_{t=0}^{\infty} \beta^t u(\widetilde{c}_t)$$

$$+\beta^t \eta_t \left(-\widetilde{c}_t + z_t \widetilde{k}_{t-1}^{\alpha} + (1-d)k_{t-1} + b_{t-1} + (1-\lambda)a_{t-1}(z_t) - \bar{c} - \widetilde{k}_t - \frac{1}{1+r} b_t - p_t' a_t \right)$$

$$+ \beta^t \sum_{j=1}^{N} \nu_t(z^j) a_t(z^j)$$

$$+\beta^t \sum_{j=1}^{N} \omega_t(z^j) \left(z^j \widetilde{k}_t^{\alpha} + (1-d)k_t + b_t + (1-\lambda)a_t(z^j) - NBL(z^j) \right) \Bigg]$$

We drop the inequality constraints for $\widetilde{k}_t$ and $\widetilde{c}_t$, as the Inada conditions guarantee an interior solution for these variables whenever $\widetilde{w}_0 > NBL(z^0)$. Differentiating the Lagrangian with respect to the choice variables, one obtains

$$\widetilde{c}_t: \; u'(\widetilde{c}_t) - \eta_t = 0$$

$$b_t: \; -\eta_t \frac{1}{1+r} + \beta E_t \eta_{t+1} + \sum_{n=1}^{N} \omega_t(z^n) = 0$$

$$a_t(z^j): \; -\eta_t p_t(z^j) + \beta\pi(z^j|z^t)\eta_{t+1}(z^j)(1-\lambda) + \nu_t(z^j) + \omega_t(z^j)(1-\lambda) = 0 \quad \text{for } j = 1, ..., N$$

$$\widetilde{k}_t: \; -\eta_t + \beta E_t\left[\left(\alpha z_{t+1}\widetilde{k}_t^{\alpha-1} + 1 - d\right)\eta_{t+1}\right] + \left(\alpha\widetilde{k}_t^{\alpha-1} + 1 - d\right)\sum_{j=1}^{N}\omega_t(z^j)z^j = 0$$

Using the FOC for consumption to replace η_t in the last three FOCs, delivers three Euler equations:

(C2a)

$$Bond: -u'(\widetilde{c}_t)\frac{1}{1+r} + \beta E_t u'(\widetilde{c}_{t+1}) + \sum_{j=1}^{N}\omega_t(z^j) = 0$$

$$Arrow: -u'(\widetilde{c}_t)p_t(z^j) + \beta\pi(z^j|z_t)u'(\widetilde{c}_{t+1}(z^j))(1-\lambda)$$

(C2b)

$$+\nu_t\left(z^j\right) + \omega_t(z^j)(1-\lambda) = 0 \quad \text{for } j = 1, ..., N$$

(C2c)

$$Capital: -u'(\widetilde{c}_t) + \beta E_t\left[\left(\alpha z_{t+1}\widetilde{k}_t^{\alpha-1} + 1 - d\right)u'(\widetilde{c}_{t+1})\right] + \left(\alpha\widetilde{k}_t^{\alpha-1} + 1 - d\right)\sum_{j=1}^{N}\omega_t(z^j)z^j = 0$$

In addition, we have the following complementarity conditions for $j = 1, ..., N$:

(C2d)

$$0 \le a_t(z^j), \; \nu_t(z^j) \ge 0, \; \text{one holds strictly}$$

(C2e)

$$0 \le z^j\widetilde{k}_t^{\alpha} + (1-d)k_t + b_t + (1-\lambda)a_t(z^j) - NBL(z^j), \; \omega_{t+1}(z^j)) \ge 0, \; \text{one holds strictly}$$

Combined with the budget constraint, the Euler equations and the complementarity conditions constitute the necessary and sufficient optimality conditions for problem (7).

C2. Proof of Proposition 1

Consider some state-contingent beginning-of-period wealth profile w_t arising from some combination of bond holdings, default decisions and capital investment $(F_{t-1}, D_{t-1}, \Delta_{t-1}, k_{t-1})$ in problem (5). We show below that one can generate the same state contingent beginning-of-period wealth profile $\widetilde{w}_t = w_t$ in problem (7) by choosing $\widetilde{k}_{t-1} = k_{t-1}$ and by choosing an appropriate investment profile (a_{t-1}, b_{t-1}). Moreover, the same amount of funds are required to purchase (a_{t-1}, b_{t-1}) in $t-1$ than to purchase (F_{t-1}, D_{t-1}) when the default profile is Δ_{t-1}. With the costs of financial investments generating a particular future payout profile being the same in both problems, identical physical investments, and identical beginning of period wealth profiles, it then follows from constraints (5b) and (7b) that a consumption path which is feasible in (5) is also feasible in (7).

To simplify notation we establish the previous claim for the case with $N = 2$ productivity states only. The extension to more states is straightforward. Consider the following state-contingent initial wealth profile

$$\begin{pmatrix} w_t(z^1) \\ w_t(z^2) \end{pmatrix} = \begin{pmatrix} z^1 k_{t-1}^{\alpha} + (1-d)k_{t-1} + F_{t-1} - D_{t-1}(1-(1-\lambda)\delta_{t-1}(z^1)) \\ z^2 k_{t-1}^{\alpha} + (1-d)k_{t-1} + F_{t-1} - D_{t-1}(1-(1-\lambda)\delta_{t-1}(z^2)) \end{pmatrix}.$$

As is easily verified, this beginning-of-period wealth profile in problem (7) can be replicated by choosing $\widetilde{k}_{t-1} = k_{t-1}$ and by choosing the portfolio

$$b_{t-1} = F_{t-1} - D_{t-1}, \tag{C3}$$

$$a_{t-1} = \begin{pmatrix} D_{t-1}\delta_{t-1}(z^1) \\ D_{t-1}\delta_{t-1}(z^2) \end{pmatrix} \tag{C4}$$

The funds f_{t-1} required to purchase and issue (F_{t-1}, D_{t-1}) under the default profile $\Delta_{t-1} = (\delta_{t-1}(z^1), \delta_{t-1}(z^2))$ are given by

$$f_{t-1} = \frac{1}{1+r} F_{t-1} - \frac{1}{1+R(z_{t-1}, \Delta_{t-1})} D_{t-1}$$

where the interest rate satisfies

$$\frac{1}{1+R(z_{t-1}, \Delta_{t-1})} = \frac{1}{1+r}\left((1-\delta_{t-1}(z^1))\pi(z^1|z_{t-1}) + (1-\delta_{t-1}(z^2))\pi(z^2|z_{t-1})\right).$$

The funds $\widetilde{f}_{t-1}$ required to purchase (b_{t-1}, a_{t-1}) are given by

$$\widetilde{f}_{t-1} = \frac{1}{1+r}(F_{t-1} - D_{t-1}) + \frac{1}{1+r}\left(\delta_{t-1}(z^1)\pi(z^1|z_{t-1}) + \delta_{t-1}(z^2)\pi(z^2|z_{t-1})\right) G^S_{t-1},$$

where we used the price of the Arrow security in (6). As can be easily seen $\widetilde{f}_{t-1} = f_{t-1}$, as claimed. This completes the proof that a consumption path which is feasible in (5) is also feasible in (7). Since $a \geq 0$, the reverse is also true, because equations (C3) and (C4) can then be solved for values $(D_{t-1}, F_{t-1}, \Delta_{t-1})$ satisfying $D_{t-1} \geq 0$, $F_{t-1} \geq 0$, and $\Delta_{t-1} \in [0,1]^N$, so that it is possible in problem (5) to obtain a portfolio with the same contingent payouts.[60] Again, this portfolio has the same costs and thus admits the same consumption choices. This completes the equivalence proof.

C3. Proof of Proposition 2

We look for fixed point solutions satisfying (13). In a first step, we derive the unique solution to (11) under the assumption that the NBLs in the constraints of (11) satisfy (13). We then show in a second step, that the NBLs implied by the objective function in (11) also satisfy (13), so that problem (11) defines a mapping from the set of NBLs satisfying (13) to the set of NBLs satisfying (13). As a last step, we show that this mapping generically has a unique solution.

Under the assumption that the NBLs in the constraints of (11) satisfy (13), we can show that the solution to (11) is given as follows. For any productivity state z^n $(n = 1, ..., N)$, define the critical future productivity state n^*

(C5)
$$n^* = \arg\max_{i \in [1,...,N]} i \quad s.t. \sum_{j=i}^{N} \pi(z^j|z^n) \geq 1 - \lambda.$$

As we establish below, the optimal choices in state state z^n are

$$\widetilde{k}_t = \left(\frac{\alpha\beta}{1 + (1-d)\beta} \left(\frac{\left(\sum_{j=1}^{n^*} \pi(z^j|z^n)\right) - \lambda}{1-\lambda} z^{n^*} + \frac{\sum_{j=n^*+1}^{N} \pi(z^j|z^n) z^j}{1-\lambda} \right) \right)^{\frac{1}{1-\alpha}}$$

(C6)
$$\widetilde{k}^n = \left(\frac{\alpha\beta}{1 + (1-d)\beta} \left(\frac{\left(\sum_{j=1}^{n^*} \pi(z^j|z^n)\right) - \lambda}{1-\lambda} z^{n^*} + \frac{\sum_{j=n^*+1}^{N} \pi(z^j|z^n) z^j}{1-\lambda} \right) \right)^{\frac{1}{1-\alpha}}$$

(C7)
$$b^n = NBL(z^{n*}) - z^{n*} \left(\widetilde{k}^n\right)^{\alpha} - (1-d)\widetilde{k}^n$$

(C8)
$$a^n(z^j) = 0 \text{ for } j \leq n^*$$

(C9)
$$a^n(z^j) = \frac{NBL(z^j) - z^j \left(\widetilde{k}^n\right)^{\alpha} - (1-d)\widetilde{k}^n - b^n}{(1-\lambda)} > 0 \text{ for } j > n^*$$

[60] Note that for $a < 0$, no such choices would exist, which shows that $a \geq 0$ is required to obtain equivalence.

The NBLs in the objective function of (11) implied by the previous solution also satisfies (13). To see this, consider two productivity states z^n and z^m with $n < m$ and the associated optimal choices. The optimal choices $a^m, b^m, \widetilde{k}^m$ for state z^m also satisfy the constraints of problem (11) for state z^n, i.e., are feasible choices in state z^n. Moreover, since $a^m(z^j)$ is increasing in j, it follows from (12) that

$$\sum_{j=1}^{N} a^m(z^j)p^n(z^j) \leq \sum_{j=1}^{N} a^m(z^j)p^m(z^j),$$

i.e., the purchase of the risky assets a^m is cheaper in state z^n than in state z^m. Since the cost of investments in capital and bonds do not depend on the productivity state, this implies that the NBL in state z^n must be weakly laxer than the one in state z^m, as claimed.

We now show that (C6)-(C9) satisfy the necessary and sufficient first order conditions of problem (11). Letting $\omega^n(z^j)$ denote the Lagrange multipliers for the first set of constraints in (11) and $\nu^n(z^j)$ the multipliers for the second set of constraints, the first order necessary conditions are given by

$$\widetilde{k}^n : 1 + \alpha \left(\widetilde{k}^n\right)^{\alpha-1} \sum_{j=1}^{N} \omega^n(z^j)z^j + (1-d)\sum_{j=1}^{N} \omega^n(z^j) = 0 \tag{C10}$$

$$b^n : \frac{1}{1+r} + \sum_{j=1}^{N} \omega^n(z^j) = 0 \tag{C11}$$

$$a^n(z^j) : p^n(z^j) + \omega^n(z^j)(1-\lambda) + \nu^n(z^j) = 0. \tag{C12}$$

We also have the constraints

$$z^j\left(\widetilde{k}^n\right)^{\alpha} + (1-d)\widetilde{k}^n + b^n + (1-\lambda)a^n(z^j) - NBL(z^j) \geq 0, \omega^n(z^j) \leq 0, \text{ one holding strictly} \tag{C13}$$

$$a^n(z^j) \geq 0, \nu^n(z^j) \leq 0, \text{ one holding strictly} \tag{C14}$$

Conditions (C12) and (C14) can equivalently be summarized as

$$p^n(z^j) + \omega^n(z^j)(1-\lambda) \geq 0, a^n(z^j) \geq 0, \text{ one holding strictly} \tag{C15}$$

so that the first order conditions are given by (C10),(C11), (C13) and (C15). Since the objective is linear and the constraint set convex, the first order conditions are necessary and sufficient. We now show that the postulated solution satisfies these first order conditions.

Since $a^n(z^j) > 0$ for $j > n^*$ for the conjectured solution, condition (C15) implies

$$\omega^n(z^j) = -\frac{p^n(z^j)}{1-\lambda}$$
$$= -\frac{1}{1+r}\frac{\pi(z^j|z^n)}{1-\lambda} < 0 \quad \text{for all } j > n^*$$

We now conjecture (and verify later) that

$$\omega^n(z^j) = 0 \quad \text{for all } j < n^* \tag{C16}$$

$$\omega^n(z^{n^*}) = -\frac{1}{1+r} - \sum_{j=n^*+1}^{N} \omega^n(z^j) \tag{C17}$$

For the previous conjecture, equation (C11) holds by construction. Also, the second inequality of (C13) holds for all $j \in \{1, ..., N\}$ because we have

$$\sum_{j=n^*+1}^{N} \omega^n(z^j) = \frac{1}{1+r}\frac{\sum_{j=n^*+1}^{N} \pi(z^j|z^n)}{1-\lambda} < \frac{1}{1+r}$$

from the definition of n^*, so that $\omega^n(z^{n*}) < 0$. Equations (C8) and (C9) then imply that (C15) hold. Furthermore, (C6) implies that (C10) holds. It thus only remains to show that the first inequality for (C13) also holds. For $j \geq n^*$ this follows from (C9). For $j < n^*$ this is also true because $-NBL(z^j)$ is increasing as j falls under the assumed ordering for the NBLs in the constraints of (11), $z^j \left(\widetilde{k}^n\right)^\alpha + (1-d)\widetilde{k}^n$ is equally increasing as j falls due to the assumed ordering of the productivity levels, and the first inequality of (C13) holds with equality for $j = n^*$ due to (C7). As a result, the first inequality in (C13) holds strictly for $j < n^*$, justifying our conjecture (C16). This proves that all first order conditions hold for the conjectured solution (C6)-(C9).

Since the solution (C6)-(C9) is linear in the NBLs showing up in the constraints of (11), the minimized objective is also a linear function of these NBLs. The fixed point problem defined by (11) is thus characterized by a system of equations that is linear in the NBLs, which generically admits a unique solution. This completes the proof.

C4. Proof of Proposition 3

Suppose that in some period t and for some productivity state z^n ($n \in \{1, ..., N\}$), beginning-of-period wealth falls short of the limits implied by the marginally binding NBL, i.e.

$$\widetilde{w}_t(z^n) = NBL(z^n) - \varepsilon, \tag{C18}$$

for some $\varepsilon > 0$. We then prove below that for at least one contingency z^j in $t+1$, which can be reached from z^n in t with positive probability, it must hold that

$$\widetilde{\omega}_{t+1}(z^j) \leq NBL(z^j) - \varepsilon(1+r), \tag{C19}$$

such that along this contingency the distance to the marginally binding NBL is increasing at the rate $1+r > 1$ per period. Since the same reasoning also applies for future periods, and since the marginally binding NBLs assume finite values, this implies the existence of a path of productivity realizations along which wealth far in the future becomes unboundedly negative, such that any finite borrowing limit will be violated with positive probability.

It remains to prove that if (C18) holds in period t and contingency z^n, this implies that (C19) holds for some contingency z^j in $t+1$ ($j \in \{1, ..., N\}$) and that z^j can be reached from z^n with positive probability. Suppose for contradiction that

$$\widetilde{\omega}_{t+1}(z^h) > NBL(z^h) - \varepsilon(1+r) \tag{C20}$$

for all $h \in \{1, ..., N\}$ that can be reached from z^n, i.e. for which $\pi(z^h|z^n) > 0$. The cost-minimizing way to satisfy the constraints (C20) for all h, when replacing the strict inequality by a weak one, is to choose the solution (C6), (C8)- (C9) and $b_t = NBL(z^{n_t^*}) - z^{n_t^*}\widetilde{k}_t^{\alpha} - (1-d)k_t - \varepsilon$. It follows from the proof of proposition 2 that this holds true whenever the NBLs in the constraint satisfy the ordering (13). Achieving this requires $NBL(z^n) - \varepsilon$ units of funds in t, which in turn implies that satisfying constraints (C20) with strict inequality requires strictly more funds than are available in t, whenever the state can be reached with positive probability. As a result, (C19) must hold for at least one j that can be reached from z^n with positive probability.

C5. Proof of Proposition 4

We first show that the proposed consumption solution (14) and investment policy (15) satisfy the budget constraint, that the inequality constraints $a \geq 0$ are not binding, and that the NBLs are not binding either. Thereafter, we show that the remaining first order conditions of problem (7), as derived in appendix C.C1, hold.

We start by showing that the portfolio implementing (14) in period $t = 1$ is consistent with the flow budget constraint and $a \geq 0$. The result for subsequent periods follows by induction. In period $t = 1$ with productivity state z^n, beginning-of-period wealth under the investment policy (15) is given by

$$\widetilde{\omega}_1^n \equiv z^n \left(k^*(z_0)\right)^{\alpha} + (1-d)k^*(z_0) + b_0 + a_0(z^n) \tag{C21}$$

To insure that consumption can stay constant from $t = 1$ onwards we again need

$$\widetilde{c} = (1-\beta)(\Pi(z^n) + \widetilde{\omega}_1^n) \tag{C22}$$

for all possible productivity realizations $n = 1, ..N$. This provides N conditions that can be used to determine the $N + 1$ variables b_0 and $a_0(z^n)$ for $n = 1, ..., N$. We also have the condition $a_0(z^n) \geq 0$ for all n and by choosing $\min_n a_0(z^n) = 0$, we obtain one more condition that makes it possible to pin down a unique portfolio (b_0, a_0). Note that the inequality constraints on a do not bind for the portfolio choice, as we have one degree of freedom, implying that the multipliers $\upsilon_1(z^n)$ in Appendix C.C1 are zero for all $n = 1, ..., N$. It remains to be shown that the portfolio achieving (C22) is feasible given the initial wealth $\widetilde{\omega}_0$. Using (C21) to substitute $\widetilde{\omega}_1^n$ in equation (C22) we obtain

$$\widetilde{c} = (1-\beta)(\Pi(z^n) + z^n \left(k^*(z_0)\right)^{\alpha} + (1-d)k^*(z_0) + b_0 + a_0(z^n)) \qquad \forall n = 1, ...N.$$

Combining with (14) we obtain

$$\Pi(z^n) + z^n \left(k^*(z_0)\right)^{\alpha} + (1-d)k^*(z_0) + b_0 + a_0(z^n) = \Pi(z_0) + \widetilde{\omega}_0$$

Multiplying the previous equation with $\pi(z^n|z_0)$ and summing over all n one obtains

$$E_0\left[\Pi(z_1) + z_1\left(k^*(z_0)\right)^{\alpha}\right] + b_0 + \sum_{n=1}^{N} \pi(z^n|z_0)a_0(z^n) = \Pi(z_0) + \widetilde{\omega}_0.$$

Using $\Pi(z_0) = -k^*(z_0) + \beta E_0\left[z_1\left(k^*(z)\right)^{\alpha}\right] + \beta(1-d)k^*(z_0) - \overline{c} + \beta E_0\left[\Pi(z_1)\right]$ and (6) the previous equation delivers

$$(1-\beta)E_0\left[\Pi(z_1) + z_1\left(k^*(z_0)\right)^{\alpha} + (1-d)k^*(z_0)\right] + b_0 + (1+r)p_0'a_0 = -k^*(z_0) + \widetilde{\omega}_0 - \overline{c}$$

Using $\beta = 1/(1+r)$ this can be written as

$$(1-\beta)\left(E_0\left[\Pi(z_1) + z_1\left(k^*(z_0)\right)^{\alpha} + (1-d)k^*(z_0)\right] + \frac{1}{\beta}p_0a_0 + b_0\right)$$

$$+\frac{1}{1+r}b_0 + p_0'a_0 = -k^*(z_0) + \widetilde{\omega}_0 - \overline{c} \tag{C23}$$

From substituting (C21) into (C22), multiplying the result with $\pi(z^n|z_0)$ and summing over all n, it follows that the terms in the first line of the previous equation are equal to

$$(1-\beta)\left(E_0\left[\Pi(z_1) + z_1\left(k^*(z_0)\right)^{\alpha}\right] + \frac{1}{\beta}p_0'a_0 + b_0\right) = \widetilde{c}$$

where we also used (6) and $1 + r = 1/\beta$. Using this result to substitute the first line in (C23) shows that (C23) is just the flow budget equation (7b) for period zero. This proves that the portfolio giving rise to (C22) in $t = 1$ for all $n = 1, ..., N$ satisfies the budget constraint of period $t = 0$. The results for $t \geq 1$ follow by induction. The result (16) follows from substituting (C21) into (C22) and noting that b_0 and $(1-d)k^*(z_0)$ are not state contingent.

From equation (C22) and the fact that $\Pi(z_t)$ is bounded, it follows that $\widetilde{\omega}_t$ is bounded so that the process for beginning-of-period wealth automatically satisfies the marginally binding NBLs. The multipliers ω_{t+1} in appendix C.C1 are thus equal to zero for all t and all contingencies. Using $\upsilon_t(z^n) \equiv 0$, $\omega_{t+1} \equiv 0$, the fact that capital investment is given by (15) and that the Arrow security price is (6), the Euler conditions (C2a) - (C2c) all hold when consumption is given by (14). This completes the proof.

C6. Proof of proposition 5

Since we start with a beginning-of-period wealth level at the marginally binding NBL, we necessarily have $\widetilde{c}_t = 0$. Otherwise one could afford an even lower initial wealth level and satisfy all constraints, which would be inconsistent with the definition of the marginally binding NBLs given in (11). Indeed, the available beginning of period wealth $\widetilde{\omega}_t$ is just enough to insure that $\widetilde{\omega}_{t+1} \geq NBL(z_{t+1})$ for all possible future productivity states z_{t+1}. The optimal choices $a_t \in R^N, b_t, \widetilde{k}_t$ are thus given by the cost-minimizing choices satisfying $a_t \geq 0$ plus the marginally binding NBLs in $t+1$ for all possible future productivity states. Formally,

$$\begin{aligned}
&\min_{a_t, b_t, \widetilde{k}_t} \bar{c} + \widetilde{k}_t + \frac{1}{1+r} b_t + \sum_{j=1}^{N} a_t(z^j) p_t(z^j) \\
&s.t. \\
&z^j \widetilde{k}_t^{\alpha} + (1-d)\widetilde{k}_t + b_t + (1-\lambda) a_t(z^j) \geq NBL(z^j) \text{ for } j = 1, ..., N \\
&a_t(z^j) \geq 0 \text{ for } j = 1, ..., N
\end{aligned}$$

The optimal choices are thus equivalent to those solving problem (11). From appendix C.C3 follows that under the stated assumptions, the optimal choices are given by (C6)-(C9).

C7. Proof of proposition 6

Using the assumed policies, $\frac{1}{1+r} = \beta$, $p_t(z^j) = \frac{\pi(z^j|z_t)}{1+r}$, and the fact that the NBLs are not binding for sufficiently high wealth levels, the Euler equations (C2a)-(C2c) for $i = 0$ imply

(C24a) $$u'(\widetilde{c}_t) = E_t u'(\widetilde{c}_{t+1})$$

(C24b) $$\nu_t(z^j) = \beta\pi(z^j|z_t)\left(u'(\widetilde{c}_t) - u'(\widetilde{c}_{t+1}(z^j))(1-\lambda)\right) \quad \text{for } j = 1, ..., N$$

(C24c) $$0 = -u'(\widetilde{c}_t) + \alpha\widetilde{k}_t^{\alpha-1}\beta E_t u'(\widetilde{c}_{t+1}) z_{t+1} + \beta(1-d)$$

We show below that the Euler equation errors for $i = 0$ converge to zero and that $\upsilon_t(z^j) \geq 0$ as the wealth $\widetilde{\omega}_t$ increases without bound. Under the assumed policies, wealth evolves according to

$$
\begin{aligned}
\widetilde{w}_{t+1} &= z_{t+1}k^*(z_t)^\alpha + b_t \\
&= z_{t+1}k^*(z_t)^\alpha + \frac{1}{\beta}\left(\widetilde{w}_t - k^*(z_t) - (1-\beta)(\Pi(z_t) + \widetilde{w}_t) - \overline{c}\right) \\
&= \widetilde{w}_t + z_{t+1}k^*(z_t)^\alpha - \frac{1}{\beta}k^*(z_t) - \frac{(1-\beta)}{\beta}\Pi(z_t) - \frac{1}{\beta}\overline{c} \\
&= \widetilde{w}_t + \Pi(z_t) + z_{t+1}k^*(z_t)^\alpha - \frac{1}{\beta}k^*(z_t) - \frac{1}{\beta}(\Pi(z_t)) - \frac{1}{\beta}\overline{c} \\
&= \widetilde{w}_t + \Pi(z_t) - E_t[\Pi(z_{t+1})] + (z_{t+1} - E_t z_{t+1})k^*(z_t)^\alpha
\end{aligned}
$$

Since the fluctuations in z_t, $k^*(z_t)$ and $\Pi(z_t)$ are all bounded, fluctuations in wealth are also bounded over any finite number of periods. Moreover, the fluctuations in wealth are independent of the initial wealth level. As a result, fluctuations in consumption are also bounded and of a size that is not dependent on the wealth level under the proposed consumption policy.

We now show that for $i = 0$ and a sufficiently high wealth level the Euler equation residuals remains below ϵ. Using the assumed consumption policy and the result from the previous equation, we have

$$
\begin{aligned}
E_t[\widetilde{c}_{t+1}] &= (1-\beta)E_t\left[(\Pi(z_{t+1}) + \widetilde{w}_{t+1})\right] \\
&= (1-\beta)\left(\widetilde{w}_t + \Pi(z_t)\right) \\
&= c_t
\end{aligned}
$$

i.e. consumption follows a random walk. Now consider equation (C24a), which requires

$$
\begin{aligned}
u'(\widetilde{c}_t) &= E_t u'(\widetilde{c}_{t+1}) \\
&= \sum_{j=1}^{N} \pi(z^j|z_t)u'(\widetilde{c}_{t+1}(z^j))
\end{aligned}
\tag{C25}
$$

From Taylor's theorem we have

$$
u'(\widetilde{c}_{t+1}(z^j)) = u'(\widetilde{c}_t) + u''(c^j)(\widetilde{c}_{t+1}(z^j) - \widetilde{c}_t)
$$

where c^j can be chosen from the bounded interval

$$
[\min\{\widetilde{c}_t, \min_j \widetilde{c}_{t+1}(z^j)\}, \max\{\widetilde{c}_t, \max_j \widetilde{c}_{t+1}(z^j)\}]
$$

whose width is independent of the wealth level $\widetilde{w}_t$ (as fluctuations in consumption do not depend on wealth as shown above). Moreover, under the assumed consumption policy, the lower bound of this interval - and thus also c^j increases without bound, as $\widetilde{w}_t$

increases without bound. Using the earlier result, (C25) can be rewritten as

$$0 = \sum_{j=1}^{N} \pi(z^j) \left(u''(c^j)(\widetilde{c}_{t+1}(z^j) - \widetilde{c}_t)\right)$$

where the sum on the right-hand side of the equation denotes the Euler equation residual whenever it is not equal to zero. For the considered consumption policies, we have that $\widetilde{c}_{t+1}(z^j) - \widetilde{c}_t$ is bounded and invariant to wealth. Moreover, for sufficiently large wealth, c^j increases without bound, therefore $u''(c^j) \rightarrow 0$ under the maintained assumption about preferences. This implies that for any given $\epsilon > 0$ we can find a wealth level w^* so that the Euler equation residual falls below ϵ. Since the fluctuations in wealth are bounded over the finite horizon T and do not depend on the initial wealth level, we can find an initial wealth level $\bar{w}$ high enough such that over the next T periods wealth stays above w^*. The Euler equation errors then remain below ε over the next $T - 1$ periods, as claimed.

Similar arguments can be made to show that (C24c) holds and that (C24b) implies $v_t(z^j) \geq 0$ for a sufficiently large initial wealth level. We omit the proof here for the sake of brevity.

C8. Default Costs Born by Lender

This appendix shows that if a consumption allocation is feasible in a setting in which default costs are borne by the borrower, then it is also feasible in a setting in which some or all of these costs are borne by the lender instead. For simplicity, we only consider the extreme alternative where all costs are born by the lender. Intermediate cases can be covered at the cost of some more cumbersome notation.

Consider a feasible choice $\{F_t \geq 0,\ D_t \geq 0,\ \Delta_t \in [0,1]^N,\ k_t \geq 0,\ c_t \geq 0\}_{t=0}^{\infty}$, i.e. a choice that satisfies the constraints of the government's problem (5), which assumes $\lambda^l = 0$ and $\lambda^b = \lambda$. Let variables with a hat denote the corresponding choices in a setting in which the lender bears all default costs, i.e., where $\lambda^l = \lambda$ and $\lambda^b = 0$. We show below that it is then feasible to choose the same real allocation, i.e., to choose $\widehat{k}_t = k_t$ and $\widehat{c}_t = c_t$, provided one selects appropriate values for $\widehat{F}_t$, $\widehat{D}_t$ and $\widehat{\Delta}_t$.

First, note that in a setting where foreign investors bear all settlement costs, the interest rate $\widehat{R}(z_t, \widehat{\Delta}_t)$ on domestic bonds satisfies

$$1 + r = \frac{1 - (1+\lambda)\sum_{n=1}^{n} \widehat{\delta}_t^n \Pi(z^n|z_t)}{\frac{1}{1+\widehat{R}(z_t,\widehat{\Delta}_t)}} \tag{C26}$$

where the denominator on the right-hand side denotes the issuance price of the bond and the numerator the expected repayment net of the lender's settlement cost. The previous equation thus equates the expected returns of the domestic bonds with the expected return on the foreign bond.

Next, consider the following financial policies:[61]

(C27a) $$\widehat{\Delta}_t = (1-\lambda)\Delta_t \frac{D_t}{\widehat{D}_t}$$

(C27b) $$\widehat{D}_t = \frac{1+\widetilde{R}(z_t, \widehat{\Delta}_t)}{\widetilde{R}(z_t, \widehat{\Delta}_t)} \frac{R(z_t, \Delta_t)}{1+R(z_t, \Delta_t)} D_t$$

(C27c) $$\widehat{F}_t = F_t + \widehat{D}_t - D_t$$

As we show below, in a setting in which settlement costs are borne by the lender, the financial policies $\{\widehat{F}_t, \widehat{D}_t, \widehat{\Delta}_t\}_{t=0}^{\infty}$ give rise to the same state-contingent financial payoffs as generated by the policies $\{F_t, D_t, \Delta_t\}_{t=0}^{\infty}$ in a setting in which default cost are borne by the borrower. Therefore, as claimed, the former policies allow the implementation of the same real allocations.

Consider the financial flows generated by the policy component $(\widehat{F}_t, \widehat{D}_t, \widehat{\Delta}_t)$. In period t, the financial inflows are given by

$$\frac{\widehat{D}_t}{1+\widehat{R}(z_t, \widehat{\Delta}_t)} - \widehat{F}_t$$

Using the definitions (C27), it is straightforward to show that these inflows are equal to

$$\frac{1}{1+R(z_t, \Delta_t)} D_t - F_t$$

which are the inflows under the policy (F_t, D_t, Δ_t) in a setting where default costs are borne by the lender.

We show next that the financial flows in $t+1$ are also identical under the two policies. The financial inflows generated by the policy choices $(\widehat{F}_t, \widehat{D}_t, \widehat{\Delta}_t)$ in some future contingency $n \in \{1, \dots N\}$ in period $t+1$ are given by

$$-\widehat{D}_t(1-\widehat{\delta}_t^n) + \widehat{F}_t$$

From the first and last equation in (C27), we determine that these flows are equal to

$$-(1-(1-\lambda)\delta_t^n)D_t + F_t$$

which are the inflows generated by the policy (F_t, D_t, Δ_t) in a setting where default costs are borne by the lender.

[61] Lengthy but straightforward calcucations, which are available upon request, show that these policies satisfy $\widehat{\Delta}_t \in [0,1]^N$, although they may imply $\widehat{F}_t < 0$, which requires the government also to issue safe bonds, i.e. bonds that promise full repayment.

Finally, since the policies $\{F_t, D_t, \Delta_t\}_{t=0}^{\infty}$ satisfy the marginally binding natural borrowing limits in the government's problem (5), it must generate bounded financial flows, with the same therefore also applying for the policies $\{\widehat{F}_t, \widehat{D}_t, \widehat{\Delta}_t\}_{t=0}^{\infty}$. These policies thus also satisfy the marginally binding natural borrowing limits, which completes the proof.

C9. Estimation of Lender's Default Costs

Consider a non-contingent one period bond that in explicit legal terms promises to repay one unit and that has an associated implicit default profile $\Delta = (\delta^1, ..., \delta^n) \in [0,1]^n$. A risk-neutral foreign lender, who bears proportional default costs λ^b in the event of default and can earn the gross return $1 + r$ on alternative safe investments, will price this bond according to equation (27). As explained below, the asset pricing equation (27) can be used to obtain an estimate for λ^l.

We start by defining the *ex post* return epr_t on a government bond

$$1 + epr_t = \frac{1 - \sum_{n=1}^{N} \delta^n \Pi(z^n|z_t)}{\frac{1}{1+R(z_t,\Delta)}},$$

which is the bond return that accounts for losses due to non-repayment but not for potential default costs. *Ex post* returns can be measured from financial market data. Using the previous equation to substitute $\sum_{n=1}^{N} (1-\delta^n)\cdot\pi(z^n|z_t)$ on the r.h.s. of equation (27) and applying the unconditional expectations operator[62], one obtains

$$\lambda^l = \frac{E\left[epr_t - r\right]}{E\left[(1 + R(z_t,\Delta))\sum_{n=1}^{N} \delta^n \Pi(z^n|z_t)\right]}. \tag{C28}$$

Information about the average excess return, which shows up in the numerator of the previous equation, can be obtained from Klingen, Weder, and Zettelmeyer 2004, who consider 21 emerging market economies over the period 1970-2000. Using data from table 3 in Klingen, Weder, and Zettelmeyer 2004, the average excess return varies between -0.2% and +0.5% for publicly guaranteed debt, depending on the estimation method used.[63,64] We use the average of the estimated values and set $E[epr_t - r] = 0.15\%$.

[62]The expectations operator integrates over the set of possible histories $z^t = \{z_t, z_{t-1}, ...\}$.

[63]As suggested in Klingen, Weder, and Zettelmeyer 2004, we use the return on a three-year US government debt instrument as the safe asset, since it approximately has the same maturity as the considered emerging market debt.

[64]The fact that *ex post* excess returns on risky sovereign debt are relatively small or sometimes even negative is confirmed by data provided in Eichengreen and Portes 1986 who compute *ex post* excess returns using interwar data. The

We now turn to the denominator on the r.h.s. of equation (C28). Using a first order approximation we obtain

$$(C29)\quad E\left[(1+R(z_t,\Delta))\sum_{n=1}^{N}\delta^n\Pi(z^n|z_t)\right]\approx E\left[1+R(z_t,\Delta)\right]E\left[\sum_{n=1}^{N}\delta^n\Pi(z^n|z_t)\right],$$

where the last term equals (again to a first order approximation)

$$E\left[\sum_{n=1}^{N}\delta^n\Pi(z^n|z_t)\right]\approx \Pr(\delta>0)E[\delta|\delta>0).$$

Using data compiled by Cruces and Trebesch 2011, who kindly provided us with the required information, we observe for the 21 countries considered in Klingen et al. 2004 and for the period 1970-2000 a total of 58 default events, thus the average yearly default probability equals 8.9%. The average haircut conditional on a default was 25%; these figures therefore imply

$$E\left[\sum_{n=1}^{N}\delta^n\Pi(z^n|z_t)\right]\approx 2.22\%.$$

The average *ex ante* interest rate $R(z_t,\Delta)$ appearing in equation (C29) can be computed by adding to the average *ex post* return of 8.8% reported in table 3 in Klingen, Weder, and Zettelmeyer 2004 for publicly guaranteed debt, the average loss due to default, which equals 2.22% to first order, such that $R(z_t,\Delta)\approx 11.02\%$. Combining these results to evaluate λ^l in equation (C28) delivers our estimate for the default costs accruing to lenders reported in the main text.

C10. Numerical Solution Approach

We solve the recursive version of the Ramsey problem from section I.B using global solution methods, so as to account for the non-linear nature of the optimal policies. The state space S of the problem is given by

$$S=\left\{z^1\times\left[NBL(z^1),w_{max}\right],...,z^N\times\left[NBL(z^N),w_{max}\right]\right\}$$

where $NBL(z^n)$ denotes the marginally binding natural borrowing limits and w_{max} is a suitably chosen and sufficiently large upper bound for the country's wealth level.

We want to describe equilibrium in terms of time-invariant policy functions that map the current state into current policies. Hence, we want to compute policies

$$\widetilde{f}:(z_t,w_t)\rightarrow(\{c_t,k_t,b_t,a_t\}),$$

negative *ex post* excess returns likely arise due to the presence of sampling uncertainty: the high volatility of the nominal exchange rate makes it difficult to estimate the mean *ex post* excess returns.

where their values (approximately) satisfy the optimality conditions derived in C.C1. We use a time iteration algorithm where equilibrium policy functions are approximated iteratively. In a time iteration procedure, tomorrow's policy (denoted by f^{next}) is taken as given and solves for the optimal policy f today, which in turn is used to update the guess for tomorrow's policy. Convergence is achieved once $||f - f^{next}|| < \epsilon$, where we set $\epsilon = 10^{-5}$. We then set $\widetilde{f} = f$. In each time iteration step we solve for optimal policies on a sufficient number of grid points distributed over the continuous part of the state space. Between grid points we use linear splines to interpolate tomorrow's policy. Following Garcia and Zangwill 1981, we can transform the complementarity conditions of our first order equilibrium conditions into equations. For more details on the time iteration procedure and how complementarity conditions are transformed into equations, see, for example, Brumm and Grill 2014. To come up with a starting guess for the consumption policy, we use the fact that at the NBLs optimal consumption equals the subsistence level. We therefore guess a convex, monotonically increasing function g which satisfies $g(z^i, NBL(z^i)) = \bar{c}\ \forall i$ and use a reasonable guess for $g(z^i, w_{max})$.

C11. Calibration of Greek Output Process: Further Details

This annex contains the details of the model specification we use in our application to the Greek economic crisis. We use realized output levels in the Greek economy for 2009 to 2013 to estimate model GDP. We obtain the five states $(z_{2009}, z_{2010}, z_{2011}, z_{2012}, , z_{2013}) = (1-8.30\%, 1-15.90\%, 1-24.70\%, 1-32.7\%, 1-37.3\%) = (0.917, 0.841, 0.753, 0.676, 0.627)$. In addition we have $z_{2008} = 1$. For 2014 we have estimated a value $z_{2014} = 0.610$. Remember that we have assumed that we start in z_{2008} and for each following year two scenarios are possible:

1) Return to trend growth of 3.6% from $t + 1$ onwards (however, no return to level)

2) Observed fall in output relative to trend growth actually happening in $t + 1$

Having defined the output process in terms of deviations from trend, the economy transitions from z_t to the absorbing state $\bar{z}_t$ whenever the first scenario realizes.

The vector of states z for our model is therefore given by

$$\begin{aligned} z &= (z_{2008}, \bar{z}_{2009}, z_{2009}, \bar{z}_{2010}, z_{2010}, \bar{z}_{2011}, z_{2011}, \bar{z}_{2012}, z_{2012}, \bar{z}_{2013}, z_{2013}, z_{2014}) \\ &= (1, 0.917, 0.917, 0.841, 0.841, 0.753, 0.753, 0.676, 0.676, 0.627, 0.627, 0.610)\,. \end{aligned}$$

To parametrize the state transition matrix we employ the following procedure: We use one year ahead OECD output forecasts to compute implied probabilities for both scenarios, i.e. we set the transition probabilities in our model such that the expected output value tomorrow is equal to the one year ahead OECD forecast. However, this is not possible for the years 2013 and 2014 where output is above the forecast. In this case, we set the probability for the second scenario to 99 per cent. This yields the following vector π of transition probabilities:

$$\begin{aligned}\pi &= (p_{2008}, p_{2009}, p_{2010}, p_{2011}, p_{2012}, p_{2013}) \\ &= (0.966, 0.732, 0.603, 0.445, 0.01, 0.01)\,.\end{aligned}$$

and the accompanying Markov transition matrix M is given by

(C32)

$$M = \begin{pmatrix} 0.966 & 0 & 0.034 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.732 & 0 & 0 & 0.268 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.603 & 0 & 0 & 0.397 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.445 & 0 & 0 & 0.555 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.010 & 0 & 0 & 0.990 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0.990 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.000 \end{pmatrix}.$$

We compute the equilibrium by starting in 2014 and solving for the optimal policies backwards in time.