

Sign-Equivalent Transformations and Equilibrium Systems: Theory and Applications*

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Abstract

We develop a novel theoretical framework—the Sign-Equivalent Transformation (SET)—to analyze the mathematical structure of equilibrium systems. SET formalizes the invariance of equilibrium solutions under specific transformations and thereby provides a unified methodology for studying a broad class of economic models. We illustrate its scope by applying it to canonical settings such as oligopoly competition, production networks, network games, and public-economics environments, as well as to new models involving nonlinear peer effects, network games with general CES norms, and team-based organizations. The SET framework not only simplifies the analysis of equilibrium systems but also yields new insights and, in some cases, enables the explicit derivation of a best-reply potential for the original game. Overall, SET offers a versatile and powerful tool for the theoretical analysis of economic interactions.

JEL classification codes: C62, C72, D85, H41, Z13.

Keywords: Network games, production network, oligopoly theory, public economics, best-response potential, variational inequality.

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