# Financial Integration with Heterogeneous Beliefs\*

Jörg Rieger<sup>†</sup>

June 2014

#### **Abstract**

In this paper we study the effects of financial integration on risk-sharing. Conventional macroeconomic theory suggests that the integration of financial markets improves welfare. In contrast to the literature we assume that households have heterogeneous beliefs. Because of the differences in beliefs, households are not only sharing the risk but also speculating. We show that with speculation, financial integration can increase the risk in the economy and that a full financial integration is not always beneficial. We also have a numerical example for a small set of countries and show that the losses due to heterogeneous beliefs are small.

**Keywords:** Heterogeneous Beliefs, Financial Integration, Incomplete Markets **JEL Classification:**D52,D53,F36,G15

<sup>\*</sup>I thank Thomas Eife, Zeno Enders, Seminar Participants at the University of Heidelberg and Participants at the Money, Macro and Finance Conference in Dublin for helpful comments. All errors are mine.

<sup>&</sup>lt;sup>†</sup>Adress for Correspondence: Alfred Weber-Institut, University of Heidelberg, Bergheimer Str. 58, 69115 Heidelberg. E-mail: joerg.rieger@awi.uni-heidelberg.de. Tel.: +49 (0)6221 542935

## 1 Introduction

Financial integration and globalization has increased since the 1980s. The common view in is that financial integration improves the risk-sharing opportunities in the economy and therefore improves welfare. In particular households are enabled to further smooth consumption across states, i.e. make consumption less volatile. Based on this argument, several authors claim that significant welfare gains could be made (see e.g. Davis et al. (2001), van Wincoop (1999)). Despite the theoretical arguments in favor of financial liberalization, the empirical evidence is mixed. Furthermore, the political support for financial liberalization is often undermined with the argument that foreign investors will increase a countries vulnerability to external shocks as well.

International asset pricing models can not deal with the arguments by the critics and the empirical evidence. Studies such as Black (1974), Stulz (1981a) or Stulz (1981b) use portfolio restrictions to study the portfolio problems of domestic and foreign investors. Subrahmanyam (1975) constructs a model that includes pricing and welfare and shows that international capital market integration is always pareto optimal. In the same vein, Errunza and Losq (1989) study a multi-country model and shows that that the removal of investment barriers generally leads to an increase in the aggregate market value of securities.

A common assumption in these models is that all households in the economy hold the same beliefs. However, there is widespread evidence that agents acting on financial markets do have heterogeneous beliefs (see e.g. Bart and Masse (1981), Frankel and

<sup>&</sup>lt;sup>1</sup>A recent example of a an attempt of financial liberalization was the 'through-train to Hong Kong' program. The goal of the program was to enable the people in mainland china to invest directly into the stock market in Hong Kong. Later that year the program was abandoned because the chinese government feared for the stability of the financial market in Hong Kong and to protect the population in mainland China.(see http://www.bloomberg.com/news/2011-03-23/china-should-revive-through-train-hong-kong-investment-plan-guotai-says.html (last access on 14.11.2013))

Froot (1987), Kandel and Pearson (1995) or Mankiw et al. (2003), Hommes (2011)). Also Mei et al. (2009) argue that heterogeneous beliefs are the cause for significant price differences of chinese dual-class shares. Furthermore, Anderson et al. (2005) show that heterogeneous beliefs are an important source of asset price volatility. Heterogeneous beliefs are also studied in the context of international financial markets, Li and Muzere (2010) and Dumas et al. (2011), however these papers do not study the effects of an increased financial integration.

In this paper we study the effects of financial integration in a two-country model, in particular the effect of financial integration on the households' risk. If households hold different beliefs then there is an additional motive to hold assets. In addition to the risk-sharing motive, driven by differences in risk aversion and endowment processes, there will also be some speculative trading, i.e. trading caused by differences in beliefs. This additional motive for holding assets might impede the gains made from risk-sharing because agents might hold too much of an asset.

We will show that financial integration can have adverse effects on the economy, i.e. after some households get access to a new asset the risk in the economy increases instead of declining. While with gaining access to more assets, households will always be able to better share the risks, households will also speculate with in these new assets. This is caused by the heterogeneity the heterogeneity in beliefs. If all households hold the same beliefs, then there is no speculation and financial integration results in a decreasing risk. If, on the other hand, households hold heterogeneous beliefs then the integration of financial markets may not be optimal.

<sup>&</sup>lt;sup>2</sup>Also surveys about macroeconomic forecasts such as the Survey of Professional Forecasters by the Federal Reserve and the European Central Bank show that there is a considerable disagreement among professional forecasters.

<sup>&</sup>lt;sup>3</sup>Other important studies on financial markets with heterogeneous beliefs include Miller (1977), Harrison and Kreps (1978), Varian (1985), Kurz and Beltratti (1997) and the recent survey by Xiong (2012).

We also provide a numerical sample for a small set of countries and study how financial integration affects the risk in these countries. The example shows that with heterogeneous beliefs the losses are smaller than 5% of the gains from financial integration.

This paper is structured as follows. In section 2 we discuss the theoretical model. The properties of a households consumption are characterized in section 3. Section 4 studies the equilibrium effects of financial integration. In section 5 we provide some numerical examples and section 6 concludes the paper.

## 2 The Model

This section describes the model and derives the basic results. We study a CARAnormal model with heterogeneous expectations and exogeneous limited participation.

## 2.1 The Economy

Let  $\mathcal{H}$  be the set of households in the economy and H the numbers of households in the economy. We assume that there are two countries in the economy, i.e. home, D and foreign F. Denote the set of households in economy D with  $\mathcal{H}^D$  and the set of households in economy F with  $\mathcal{H}^F$ . The number of households in economy D is  $H^D$  and the number of households in economy F is  $H^F$  and we have  $H = H^D + H^F$ . The economy has two periods, i.e. today (t = 0) and tomorrow (t = 1). The preferences of a household  $h \in \mathcal{H}$  can be represented by a CARA-utility function, i.e.

$$U^{h}(c_{0}^{h}, c_{1}^{h}) = -\frac{1}{\gamma^{h}} \exp\{-\gamma^{h} c_{0}^{h}\} + \beta E_{Q^{h}} \left[ -\frac{1}{\gamma^{h}} \exp\{-\gamma^{h} c_{1}^{h}\} \right], \tag{1}$$

with  $\gamma^h$  as the coefficient of absolute risk aversion of the households and  $c_t^h$  as consumption and  $E_{Q^h}[\cdot]$  as the subjective expectation of household h.

We assume that there are three assets in the economy, representing the stock market of each country. The returns of each assets are denoted by  $r_d$  and  $r_f$  and denote the vector of both returns with  $\mathbf{r} = [r_d, r_f]^{\top}$ . The expected returns are denoted by  $\bar{r}_d$ ,  $\bar{r}_f$  and the vector of mean returns is given by  $\bar{\mathbf{r}}$ . In our model households disagree about the mean returns of the assets and the subjective expectations of the households are given by  $r_d^h$ ,  $r_f^h$  and  $\mathbf{r}^h$ . Let  $\sigma_d$  and  $\sigma_f$  the standard deviation of the returns and  $\sigma_{df}$  be the covariance between the two returns. Hence, the covariance matrix of the returns is given by

$$\Sigma = \left( egin{array}{cc} \sigma_D & \sigma_{DF} \ \sigma_{DF} & \sigma_F \end{array} 
ight).$$

Furthermore we assume that the asset returns are jointly normal, i.e.

$$r \sim \mathcal{N}\left(\bar{r}^h, \Sigma\right)$$
 (2)

In addition to the two risky assets there is also a risk-free bond. We assume that this bond has a totally elastic supply and the rate of return is normalized to zero.

Note that in this economy the households disagree on the means of r while they agree on the variance and covariance of the shocks. This assumption is justified by the following observation (see e.g. Merton (1980)). If the households would know the true mean then they could estimate the variances and covariances up to an arbitrary precision, while with knowing the true variance and covariances the means cannot be exactly estimated.

Furthermore, we assume that all information in the economy is public and that households have different opinions. In an economy with asymmetric information households use the price to infer the information other households have (see e.g. Brunnermeier (2001)). In an economy in which households have different opinions and not different information prices do not reveal any private information and households do not infer anything by observing prices in the economy.

As we have different opinions and not information one may ask how does these differences in the opinion arise. One theory which explains why households have differences in opinion is the theory of Rational Beliefs by Kurz (1994, 1997). In his theory, households do have information about all past prices but may form different theories about the underlying stochastic process and because they believe that the economy is not stationary their beliefs never converge.

All households in the economy receive some endowment  $e^h_t$  today and tomorrow. While the endowment today is known to the agents, tomorrows' endowment is random. In particular, we assume that the households endowment is also normally distributed with mean  $\bar{e}^h$  and variance  $\sigma^h_e$ . We also assume that the endowments are correlated with the asset returns. Denote the covariances with  $\sigma^h_{eD}$  and  $\sigma^h_{eF}$ . Then the vector of the covariances is given by  $\sigma^h_e = [\sigma^h_{eD}, \sigma^h_{eF}]^\top$ .

#### 2.2 Financial Liberalization

We are now describing the trading restrictions in the economy. In particular we follow the definition of Errunza and Losq (1985) and consider three types of economies:

• Full Segmentation: Households can only trade in country-specific assets, i.e. households in economy F can only trade in assets of economy F and households

in economy D can only trade in asset of economy D.

- Partial Liberalization: Country D opens its stock market for investors from country F, i.e. households in country F can invest in assets from country D and F whereas households from country D can only trade in assets from country D.
- Full Liberalization: Households from both countries can trade in both markets.

Although we take these investment barriers as exogeneous given, one should note that there can be many reasons for the existence of an explicit investment barrier. Examples include differential differential taxation of foreigners, restrictions on currency conversions, ownership restrictions or high transaction costs.

#### 2.3 The Equilibrium

We denote the price of the risk-free bond by  $q_b$  and  $q_j$  as the price of asset j. The problem of agent h is to choose, consumption and portfolio to maximize equation (1) subject to the following budget constraints:

$$c_0^h = e_0^h - q_D \theta_D^h - q_F \theta_F^h, (3)$$

$$c_1^h = e_1^h + \theta_D^h r_D + \theta_F^h r_F. (4)$$

(5)

Denote the set of assets a household  $h \in \mathcal{H}$  is allowed to trade in with  $\mathcal{J}^h$ .

In particular  $\theta_j^h$ , j=D,F is unrestricted in the case of complete liberalized markets. For the case of partially liberalized financial markets we have the following restriction:

$$\theta_D^h = 0, \quad \forall h \in \mathcal{H}^D.$$
 (6)

For the case of segmented markets we have the following restrictions:

$$\theta_F^h = 0, \quad \forall h \in \mathcal{H}^D$$
 (7)

$$\theta_D^h = 0, \quad \forall h \in \mathcal{H}^F$$
 (8)

#### 2.3.1 Full Liberalization

In the case of fully liberalized markets households trade in all assets. Thus we have the following demand for the risky asset by households in country L=D,F:

$$\theta^h = \mathbf{\Sigma}^{-1} \left[ \frac{\mathbf{r}^h - \mathbf{q}}{\gamma^h} - \sigma_e^h \right] \quad \forall h \in \mathcal{H}^L.$$
 (9)

Thus, the asset prices are given by

$$\mathbf{q} = \frac{1}{H} \sum_{h \in \mathcal{H}} \left( \frac{\bar{\gamma}}{\gamma^h} r_j^h - \bar{\gamma} \sigma_e^h \right), \tag{10}$$

and with 
$$\bar{\gamma} = \left(\frac{1}{H} \sum_{h \in \mathcal{H}} \frac{1}{\gamma^h}\right)^{-1}$$
.

In the case of full liberalization the asset prices are jointly determined by the beliefs of households in country D and F as well as the covariance between the asset returns and the endowments of the households. Using (10), we are able to rewrite (9) as

$$\theta^h = \Sigma^{-1} \left[ \frac{\widetilde{\mathbf{r}}^h}{\gamma^h} - \widetilde{\sigma}^h \right] \tag{11}$$

With  $\widetilde{\mathbf{r}}^h$  defined as  $\widetilde{\mathbf{r}}^h = \mathbf{r}^h - \frac{1}{H} \sum_{h \in H} \frac{\bar{\gamma}}{\gamma^h} r^h$  and  $\widetilde{\sigma}$  defined as  $\widetilde{\sigma} = \sigma^h - \frac{1}{\gamma^h H} \sum_{h \in H} \bar{\gamma} \sigma^h$ .

#### 2.3.2 Partial Liberalization

Under partial liberalization the domestic investor can only invest into domestic securities, hence his optimal portfolio is given by

$$\theta_D^h = \frac{1}{\sigma_D} \left[ \frac{r_D^h - q_D}{\gamma^h} - \sigma_{eD}^h \right] \quad \forall h \in \mathcal{H}^D.$$
 (12)

On the other hand for the foreign investor the opportunity set does not change, i.e. he is still able to invest in stocks D and F and hence his asset demand is given by equation (9). Define the matrix V as follows:

$$\begin{bmatrix} V_D & V_{DF} \\ V_{DF}^{\top} & V_F \end{bmatrix} = V = \begin{bmatrix} \sigma_D & \sigma_{DF} \\ \sigma_{DF}^{\top} & \sigma_F \end{bmatrix}^{-1}$$

Then the we can rewrite the optimal demand of an asset D or F for an foreign investor as follows:

$$\theta_D^h = \frac{1}{\gamma^h} [V_D \quad V_{DF}] \begin{bmatrix} r_D^h - q_D \\ r_F^h - q_F \end{bmatrix} - \gamma^h \sigma_e$$

$$\theta_F^h = \frac{1}{\gamma^h} [V_{DF}^\top \quad V_F] \begin{bmatrix} r_D^h - q_D \\ r_F^h - q_F \end{bmatrix} - \gamma^h \sigma_e$$

Using the latter equations we can write the market clearing conditions for both assets as follows:

$$0 = \sum_{h \in \mathcal{H}_D} \frac{1}{\sigma_A} \left[ \frac{r_A^h - q_A}{\gamma^h} - \sigma_a^h \right] + \sum_{h \in \mathcal{H}_F} \frac{1}{\gamma^h} [V_D \quad V_{DF}] \left[ \begin{pmatrix} r_D^h - q_D \\ r_F^h - q_F \end{pmatrix} - \gamma^h \sigma_e \right]$$

$$0 = \sum_{h \in \mathcal{H}_F} \frac{1}{\gamma^h} [V_{DF}^{\top} \quad V_F] \left[ \begin{pmatrix} r_D^h - q_D \\ r_F^h - q_F \end{pmatrix} - \gamma^h \sigma_e \right]$$

The price of the foreign asset is solely determined by the preferences and beliefs of the foreign investors, i.e. the risk aversion of the household as well as the covariance of the asset and future endowments and their expectations about futures. On the other hand, the price of the domestic asset is now determined additionally by the preferences and beliefs of the foreign investors.

A special case would be if the asset returns are not correlated with each other. In that case we can write a simple closed form solution of both asset prices. These would be:

$$q_D = \frac{1}{H} \sum_{h \in \mathcal{H}} \left[ \frac{\bar{\gamma}}{\gamma^h} r_D^h - \bar{\gamma} \sigma_e^h \right]$$

$$q_F = \frac{1}{H_F} \sum_{h \in \mathcal{H}_F} \left[ \frac{\bar{\gamma}_F}{\gamma^h} r_F^h - \bar{\gamma} \sigma_e^h \right]$$

And 
$$\bar{\gamma}_F$$
 defined as  $\bar{\gamma}_F = \left(\frac{1}{H} \sum_{h \in \mathcal{H}_F} \frac{1}{\gamma^h}\right)^{-1}$ .

#### 2.3.3 Full Segmentation

In the case of full segmentation the demand for the assets are given by

$$\theta_D^h = \frac{1}{\sigma_D} \left[ \frac{r_D^h - q_D}{\gamma^h} - \sigma^e \right]$$

$$\theta_F^h = \frac{1}{\sigma_F} \left[ \frac{r_F^h - q_F}{\gamma^h} - \sigma^e \right]$$

And the asset prices are as follows:

$$q_D = \frac{1}{H_D} \sum_{h \in \mathcal{H}_D} \left[ \frac{\bar{\gamma}_D}{\gamma^h} r_D^h - \bar{\gamma} \sigma_e^h \right]$$

$$q_F = \frac{1}{H_F} \sum_{h \in \mathcal{H}_F} \left[ \frac{\bar{\gamma}_F}{\gamma^h} r_F^h - \bar{\gamma} \sigma_e^h \right]$$

In the case of full segmentation the asset prices are only priced taking into account the beliefs of the residents of the country D and F and the covariance between their endowment and the asset returns.

## 3 Consumption

We are now studying the properties of the consumption of the households. In particular we are interested in the variance of consumption in t = 1. In light of equation (11) we provide the following characterization of the consumption volatility:

$$var(c_1^h) = var(e_1^h) - \sum_{j \in \mathcal{J}^h} \frac{(\widetilde{\sigma}^{h,j})^2}{(\sigma_j)^2} + \sum_{j \in \mathcal{J}^h} \frac{1}{(\gamma^h)^2} \frac{(\widetilde{r}_j^h)^2}{(\sigma_j)^2}.$$
 (13)

As mentioned in the introduction, with heterogeneous beliefs at most one belief can be correct. Furthermore, we do not know the true or objective proability measure. Hence, we don't have a probability measure to make welfare calculations. Here, we are taking a different approach and look at the risk in the economy. In particular, we follow Simsek

(2011) observe that by averaging the certainty equivalents of all households, we have

$$\frac{1}{H} \sum_{h \in \mathcal{H}} \left( E_{Q^h}[c_1^h] - \frac{\gamma^h}{2} var(c_1^h) \right) = \frac{1}{H} \sum_{h \in \mathcal{H}} \left( E_{Q^h}[e_1^h + \sum_{j \in \mathcal{J}^h} \theta_j^h r_j^h + \theta_b^h] - \frac{\gamma^h}{2} var(c_1^h) \right),$$

$$= \frac{1}{H} \sum_{h \in \mathcal{H}} \left( E_{Q^h}[e_1^h] - \frac{\gamma^h}{2} var(c_1^h) \right).$$

The first equality follows from the budget constraint and the second equality from the market clearing conditions.

While financial integration does not affect the average endowment in the economy, they will affect the average variance in the economy. While the average variance provides a description of the risk the households in the economy face, it should not be considered as a measure of welfare. In fact, it is still possible for households to be better off, although the risk in the economy increases. However, the volatility of aggregate macroeconomic variables such as consumption or the gross domestic product play a key role in the empirical literature.

With the alternative presentation (13) can define the average variance in the economy as follows:

$$APR = \frac{1}{H} \sum_{h \in \mathcal{H}} \left( var(e_1^h) - \sum_{j \in \mathcal{J}^h} \frac{(\widetilde{\sigma}^{h,j})^2}{(\sigma_j)^2} + \sum_{j \in \mathcal{J}^h} \frac{(\widetilde{r}_j^h)^2}{(\gamma^h \sigma_j)^2} \right). \tag{14}$$

As with the consumption variance, we can split up the average variance into two parts. The first part is the average uninsurable risk, i.e.  $APR^R = \frac{1}{H} \sum_{h \in \mathcal{H}} var(e_1^h) - \sum_{j \in \mathcal{J}^h} \frac{(\tilde{\sigma}_j^h)^2}{\sigma_j^2}$ . The second part is the average speculative risk, i.e.  $APR^S = \frac{1}{H} \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{J}^h} \frac{(\tilde{r}_j^h)^2}{(\gamma^h \sigma_j)^2}$ . Hence, the average risk in the economy is  $APR = APR^R + APR^S$ .

However, for a national government the important decision criterion is not that the

aggregate risk in both countries declines but that the risk at home declines, i.e. if liberalization would increase the risk in their country liberalization the national government might view liberalization as detrimental to their economy and hence oppose liberalization. As we assume that country D makes the decision, we also introduce the *local portfolio risk* for country D. If the asset returns are uncorrelated<sup>4</sup>, we can write the local portfolio risk as follows.

$$LPR = \frac{1}{H} \sum_{h \in \mathcal{H}_D} \left( var(e_1^h) - \sum_{j \in \mathcal{J}^h} \frac{(\widetilde{\sigma}^{h,j})^2}{(\sigma_j)^2} + \sum_{j \in \mathcal{J}^h} \frac{(\widetilde{r}_j^h)^2}{(\gamma^h \sigma_j)^2} \right). \tag{15}$$

With  $\widetilde{\sigma}$  and  $\widetilde{r}$  depending on the degree of market integration. In the case of partial as well as full liberalization we have:

$$\widetilde{r}_{D}^{h} = r_{D}^{h} - \frac{1}{H} \sum_{h \in \mathcal{H}} \frac{\overline{\gamma}}{\gamma^{h}} r_{D}^{h} 
\widetilde{\sigma}_{eD}^{h} = \sigma_{eD}^{h} - \frac{1}{\gamma^{h} H} \sum_{h \in \mathcal{H}} \sigma_{eD}^{h}$$

with  $\bar{\gamma} = \left(\sum_{h \in \mathcal{H}} \frac{1}{\gamma^h}\right)^{-1}$ . In the case of full segmentation, we have

$$\widetilde{r}_{D}^{h} = r_{D}^{h} - \frac{1}{H} \sum_{h \in \mathcal{H}^{D}} \frac{\overline{\gamma}}{\gamma^{h}} r_{D}^{h} 
\widetilde{\sigma}_{eD}^{h} = \sigma_{eD}^{h} - \frac{1}{\gamma^{h} H^{D}} \sum_{h \in \mathcal{H}^{D}} \sigma_{eD}^{h}$$

with 
$$\bar{\gamma} = \left(\sum_{h \in \mathcal{H}^D} \frac{1}{\gamma^h}\right)^{-1}$$

<sup>&</sup>lt;sup>4</sup>For risk sharing only the uncorrelated parts of the asset returns would be important. Thus, assuming that the returns are uncorrelated does not change the key results.

# 4 Equilibrium Effects of Financial Integration

In this section, we study the effects of relaxing the participation constraint. We first discuss the effects of financial liberalization on the aggregate risk and then discuss the effects of financial liberalization on a countries risk.

#### 4.1 Financial Integration and Aggregate Portfolio Risk

The following proposition shows how financial liberalization affects the aggregate risk in the economy:

**Proposition 1.** If the financial market structure changes from full segmentation to partial liberalization, then:

- (i) The aggregate speculative risk increases;
- (ii) The aggregate uninsurable risk decreases.

Similarly, if the market structure changes from partial liberalization to full liberalization. then:

- (iii) The aggregate speculative risk increases;
- (iii) The aggregate uninsurable risk decreases.

However, nothing can be said about the aggregate portfolio risk, i.e. it can actually decrease or increase. Whether it increases or decreases depends on the distribution of beliefs. For example consider that all households in country D have the same expectations about future returns on the asset and all households in country F hold the same expectations. Now, if the beliefs in country P and F coincide, then there would be no

speculative risk and financial liberalization would decrease aggregate portfolio risk as there would be no speculative risk. On the other hand, if the households in country D disagrees with the households in country F about the returns on the assets, then there is the possibility that the speculative risk increases.

#### 4.2 Financial Integration and Local Portfolio Risk

Now we are turning our attention to the effect of financial liberalization on the local portfolio risk.

**Proposition 2.** If the financial market structure changes from full segmentation to partial liberalization, then:

- (i) the speculative risk of country F increases;
- (ii) the uninsurable risk of country F decreases;

Furthermore, if the financial market structure changes from partial liberalization to full liberalization, then:

- (iii) the speculative risk of country D increases;
- (iv) the uninsurable risk of country D decreases;

Given the properties of  $var(c_1^h)$  provided earlier it is clear that if participation constraints are relaxed, then the opportunities to share risk become greater. Thus, reducing the uninsurable risk which proves part (i). On the other hand, if a household is able to trade in more assets his possibilities to speculate become greater. Hence, the speculative risk of the household becomes greater as well and proving part (ii). The argument for (iii) and (iv) is similar.

Similarly to the previous proposition, country F increases its speculative risk but also decreases its risk while the aggregate effects ambigouos by going from full segmentation to partial liberalization. On the other hand the effects on country D are not clear.

However, the effects on countries D and F when opening their markets are ambiguous. For example, if country D opens its financial market then the speculative risk of the country might increase or decrease, similar for the uninsurable risk. In the case of the speculative risk, if the beliefs of the households in country D are very different from the beliefs of country F, e.g. the households in one country are very optimistic and the other ones are very pessimistic. Then the speculative risk for country D increases, because the mean belief on one asset will change and hence the local portfolio risk of country D increases.

The reverse argument holds true for the uninsurable risk, i.e. the further away the covariances of the new households are the better the risk sharing across countries.

## 5 A Numerical Example

So far we have studied the effects of financial integration theoretically. We are now turning to a simple numerical example to study the effects of financial integration in the Euro-Area. Several studies have shown that the introduction of the Euro lead to a deeper financial integration.

#### 5.1 Data and Parameters

We focus our attention on the following 7 countries from the Euro Area: Austria, Belgium, France, Germany, Italy, Netherlands, and Spain. For these countries MSCI pro-

	Autarchy	Integration						T
Austria	832.56	808.94		Autarky	Integration		Autarky	Integration
			Austria	832.46	808.10	Austria	13.12	36.98
Belgium	779.12	766.97	Belgium	778.83	766,21	Belgium	21.29	34.16
France	660.52	652.96	France	660.23	652.05	France	19.32	34.41
Germany	667.84	658.02						
Italy	739.45	720.04	Germany	667.44	657.01	Germany	23.16	36.46
			Italy	739.31	719.32	Italy	14.29	32.15
Netherlands	850.94	775.31	Netherlands	850.30	774.48	Netherlands	32.91	35.78
Spain	511.07	469.48						
Average	683.54	663.59	Spain	510.88	468.66	Spain	13.94	27.73

<sup>(</sup>a) Average Standard Devia-

Table 1: Average Standard Deviation, Uninsurable and Speculative Risk with Financial Autarky and Financial Integration

vides a national stock index starting from 1969. In particular, we use the gross index denominated in US-Dollar. To convert the nominal indices to real indices we use the Consumer-Price Index provided by the Federal Reserve Economic Database from the Federal Reserve Bank of St. Louis. Data for GDP, Consumption and Population are from the Penn World tables (Heston et al. (2012)).

In contrast to the model studied in the previous sections, we assume that there exists a continuum of households. The absolute risk-aversion of the households in all countries is 3. We divide the relative risk-aversion by the annual consumption in 1998 to approximate relative risk-aversion. We chose the year 1998, because in 1999 the Euro was introduced as an accounting currency in the seven countries.

For the beliefs, we assume that in each country the beliefs are normally distributed with a mean of 0 and a standard deviation of  $\sigma^{\mu}$ . We assume that  $\sigma^{\mu} = 9bp$ .

## 5.2 Financial Integration and Consumption Risk

In table 1a we depict average consumption risk for the seven countries in the Euro-Area. One can see that after financial integration the consumption risk declines in all countries. However, with the exception of Netherlands and Spain, the consumption risk is reduced by less than 5%.

<sup>(</sup>b) Uninsurable Risk

<sup>(</sup>c) Speculative Risk

In table 1b we show the uninsurable risk of the coutnries. In particular the table shows the square root of the uninsurable risk, i.e.  $\sqrt{APR^R}$ . As pointed out in the theoretical section, the uninsurable risk decreases in all 7 countries. It can be seen from this table that the gains from risk-sharing are of the same magnitude as the gains from integration with heterogeneous beliefs.

The speculative risk is shown in table 1c. Similar to the previous table we display the square root of the speculative risk, i.e.  $\sqrt{APR^S}$ . And although the speculative risk strongly increases in all seven countries. The losses for the countries are negligible. For example, in the case of Spain, the speculative risk more than doubles. Without heterogeneous beliefs, the consumption risk is 468.66 after financial integration. On the other hand, with heterogeneous beliefs the consumption risk is 469.48 after financial integration. Hence, the increase from speculative risk is less than 0.5%.

So far, we have studied the effects of financial integration on the risk in the economy. We have seen that the average consumption risk in the economy decreases as well as the consumption risk in all 7 countries and that the losses from speculation in all countries are very small. However, two thing should be kept in mind. Prior to the Introduction of the Euro-Area there was already some risk-sharing across countries and the gains from risk-sharing should be seen as an upper bound for the gains. Furthermore, beliefs are not very divergent and the losses from speculation could be greater if the beliefs are more diverse.

### 5.3 Impact of Beliefs

In the previous subsection we had the dispersion of beliefs fixed. However, as the analysis of survey forecast shows the dispersion of beliefs varies over time and different

	Autarky	Integration						
	,			Autarky	Integration		Autarky	Integration
Austria	832.89	811.62	Austria	832.46	808.10	Austria	26.80	75.52
Belgium	780.05	769.38						
France	661.41	655.83	Belgium	778.83	766.21	Belgium	43.49	69.79
		661.22	France	660.23	652.05	France	39.46	70.28
Germany	669.12		Germany	667.43	657.01	Germany	47.31	74.48
Italy	739.89	722.32						
Netherlands	852.95	777.92	Italy	739.31	719.33	Italy	29.18	65.66
		472.07	Netherlands	850.30	774.48	Netherlands	67.23	73.08
Spain	511.67		Spain	510.88	468.66	Spain	28.46	56.64
Average	684.476	666.33	Брин	510.00	100.00	Spain	20.10	50.01

(a) Average Standard Deviation

(c) Speculative Risk

Table 2: Average Standard Deviation, Uninsurable and Speculative Risk with Financial Autarky and Financial Integration with  $\sigma^{\mu} = 18bp$ .

survey often have a different dispersion of the beliefs (cf. Mankiw et al. (2003)). If the dispersion of beliefs is increased to 18bp all countries still benefit from financial integration but the aggregate gains from financial gains are now less than 0.1%.

In table 2 we show the average standard deviation, the square root of the uninsurable risk and speculative risk if the dispersion of beliefs is increased to 18bp.

As it can be seen in table 2a all countries will still profit from financial integration. And although the change in the dispersion of beliefs had no impact on the uninsurable risk in the economy (table 2b) it did affect the speculative component of the consumption risk as it is depicted in table 2c. As it can be seen the speculative risk for all countries more than doubled, e.g. in the case of italy the speculative risk increased from 32.15 to 65.66.

### 5.4 Impact of Risk Aversion

In this subsection we are studying the impact of risk-aversion on the average risk. A change in the risk-aversion changes the speculative risk of the households. With a lower risk-aversion, households speculate more and hence the speculative risk increased.

We are now setting the relative risk-aversion of the households to 1.5 and table 3 shows the impact of financial integration. As before we have the standard deviation of

<sup>(</sup>b) Uninsurable Risk

	Autarky	Integration						
Austria	832.87	811.48		Autarky	Integration		Autarky	Integration
			Austria	832,46	808.10	Austria	26.24	73.95
Belgium	780.00	769.25	Belgium	778.83	766,21	Belgium	42.58	68.33
France	661.36	655.67	France	660.23	652.05	France	38.64	68.81
Germany	669.05	661.05						
Italy	739.86	722.19	Germany	667.44	657.01	Germany	46.32	72.92
Netherlands	852.84	777.78	Italy	739.31	719.33	Italy	28.57	64.29
Spain	511.64	471.93	Netherlands	850.30	774.48	Netherlands	65.82	71.56
			Spain	510.88	468.66	Spain	27.87	55.46
Average	684.42	666.18						

(a) Average Standard Devia-

(c) Speculative Risk

Table 3: Average Standard Deviation, Uninsurable and Speculative Risk with Financial Autarky and Financial Integration with RRA = 1.5.

the consumption risk and the square root of the uninsurable and speculative risk.

As it can be seen from the table 1b, the average variance has increased compared to the base case, because the speculative risk has increased. However, financial integration is now desirable for all countries.

In table 2b we show the uninsurable risk in the economy and we see that a change in the risk-aversion does not affect the uninsurable risk in the economy.

The speculative risk is shown in table 3b. We see that the speculative risk has increased for all countries. In particular it doubled for all countries.

## 6 Conclusion

In this essay we studied an economy with restricted participation and heterogeneous beliefs and we showed that because of speculation the integration may have adverse effects on the economy. Whether the financial market integration has adverse effects depends on the distribution of the covariance and beliefs. The more dispersed the covariance between endowment and returns the more gains can be made by sharing the risk and a low dispersion of the beliefs implies a low speculation.

In addition, we studied the gains from financial integration for seven countries from

<sup>(</sup>b) Uninsurable Risk

the Euro-Zone. The example showed that all countries would profit financial integration and that the losses from speculation are relatively small. The caveat of this example is that the result depends heavily on the diversity of opinions in the economy and if the diversity of opinions is far greater than in the example this could overturn the results and financial integration might not be beneficial.

This research could be extended into several directions. A simple direction is to extend the static model to a dynamic model. With a dynamic model the value of reselling the asset has to be taken into account. This can lead to a much larger speculation and hence to bigger losses. On the other hand, households betting into the right direction can gain a lot.

Another line of research would be to endogenize the participation on financial markets. The literature on the home-bias puzzle has identified several frictions which could cause the home-bias puzzle and it would be investigating how these frictions interact with heterogeneous beliefs.

## References

Evan W. Anderson, Eric Ghysels, and Jennifer L. Juergens. Do heterogeneous beliefs matter for asset pricing. *Review of Financial Studies*, 18:875 – 924, 2005.

John Bart and Isidore J. Masse. Divergence of opinion and risk. *The Journal of Financial and Quantitative Analysis*, 16:23–34, 1981.

Fischer Black. International capital market equilibrium with investment barriers. *Journal of Financial Economics*, 1:337 – 352, 1974.

- Markus Brunnermeier. Asset Pricing under Asymmetric Information: Bubbles, Crashes, Technical Analysis, and Herding. Oxford University Press, 2001.
- Steven J. Davis, Jeremy Nalewaik, and Paul Willen. On the gains to international trade in risky financial assets. Working paper, NBER, 2001.
- Bernard Dumas, Karen Lewis, and Emilio Osambela. Differences of opinion and international equity markets. Working paper, 2011.
- Vihang Errunza and Etienne Losq. International asset pricing under mild segmentation. *The Journal of Finance*, 40:105 124, 1985.
- Vihang Errunza and Etienne Losq. Capital flow controls, international asset pricing and investors welfare: A multi-country framework. *The Journal of Finance*, 44:1025 1037, 1989.
- Jeffrey A. Frankel and Kenneth A. Froot. Using survey data to test standard propositions regarding exchange rate expectations. *American Economic Review*, 77:133–153, 1987.
- J Michael Harrison and David M. Kreps. Speculative investor behavior in a stock market with heterogeneous expectations. *Quarterly Journal of Economics*, 92:323–336, 1978.
- Alan Heston, Robert Summers, and Bettina Aten. *Penn World Table Version 7.1*. Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, July 2012. URL http://pwt.econ.upenn.edu/.
- Cars Hommes. The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic Dynamics and Control*, 35:1–24, 2011.

- Eugene Kandel and Neil D. Pearson. Differential interpretation of public signals and trade in speculative markets. *Journal of Political Economy*, 103(4):831–872, August 1995.
- Mordecai Kurz. On rational belief equilibria. Economic Theory, 4:859–876, 1994.
- Mordecai Kurz. Endogeneous economic fluctuations and rational beliefs: A general perspective. In *Endogeneous Economic Fluctuations: Studies in the Theory of Rational Beliefs*, chapter 1, pages 1 36. Springer, 1997.
- Mordecai Kurz and Andrea Beltratti. The equity premium is no puzzle. In *Endogeneous Economic Fluctuations: Studies in the Thoery of Rational Beliefs*. Springer, 1997.
- Tao Li and Mark L. Muzere. "heterogeneity and volatility puzzles in international finance". *Journal of Financial and Quantitative Analysis*, 45(6):1485–1516, 2010.
- N. Gregory Mankiw, Ricardo Reis, and Justin Wolfers. Disagreement about inflation expectation. Working Paper 9796, NBER, 2003.
- Jianping Mei, Jose Scheinkmann, and Wei Xiong. Speculative trading and stock prices: Evidence from chinese a-b share premia. *Annals of Economics and Finance*, 10:225 255, 2009.
- Robert C. Merton. On estimating the expected return on the market. *Journal of Finan- cial Economics*, 8:323 361, 1980.
- Edward M. Miller. Risk, uncertainty and the divergence of opinion. *The Journal of Finance*, 32(4):1151–1168, 1977.
- Alp Simsek. Speculation and risk sharing with new financial assets. Working paper, Harvard University, 2011.

- Rene M. Stulz. A model of international asset pricing. *Journal of Financial Economics*, 9:383 403, 1981a.
- Rene M. Stulz. On the effects of barriers to international investment. *The Journal of Finance*, 36:923 934, 1981b.
- Marti G. Subrahmanyam. On the optimality of international capital market integration. *Journal of Financial Economics*, 2:3 28, 1975.
- Eric van Wincoop. How big are potential welfare gains from international risksharing. *Journal of International Economics*, 47:109 135, 1999.
- Hal R. Varian. Divergence of opinion in complete markets: A note. *The Journal of Finance*, 40:309–317, 1985.
- Wei Xiong. Bubbles, crises and heterogeneous beliefs. In Jean-Pierre Foque and Joe Langsam, editors, *forthcoming in Handbook on Systemic Risk*. Cambridge University Press, 2012.

## A Proof of Proposition 1

(i) First, we need to show that the risk-sharing portfolio minimizes the average variance in the economy. Assume for now that all households have the same beliefs. Then the optimization problem can be rewritten as

$$\min_{\theta_j^h \forall j,h} \quad \frac{1}{H} \left( \sum_{h \in \mathcal{H}} \left( var(e_1^h) - \sum_{j \in \mathcal{J}^h} \frac{\widetilde{\sigma^{h,j}}^2}{(\sigma_j)^2} \right) \right)$$

subject to the participation restrictions under full integration, partial integration or segmented markets.

After some calculations, one can see that the risk-sharing portfolio is the solution to the above minimization problem.

Relaxing participation constraints expands the constraints set of the minimization problem. Hence, the consumption variance decreases.

(ii) The argument, that relaxing the participation constraint increases the speculative variance, is similar to part (i), i.e. relaxin the participation constraint relaxes the constraint of an optimization problem. In particular, the speculative portfolio is the solution of the optimization problem but this time it is a pure speculative portfolio. In particular, given a budget constraint it solves the following maximization problem for each trader:

$$\max_{\theta_j^h \in \mathbb{R} \forall j \in \mathcal{J}} E_{Q^h}[c_1^h] - \frac{\gamma^h}{2} var(c_1^h)$$
(16)

This is the maximization problem of the household if there is no risk-sharing motive in the economoy, i.e.  $\sigma^{h,j}=0$  for all  $h\in\mathcal{H}$  and the set of tradable assets given full

integration, partial integration or segmented markets. Relaxing the participation constraints, relaxes the constraint set of the optimization problem. Hence, the possibilities of speculative trades for all households increases. As we have an quadratic optimization problem, expected payoffs and variance are proportional. Hence, a relaxed participation constraint increases the speculative variance. The result now follows from averaging over all households.

# **B** Proof of Proposition 2

The argument for this proof is similar to the proof of proposition 1. In particular, moving from full segmentation to partial liberalization relaxes the constraints of the foreign investor. Thus increasing the speculative risk and decreasing the uninsurable risk. On the other hand, going from partial liberalization to full liberalization relaxes the constraint of the domestic investor.