Endogenous Banks' Networks, Cascades and Systemic Risk*

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Abstract

We develop a network model whose links are governed by banks' optmizing decisions and by an endogenous tâtonnement market adjustment. Banks in our model can default and engage in fire-sales: risk is transmitted through direct and cascading counterparty defaults as well as through indirect pecuniary externalities triggered by fire-sales. We use the model to assess the evolution of the network configuration under various prudential policy regimes, to measure banks' contribution to systemic risk (through Shapley values) in response to shocks, and to analyze the effects of systemic risk charges. We complement the analysis by introducing the possibility of central bank liquidity provision.

Keywords: Network formation, tâtonnement, contagion.
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1 Introduction

Interconnections in the banking system, as fostered by fast developments in financial innovation, increased degree of complexity in modern financial systems, and the diffusion of over-the-counter derivatives, made systemic risk endemic and epidemic at crises times. Interconnections, initially set-up to facilitate risk sharing, have created channels whereby financial distress is quickly spread onto the entire system. Not surprisingly, the rationale behind government intervention and bank bail out programs in the aftermath of the recent financial crisis was to be found not in the too-big-to-fail argument but in the too-interconnected-to-fail argument. The dangers associated with highly interconnected systems come from the possibility that the financial distress, experienced by one bank, might turn through cascading effects into full-fledged systemic risk, whose monitoring, assessment, and prevention has become paramount. Indeed one of the most important legacies of the 2007-2008 crisis has been the creation and development of a number of institutions whose mission is that of measuring systemic risk, monitoring financial vulnerabilities and safeguarding the financial system.[1]

Against this background the literature offered no concrete paradigm to account for network externalities in combination with micro-founded decisional rules and financial (mis)-incentives, to quantify systemic risk and to forecast the development of financial contagion. We do a step forward in that direction by constructing a dynamic network model with *heterogenous* and *micro-founded* banks, whose links emerge endogenously from the interaction of intermediaries' optimizing decisions and an iterative *tâtonnement* process which determines market prices endogenously. The financial system featured by our model consists of

[1]In the U.S. the Dodd–Frank Wall Street Reform and Consumer Protection Act (See Financial Stability Oversight Council [17]) had created the Financial Stability Oversight Council, whose statute states in Title 1 that the primary objective of this institute is that of monitoring systemic risk. The main mission of the European Systemic Risk Board, established 16 December 2010, is the prevention or mitigation of systemic risks to financial stability in the Union that arise from developments within the financial system. The Financial Stability Board (FSB) has been established to coordinate, at the international level, the work of national financial authorities in addressing vulnerabilities and to develop and implement strong regulatory and supervisory policies.

a *network* with a finite number of financial institutions which solve an optimal portfolio allocation taking into account liquidity and capital constraints. Banks hold different amounts of equity capital and differ for the returns on non-liquid assets due to different information and administrative cost. Such differences in returns gives rise to heterogenous optimal portfolio allocation on banks assets and remainder liabilities, hence to excess demand or supply of bank borrowing and lending. Banks' links are given by lending and borrowing that takes place in the interbank market. A crucial feature of our model is that the links in the adjacency matrix characterizing the network are not assigned randomly as in random network models but emerge endogenously from the combination of the optimal banks' decision.[2] Network externalities thus emerge as a manifestation of individual optimizing behavior and market adjustment. Since non-liquid assets are marked-to-market, the model also features pecuniary externalities via the occurrence of fire-sales.

Contagion manifests itself through direct and indirect effects. The direct effects comprise *common exposure to risky assets* and *local network externalities*. First, if banks invest in the same financial products their balance sheets are correlated due to the multinomial nature of the shocks. Second, as banks are interlinked through counterparty exposure in the interbank market, a defaulting bank transmits losses to creditor banks. The cascading sequences of defaults effectively constitute an endogenous risk propagation mechanism. Indirect contagion effects manifest through fire-sales (*pecuniary externalities*). A negative shock in the value of non-liquid assets induces several banks to de-leverage in order to satisfy their capital and liquidity requirements: this is a credit event that produces a fall in the market price and a *cascade* of losses in marked-to-market balance sheet of all other banks.

We use our model to evaluate the effects of credit events on the configuration of the network, to asses the role of a number of prudential policies, and to quantify systemic risk. To this purpose we simulate our model, using a sequential clearing algorithm,[3] in response to

[2] Furthermore, dynamic adjustment in our model emerges as an intrinsic feature of the market adjustment even in absence of an initial shock impulse.

[3] Our algorithm is affine to the clearing algorithms implemented in presence of sequential defaults as in Eisenberg and Noe [13]. The algorithm is consistent with general properties on clearing vector such as

adverse shocks to non-liquid assets, interpreted as a credit event, and analyze the evolution of the banking network. Using Shapley values[4] we compute the contribution of each bank to systemic risk, defined as the aggregate sum of imputed asset losses under sequential default over total assets in the banking system. The contribution of each bank to systemic risk crucially depends upon the bank's position in the network: a large over-leveraged bank linked to many other lenders can more likely contaminate the system when subject to shocks to its non-liquid assets.

We analyze overall systemic risk and the contribution of each bank to it under different parameter configurations for various prudential policies. In this respect our paper contributes to the discussion on the role of prudential regulation in taming systemic risk in financial systems. Prudential policies are investigated along two lines: first of all, liquidity requirements, changes in the capital requirement, and changes in asset risk weights are considered. These policies all directly affect the *constraints* in banks' portfolio optimization. Second, we investigate systemic risk charges as introduced, for example, in Germany in 2011, which affect banks' *objective function* in banks' portfolio optimization. In the spirit of a Pigouvian tax, these risk charges on a bank's derivative investments and interconnectedness shall incentivize to financial institutions to lower their contribution to systemic risk.

Generally speaking changes in policy and regulations affect the strength of the cascade in response to shocks and the extent of both the network and pecuniary externalities. We find for instance that an increase in the capital requirement, as well as an increase in the risk weights, induce a bell shaped dynamic of overall systemic risk. At low levels of capital requirements banks endowed with high return investment tend to leverage up, therefore increasing the demand for liquidity. The ensuing increase in the lending rates in the interbank market induces banks which feature low returns on non-liquid assets to invest in interbank lending due asset substitution. The market then clusters the connections around the highly leveraged banks –which end up contributing heavily to systemic risk. As the requirement

proportional repayment under default and limited liability.

[4]See Bluhm and Krahnen [24] and Borio, C., N. Tarashev and K. Tsatsaronis [8].

raises (say beyond a capital requirement of 10% of risk weighted assets), the capital constraint becomes binding, and banks start to hoard liquidity: the banking network becomes sparse and systemic risk decreases. Increases in liquidity requirement instead tend to decrease overall systemic risk monotonically: higher liquidity requirements force all banks to retain buffer savings. As a result the size of each interconnection (as captured by the amount of money lent/borrowed for each pair of banks) decreases and robustness tends to prevail on fragility making the network safer. At last, risk charges, namely taxes on non-liquid asset returns and on interbank lending returns, lead to a decrease in systemic risk and banks' contribution to it: banks' incentives to participate in the interbank market decrease and so do banks' interconnections. The downside of this is that the overall investment in non-liquid asset decreases due to limited availability of liquidity and to expropriation of non-liquid asset returns due to taxation.

All our experiments are also repeated in the case in which a central bank intervenes by providing liquidity in the interbank market: overall the presence of the central banks improves liquidity provision, hence investment prospects, and reduces the extent of interbank interconnections, as banks need to rely less on market funds.

The rest of the paper is structured as follows. Section 2 compares our model to some close literature on systemic risk. Section 3 describes the model, the equilibrium, the shock transmission, and the measure of systemic risk. Section 4 describes the numerical results and analyzes the policy designs. Section 6 concludes.

2 Relation to the Literature

This paper is related to different strands of the literature. It is related to the literature on economic networks, it contributes to the literature on market mechanisms, and to an emerging literature on measurement of systemic risk.

Over the last decade network models have emerged as an alternative paradigm to analyze a variety of economic and social problems ranging from the formation of contacts and links in labour, financial and product markets to the formation and evolution of research networks (see Jackson [22]). The recent financial crisis has conveyed increased attention toward models featuring pecuniary and network externalities. Allen and Gale [2] exploit network externalities as banks in their model hold cross-deposit whose connections expose them to contagion. Recently Gai, Haldane and Kapadia [18] have developed a random network model for the inter-bank market and have analyzed the effects of complexity and concentration onto financial fragility. In their model inter-linkages are driven by Poisson distributions and evolve in response to shocks: their model therefore belongs to the class of random networks. More recently Elliot, Golub and Jackson [14] analyze integration and diversification in payment systems for banks subjects to default. In those models payments or financial transactions are obtained through heuristics, tipping point, or are randomly assigned: relatively to those contributions in our model network links are the result of micro-founded optimizing banks' decisions and of an endogenous market process. Dynamic adjustment in our model results from the endogenous response to shocks of optimizing banks and of the tâtonnement equilibrium process characterizing market adjustment.[5] The endogeneity of market price adjustment is most closely related to Cifuentes, Ferrucci and Shin [11] which also analyzes network and pecuniary externalities, although banks in their model do not form optimizing decisions. Caballero and Simsek [10] focus on the role of complexity in network models: given the intricate structure of inter-linkages, banks face ambiguity when trading in the interbank market. This might amplify fire-sale when rumors of financial vulnerabilities are released. Krahnen and Bluhm [24] analyze the formation of systemic risk, through Shapley values, in a model with three interconnected banks. In their model tipping points for the diffusion of systemic risk are determined by exogenously given heuristics, hence contrary to us they do not analyze optimizing banks decisions. Anand, Gai and Marsili [3] analyze the effects of rollover risk in a model combining features from the global game theory and from the random networks. Finally, Georg [15] uses a dynamic multi-agent model of a banking

[5] See also Cifuentes, Ferrucci and Shin [11].

system with central bank to compare different interbank network structures. He provides evidence that money-center networks are more stable than random networks and that the central bank stabilizes interbank markets in the short run only

Our model also contributes to the literature on market mechanisms, by analyzing the quantitative implications of centralized tâtonnement. Experimental evidence on the effects of tâtonnement pricing mechanism is reported in Lugovskyy, Puzzello and Tucker [26] and Baghestanian [5]. The price convergence process featured in their works is in line with the one obtained in our model. While our model uses a centralized market mechanism (see also Cifuentes, Ferucci and Shin [11] or Duffie and Zhu [12] for other centralized mechanisms), other models of financial networks use bilateral trading (see for instance Atkenson, Eisfeldt and Weill [4]). The algorithm developed to analyze the tâtonnement process of our model follows the traditions of clearing mechanisms that rely on lattice theory, most notably Eisenberg and Noe [13] who however take the banks' asset and liability structure as given.

A number of other papers have dealt with the analysis of systemic risk: among others see Lagunof and Schreft [25], Rochet and Tirole [29], Eisenberg and Noe [13], Billio, Getmansky, Lo and Pellizon [7], Geanakoplos [19].

3 The Model

The financial system is made up with a population of N banks. Let $N \in \{1, ..., n\}$ represent a finite set of individual banks, each of whom is identified with a node of the network. We define ex-ante for this population a network $g \in \mathcal{G}$ as the set of links among heterogenous banks N, with $\mathcal{G}$ being the set of all possible networks. An arc or a link between two banks i and j is denoted by $g_{i,j}$ where $g_{i,j} \in \mathbb{R}$. Here $g_{i,j} \neq 0$ reflects the presence of a link (directed network), while $g_{i,j} = 0$ reflects the absence of it. Later on we shall specify the link $g_{i,j}$ as either borrowing or lending from bank i to bank j, therefore the real valued link could take either a positive or a negative value. A crucial aspect of our analysis lies in the fact that those cross investment positions (hence the network links) result endogenously from the banks' optimizing decision and the markets' tâtonnement processes. An important dimension in the diffusion of risk concerns the number of direct links held by each bank: a loss of value in the balance sheet of bank i will affect immediately all banks directly connected with bank i. For this reason it is instructive to define $N^d(i;g) = \{k \in N \mid g_{i,k} \neq 0\}$ as the set of banks with whom bank i has a direct link in the network. The cardinality of this set is given by $\mu_i^d(g) = \left|N^d(i;g)\right|$, namely the number of banks with whom bank i is directly linked in the network g. The $n-square$ adjacency matrix $\mathbf{G}^{(t)}$ of the network g describes the connections which arise after (t) iterations of the tâtonnement process. Given that our model features a directed weighted network, banks i and j are directly connected if $g_{ij} \neq 0$.

Our network features optimizing banks which undertake an optimal portfolio allocation by maximizing profits subject to liquidity and capital requirement constraints and a non-zero non-liquid asset constraint. Banks decide about the optimal amount of liquid assets (cash), the optimal amount of lending and borrowing in the interbank market, and the optimal investment in non-liquid assets (bonds or collateralized debt obligations). Network externalities materialize through the lending and borrowing taking place in the interbank market, while pecuniary externalities materialize since non-liquid assets are marked-to-market.

Banks differ for their equity endowment and return on non-liquid asset investments, which result, after optimization has taken place, in heterogenous optimal portfolio allocations. The optimizing decision together with the dynamic adjustment taking place in asset and interbank markets determines the final portfolio allocations and the final borrowing and lending positions in the interbank market: the latter represent the entry of the adjacency matrix $\mathbf{G}$ characterizing the interbank network.

The clearing mechanism in our model is achieved through a sequential tâtonnement process[6] that takes place first in the interbank market (for given price of non-liquid assets) and subsequently in the market for non-liquid assets (for given clearing price in the interbank market). Central walrasian auctioneers (see also Cifuentes, Ferucci and Shin [11] or

[6]See Mas-Colell and Whinston [28], and Mas-Colell [27].

Duffie and Zhu [12]) receive individual demand and supply of interbank lending and adjust prices until the distance between aggregate demand and supply has converged to zero:[7] the price adjustment in each market is done in fictional time during which trade does not take place. Once a clearing price has been achieved, actual trade in the interbank market takes place according to the criterion of the *closest matching partner*: to put it simply, banks wishing to borrow are matched with banks wishing to lend the closest possible amount. This matching mechanism is compatible with pair-wise efficiency and is in line with actual practice. Once equilibrium, both in price and quantities, has been achieved we can analyze the final network configuration. The latter can however change once the asset portfolio of one bank is subject to shocks to non-liquid assets: the shock indeed triggers a new round of tâtonnement adjustments which results in fire-sales of non-liquid assets for banks wishing to adjust their equity ratios and in possible cascading defaults for banks which are unable to repay interbank debts.

3.1 Banks' Optimization

A bank's balance sheet consists of the elements displayed on Table 1.

Assets	Liabilities
Cash (c)	Deposits (d)
Bank lendings (l)	Bank borrowings (b)
Non-liquid assets (e)	Equity (q)

Table 1: Banks' Balance Sheets

Banks hold deposits, d, and choose cash, c, investment in non-liquid assets, e, and the amount of borrowing, b, or lending, l.[8] We use the index i to indicate each individual bank, which can be either a borrower or a lender, we use the index j to indicate the trading partner of each bank. Banks' solve a static optimization problem which is detailed as follows. Bank i's objective function is given by:

[7] The convex banks' optimization problem and an exponential aggregate supply guarantee that individual and aggregate excess demands behave according to Liapunov convergence. See details below.

[8] Banks in our model can either borrow or lend.

$$E(\pi^i) = l^i \cdot r^{rf} + \frac{r^i}{p} \cdot e^i - b^i \cdot r^{rf} \cdot \frac{1}{1 - \xi PD^i}, \tag{1}$$

where π denotes profit, $l^i = \sum_{j=1}^{N} l^{i,j}$ is bank i's lending vis a vis all counterparts, $b^i = \sum_{j=1}^{N} b^{i,j}$ is bank i lending vis-à-vis all counterparts, r^{rf}, is the risk-free interest rate on the interbank market which will later on be determined through the centralized tâtonnement process in the interbank market, e^i is bank's i holding of non-liquid assets, p is the market price of the non-liquid asset, later determined through the centralized tâtonnement process in the market for non-liquid assets, r^i is the return on non-liquid assets, which is bank specific and set exogenously according to a uniform distribution. Heterogeneity in assets returns is meant to capture the fact that banks have access to investment opportunities with different profitability: this generates heterogeneity in asset and portfolios' positions and justifies the desire for trade in both interbank and asset markets. Finally the parameter ξ is the loss-given-default ratio: only a fraction of the outstanding amount is paid back in case of the debtor's default. Two considerations are in order. First, notice that while non-liquid assets are traded at a single centralized price, whose changes trigger fire sale externalities on banks' asset portfolios, the return on bank borrowing features two components, a central clearing price, r^{rf}, common to all banks and an additional risk premium, $\frac{1}{1-\xi PD^i}$, which is bank specific. The latter is determined based on equilibrium consistent expectations of individual banks' default probabilities, which are obtained through a least square iterative process, as detailed in section 3.2. This assumption captures the idea that bank borrowing typically features heterogenous prices linked to individual bank's health. Second, the profit function takes into account the fact that in every period a fraction of banks might default on repayment. The possibility of sequential default is also the reason for which the return on bank lending does not include the premium: each lending bank charges premia to different counterparts; ex-post however some counterparts default and the return on bank lending is set to satisfy arbitrage on risky assets. A detailed derivation of Equation (1) that takes into account this mechanism can be found in Appendix A.

Banks face a *liquidity constraint*, of the type envisaged in Basel III agreements, due to which they have to hold at least a percentage, α, of their deposits in cash:[9]

$$c^i \geq \alpha \cdot d \tag{2}$$

where c^i is the bank's holding of cash and d is an exogenous amount of deposits. Furthermore, banks face a *capital requirement constraint*, as they must maintain an equity ratio, er^i, of at least $\gamma + \tau$:

$$er^i = \frac{c^i + p \cdot e^i + l^i - d^i - b^i}{\chi_1 \cdot p \cdot e^i + \chi_2 l^i} \geq \gamma + \tau \tag{3}$$

where χ_1 and χ_2 are risk weights assigned respectively to the two risky assets, namely non-liquid investment and bank lending. The parameter γ identifies the regulatory requirement, while the parameter τ reflects banks preference for capital buffer. The risk coefficients are set exogenously as part of the regulatory system. Realistically we assume that banks need to hold less capital for bank lending than for investments in non-liquid assets, i.e. $\chi_1 \succ \chi_2$. More details on the exact numbers chosen in simulations are given in the calibration section below. If banks' equity ratio, er^i, is lower than the minimum capital requirement, γ, banks can reduce their exposure to bank lending (or to non-liquid assets): effectively this results in a reduction of the denominator of Equation 3, relatively to the numerator, until the required ratio is achieved. This implies for instance, as we shall see later on, that any change in the regulatory capital requirement, γ, will result in a change of the demand (or supply) of bank lending in the interbank market, hence in a change of the cross-exposure of the network. Changes in the regulatory levels of the risk weights parameter χ_1 and χ_2 will also trigger an adjustment in the interbank and non-liquid asset markets. The higher are those weights, the larger is the extent to which banks have to re-adjust their non-liquid asset and bank lending positions in order to satisfy the capital requirement. As losses materialize due to cascading lack of repayments from counterparties, banks will sell non-liquid assets to rebalance portfolios and meet capital requirements. Banks which cannot fulfill regulatory requirements default.

[9]For simplicity this fraction is assumed constant.

Three further observations are worth noticing. First, note that liquid assets do not appear in the denominator of Equation 3; this is so since banks do not have to hold capital for their liquid asset holdings. Second, similar to Cifuentes, Ferucci, and Shin [11], non-liquid assets are marked to market, which gives the potential for fire-sale spirals in the model: as the price of non-liquid assets falls due to fire-sales, the asset values of all banks investing in non-liquid assets falls. Third, banks face a *no-short sales constraint*:

$$e^i \geq 0. \tag{4}$$

The latter is needed for the problem to be well-behaved: this indeed rules out the possibility of negative prices for non-liquid assets.

Individual banks' constrained optimal solution to their profit function which determines their optimal asset and liability allocations is found via maximizing Equation 1 subject to constraints 2, 3, and 4, using linear programming techniques. We also add four further constraints which make sure the solution is feasible. Due to the linear nature of both the objective and the constraints in the portfolio optimization problem and according to the *Duality Theorem of Linear Programing* we can reformulate the maximization problem as a minimization problem for the $i^t h$ bank subject to smaller equal constraints. The new constrained minimization optimal problem reads as follows:

$$min_{l^i,b^i,e^i,c^i} \; - E(\pi^i) = -e^i \cdot \frac{r^i}{p} - l^i \cdot r^{rf} + b^i \cdot r^{rf} \cdot \frac{1}{1 - \xi PD^i} \tag{5}$$

s.t.

$$-c^i \leq -\alpha \cdot d$$

$$-c^i - e^i(p(1-(\gamma+\tau)\chi_1)) - l^i(1-(\gamma+\tau)\chi_2) + b^i \leq -d^i$$

$$e^i \geq 0; c^i \geq 0; b^i \geq 0; l^i \geq 0; c^i + e^i p + bl^i - bb^i = d^i + e^i$$

The next section describes the sequential tâtonnement processes, and the respective clearing mechanisms, taking place first in the interbank market (for given price of non-liquid asset) and then in the market for non-liquid assets.

3.2 Tâtonnement in the Interbank Market and Clearing Mechanism

The equilibrium allocation on the interbank market is found in two steps. The first step consists of finding the market clearing interest rates as well as the aggregate supply/demand of interbank funds. The second step consist of finding the allocation of interbank funds supplied in equilibrium, which then determines the structure of interlinkages between lending and borrowing banks.

The market clearing rates $r^{rf} + r^{PD^i}$ are found via a discrete tâtonnement process as follows. Given a set of parameters,[10] including r^{rf} and r^{PD^i}, banks optimize their portfolio via maximizing Equation (1) subject to the set of regulatory constraints (Equations (2) to (4)). Banks submit their optimal demand and supply of funds to an auctioneer, which then sums them up to obtain the aggregate excess demand or supply in the interbank market and to adjust the price accordingly. The interbank centralized rate, r^{rf}, is increased if $F^{supply} < F^{demand}$ and decreased in the opposite case, where F^{supply} and F^{demand} are the overall amounts of funds supplied and demanded, respectively. The rates are adjusted in fictional time until equilibrium is achieved and then actual trade takes place.

The exact implementation of the tâtonnement process is as follows. At time zero, there are three reference points: an upper interest bound, $\overline{r}_0^{rf}$, a lower interest bound, $\underline{r}_0^{rf}$, and the actual risk-free rate, r_0^{rf}. It is assumed that $\underline{r}_0^{rf} \leq r_0^{rf} \leq \overline{r}_0^{rf}$. Given those bounds and banks' initial optimal portfolio allocation there might be excess demand or supply on the interbank market. To fix ideas let's assume that it results in an excess supply of bank lending. In this case the lending rate adjusts downwards to re-equilibrate bank lending. The new lending rate is set to $r_1^{rf} = \frac{r_0^{rf} + \underline{r}^{rf}}{2}$ and the new upper bound is set to $\overline{r}_1^{rf} = r_0^{rf}$. Given the new lending rates, banks re-optimize their portfolio allocation, which then results in new bank lending positions. Gradually, the excess supply of bank lending is absorbed through this

[10] This set of parameters includes specific values for all regulatory requirements, in particular for γ (capital requirement ratio), χ_1 and χ_2 (risk weights on interbank assets and non-liquid assets), α (the liquidity ratio requirement); and banks capital endowment, in particular d (amount of deposits bank start with), and e^i (banks equity endowment).

sequential adjustment of the lending rate. The opposite adjustment takes place if demand for liquidity exceeds supply. The process converges when the interest rate adjustment is below a tolerance value ϖ.

Once equilibrium amounts of funds exchanged on the interbank market have been obtained, it remains to determine the actual allocation of funds across banks, namely the interlinkages in the interbank market. Notice that banks are indifferent among different counterparts as ex-post they can charge different risk premia based on individual banks' default risk An efficient allocation is then achieved simply by identifying the *closest matching partners*. Closest matching partners are lender-creditor pairs of banks which, within a specified set, feature the smallest distance between funds demand and supply. Consider for instance the following example: at market clearing prices the system consists of 4 banks wishing to lend and 2 banks wishing to borrow. Upon ordering of the respective demand and supply vectors, we can immediately identify two matching partners: two banks that demand money and the two banks which provide the largest amounts of funds. For each of those matching partners, the amount given by the minimum between demand and supply is exchanged. Given these transactions, two banks have satisfied their desired fund allocation and therefore become inactive: the matching process continues by sorting demand and supply vectors for the remaining banks until all transactions have been concluded. Note that the equilibrium set up of a financial system outlined in this sub-section is obtained for given individual probabilities of default. However, the probabilities of default which banks have assumed in their portfolio optimization might differ from actual probabilities of default in the financial system which emerges. The next sub-section outlines how equilibrium probabilities of default are determined in our model.

3.2.1 Model Equilibrium Consistent Expectations of Default

As explained above the rate charged for borrowing includes a premium to cover for expected default probabilities: to this purpose we shall formulate a process through which banks form expectations about cross-sectional probabilities of banks' default. Beyond the recovery rate,

ξ, which we assume to be a common parameter across banks,[11] in our model bank equilibrium probabilities of default, PD^i, are derived endogenously via an iterative algorithm. First, a bank goes bankrupt when its liquidity and the proceeds from selling non-liquid assets are not sufficient for repaying its debts in the interbank market; if we define s_i as bank i sales of non-liquid assets, the default probability of bank i is defined as follows:

$$PD^i = prob\left\{er^i < \gamma | c^i < \alpha \cdot d\right\} \tag{6}$$

We assume that agents form beliefs relatively to each bank's default probability by learning over time from the equilibrium of the financial systems subject to repeated shocks. As agents learn the adjacency matrix describing the system reaches a stable configuration compatible with the limiting distribution for the vector of the default probabilities. Hence the underlying assumptions is that banks' expectations are consistent with a long run equilibrium of the model. Note that all agents share the same beliefs, that is, banks probabilities of default are common knowledge.

In the numerical examples default probabilities are computed as follows. Banks' default probabilities are initially set to zero. First, for a given set of model parameters, a financial system forms as outlined in the previous sub-section, based on banks' individual portfolio choices, the tâtonnement process, and the interbank market allocation. Second, this specific financial system is then exposed to a large number of shocks.[12] Third, bank i's conditional probability of default is computed as the fraction of defaults of that bank in all shock scenarios. By the law of large numbers this percentage can be used to approximate the probability of default of bank i. These updated probabilities are then used as guesses for the default probabilities in computing a new financial system, that is, the first step outlined above is repeated.

This iterative procedure is repeated until we detect a financial system cycle. A financial

[11] Following Grunert and Weber [20] this parameter is set to 0.75.

[12] We set this number being 1000. Each shock is drawn from a multivariate normal distribution. Mean and variance are set to two and four, respectively. Correlation is assumed to be zero. The moments of distribution are chosen so as to rule out large tail events.

system cycle is detected when the adjacency matrix describing the network of interlinkages becomes recurrent or equivalently when all banks in the system repeatedly choose the same portfolio allocation. When a cycle is detected, the probabilities of default are calculated as the average probabilities of default over a cycle, assuming that banks assign the same probability to each financial system in a given cycle.

3.3 Tâtonnement in the Market for Non-Liquid Assets

In the model, the market price of the non-liquid asset is found via a continuous time tâtonnement process (see also Cifuentes, Ferucci and Shin [11]). Sales and purchases in non-liquid asset markets are triggered by shocks that prevent banks from fulfilling its regulatory requirements. The bank's supply (or demand), s_i, of non-liquid assets is obtained by solving Equation 3 for the amount of non-liquid assets that would allow bank i to fulfill the capital requirements. Since each s_i is decreasing in p, the aggregate sales function, $S(p) = \sum_i s_i(p)$, is also decreasing in p. An equilibrium price is such that total excess demand equal supplies, namely $S(p) = D(p)$. The price at which total aggregate sales are zero, namely $p = 1$ can certainly be considered one equilibrium price. We can define an aggregate demand function $\Theta : [p, 1] \to [p, 1]$: given this function an equilibrium price solves the following fixed point:

$$\Theta(p) = d^{-1}(s(p)) \tag{7}$$

The price convergence process in this case is guaranteed by using the following inverse demand function:[13]:

$$p = exp(-\beta \sum_i s_i), \tag{8}$$

where β is a positive constant to scale the price responsiveness with respect to non-liquid assets sold, and s_i is the amount of bank i's non-liquid assets sold on the market. Integrating back the demand function in Equation 8 yields the following:

$$\frac{dp}{dt} = \beta S(p) \tag{9}$$

[13]See also Cifuentes, Ferrucci, and Shin [11].

which states that prices will go up in presence of excess demand and downward in presence of excess supply. In the above differential equation β represents the rate of adjustment of prices along the dynamic trajectory.

Tâtonnement on the market for non-liquid assets can be described by the following iterative process. Prior to any shock, the market price for non-liquid assets equals 1, which is the initial price when all banks fulfill their regulatory requirements, and sales of the non-liquid asset are zero. A shock to bank i, say a certain loss of assets, shifts the supply curve upwards, resulting in $S(1) = s_i \succ 0$ because bank i starts selling non-liquid assets to fulfill its capital ratio. However, for $S(1)$ the bid price, given by the inverse demand function, Equation (8), equals only $p(S(1))^{bid}$, while the offer price is one. The resulting market price is $p(S(1))^{mid}$, the price in the middle between bid and offer prices. Since the market price thus decreases and banks have to mark their non-liquid assets to market, additional non-liquid asset sales may be needed to fulfill the capital requirement. The step-wise adjustment process continues until the demand and the supply curves intersect at p^*. Note that the supply curve may become horizontal from some value of non-liquid assets sold onwards, as the total amount of non-liquid assets on the banks' balance sheets is limited. Since a shock to a bank will always result in an upward shift of the supply curve, and the maximum price of the non-liquid asset equals 1, while the initial equilibrium prior to the shock equals zero, a market price $p \in (0, 1)$ always exists. The tâtonnement process on the market for non-liquid assets is displayed on figure 1.

3.4 Equilibrium

Definition. An equilibrium in our model is defined as follows:

(i) A quadruple (l^i, b^i, e^i, c^i) for each bank i that maximizes Equation 1 subject to Equations 2, 3, 4.

(ii) A price in the interbank market, r^{rf}, which is set to equilibrate aggregate supply and demand of funds: $F^{supply} = F^{demand}$.

(iii) A *closest matching partners* clearing mechanism for the interbank market.

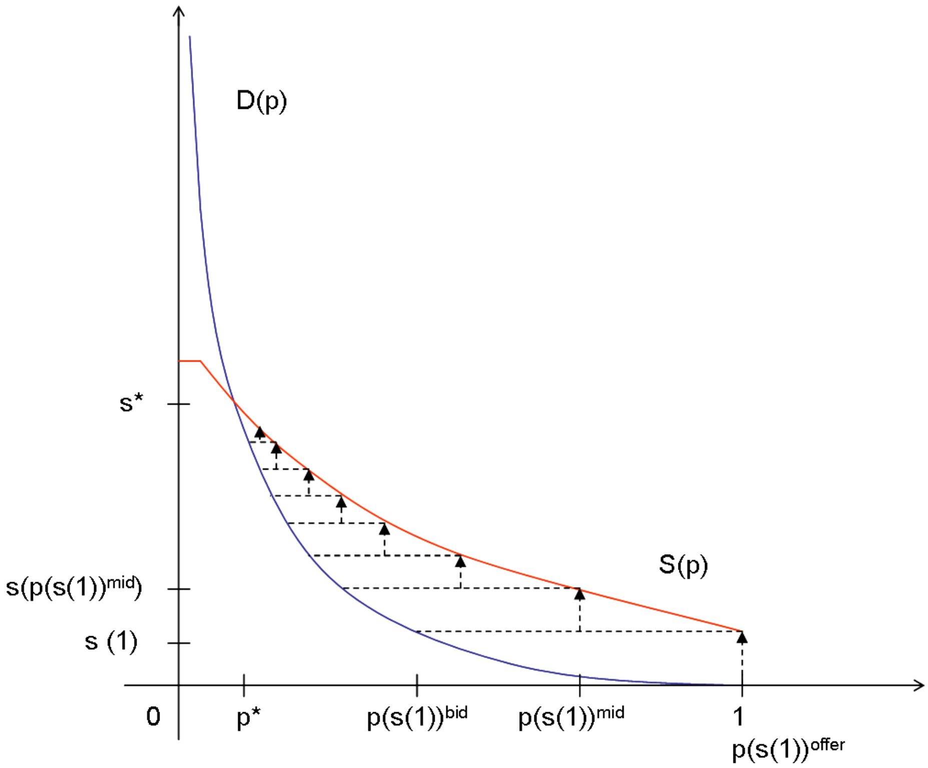

Figure 1: Tâtonnement Process on the Market for non-liquid Assets

(iv) Banks form model equilibrium consistent expectations about $PD^i = prob\left\{er^i < \gamma | c^i < \alpha \cdot d\right\}$.

(v) The price of non-liquid assets solves the fixed point: $\Theta(p) = d^{-1}(s(p))$.

3.5 Systemic Risk Measure

Generally speaking systemic risk occurs in the event in which a shock to one or several institutions spreads to the system in a way that determines the collapse of a large part or the entire system. A prerequisite for the emergence of systemic risk is the presence of inter-linkages and interdependencies in the market, so that the default (or a run) on a single intermediary or on a cluster of them leads to a cascade of failures, which could potentially undermine the functioning of the financial system. The Financial Stability Board, International Monetary Fund, and Bank for International Settlements [16] define systemic risk as "disruption to financial services that is (i) caused by an impairment of all or parts of the financial system, and (ii) has the potential to have serious negative consequences for the real economy." Following this definition, systemic risk is the risk that large parts of the financial system default leading to negative repercussions on the real economy because of a subsequent lack of financial services provision and credit. In our paper we define systemic risk as the proportion of the financial system in default subsequent to a shock which hit banks' assets. As explained above a bank defaults when it is unable to meet regulatory requirements. Recall that banks might default either because they are directly hit by a shock to their asset portfolio which forces them into fire sale spirals or because they have suffered losses to their portfolios due to lack of repayment from other defaulting banks (*cascades*). Systemic risk is then computed as the ratio of assets from all defaulting banks subsequent to a shock to non-liquid assets as from Equation (10)

$$\Phi = \frac{\sum_{def} assets_{def}}{\sum_{i} assets_{i}}, \tag{10}$$

where $def \in i$ indexes banks that are in default after the financial system has absorbed the shock.[14]

Since we are also interested in how much each bank contributes to systemic risk, we need a metric to measure their impact. While there is much agreement about the general definition of systemic risk, there is much less agreement upon quantitative measures for individual contributions. The traditional analysis for measuring contribution to systemic risk was based upon the judgement of whether the defaulting bank or group of intermediaries was *too-big-to-fail:* such an assessment is based on indicators such as the institution's size relative to the system, market share concentration indices such as the Herfindahl-Hirschman Index, the oligopolistic structure of the market and the presence of barriers to entries. Recently and due to the emergence of complex financial relations, the focus of contribution to systemic risk measures has been shifted toward an assessment of the *too-interconnected-to-*

[14]Note that the amounts of assets used to compute this measure for systemic risk are taken from the financial system set-up prior to the shock. The reason for this is that the dynamic absorption of the shock in the financial system changes the allocation of assets, potentially resulting in banks having no assets at all when they default.

fail. It is on both concepts that we focus. One measure which has been recently proposed to determine contribution to systemic risk is the Shapley value.[15] Generally speaking the Shapley value is affected by banks' sizes and the degree of bank interconnections. In our model interconnection occurs through both, direct and indirect links. Direct links are given by the correlations of shocks to non-liquid assets and the exposure to others' banks balance sheets. Indirect links are given by the effects that a fall in the market price of non-liquid assets has on the balance sheet of the entire system. Note that the overall degree of interconnections in our model is affected by the parameters characterizing the optimizing decision. The link between size and interconnections with systemic risk implies that any parameter change which affects these metrics in the network structure will eventually have an impact on systemic risk as well. In game theory the Shapley value is used to find the fair allocation of gains obtained both under cooperative and non-cooperative games.[16] It can be defined in terms of all possible orders of the players N.[17]

Define $O : 1, \ldots, n \to 1, \ldots, n$ to be a permutation that assigns to each position k the player $O(k)$. Furthermore denote by $\delta(N)$ the set of all possible permutations with player set N. Given a permutation O, and denoting by $Pre^i(O)$ the set of predecessors of player i in the order O, the Shapley value can be expressed in the following way:

$$\Omega_i(v^\Psi) = \frac{1}{N!} \sum_{O \in \pi_N} \left(v^\Psi(Pre^i(O) \cup i) - v^\Psi Pre^i((O)) \right) \tag{11}$$

where $v^\Psi Pre^i((O))$ is the value obtained in permutation O by the players preceding player i and $v^\Psi(Pre^i(O) \cup i)$ is the value obtained in the same permutation when including player i. That is, $\Omega_i(v^\Psi)$ gives the average marginal contribution of player i over all permutations of player set N. Note that the index Ψ denotes different possible shock scenarios,

[15]See Shapley [30]. See also Tarashev, Borio, and Tsatsaronis [8] and Bluhm and Krahnen [24]. Alternative measures of systemic risks are proposed for instance in Adrian and Brunnermeier [1] through a CoVaR methodology.

[16]Gul [21] proves that Shapley values are a good approximation of agents' payoff in efficient equilibria also under non-cooperative games.

[17]The following exposition draws upon Castro, Gomez, and Tejadab [6] and Stanojevic, Laoutaris, and Rodriguez [31].

that is, banks' contribution to systemic risk is computed conditional on a shock vector to the banking system.

Lemma. *The Shapley value is characterized by the following properties: a. Pareto efficiency. The total gain of a coalition is distributed. b. Symmetry. Players with equivalent marginal contributions obtain the same Shapley value. c. Additivity. If one coalition can be split into two sub-coalitions then the pay-off of each player in the composite game is equal to the sum of the sub-coalition games. d. Zero player. A player that has no marginal contribution to any coalition has a Shapley value of zero.*

Since the number of permutations involved in calculating the Shapley value increases strongly with the number of banks, the analysis is subject to the curse of dimensionality. The Shapley value can then be approximated by the average contribution of banks to systemic risk over k randomly sampled permutations as displayed in Equation 12:

$$\Omega_i(v^\Psi) \approx \overset{\wedge}{\Omega}_i(v^\Psi) = \frac{1}{k}\sum_{O\in\pi_k}\left(v^\Psi(Pre^i(O)\cup i) - v^\Psi Pre^i((O))\right). \tag{12}$$

Proposition 1. *Given that each permutation has the same probability of being sampled in δ_k, the sample mean $\overset{\wedge}{\Omega}_i(v^\Psi)$ is an unbiased estimator of the population mean $\Omega_i(v^\Psi)$.*

Proof.

$$\begin{aligned} E(\overset{\wedge}{\Omega}_i(v^\Psi)) &= E(\frac{1}{k}\sum_{O\in\pi_k}\left(v^\Psi(Pre^i(O)\cup i) - v^\Psi Pre^i((O))\right)) \\ &= \frac{1}{k}(k\Omega_i(v^\Psi)) \\ &= \Omega_i(v^\Psi). \end{aligned} \tag{13}$$

We will investigate systemic risk and banks' contribution to it using a distribution of shock scenarios, that is, the shock vector m with dimension (N by 1) is drawn from the multivariate normal distribution $\Psi \sim \mathcal{N}(\mu, \Sigma, \Gamma)$,with μ being the mean vector, while Σ, Γ are the variance and correlation matrix, respectively. Given that each draw from Ψ has the same probability of being sampled, the first two moments of $\overset{\wedge}{\Omega}_i(v^\Psi)$ can be computed as

$$\hat{\mu}_i^{\hat{\Omega}_i(v^\Psi)} = \frac{1}{M}\sum_m \hat{\Omega}_i(v^{m\in\Psi}) \tag{14}$$

and

$$\hat{Var}_i^{\hat{\Omega}_i(v^\Psi)} = \frac{1}{M}\sum_m (\mu^{\hat{\Omega}_i(v^\Psi)} - \Omega_i(v^{m\in\Psi}))^2. \tag{15}$$

In the numerical stress test analysis Equations 14 and 15 will be computed with 1000 random draws for k and m. In Appendix B we show, using a Monte Carlo study, that in our model the approximation becomes precise enough beyond 400 draws.

Given the pareto efficiency and additivity properties of the Shapley value, Equation (16) shows overall expected systemic risk which can be computed as the sum of all banks' contribution to systemic risk as outlined in Equation (14):

$$\hat{SR}_i^\Psi = \sum_i \hat{\mu}_i^{\hat{\Omega}_i(v^\Psi)}. \tag{16}$$

3.6 Numerical Algorithm

In the model shocks take the form of a loss in banks' non-liquid asset holdings.[18] If subsequent to a shock realization, a bank cannot fulfill its capital requirement, it will sell non-liquid assets,[19] thereby indirectly transmitting the shock to other banks, via downward pressure on the market prices of non-liquid assets. If upon re-adjustment the capital requirement is still non satisfied, the bank will default. The clearing algorithm for shock transmission is similar to the algorithm used in Cifuentes, Ferruci, and Shin [11] based on the Eisenberg and Noe [13] clearing algorithm.

If the shock has been transmitted, systemic risk is computed as displayed in Equation 10.

[18] We follow Bluhm and Krahnen [24] to model the shock transmission process. Other shocks are possible, for example a sudden drop in non-liquid asset prices or the default of a bank in the system.

[19] Note that at the shock transmission stage the interbank links are taken as given, that is, banks do not adjust their lendings and borrowings except for the case of a counterparty default.

3.7 Calibration

The model parameters are chosen to match values observed in the financial system and/or imposed by supervisory policy. The parameter α, the amount of liquid assets banks have to hold as a function of the amount of deposits, is set to 0.1, thus being equivalent to the cash reserve ratio in the U.S. The parameter χ_1, the risk weight for non-liquid assets, is set to 1: this value reflects the risk weight applied in Basel II to commercial bank loans. The parameter χ_2, the weight for interbank lending, is set to 0.2, which is also the risk weight actually applied to interbank deposits between banks in OECD countries. The amount of equities and deposits that banks have initially on their balance sheets is set to 65 billions (mean with variance 10) and 600 billions which is the figure actually found on the balance sheet of the Deutsche Bank in the second quarter of 2012. Following federal reserve bank regulatory agency definitions, banks must hold a capital ratio of at least 8%. Finally, banks return on non-liquid assets is uniformly distributed on the interval between 0% to 15%. The vector of shocks to non-liquid assets is drawn from the multivariate normal distribution Ψ with mean 5, variance of 25 and zero covariance. Note that the variance is set high enough to mimic stress test scenarios. Having a large range is important to capture the effect of all risk channels, in particular the direct interconnection channel.[20] The model parameters are displayed on Table 2.

α	χ_1	χ_2	γ	Deposits	ς	Equity	Yield on NLA	Ψ
0.1	1	0.2	0.08	500	0.01	$N(65, 10)$	$U(0, 0.15)$	$-abs(N(\boldsymbol{mean}, \boldsymbol{\sigma}^2, \boldsymbol{\rho}))$

Table 2: Parameter Values in the Baseline Setting

The table displays the parameter values in the baseline setting. α is banks' liquidity requirement, χ_1 is the risk weight for non-liquid asset investments, χ_2 is the risk weight for interbank lending, γ is the capital requirement ratio, ς is the amount by which banks overfulfill regulatory requirements, and Ψ is the multivariate normal distribution of the shocks to the financial system (note that shocks between banks are uncorrelated, that is, the covariances between vector elements are zero), with $\boldsymbol{mean} = \boldsymbol{\iota} \cdot 5$, $\boldsymbol{\sigma}^2 = diag(\boldsymbol{\iota} \cdot 25)$, and $\boldsymbol{\rho} = \boldsymbol{\iota} \cdot \boldsymbol{\iota}' \rho - diag(\boldsymbol{\iota} \cdot \rho))$, where $\boldsymbol{\iota}$ is an identity vector of dimension N by 1. N and U designate normal and uniform distributions, respectively.

[20] See Bluhm and Krahnen [24].

4 Network Evolution and Shapley Values

In this section we analyze in conjunction the evolution of the network configuration as well as the Shapley value of each bank. First we provide a network configuration for our baseline calibration as from Table 2. Second we analyze the evolution of the network for different values of policy parameters: this will provide the basis for evaluating the impact of different policy regimes on the degree of interconnections and on the overall level of investment in non-liquid assets. Higher interconnections always imply more liquidity provision, but also higher degree of risk diffusion due to endogenous cascading defaults. The baseline policy experiments will be done by changing the capital and liquidity requirements: the configuration for different values of the risk factors in the capital requirements are shown in the technical appendix C. Next, we analyze Shapley values in response to random shocks to non-liquid assets and for different parameter configurations of the prudential policy regimes. The numerical analysis is akin to those performed for banks' stress testing. Our analysis carries an additional dimension in that we analyze the results under different policy regimes and provide a theoretically founded metric of banks' contribution to systemic risk.

The diffusion of systemic risk under different policy regimes will be tightly linked to the dynamic evolution of the network: a network with strong interconnections, particularly around highly leveraged or fragile banks, will feature a higher level of systemic risk. Banks highly exposed on the interbank market and prone to defaults will be the main risk spreaders.

In our simulations we will fix the number of banks to $N = 15$. We consider this number as representative of a mildly concentrated banking system. Changes in several of the policy parameters (capital and liquidity ratios, risk factors), which implicitly affect the banks' optimization constraints, will indirectly also provide robustness checks of the results of the baseline scenarios. To fully assess the role of prudential regulation, with a particular focus on the current debate over financial levies,[21] we will investigate the impact on the network

[21]In Europe there is currently a vivid debate on various forms of financial taxes, ranging from Pigouvian type of taxation (linked to risk contribution) to Tobin taxes (aimed at curbing financial transactions of various sort). An example is represented by the German Restrukturierungsfondsgesetz, a regulation according to

and the individual banks' Shapley values of risk charges, in the form of taxes on interbank borrowing and investment in non-liquid assets. Both types of taxes can be considered an approximate of Pigouvian charges as they directly tackle system wide externalities: taxes on borrowing are aimed at reducing the extent of interconnections and the ensuing network externalities; taxes on non-liquid investment can instead boost available market liquidity.

Note that all results reported as well as confidence intervals given are based on the outcomes from 1000 multivariate normally distributed random shocks drawn from $\mathbf{\Psi}$.

4.1 Alternative Prudential Regulations

At time zero the financial system is represented by the solution of the model (banks' optimization and market clearings) using baseline parameters outlined on Table 2. The equilibrium linkages of the time-zero financial network are displayed on Table 3. The letter B in the table is used to represent banks, banks' assets are displayed in the respective rows and banks' liabilities are displayed in the respective columns. Equilibrium non-liquid (NLA) and liquid (LA) assets are represented in the last two columns. The last row are liabilities vis-à-vis outside investors, namely deposits.

which banks are charged a levy which depends upon their degree of interconnectedness with other banks and upon the extent of their derivative investments. The proceeds of these levies are used to finance a resolution fund to stabilize the financial system.

BANKS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	NLA Investment	Cash
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	696	50
2	0	0	0	0	174	0	9	0	0	0	0	317	0	0	0	12	50
3	183	0	0	0	0	0	2	0	0	0	324	0	0	0	0	12	50
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	829	50
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	685	50
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	851	50
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	725	50
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	696	50
9	0	0	0	304	0	0	194	0	0	0	0	0	0	0	0	12	50
10	0	0	0	0	0	325	3	184	0	0	0	0	0	0	0	12	50
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	851	50
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	843	50
13	0	0	0	0	0	0	2	0	0	0	0	0	0	182	318	12	50
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	694	50
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	844	50
Deposits	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	0	0

Table 3: Financial System in Baseline Setting
The table displays the financial system for the baseline setting. Banks are designated with letter B. Each bank's assets are displayed along the according row and banks liabilities are displayed along the according column. For example, matrix element (2,5) shows how much Bank 1 has lent to Bank 5. The last two columns designate non-liquid assets (NLA) and liquid assets (LA), that is, cash, respectively. The last row are assets from the rest of the world, that is, deposits.

Figure 2 displays a visual outline of the financial system displayed on Table 3. Each bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' average equity. Below each of the stylized financial systems there are four further indicators. First, the representative red ball provides the basic measurement unit for banks' size. Second, the thickness of the representative black line provides the basic measurement unit for the size of the lending linkage. Third, the interbank rate is the equilibrium interest rate resulting from the tâtonnement process in the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) indicates the average investment (across banks) in non-liquid assets relative to equities.

Recall that banks start with different returns on non-liquid assets. Banks with relatively higher returns (banks 4,6,11,12, and 15) will invest more in non-liquid assets, thereby strongly leveraging in the inter-bank market. Banks with low returns will invest less, while they will find it more profitable to lend in the interbank market: a form of asset substitution takes place as for those banks the return on lending is higher than the return on non-liquid investment.

Table 4 displays systemic risk (SR) as well as banks' contribution to it ($B1$ - $B15$) computed according to Equation 14, in the baseline parameter setting. Notice that more

B 1	B 2	B 3	B 4	B 5	B 6	B 7	B 8
0.042	0.001	0.001	0.051	0.046	0.048	0.048	0.040
B 9	B 10	B 11	B 12	B 13	B 14	B 15	SR
0.002	0.002	0.041	0.044	0.002	0.046	0.054	0.469

Table 4: Systemic Risk and Banks' Contribution in the Baseline Setting Without Central Bank Intervention

The table displays banks' contribution to systemic risk computed according to Equation (14), as well as overall systemic risk (SR) computed according to Equation (16), in the baseline setting. Note that values have been rounded.

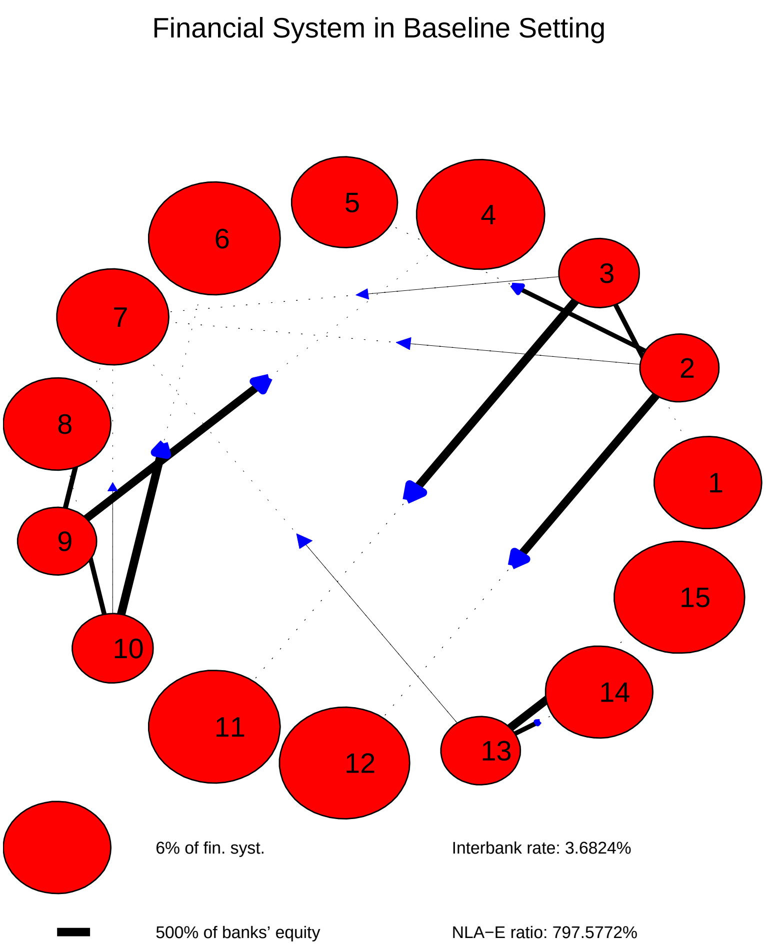

Figure 2: Financial System in Baseline Scenario

The figure displays an outline of the financial system emerging in the baseline setting. Each bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial system there are four further indicators. First, the red ball gives an indication about the percentage of the financial systems a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

leveraged banks do contribute more to systemic risk: when hit by a shock those banks might find themselves unable to repay lending in interbank markets. Therefore they contribute more in spreading risk through direct links in the interbank market. Moreover and in response to the shock, those banks would need to sell non-liquid assets to meet the capital requirement.

Thereby they contribute to the fall in the price of non-liquid assets, implicitly triggering a reduction in the portfolio values of other banks, thus spreading risk of insolvency indirectly.

Figure 3 displays selected financial system realizations at different values for the liquidity requirements, α. By increasing values of the liquidity requirement several effects emerge.

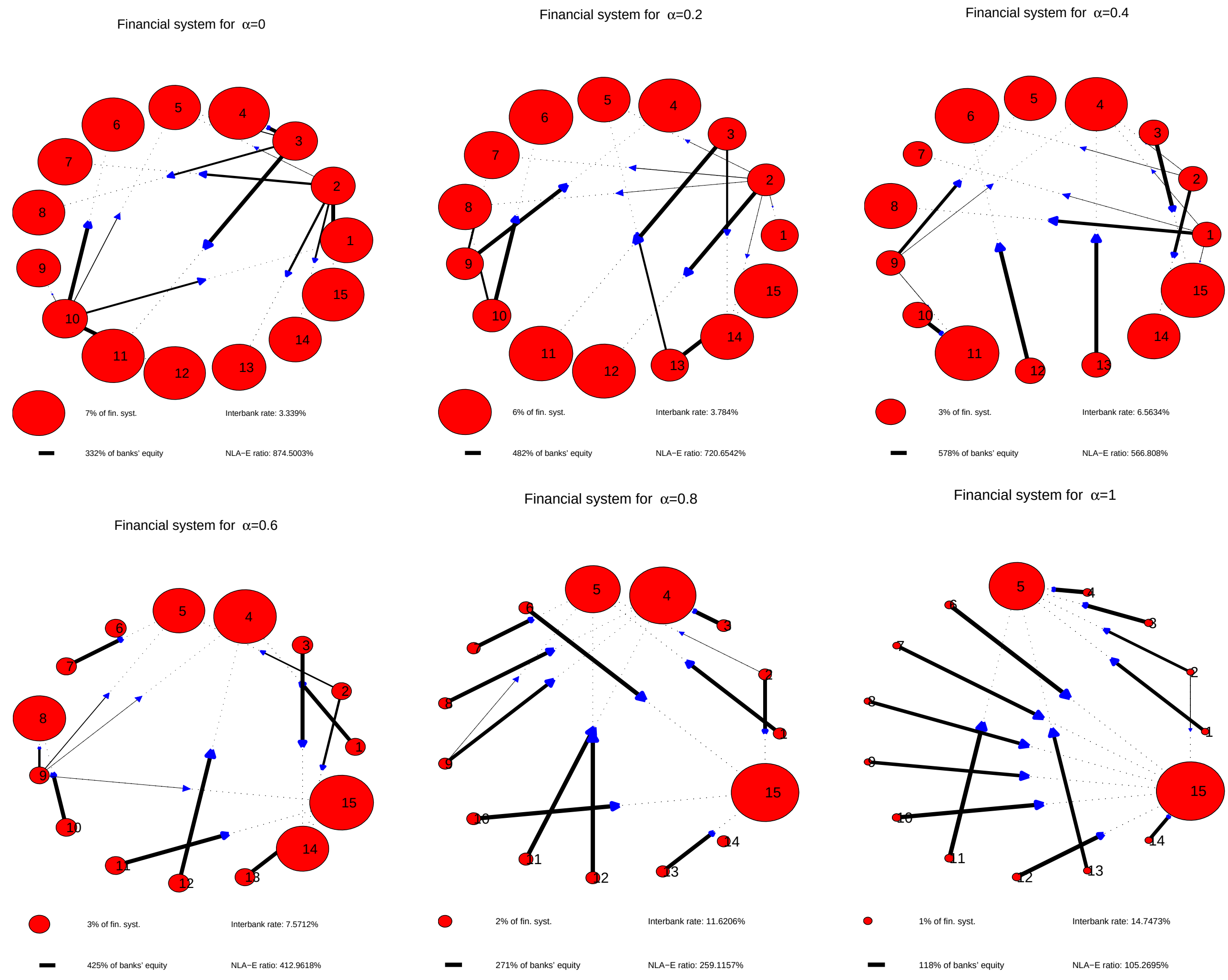

Figure 3: Financial System Structures and Liquidity Requirement

The figures displayed on the panel are financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

First, the financial system becomes more concentrated: the few banks with higher returns on non-liquid assets invest more (their ball grows). Due to the tighter liquidity requirements (a larger fraction of deposits has to be kept in cash), they increase the demand for liquidity in the interbank market. As the demand for liquidity in the interbank market increases, the equilibrium return on lending raises. This induces banks, with low returns on non-liquid assets, to engage in interbank lending as a result of asset substitution. Finally, as the fraction of banks lending increases, the relative amount of overall non-liquid asset investments to banks' equity falls.

Figure 4 shows the evolution of systemic risk and banks' contribution to it for increasing levels of the liquidity ratio, α. The figure displays systemic risk (y-axis on panel 16, bottom right, computed using Equation 16) and banks' contribution to it (y-axis on panels 1 to 15, computed using Equation 14) as solid lines over different values of the liquidity requirement ratio. The dotted lines are the two standard deviation error bands, where thresholds are the 5% cut-off points of the most extreme observations of μ_i obtained conditional on the shock vectors drawn. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, namely the sum of all interbank lending relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, namely the sum of all non-liquid assets held by banks relative to the sum of all banks' equity. Several effects emerge. First, overall systemic risk (panel 16) decreases. Higher liquidity requirements force banks to invest less in non-liquid assets: this implies less leverage in the interbank market, hence fewer cascades, and lower probability of fire-sales. Second, the evolution of the relative amount of interbank lending is bell-shaped. Initially, as described above, highly leveraged banks replace their liabilities which they have to hold in cash with interbank lending. This increases the amount of lending and correspondingly the loan-to-equity ratio. However, as the liquidity requirement becomes more and more restrictive, supplying banks on the interbank market have to reduce their supply more and more to meet requirements. This ultimately leads to a decrease of interbank lending. Notice that the liquidity hoarding in this case also produces

Systemic Risk at Varying Degrees of α

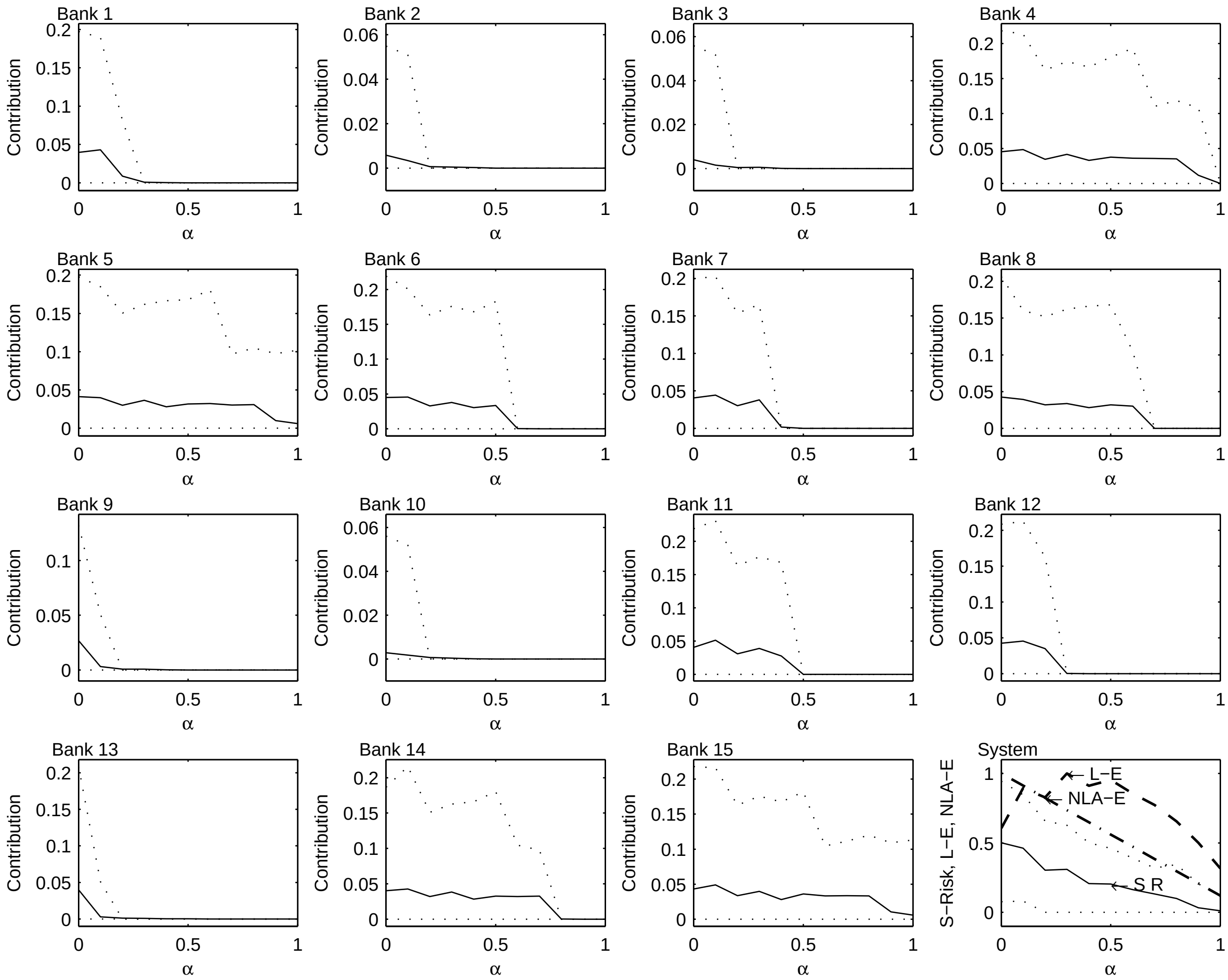

Figure 4: Systemic Risk and Liquidity Requirement

The figure displays systemic risk (y-axis on panel 16, bottom right, computed following Equation (16)) and banks' contribution to it (y-axis on panels 1 to 15, computed following Equation (14)) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. The dotted lines are the two standard deviation error bands (where thresholds are the 5% cut-off points of the most extreme observations of μ_i obtained conditional on the shock vectors drawn). On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

a destruction of investment in non-liquid assets. This reduces the extent and the probability of cascades. As noticed earlier banks' contribution to systemic risk increases for banks which invest more and leverage more.

Next, we turn to investigating the effects of increasing the capital requirement ratio, γ. Figure 5 displays the evolution of the financial network for increasing values of capital requirement. As the capital requirement increases, all banks are forced to leverage less: the effects of difference in the returns for non-liquid assets fades away and as a result banks acquire similar sizes. The scope for leveraging diminishes also for banks with high returns on non-liquid assets: as a result overall interbank activity and the equilibrium lending rate decline. Interestingly and as an effect of asset substitution, the decline in lending rates induces even less profitable banks (with low non-liquid asset returns) to shift by investing more in non-liquid assets and to provide less liquidity in the interbank markets. Therefore the system features two counterbalancing effects: when γ increases, on the one side more profitable banks leverage less and invest less in non-liquid assets, on the other side less profitable banks tend to invest more. Ultimately, at high values of the capital requirement ratio, banks have less and less scope to invest in non-liquid assets, resulting in a decline of overall non-liquid investment. Eventually the latter effect prevails and the overall amount of investment in non-liquid assets (relative to banks' equity), which is initially constant, falls.

Figure 6 shows the effect of changes in the capital requirement ratio on systemic risk and banks' contribution to it. Overall systemic risk has a bell-shaped dynamic: this is due to the evolution of the two counterbalancing effects described above. For low levels of γ the extent of market interconnections is still large and increasing: this renders the financial system more vulnerable as it triggers more cascades in the event of negative shocks. Thereby overall systemic risk and the contribution of highly leveraged banks tend to initially increase. For high levels of γ the financial network becomes sparse: hence the potential for sequential defaults falls.

Appendix C reports the evolution of the network configuration and the Shapley values for different values of the risk factors, χ_1 and χ_2 : their effects are actually pretty much akin to the effects observed for different values of γ.

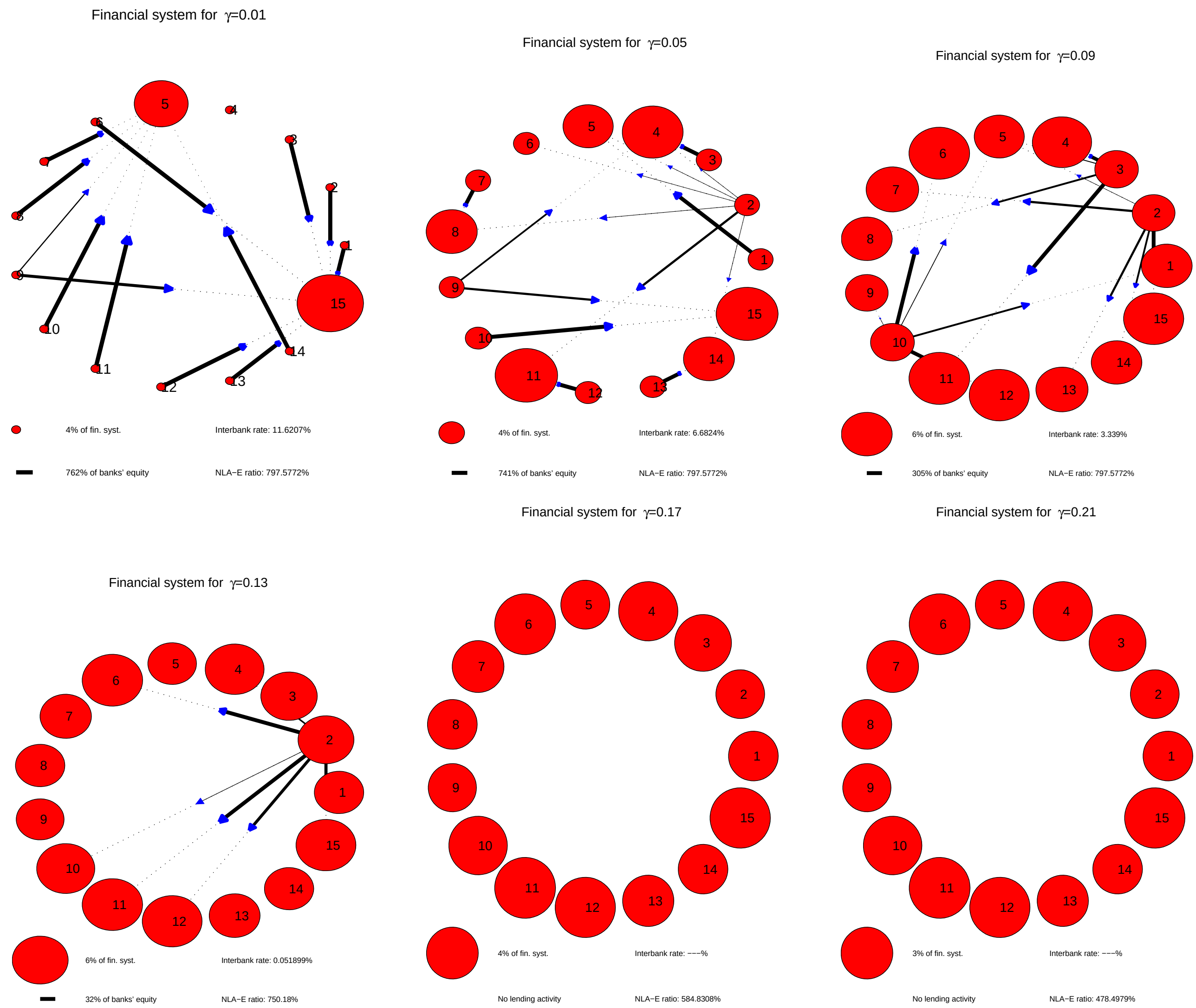

Figure 5: Financial System Structures and Capital Requirement Ratio

The figures displayed on the panel are selected financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

Systemic Risk at Varying Degrees of γ

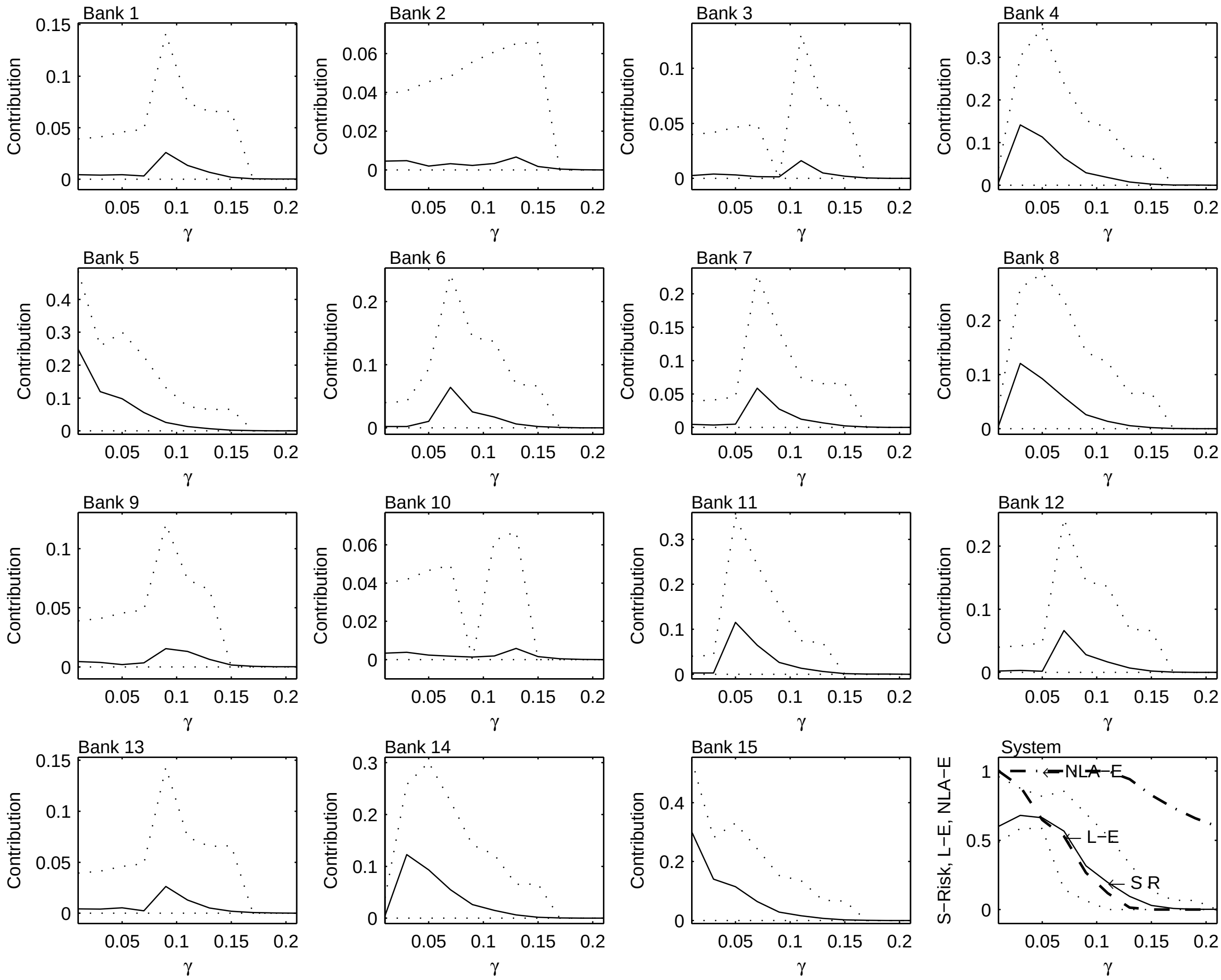

Figure 6: Systemic Risk and Capital Requirement Ratio

The figure displays systemic risk (y-axis on panel 16, bottom right, computed following Equation (16)) and banks' contribution to it (y-axis on panels 1 to 15, computed following Equation (14)) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. The dotted lines are the two standard deviation error bands (where thresholds are the 5% cut-off points of the most extreme observations of μ_i obtained conditional on the shock vectors drawn). On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

4.1.1 Risk Charges

In light of the recent debate on the need to introduce taxes on financial transactions and/or systemic risk charges we use our network model to analyze the impact of two types of taxes, respectively on banks' borrowing and investment in non-liquid assets. Banks' borrowing raises the potential for network externalities as exemplified by sequential cascades, investment in non-liquid assets reduces the scope for liquidity provision and raises the potential for fire-sale externalities. Our analysis is positive in that taxes are not optimally chosen by a regulator that could internalize both type of externalities, but it does provide an important feedback on how those taxes can curb the disruptive power of financial instability.

After including taxes, the banks' profit function changes as follows:

$$E(\pi^i) = l \cdot r^{rf} + \frac{(r^i - \tau_2)}{p} \cdot e^i - b^i \cdot \left(r^f \cdot \frac{1}{1 - \xi PD^i} + \tau_1 \right), \tag{17}$$

where τ_1 is the risk levy for interconnectedness and τ_2 is the risk levy for derivative investments.

Figure 7 displays the development of the financial system along increasing values of both taxes.

The effects of those taxes on the evolution of the network are fairly intuitive. The penalty parameter on banks' borrowing reduces the number of banks leveraging on the interbank market, therefore also lowering overall investment in non-liquid assets. The fall in the demand for liquidity reduces the lending rate. The reduced availability of liquidity reduces the overall amount of non-liquid investment.

More intricate is the interpretation of the behavior of banks featuring different returns on non-liquid assets. The penalty parameter on non-liquid assets lowers banks' yield in this asset class for all banks: the fraction of banks which then engages in interbank lending activity increases compared to the case with no levies. As a result overall investment in non-liquid assets decreases, while the supply of funds on the interbank market increases. The latter pushes down the interest rate on the interbank market.

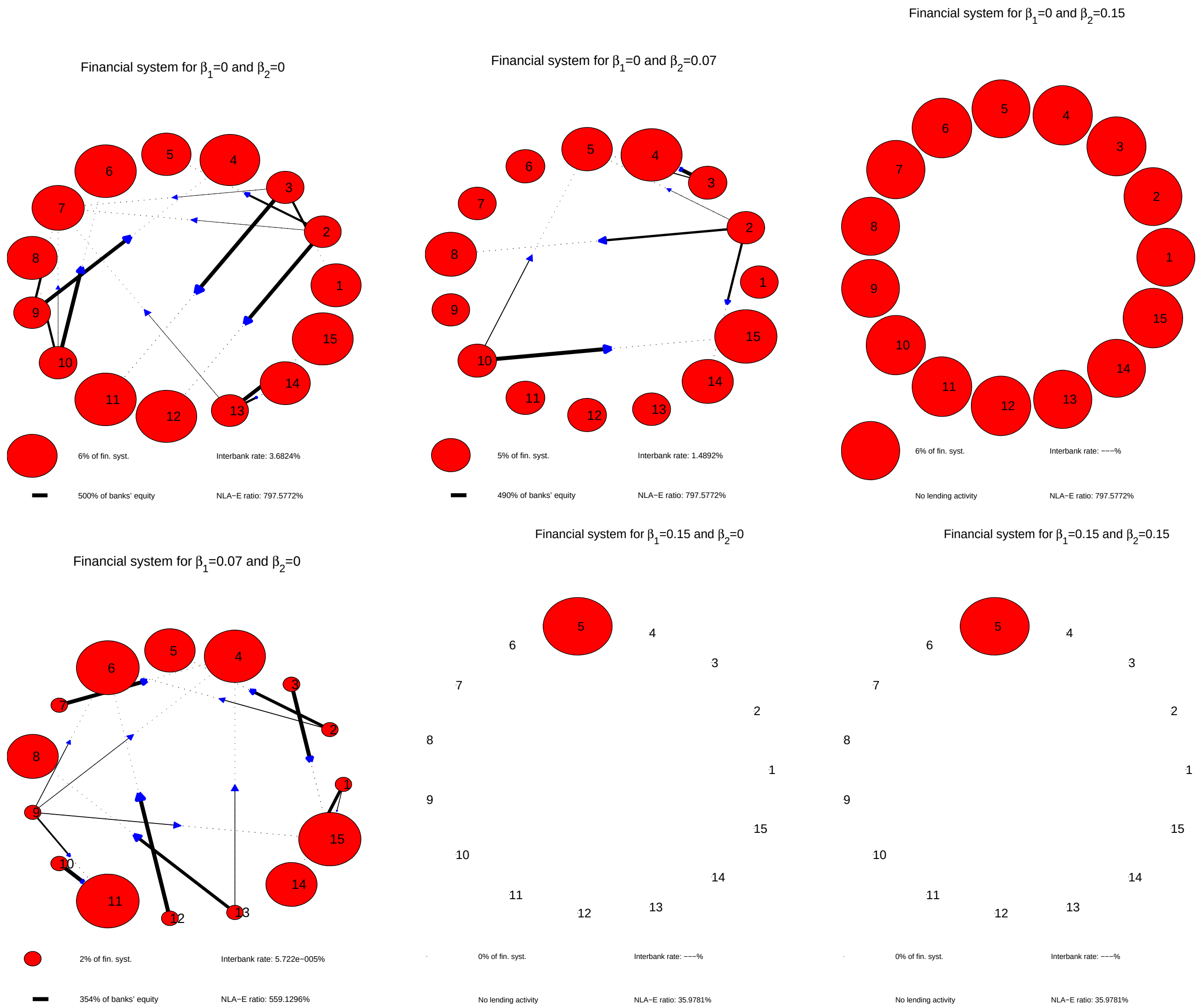

Figure 7: Financial System Structures and Risk Charges

The figures displayed on the panel are financial system realizations at different values for systemic risk charges for derivative investments (β_1) and interbank lendings (β_2), with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

Figure 8 displays the effect of the penalty parameters on systemic risk and banks' contribution to it.

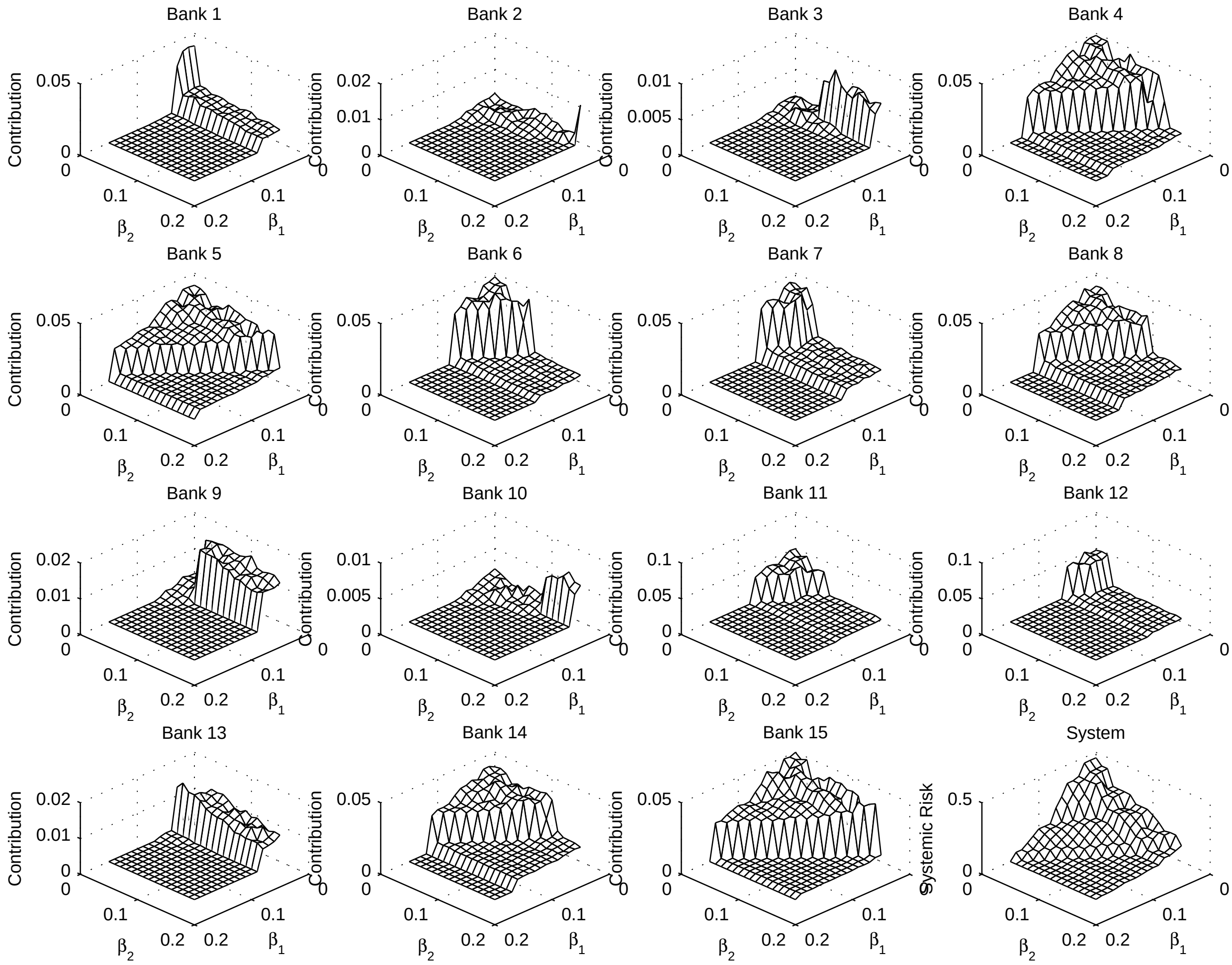

Figure 8: Systemic Risk and Risk Charges

The figure displays systemic risk (y-axis on panel 16, computed following Equation (16)) and banks' contribution to it ((y-axis on panels 1 to 15, computed following Equation (14))) over a range of increasing values for the penalty parameters for derivatives (β_1) and interbank lendings (β_2) on the z- and x-axes, respectively.

The taxes reduce both, incentives to borrow and to invest: as a result the interbank market dries out. Overall results are mixed: the extent of direct interconnection falls and so does systemic risk. On the other side the taxes reduce the scope for investment, thereby affecting adversely the growth prospects of the real side of the economy.

4.2 Adding Central Bank Intervention

Central banks intervene in the interbank market both as part of the normal activity of their operational system as well as for unconventional interventions. Both the New York Fed and the ECB achieve the target policy rate by supplying or withdrawing liquidity from the market as part of their normal operational procedures. In times of financial crises and following the disruption of trust in the interbank market as well as the ensuing liquidity hoarding, central banks around the globe have taken unconventional measures also with direct borrowing and lending to individual banks. We therefore want to reconsider the results obtained so far under the assumption that a central bank intervenes in the interbank market.

The central bank is defined as the n^{th} bank, where $n = 15$. This bank will neither hold cash nor non-liquid assets, but will solely supply or demand liquid funds on the interbank market with the goal of achieving the desired interest rate target. We assume that the central bank has unlimited funds and thus cannot default.

Prior to any shock central bank interventions can be characterized as follows. If the target interest rate, r^{rf} is below the equilibrium interest rate on the interbank market,[22] the central bank supplies money until the target is achieved. It demands money in the opposite case. Following endogenous changes in the financial system structure (e.g. through supervisory intervention) the equilibrium interest rate will deviate from the central bank's target: in this case the central bank intervenes via supplying/drawing liquidity to/from the market until the interest rate on the interbank market is within an interval band around its desired rate (the bands are set to 100 basis points).

The parameters in the baseline setting with central bank are the same as displayed on Table 2, with the addition, that the target interest rate of the central bank equals 3.59% which is the equilibrium interest rate in absence of the central bank. Given the interest rate equilibrium value the bands of the intervention corridor are set to .5 percentages points.

[22]This corresponds to the equilibrium interest rate obtained in absence of any central bank intervention.

BANKS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	NLA Investment	Cash
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	696	50
2	132	0	0	304	0	0	0	0	0	0	0	0	0	0	0	75	50
3	52	0	0	0	10	0	0	0	0	0	324	0	0	60	0	77	50
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	829	50
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	685	50
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	851	50
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	725	50
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	696	50
9	0	0	0	0	42	0	210	184	0	0	0	0	0	0	0	75	50
10	0	0	0	0	122	325	0	0	0	0	0	0	0	0	0	77	50
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	851	50
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	843	50
13	0	0	0	0	0	0	0	0	0	0	0	317	0	122	0	76	50
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	694	50
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Deposits	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	0	0

Table 5: Financial System in Baseline Setting With Central Bank Intervention
The table displays the financial system for the baseline setting with central bank intervention. Banks are designated with letter B. Each bank's assets are displayed along the according row and banks liabilities are displayed along the according column. For example, matrix element (4,2) shows that Bank 3 has lent 52 to Bank 1. The last two columns designate non-liquid assets (NLA) and liquid assets (LA), that is, cash, respectively. The last row are assets from the rest of the world, that is, deposits.

Figure 9 outlines the equilibrium financial system for the baseline scenario with central bank intervention. The complete financial system matrix is outlined on Table 5.

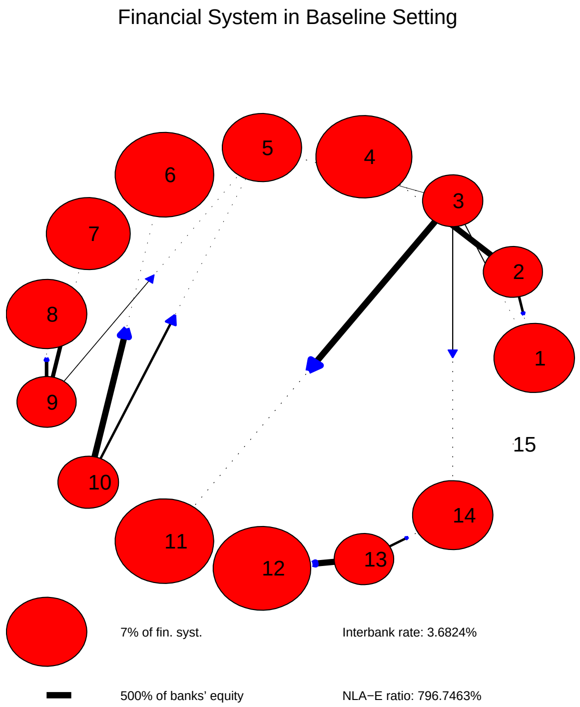

Figure 9: Financial System in Baseline Scenario With Central Bank Intervention

The figure displays an outline of the financial system emerging in the baseline setting with central bank intervention. Each bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below the stylized financial system there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

The considerations done earlier for the case with no central bank interventions are generally valid here. One noteworthy difference arises: the financial network linkages are now much weaker, despite the overall investment in non-liquid assets remains roughly the same as in absence of interventions.

Figure 11 displays selected financial networks at different values of the liquidity require-

B 1	B 2	B 3	B 4	B 5	B 6	B 7	B 8
0.044	0.002	0.001	0.051	0.045	0.052	0.056	0.043
B 9	B 10	B 11	B 12	B 13	B 14	B 15	SR
0.002	0.003	0.041	0.050	0.002	0.046	0.000	0.438

Table 6: Systemic Risk and Banks' Contribution in the Baseline Setting

The table displays banks' contribution to systemic risk, as well as overall systemic risk (SR), in the baseline setting with central bank intervention. Note that values have been rounded.

ment, α. Much of the qualitative developments outlined in absence of central banks remain valid: few profitable banks invest and borrow, while the rest initially engages in bank lending and then exits the market. As before increasing the liquidity requirement drives up the interest rate on the interbank market. However, with central bank intervention this effect only applies within the central bank interest corridor, that is, between 3.09% and 4.09%. If the interest rate hits the upper boundary of the corridor, the central bank starts supplying liquidity to prevent the interest rate from increasing further. The central bank supplies much of the liquidity demanded in the market, hence, contrary to before, a lower fraction of banks engages in interbank lending and overall investment in non-liquid assets is larger: as the lending rate is now smaller also the less profitable banks tend to invest more in non-liquid assets.

Figure 11 displays the effect of changes in the liquidity requirement on systemic risk and banks' contribution to it. As for the case with no central bank intervention, the pattern of the non-liquid asset to equity ratio is bell shaped. Now, however the contribution of each bank to systemic risk is smaller since much of the inter-bank liquidity is provided by the central bank. This reduces the adverse consequences of default losses.

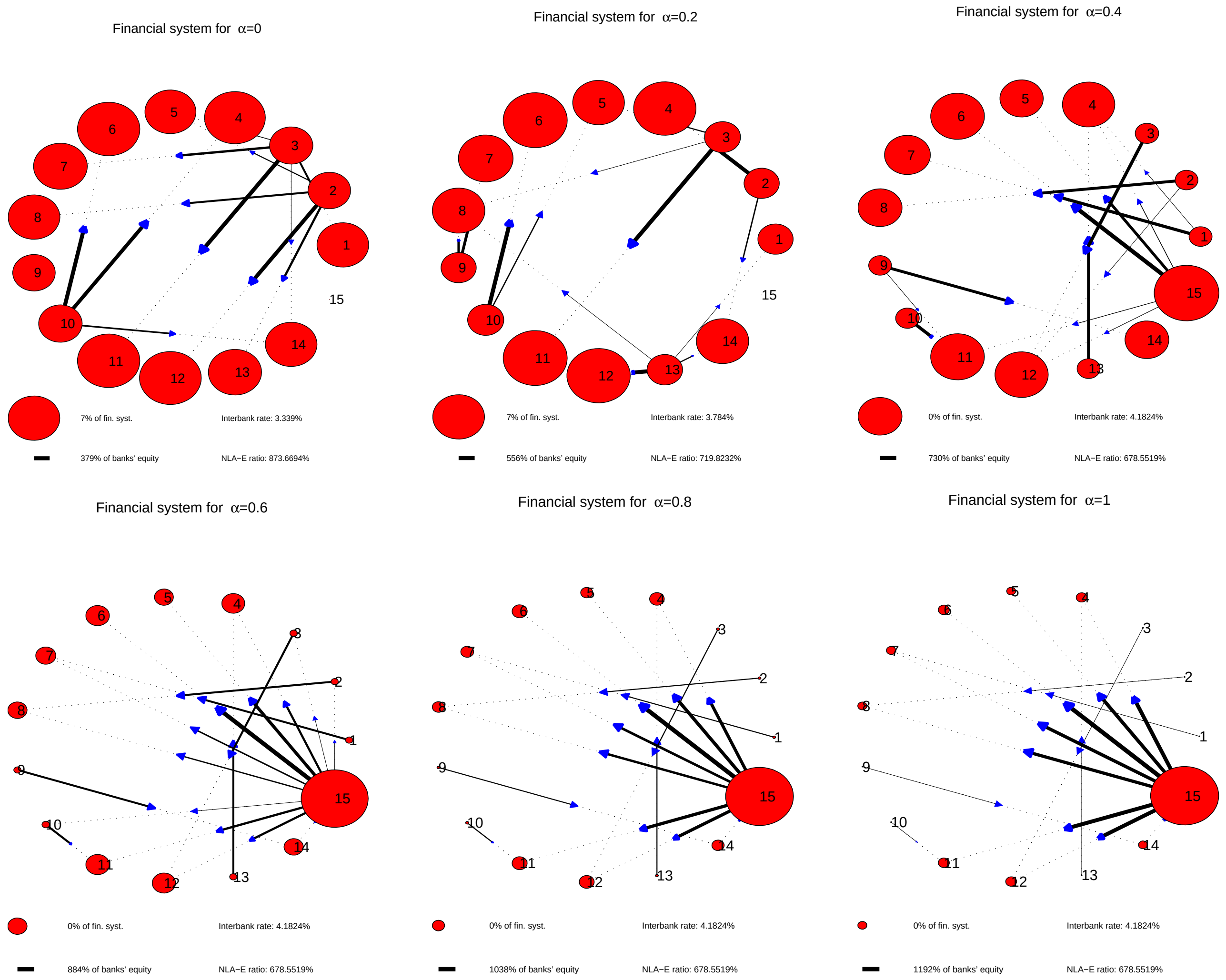

Figure 10: Financial System Structures and Liquidity Requirement With Central Bank Intervention

The figures displayed on the panel are financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. Note that this set up includes central bank intervention which is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

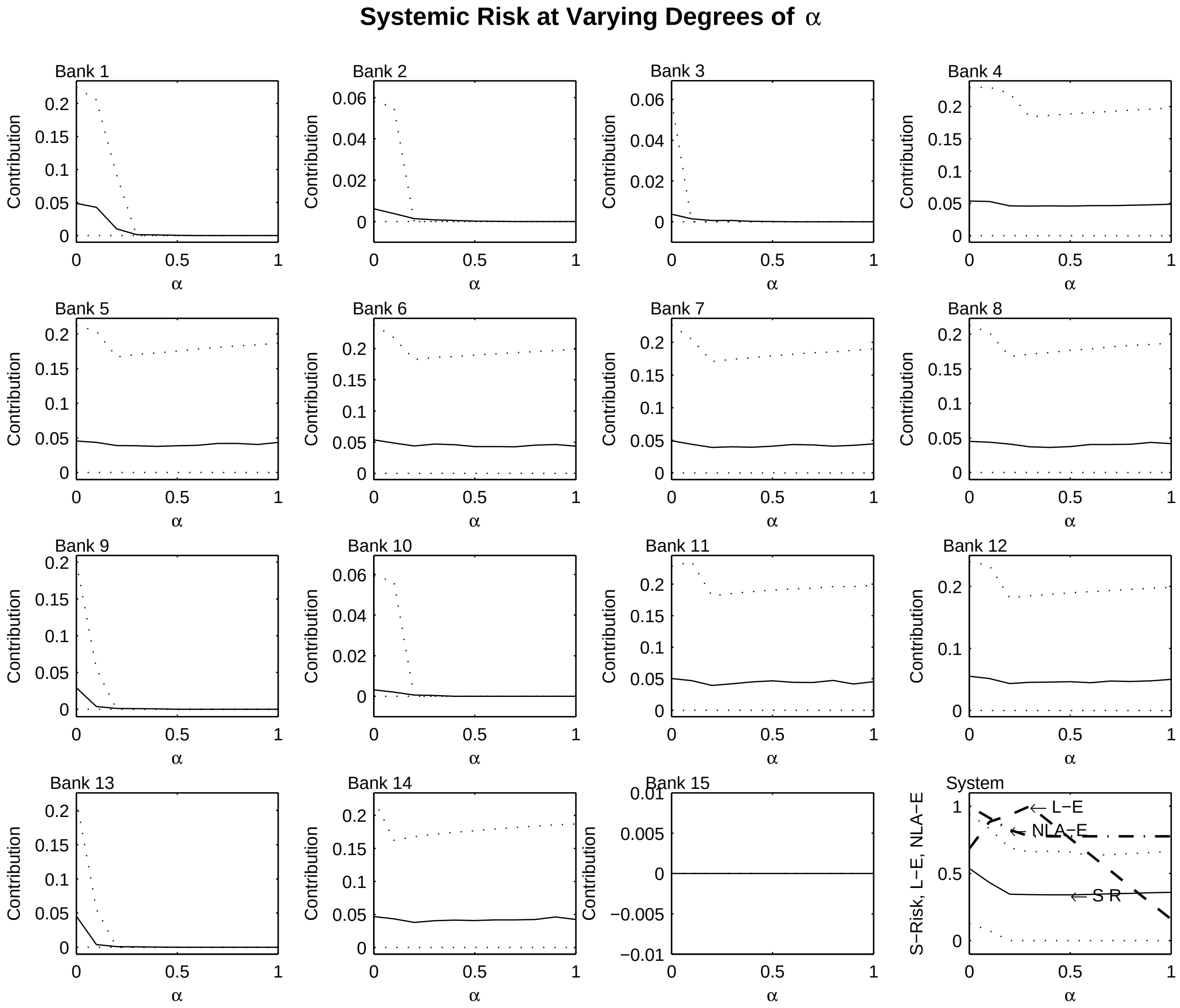

Figure 11: Systemic Risk and Liquidity Requirement With Central Bank Intervention

The figure displays systemic risk (y-axis on panel 16, bottom right, computed following Equation (16)) and banks' contribution to it (y-axis on panels 1 to 15, computed following Equation (14)) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. Note that this set up includes central bank intervention which is by default bank 15. The dotted lines are the two standard deviation error bands (where thresholds are the 5% cut-off points of the most extreme observations of μ_i obtained conditional on the shock vectors drawn). On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Figure 12 displays selected financial network configurations at various capital requirement ratios, γ. At low levels of γ, profitable banks have an incentive to leverage in the inter-bank market, the more so as with the central bank the lending rate is kept stable at low levels. The demand for liquidity raises the interbank rate: this triggers an intervention from the central bank which starts providing liquidity. As the capital requirement increases, all banks reduce their demand for funds. The interbank rate falls (below the lower bound of the corridor) and the central bank starts to drain liquidity from the market.

Figure 13 displays systemic risk and banks' contribution to it when the capital requirement ratio increases. Systemic risk decreases because the system develops from a highly leveraged one to an un-leveraged one as described above.

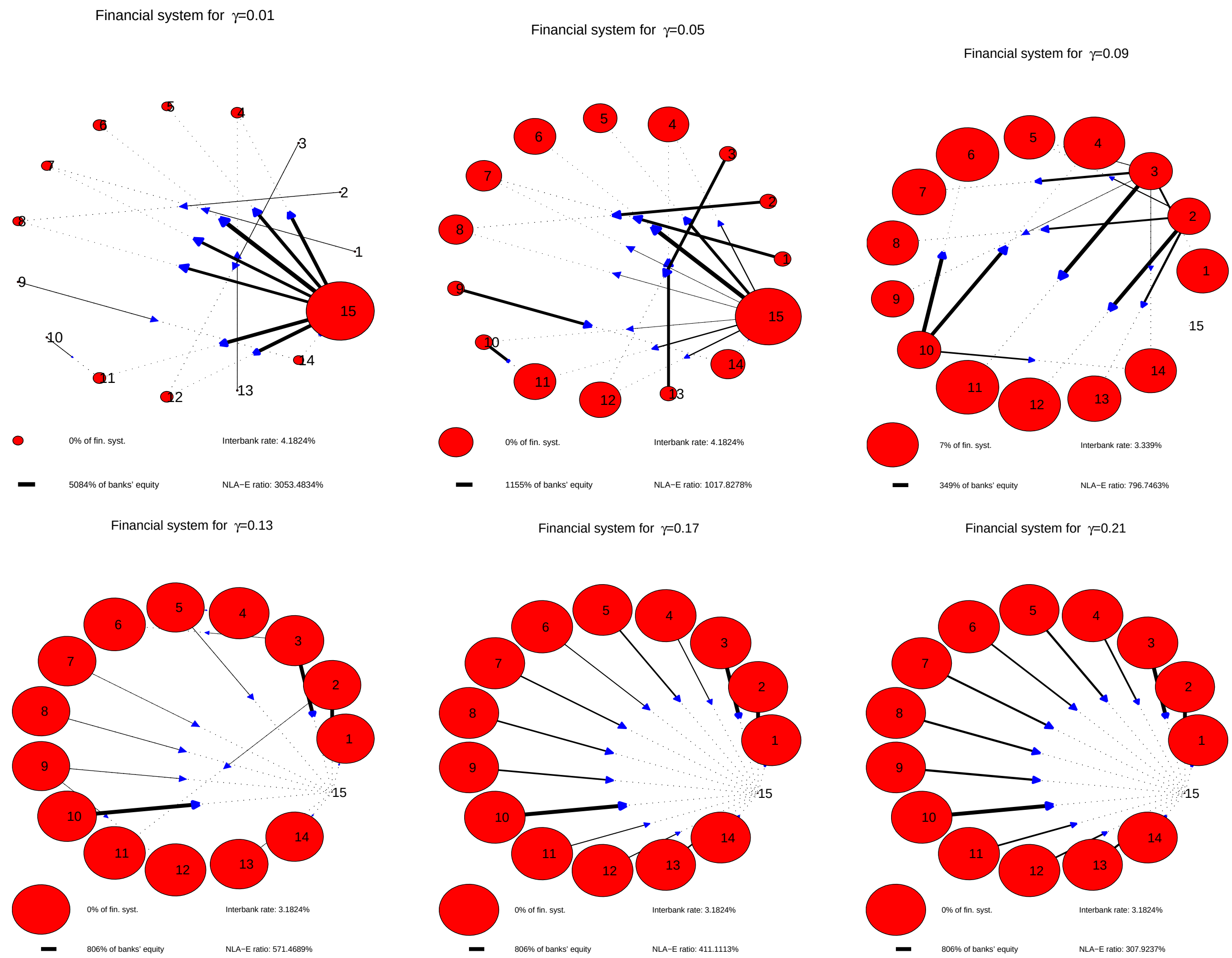

Figure 12: Financial System Structures and Capital Requirement Ratio With Central Bank Intervention

The figures displayed on the panel are financial system realizations at different capital requirement ratios, with all remainder model parameters kept at their baseline value. Note that this set up includes central bank intervention which is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

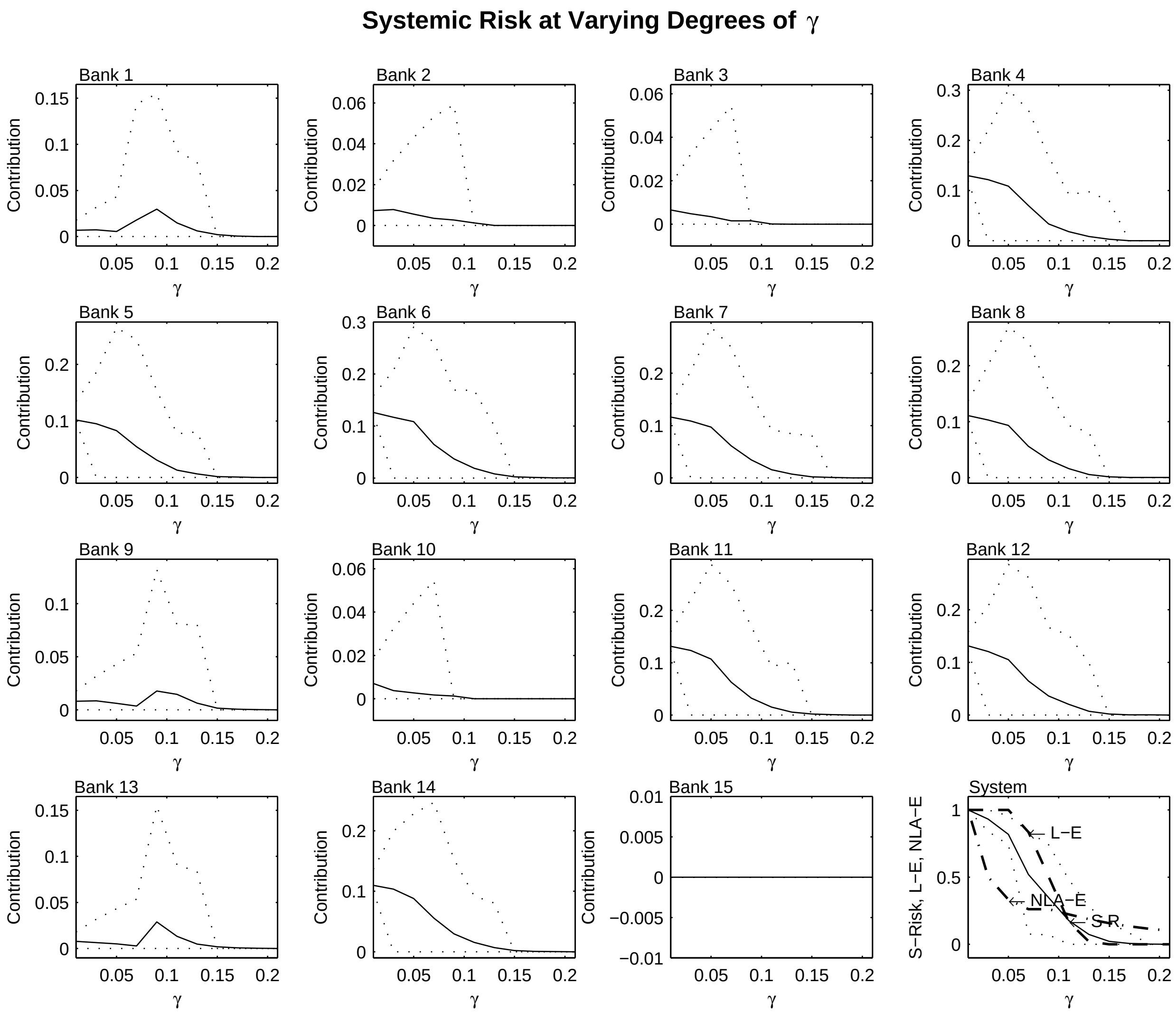

Figure 13: Systemic Risk and Capital Requirement Ratio With Central Bank Intervention

The figure displays systemic risk (y-axis on panel 16, bottom right, computed following Equation (16)) and banks' contribution to it (y-axis on panels 1 to 15, computed following Equation (14)) as solid lines over different values of the capital requirement ratio, with all other model parameters kept as in the baseline setting. Note that this set up includes central bank intervention which is by default bank 15. The dotted lines are the two standard deviation error bands (where thresholds are the 5% cut-off points of the most extreme observations of μ_i obtained conditional on the shock vectors drawn). On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

In presence of a central bank it is also of interest to analyze the impact on the interbank market of changes in the target rate. Figures 14 displays selected financial networks under increasing central bank's target rates.

The central banks increases the target rate by draining liquidity from the market. The higher lending rate induces a higher fraction of banks to supply funds in the interbank market at the expenses of investment in non-liquid assets. Eventually however the reduction in non-liquid investment and the increase in the lending to equity ratio reduces the equilibrium lending rate. Figure 15 indeed shows that the loan to equity ratio has a bell-shaped dynamic. While the loan to asset ratio rises systemic risk and the Shapley values remain high: they start to decrease when interbank lending falls.

Appendix D shows the effect of changing the risk weights for interbank lending and investment into non-liquid assets as well as that of financial levies, all in presence of a central bank's interventions.

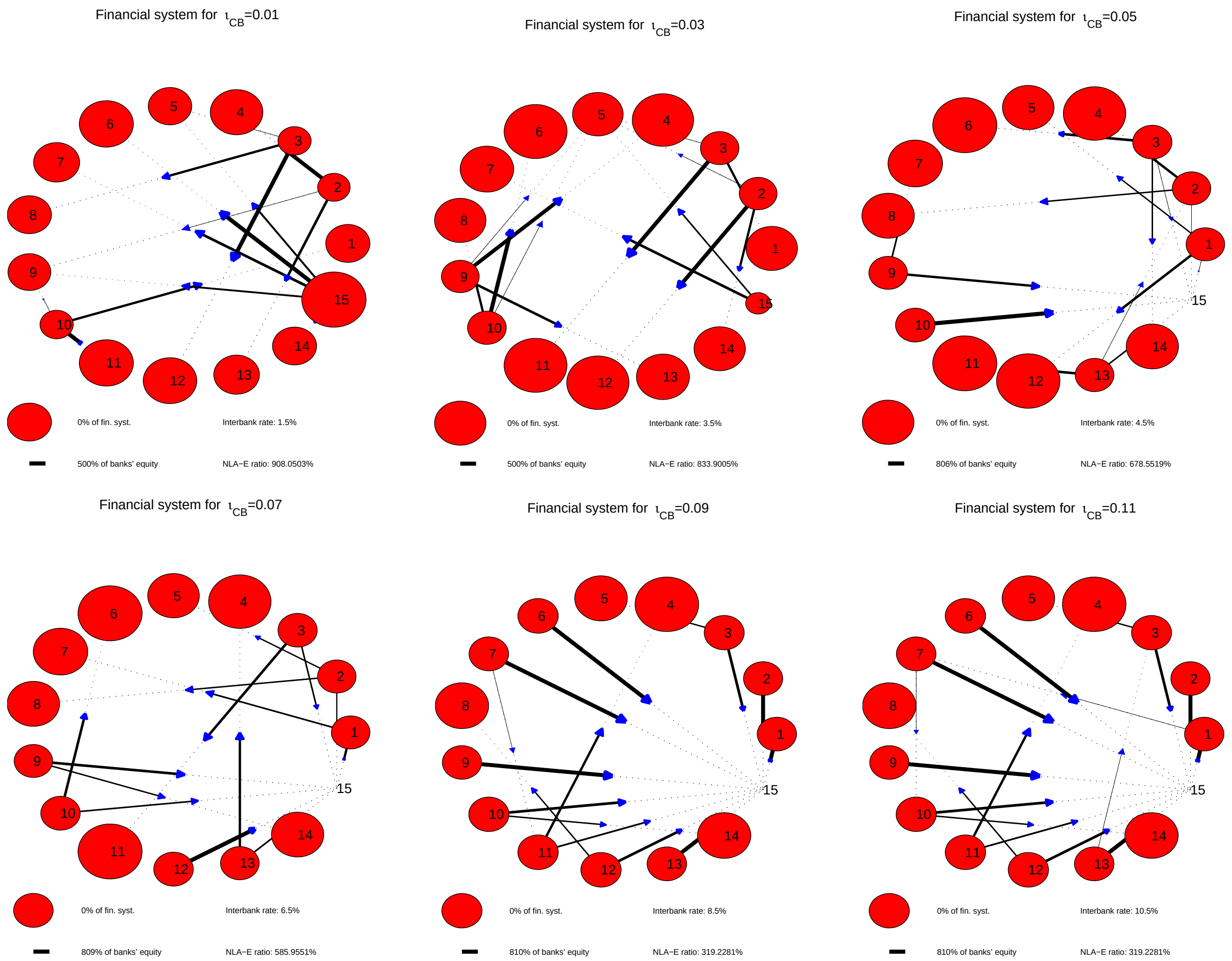

Figure 14: Financial System Structures and Central Bank Target Rate

The figures displayed on the panel are financial system realizations at interest rate targets for the central bank, with all remainder model parameters kept at their baseline value. The central bank is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

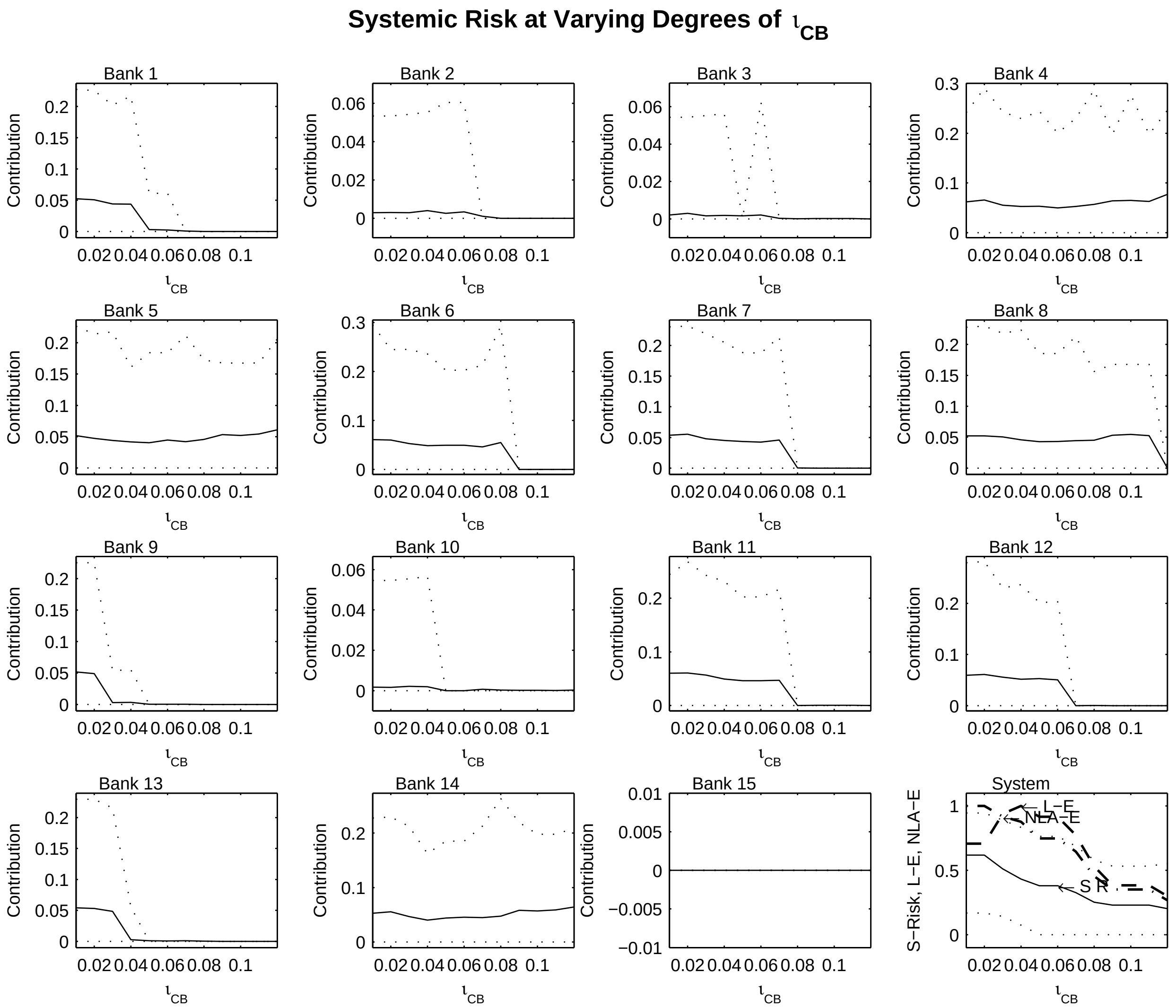

Figure 15: Systemic Risk and Central Bank Target Rate

The figure displays systemic risk (y-axis on panel 16, bottom right, computed following Equation (16)) and banks' contribution to it (y-axis on panels 1 to 15, computed following Equation (14)) as solid lines over different values of the central bank's target rate, with all other model parameters kept as in the baseline setting. Note that this set up includes central bank intervention which is by default bank 15. The dotted lines are the two standard deviation error bands (where thresholds are the 5% cut-off points of the most extreme observations of μ_i obtained conditional on the shock vectors drawn). On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

5 Conclusions

One of the major legacies of the recent financial crisis is the quest for measuring, assessing and monitoring systemic risk. So far, this task was made difficult by the mounting complexity of the modern financial systems, all characterized by extensive degrees of interconnections, and the lack of models apt to perform such tasks. We laid down a dynamic network model of banks, in which heterogeneity, network externalities and fire-sale effects contribute to propagate financial shocks through cascades. The model displays a rich pattern for the dynamic of network configuration and the diffusion of systemic risk, thereby contributing to the understanding of market mechanism in models with interacting agents. The impact of prudential regulation on financial stability and asset investment depends upon a number of factors, such as asset substitution and market concentration. Note that in most cases there seems to be a trade-off between curbing the potential for sequential cascades and fostering banks' investments in non-liquid assets –which can be taken as our models' proxy for banks' links with the (exogenous) real economy. Results thus indicate that higher stability might come at the cost of a lower provision of financial products and services to the real economy. Whether this has welfare effects would be interesting to analyze but is beyond the scope of our current model.

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Appendix A: Banks' Objective Function

Bank i's expected profit is outlined in Equation (18)

$$E(\pi^i) = E(\pi^{lending^i}) + E(\pi^{e^i}) - E(cost^{borrowing^i}), \tag{18}$$

where

- $E(\pi^{lending^i})$ is bank i's expected profit from lending funds on the interbank market,
- $E(\pi^{e^i})$ is bank i's expected profit from investments into non-liquid assets, and
- $E(cost^{borrowing^i})$ is bank i's expected cost for borrowing funds on the interbank market.

Bank i's expected profit is thus related to two different asset classes: derivative investments (non-liquid assets) and interbank lending. Consider first the interbank market. In our model the interest rate on the interbank market consists of two components: the first component is the risk-free rate, r^f, which purely reflects the cost of intertemporal transfer of funds between counterparts, regardless of any insolvency risk. The second component is a premium, r^{PD}, which reflects the probability of default of the borrowing bank. Thus, the overall cost for bank j to borrow an amount b^j on the interbank market is

$$E(cost^{bb^j}) = (r^f + r^{PD^j}) \cdot b^j. \tag{19}$$

To shed more light on the risk premium charged for borrowing money consider banks' lending decision. Note that lending banks charge a fair risk premium which reflects the counterpart's actual probability of default. A bank i engaging in interbank lending has the following expected profit from providing an amount of money, l^{ij}, on the interbank market to bank j:

$$E(\pi^{bl^{ij}}) = (1 - PD^j) \cdot l^{ij} \cdot (r^{rf} + r^{PD^j}) + PD^j \cdot (l^{ij} - \xi l^{ij}) \cdot (r^{rf} + r^{PD^j}) \tag{20}$$

where PD is a bank's probability of default and ξ, $0 \leq \xi \leq 1$ is the loss-given-default ratio which captures that only a fraction of the outstanding amount is paid back in case of the debtor's default. The first product in Equation (20) reflects the lender's profit in case the debtor does not default, and the second term reflects the case when the debtor defaults.

Since creditors charge a fair risk premium for debtors' probability of default, their expected profit from lending must be equal to the profit they obtain in the absence of risk, that is,

$$E(\pi^{bl^{ij}}) = l^{ij} \cdot r^{rf}. \tag{21}$$

Replacing $E(\pi^{bl^{ij}})$ by $l^{ij} \cdot r^{rf}$ in Equation (20) and solving for r^{PD} yields

$$r^{PD^j} = \frac{\xi PD^j}{1 - \xi PD^j} \cdot r^{rf} \tag{22}$$

which is the fair premium charged on the interbank market for banks' individual default risk.

We assume that banks' individual probability of default is publicly known. Using Equations (19) and (22), Bank i's expected cost of borrowing is thus equal to

$$E(cost^{b^i}) = (r^f + r^{PD^i}) \cdot b^i = b^j \cdot r^f \cdot \frac{1}{1 - \xi PD^i}. \tag{23}$$

Next, bank i's overall expected profit from lending is given by the sum of individual amounts lent to its counterparties:

$$E(\pi^{l^{i\cdot}}) = \sum_{1:h \in J} E(\pi^{l^{ih}}), \tag{24}$$

where $\cdot$ indicates several counterparties, $1 : h \in J$ are the h banks bank i has lent money to, from the set of all banks J not including bank i. Taking the sum over h in Equation (20) and using Equation (22), it can be shown that Equation (24) simplifies to

$$E(\pi^{l^{i\cdot}}) = l \cdot r^{rf}, \tag{25}$$

where $l = \sum_{1:h \in J} \cdot l^{ih}$. Equation (25) reflects that banks charge fair risk-premia, that is, in expectation the losses resulting from the default of some counterparties are compensated by risk premia paid by banks that actually do not default. As a result, the expected yield from bank lending is equal to the risk free rate.

Finally, bank i's expected return is also linked to its non-liquid asset investments which is related to derivative investments. Bank i's expected return from investments into non-liquid assets is given by

$$\frac{r^i}{p} \cdot e^i, \tag{26}$$

where r^i is bank i's yield on non-liquid asset investments, e^i is bank i's investment in non-liquid assets, and p is the market price of the non-liquid asset. Note that banks' yield is divided by the market price of the non-liquid asset –which is initially set to 1– to reflect that the yield has an inverse relation with the market price. This is the case for financial products which feature fixed payoffs such as bonds. Since the market price of non-liquid assets can change in our model and banks can re-optimize their portfolio, we include this feature in the objective function.

Using Equations (23), (25), and (26) banks i's objective function, Equation (18), can be expressed as

$$E(\pi^i) = l \cdot r^{rf} + \frac{r^i}{p} \cdot e^i - b^i \cdot r^f \cdot \frac{1}{1 - \xi PD^i}. \tag{27}$$

Note that in expectation banks' return from lending, r^f, is smaller than their cost of borrowing, $r^f \cdot \frac{1}{1-\xi PD^i}$. This difference emerges because because banks always have to pay a fair risk premium for borrowing (as long as they do not default) but do not expect to get back all the funds they lend because in expectation some of their counterpart debtors will default. In case all borrowing banks' probability of default is zero, expected borrowing and lending cost are the same.

Appendix B: Reliability of Shapley Value Approximation

To assess the reliability of our approximation of the Shapley value, Equation 12, we compute the relative standard error, based of 100 replications each, over different sample sizes. The relative standard error is obtained via dividing the standard deviation of a random variable by its mean, expressing it as a percentage. Figure 16 displays the relative standard errors (y-axis) over sample sizes ranging from 10 to 4500 draws (x-axis). The constant solid line at a relative standard error of 30% is the threshold below which relative standard errors indicate reliability of estimates,[23] with smaller values indicating higher reliability. The thin dash-dotted line is the the relative standard error of the financial institution with the smallest mean in the sample. The thin dotted line is the average relative standard error of financial institutions with sample means in the bottom quartile of all financial institutions. The solid medium sized line is the average relative standard error of all financial institutions. The thick dotted line is the average relative standard error of financial institutions with sample means in the top quartile of all financial institutions. The thick dash-dotted line is the relative standard error of the financial institution with the highest mean in the sample.

There are three key points to be highlighted on the figure. First, when increasing the sample size, the estimates become more reliable. Second, estimates of financial institutions with a high contribution to systemic risk are more precise relative to institutions with a lower contribution to systemic risk. Third, from a sample size of 400 draws and larger, all estimates are below the reliability threshold of a relative standard error of 30%.

[23]The relative standard error is mainly used in the context of survey analyses where the true population moments are unknown. See, for example, National Center for Health Statistics [23] for an assessment of the reliability of relative standard errors below a threshold of 30%.

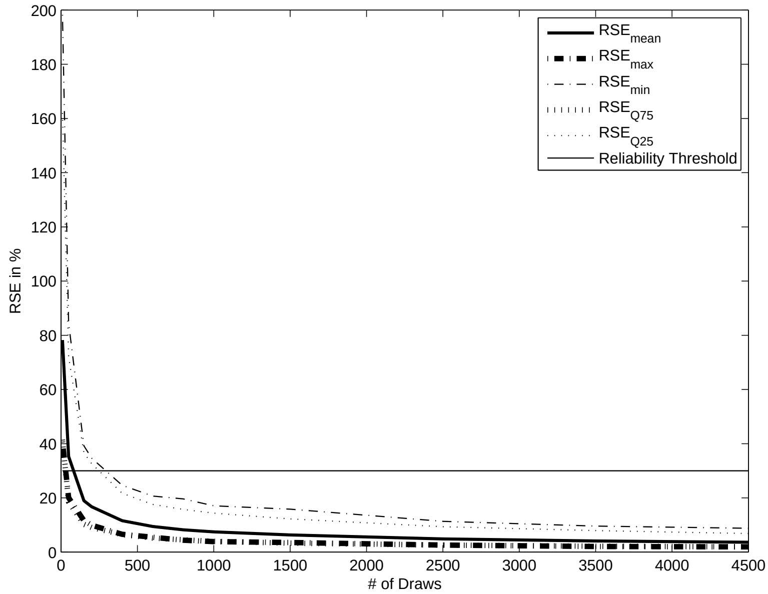

Figure 16: Relative Standard Errors at Various Sample Sizes

The figure displays the relative standard errors (y-axis) over sample sizes rangeing from 10 to 4500 draws (x-axis), based on 100 replications each. The constant solid line at a relative standard error of 30% is the threshold below which relative standard errors indicate reliability of estimates, with smaller values indicating higher reliability. The thin dash-dotted line is the the relative standard error of the financial institution with the smallest mean in the sample. The thin dotted line is the average relative standard error of financial institutions with sample means in the bottom quartile of all financial institutions. The solid medium sized line is the the average relative standard error of all financial institutions. The thick dotted line is the average relative standard error of financial institutions with sample means in the top quartile of all financial institutions. The thick dash-dotted line is the the relative standard error of the financial institution with the highest mean in the sample.

Appendix C. Network Configuration and Systemic Risk for Different Values of Risk Factors

Figure 17 displays the evolution of the financial systems at increasing values of the risk weight on non-liquid assets, χ_1.

Overall the financial system becomes less concentrated, interest rates on the interbank market decrease, the non-liquid-asset-to-equity ratio, which is initially stable, falls beyond a certain level and interconnectedness in interbank lending decreases.

Figure 18 shows the effect of increasing the risk weight on non-liquid asset investments, χ_1, on systemic risk and banks' contribution to it.

As for the case of changes in the capital requirements parameter, γ, here we observe a

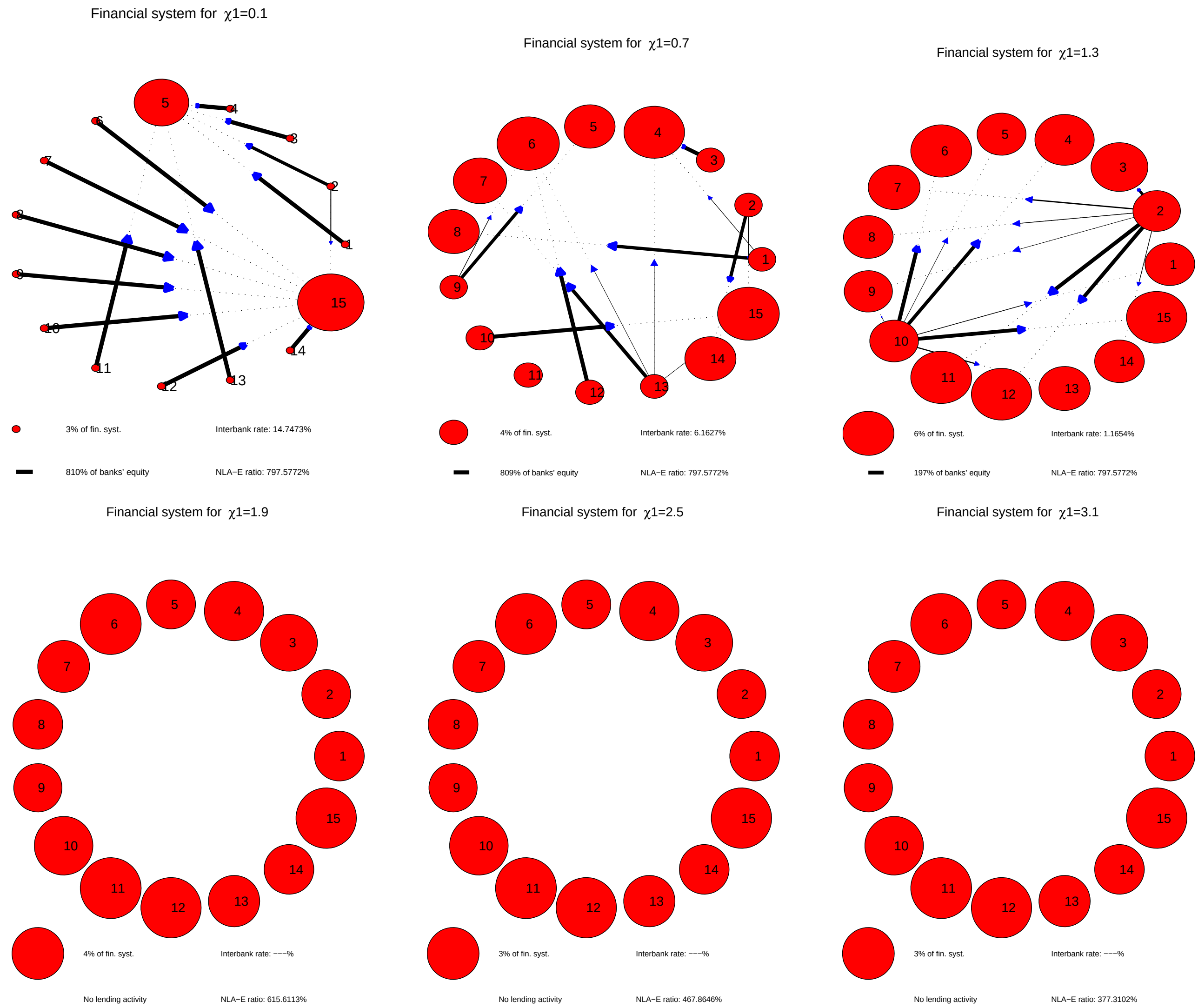

Figure 17: Financial System Structures and Derivative Risk Weights

The figures displayed on the panel are financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

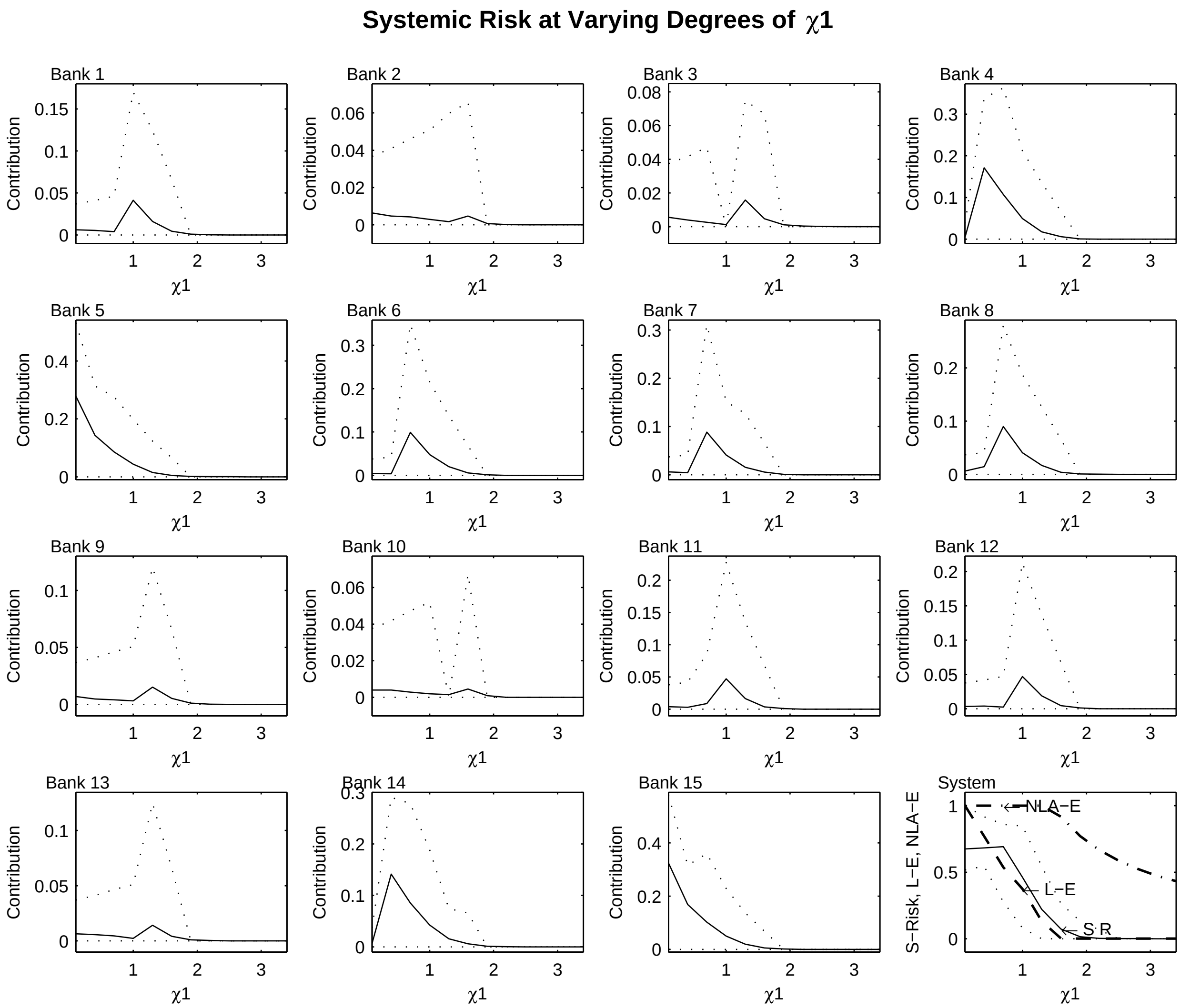

Figure 18: Systemic Risk and Derivative Risk Weights

The figure displays systemic risk (y-axis on panel 16, bottom right, computed following Equation (16)) and banks' contribution to it (y-axis on panels 1 to 15, computed following Equation (14)) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. The dotted lines are the two standard deviation error bands (where thresholds are the 5% cut-off points of the most extreme observations of μ_i obtained conditional on the shock vectors drawn). On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

bell-shaped dynamic of systemic risk.

Figure 19 displays the evolution of the financial system along increasing values of the risk weight on interbank lending, χ_2.

The interbank market interest rate is stable and then increases, the ratio of non-liquid assets to banks' equity is first stable and then decreases, the number of banks engaging in interbank lending increases.

Figure 20 shows the effect of increasing the risk weight on interbank lending on systemic risk and banks' contribution to it.

Increasing the risk weight on interbank lending reduces supply of funds on the interbank market and thus the interbank interest rate tends to increase: more banks engage in interbank lending. The increasing level of interconnectedness raises systemic risk. Beyond a certain value however banks' lending supply falls and so does systemic risk.

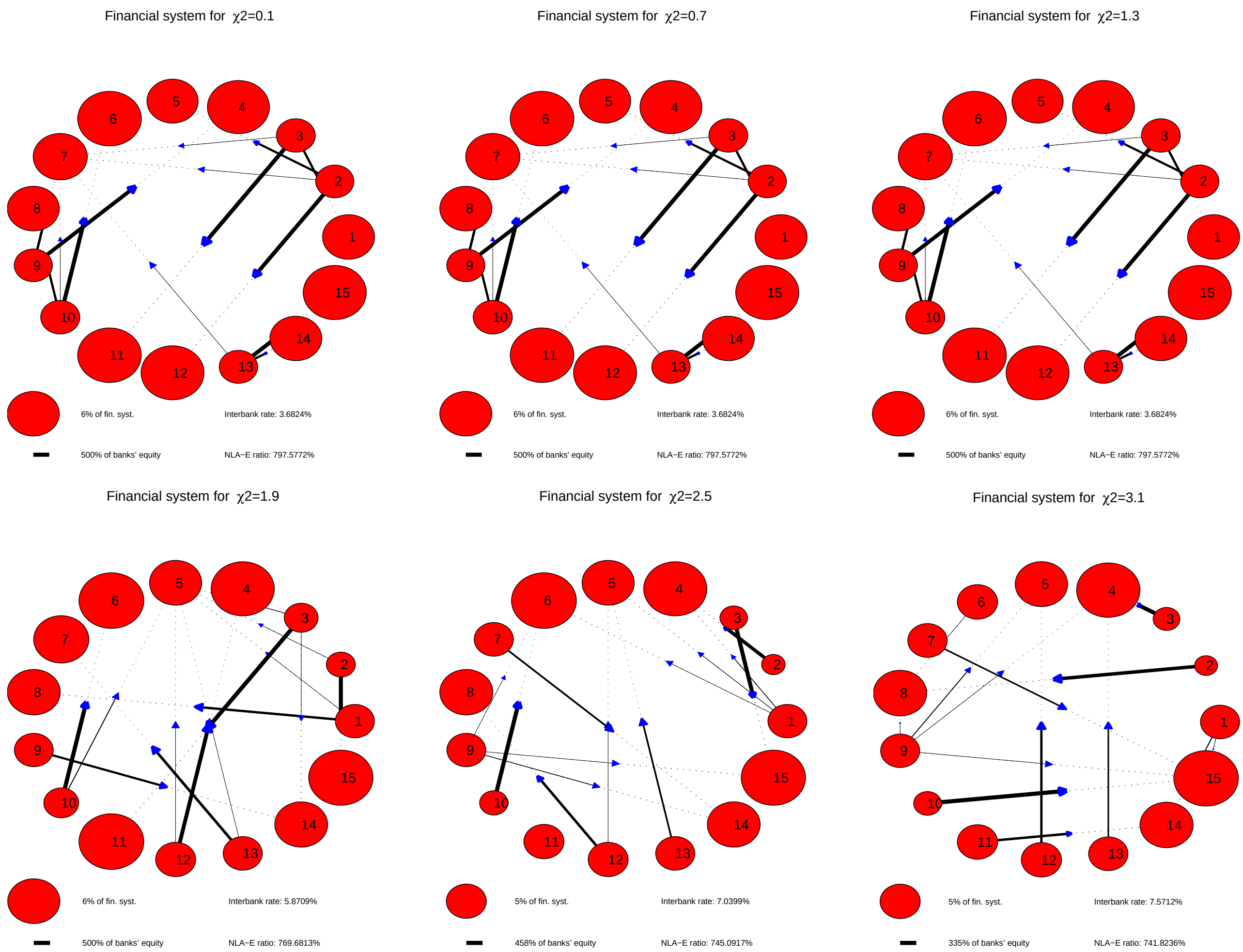

Figure 19: Financial System Structures and Interbank Lending Risk Weights

The figures displayed on the panel are financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

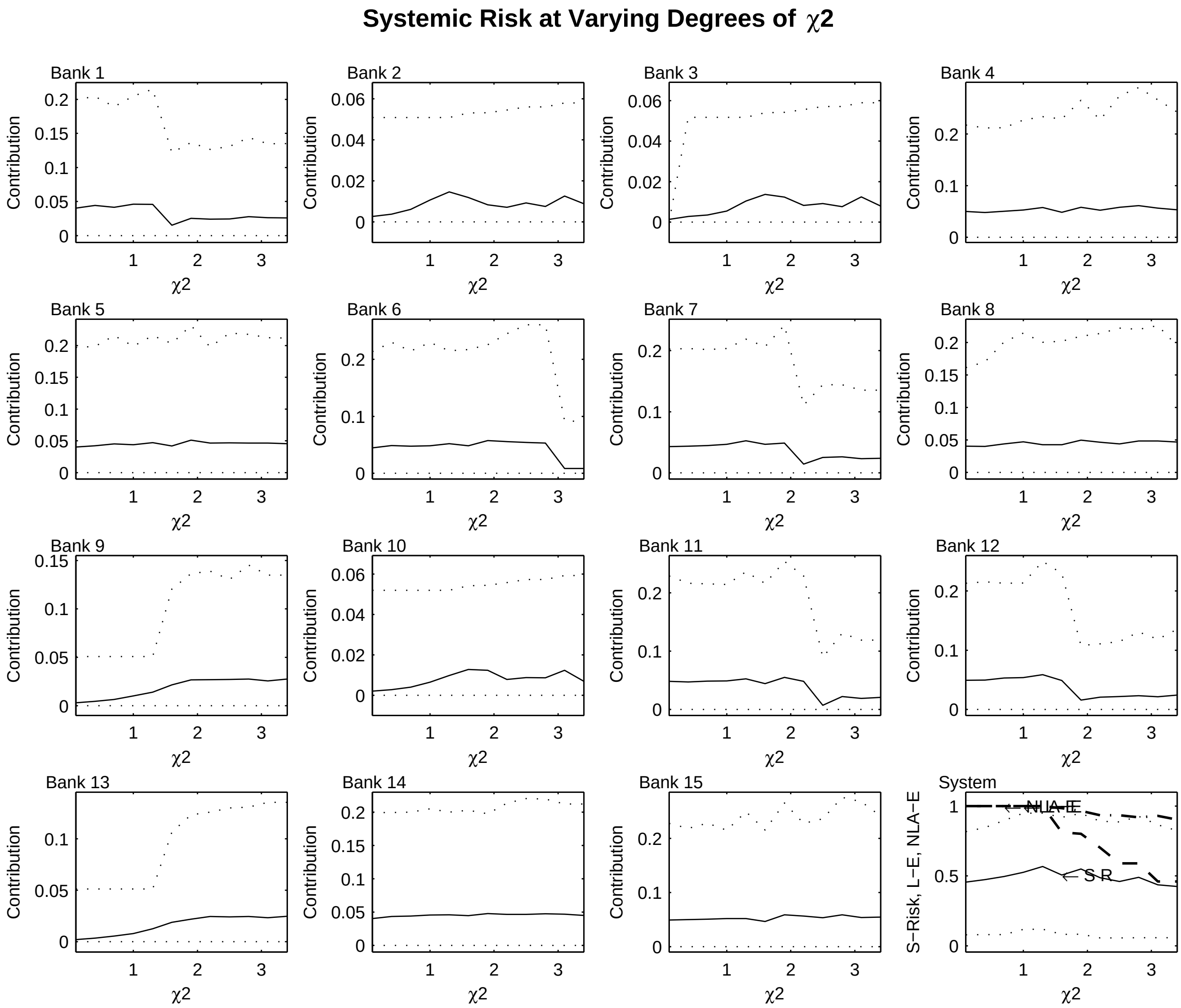

Figure 20: Systemic Risk and Interbank Lending Risk Weights

The figure displays systemic risk (y-axis on panel 16, bottom right, computed following Equation (16)) and banks' contribution to it (y-axis on panels 1 to 15, computed following Equation (14)) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. The dotted lines are the two standard deviation error bands (where thresholds are the 5% cut-off points of the most extreme observations of μ_i obtained conditional on the shock vectors drawn). On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Appendix D. Adding Central Bank Intervention

Figure 21 displays selected financial network configurations at various levels of the risk weight for non-liquid asset investments, χ_1.

The interest rate decreases from the upper to the lower bound of the corridor, the banking system becomes more homogenous in terms of banks' size, and investments into non-liquid assets rapidly go down.

Figure 22 displays systemic risk and banks' contribution to it: increasing the risk weight results in lower non-liquid asset investment, lower interbank lending, hence lower systemic risk.

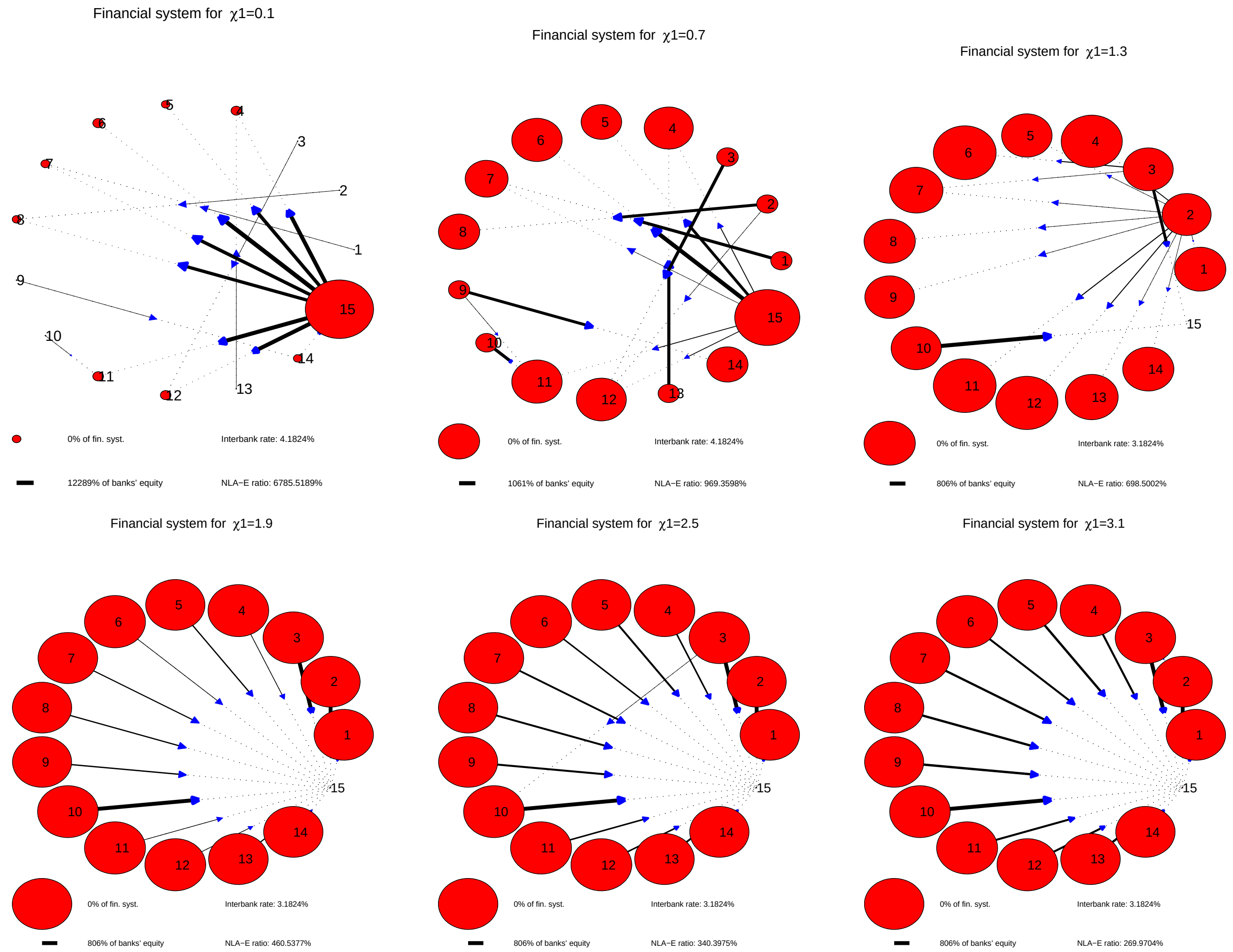

Figure 21: Financial System Structures and Derivative Risk Weights With Central Bank Intervention

The figures displayed on the panel are financial system realizations at different risk weights for derivative investments, with all remainder model parameters kept at their baseline value. Note that this set up includes central bank intervention which is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

Systemic Risk at Varying Degrees of $\chi 1$

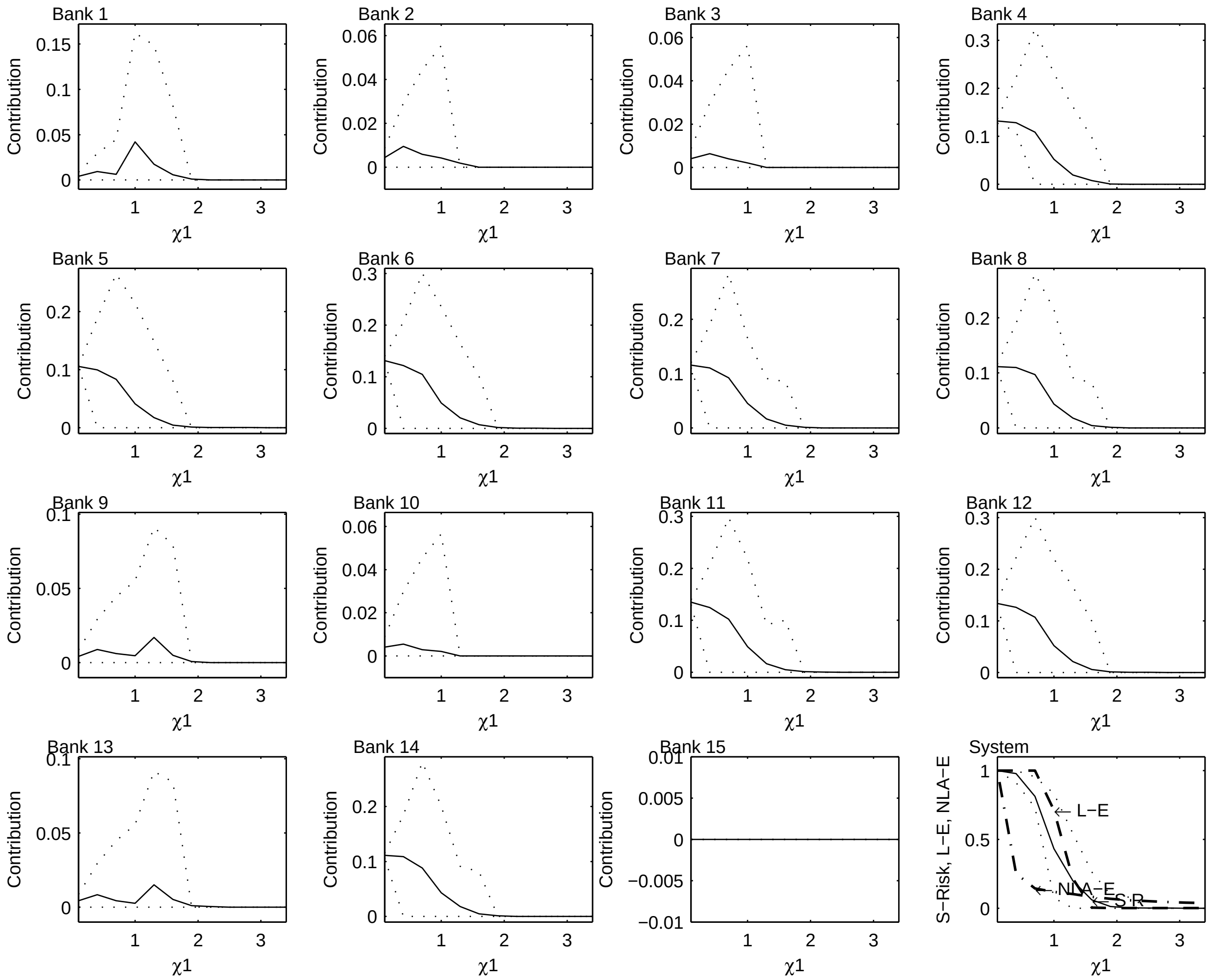

Figure 22: Systemic Risk and Derivative Risk Weights With Central Bank Intervention

The figure displays systemic risk (y-axis on panel 16, bottom right, computed following Equation (16)) and banks' contribution to it (y-axis on panels 1 to 15,, computed following Equation (14)) as solid lines over different values of the risk weight for derivatives, with all other model parameters kept as in the baseline setting. Note that this set up includes central bank intervention which is by default bank 15. The dotted lines are the two standard deviation error bands (where thresholds are the 5% cut-off points of the most extreme observations of μ_i obtained conditional on the shock vectors drawn). On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Figure 23 displays selected financial network configurations at various levels of the risk weight for banks' interbank lending, χ_2.

Investments in non-liquid assets remains largely unaffected in this case, since the interest rate increase is dampened within the bounds of the corridor.

Figure 24 displays systemic risk and banks contribution to it when the risk weight on interbank lending is increased. When increasing the risk weight on interbank lending, systemic risk increases slightly, the loan-to-equity ratio decreases and the non-liquid assets-to-equity ratio decreases slightly.

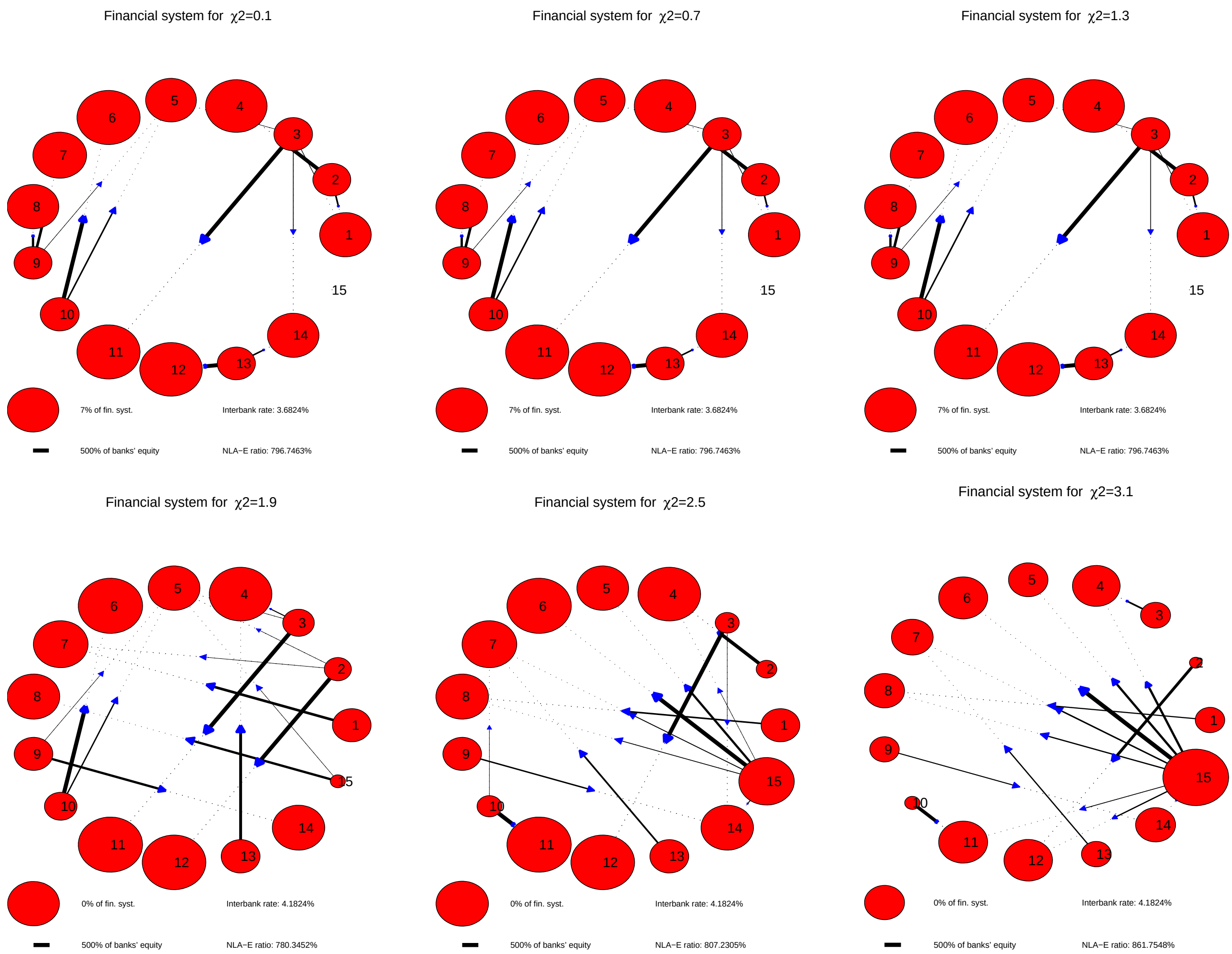

Figure 23: Financial System Structures and Interbank Lending Risk Weights With Central Bank Intervention

The figures displayed on the panel are financial system realizations at different risk weights for interbank lendings, with all remainder model parameters kept at their baseline value. Note that this set up includes central bank intervention which is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

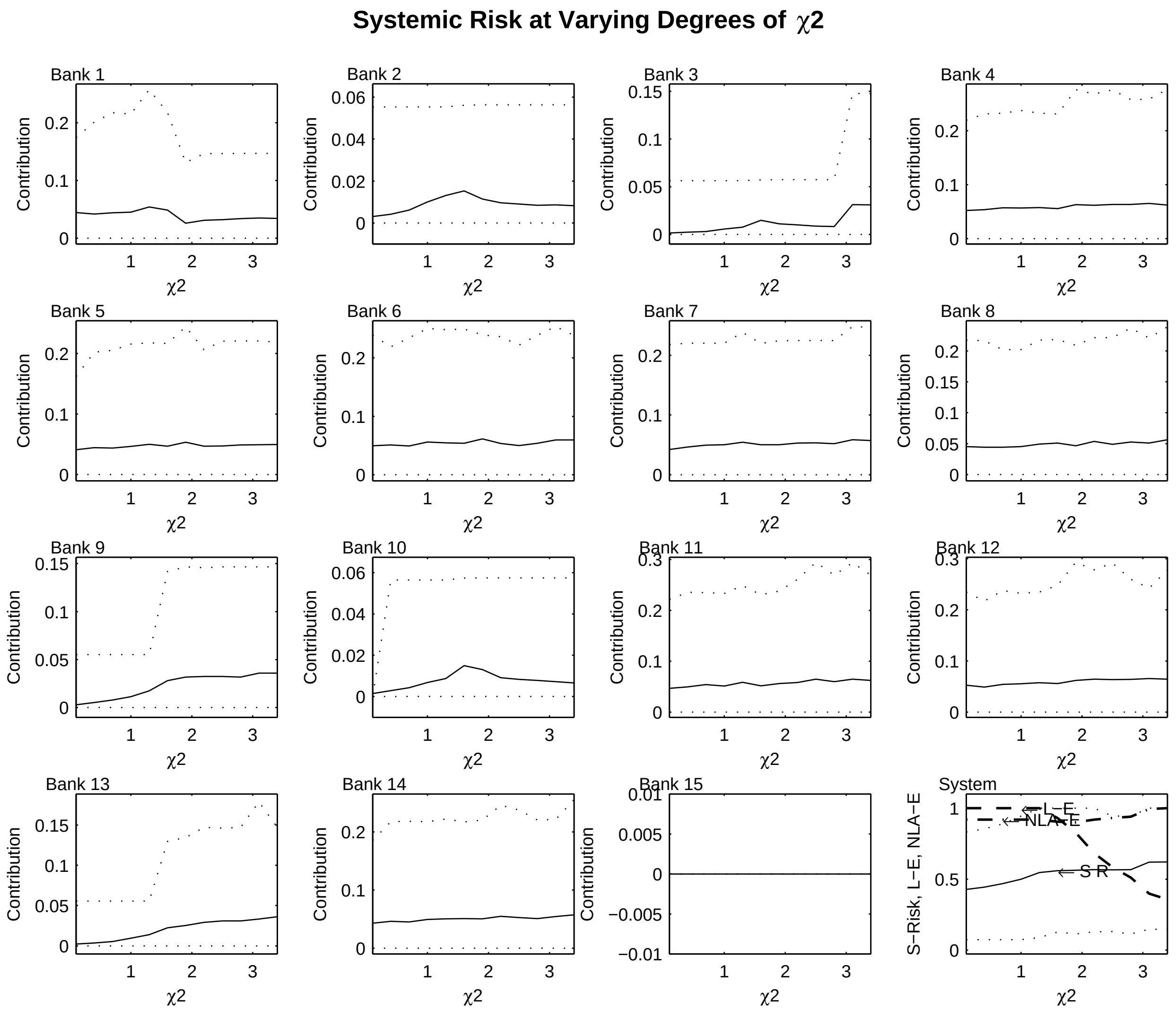

Figure 24: Systemic Risk and Interbank Lending Risk Weights With Central Bank Intervention

The figure displays systemic risk (y-axis on panel 16, bottom right, computed following Equation (16)) and banks' contribution to it (y-axis on panels 1 to 15,, computed following Equation (14)) as solid lines over different values of the risk weight for interbank lending, with all other model parameters kept as in the baseline setting. Note that this set up includes central bank intervention which is by default bank 15. The dotted lines are the two standard deviation error bands (where thresholds are the 5% cut-off points of the most extreme observations of μ_i obtained conditional on the shock vectors drawn). On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Figure 25 displays selected financial network configurations at different levels of the levies on non-liquid investment and banks' borrowing.

As in absence of central bank's interventions, investments in non-liquid, interbank lending and the lending rate fall. The financial system becomes more homogenous with respect to banks' size.

Figure 26 displays systemic risk and banks' contribution to it when changing the financial levies. Systemic risk and banks' contribution to it fall, though the decline is not monotonous for all banks.

Generally speaking the increase in financial levies reduces the demand for interbank liquidity and the lending rate: in presence of a central bank's interventions however the fall in the lending rate is limited by the lower bound of the corridor. The direct shock transmission channel is therefore dampened relative to the case without central bank intervention.

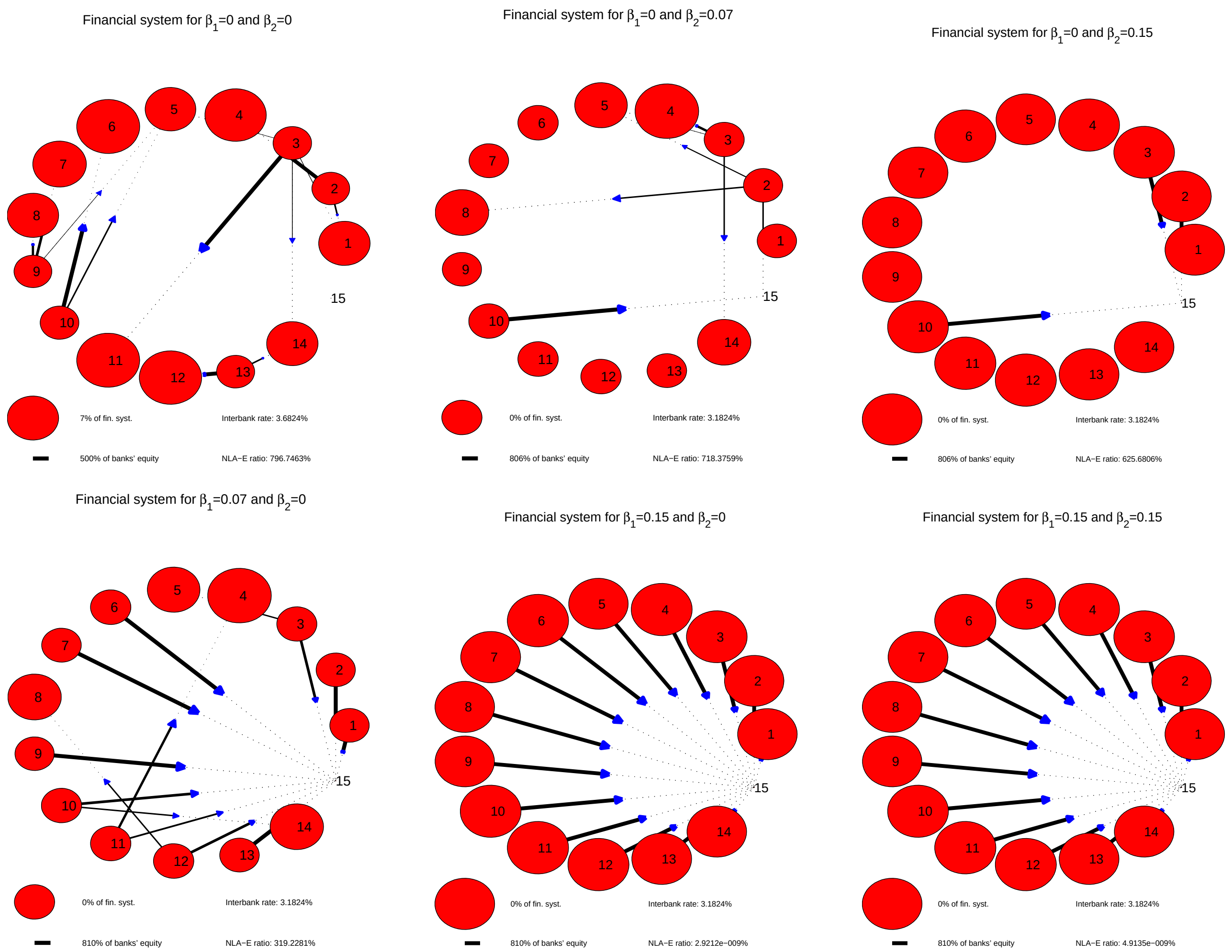

Figure 25: Financial System Structures and Risk Charges With Central Bank Intervention

The figures displayed on the panel are financial system realizations at different values for systemic risk charges for derivative investments (β_1) and interbank lendings (β_2), with all remainder model parameters kept at their baseline value. The central bank is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

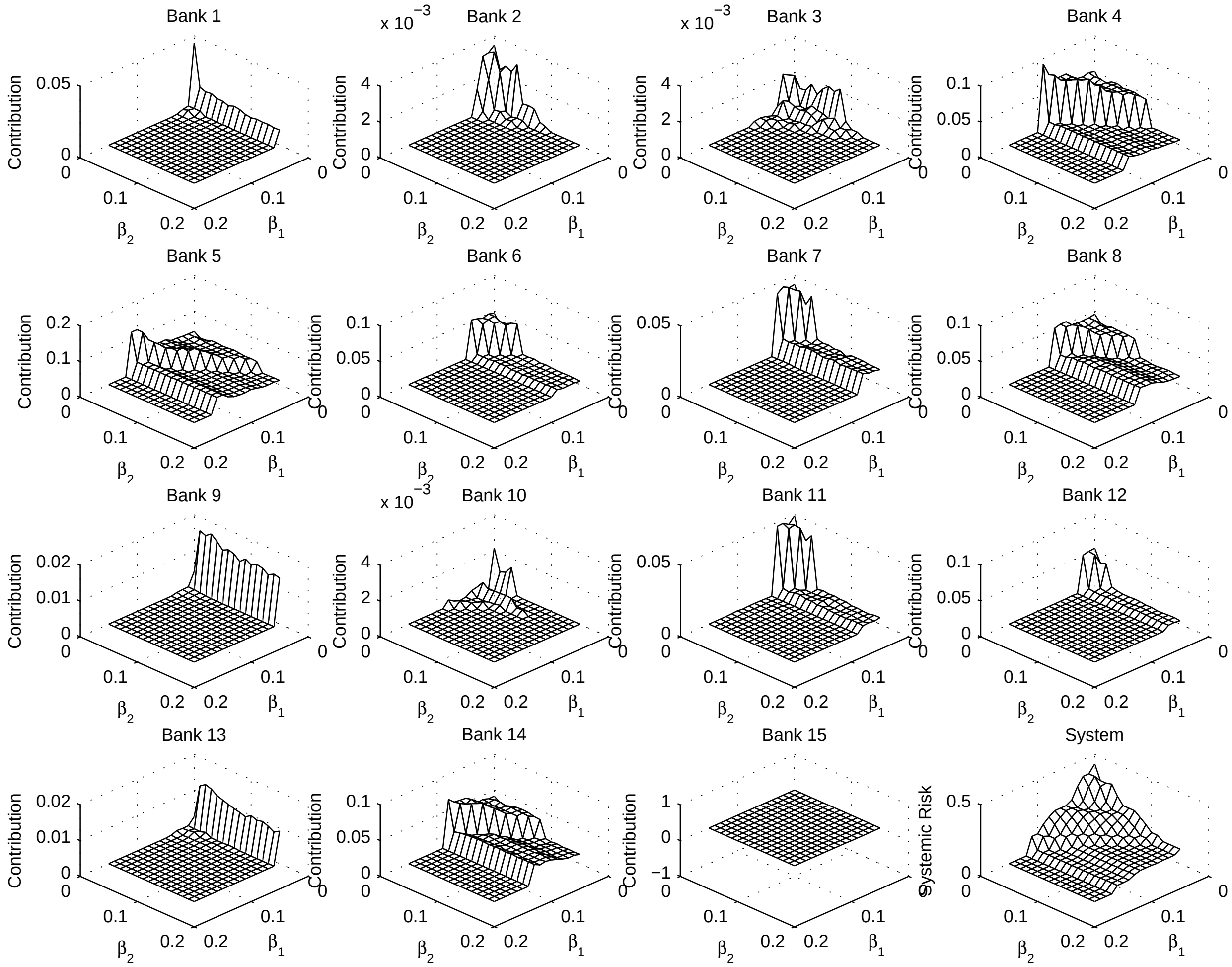

Figure 26: Systemic Risk and Risk Charges With Central Bank Intervention

The figure displays systemic risk (y-axis on panel 16, computed following Equation (16)) and banks' contribution to it ((y-axis on panels 1 to 15, computed following Equation (14))) over a range of increasing values for the penalty parameters for derivatives (β_1) and interbank lendings (β_2) on the z- and x-axes, respectively. Note that there is central bank intervention which is by default bank 15.